

THE
PHYSICAL REVIEW.

A BOLOMETRIC STUDY OF LIGHT STANDARDS.

BY CLAYTON H. SHARP AND W. R. TURNBULL.

I.

Introduction.

THE two great essentials for a light standard are constancy during use and exact reproducibility.

It is evident even to a most casual observer that the standard candles which are in use do not possess the first of these essentials. The English standard candle has been charged with fluctuations of 40 per cent, and it is only by snuffing and measuring carefully the height of flame that anything approaching uniform results has been obtained.

The Carcel lamp has stood the test of time but little better than the best of standard candles. It has disadvantages entailed by the use of a light-absorbing chimney and a rapidly charring wick; while of its reproducibility, within a reasonable margin of error, there is serious question. Although the Carcel lamp furnishes a very steady source of light, it is known to show slow changes in the amount of light emitted, rising slowly to a maximum as the lamp and oil become heated, and then falling off with the charring of the wick.

Among the more modern standards that of Methven is faulty in that it requires a chimney, employs as an illuminant an indeter-

minate mixture of gases, and is affected also by variations in the pressure of the gas and by draughts of air in the room. The Vernon-Harcourt pentane burner and the Hefner-Alteneck amyl-acetate lamp, while avoiding this difficulty, run into others perhaps equally serious. The open flame of the latter is not only of a decidedly reddish hue, but is also extremely sensitive to draughts and vibrations of the air. On this account it is only with the greatest care that the height of the flame can be adjusted according to the prescription.

Many attempts have been made to determine the fluctuations of the various standard lights by comparing them by means of a photometer with a source of constant light, as, for example, a glow-lamp at constant voltage. This method is very faulty on account of fundamental difficulties of a physiological or psychological nature which are met in the use of even the most perfect photometer. It requires care and deliberation to make a photometer setting which can be relied upon as being correct within a narrow limit of error, while the fluctuations in the intensity of the light may be quite rapid.

If one seeks to determine these fluctuations by independent settings, succeeding each other as rapidly as reliable settings can be made, the extreme fluctuations may escape observation. The consequence is that no very definite idea can be obtained of the form of the curve which would be traced by the changes of the light intensity. On the other hand, if continuous observations are made with the photometer, it is possible to follow up the fluctuations only very tardily, irregularly, and imperfectly.

In any case the personal error of the observer enters so largely into photometric results that the perplexing question is continually arising, Have I really observed a fluctuation of the light, or have I merely made a bad setting? This remark applies, of course, only to the smaller fluctuations, and not to the large ones commonly observed in the case of candles.

In order to determine these fluctuations, an instrument is needed which will not only serve to measure radiant energy, but which will also respond instantly to any change in the amount of radiation meeting it. Such an instrument is found in the bolometer.

Suggestions as to the use of the bolometer in photometry have not been infrequent. Lummer and Kurlbaum¹ have described a large double-faced bolometer of platinum foil, constructed by them for photometric work. It is difficult to see how this bolometer, intended as it was to be used among the fickle draughts and changing temperatures of a photometer room, would serve to get a continuous curve of the variations of a light standard, by the method described below.

In order to get a complete knowledge of the performance of any light source, it is necessary to obtain a continuous record of its behavior during a considerable period of time. If we make the assumption that light radiation from a given source varies in direct ratio to the total radiation, a properly constructed bolometer furnishes a suitable instrument for obtaining the record desired. The bolometer strip must be so thin that its temperature will follow almost instantly all changes in the amount of radiation meeting it, and the galvanometer must be nearly dead beat. Then if the working conditions are such that it is certain that all observed galvanometer deflections are due to changes in the radiation from the light source, and to nothing else, a plot of galvanometer deflections will represent very truly the variations in the intensity of the light.

The proper adjustment of conditions involves the very careful exclusion of draughts of air from the bolometer strips and from the other parts of the apparatus, and the maintenance of a nearly constant temperature in the room. Moreover, it necessitates the taking of observations at such a time or place that the galvanometer shall not be subjected to any irregular magnetic influences. In order to fulfil these conditions, the apparatus described below was constructed.

II.

Description of Apparatus.

After numerous unsatisfactory experiments with gold leaf lightly plated with nickel, with pure nickel, and with the thinnest tin foil, the bolometer strips were made as follows :—

¹ Lummer und Kurlbaum, Weid. Ann., Vol. 46, p. 204; Zeitsch. für Instrumentenkunde, Vol. 12, p. 81.

A piece of Swedish iron wire, of No. 30 B. & S. gauge, was passed through jeweler's rollers until its thickness was about 0.045 mm., and its width 1.5 mm. It was then placed in dilute sulphuric acid, in which potassium bichromate had been dissolved, and a current was passed through it in such a manner as to dissolve the iron. The potassium bichromate was introduced into the solution to dispose of the hydrogen bubbles which would ordinarily have clung to the metallic surface and which would have caused it to be dissolved unevenly.

In this way a strip was obtained which was about 0.025 mm. in thickness and still moderately strong. From this strip were cut two pieces, each about 6 cm. in length, to constitute two arms of a Wheatstone's bridge.

To carry the strips so obtained, a light oblong frame, F (Fig. 1), of thin wood was made, and to it were fastened small bits of sheet

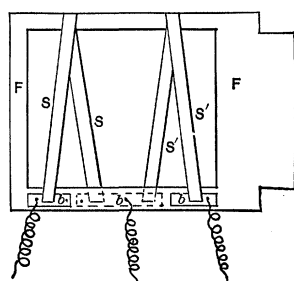


Fig. 1.

brass, b, b, b , to which the strips and the copper wires intended to connect them with the other arms of the bridge could be soldered. The strips, S, S' , were then bent and placed over the frame, so that each strip crossed the frame twice. The free ends of each strip were displaced laterally from each other so that, when viewed from the front, the portion of the strip on one side of the frame hid only very little of the portion on the other side of the frame.

After the two strips had been arranged on the frame symmetrically with respect to each other, the one which was to receive radiation was carefully smoked on both sides. To accomplish this smoking without undue heating of the strip, a piece of sheet metal, through which a small hole had been punched, was held over a candle flame so that the flame was caused to smoke. The smoke passed through the hole, over which a tube was held to direct the current. The strip was passed back and forth over the top of the tube. In this way a very delicate strip can be blackened without injury. In their completed state the strips had a resistance of about 0.5 ohm each.

The frame holding the strips was mounted in a wooden box about 20 cm. long and 5 cm. \times 6 cm. in cross-section. A number of suitable cardboard screens were placed in the front portion of the box to shield one of the strips from radiation and to protect both of them from draughts. To close the box at its other end a piece of bright tin was cut equal in size to the cross-section of the box. This was soldered fast to a heavy block of brass, to the other end of which was soldered another piece of tin which covered the end of the box. When this arrangement was placed in the box, the first piece of tin came close behind the bolometer strips, and its bright surface acted as a mirror in reflecting upon the back of the exposed strips many rays which would otherwise have been lost.

By this means the efficiency of the bolometer was nearly doubled. The tinned surface did not tarnish perceptibly during the course of the investigation. It was not smooth enough to reflect a distinct image, and the light reflected from it was to a large extent scattered. The use of a plane-surfaced mirror in such a position would not be allowable, since any slight change in the angle of incidence would cause a different amount of light to be reflected upon the bolometer strip. The use of the irregular-surfaced plate, since it diffuses the light, can scarcely affect the accuracy of the results to an appreciable degree in such work as has been done with this bolometer. Nevertheless, this arrangement is to be recommended only where great sensitiveness is desired rather than the most exact comparison of results.

Other sources of error which might affect the exact comparability of results have also been ignored. Among these may be mentioned the error which is introduced in assuming that the sensitiveness of the bolometer is independent of the absolute temperature of the strips. The brass block served to keep the temperature of the tin screen from rising to any marked degree above that of the air in the room, by conducting any excess of heat to the outer piece of tin, the surface of which was smoked.

The compensating resistance, forming the other arms of the bridge, was made of german-silver wire arranged inside a small wooden box. Fig. 2 gives the top and side views of this box.

The german-silver wire was strung back and forth on tacks nailed in the two sides of the box, as shown in the side view. The middle portion, *S*, of this wire was drawn tightly, lengthwise of the box, and a sliding contact piece, *C*, was arranged to press against this wire. This contact piece was carried on a little block of wood, *A*, running on ways, *F*, *F'*, and driven by a fine-threaded screw, *E*, which could be turned by means of a large disk head, *D*, outside of the box. One terminal of the battery (which consisted of two

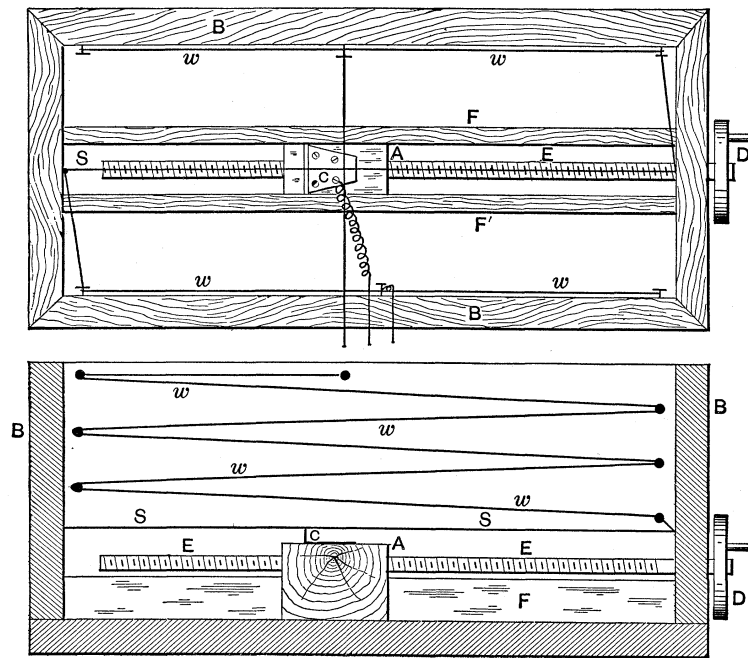


Fig. 2.

gravity cells in multiple with each other) was connected with corresponding ends of the bolometer strips. The other terminal was connected with the sliding contact piece just mentioned. In order to adjust the balance of the bridge, it was necessary only to turn the screw and so to move the contact piece along the stretched wire, until a position was found where no current flowed through the galvanometer. This method was found to be very delicate and convenient.

The compensating resistance was connected with the bolometer strips by means of copper wires. All metallic junctions which might give rise to thermal currents were, as far as possible, so arranged that currents produced in this way in any part of the bridge would be opposed by similar thermal currents produced in the opposite part of the bridge; this condition being secured by bringing corresponding thermo-junctions as near to each other as it was convenient to do.

The galvanometer employed was of the four-coil type. It was constructed by Professor W. S. Franklin, after the same general plan as has been followed by Snow,¹ Paschen,² and others. When the two front coils were in multiple with each other and the two rear coils similarly connected, and the two pairs were connected in series; the resistance was found to be 190 ohms. The moving parts consisted of four little magnets of piano wire, each about 5 mm. long, and a mirror of thin cover glass, 4 mm. wide by 7 mm. long, all mounted on a slender rod of glass and suspended by a very fine quartz fiber. Any oscillations of the needles were very strongly and effectively damped by the air resistance to the light-moving parts, — a very essential condition to the correct operation of the instrument when used to get the variations of a rapidly fluctuating source of radiant energy.

The scale was divided into 100 half-inch divisions, each of which was, in turn, divided into tenths. The distance of the scale from the galvanometer was 100 scale divisions, *i.e.* 50 inches. With the telescope used, fifths of the smallest divisions could be estimated. In speaking of scale divisions, the half-inch divisions will always be meant.

To determine at any time the sensitiveness of the arrangement, a plan was followed which was first proposed by Knut Ångström.³ In multiple with the exposed bolometer strip was placed a resistance of 1000 ohms. By removing a plug and throwing 5000 ohms more into the shunt circuit, the galvanometer needle was deflected in the same direction as when the strip was heated. Only the

¹ B. W. Snow, *Wied. Ann.*, Vol. 47, p. 213; *Phys. Review*, Vol. 1, p. 2.

² F. Paschen, *Wied. Ann.*, Vol. 48, p. 272; *Zeitschr. für Instrumentenkunde*, Vol. 13, 1893, p. 13.

³ Ångström, *Oefvers. af kongl. Vetenskaps-Akad. Förhandl.*, 1888, Vol. 6, p. 379.

first throws of the needle were taken in observations for sensitiveness, — a procedure which a comparison of first throws and final deflections, at different degrees of sensitiveness, showed to be justifiable. The ratio of first throws to permanent deflections was found to be 1.5.

No attempt has been made to correct the determinations of sensitiveness for any variation of the ratio of resistance of the bolometer strip and that of the shunt, due to temperature changes. This would be a simple correction to apply if only one could ascertain the temperature of the strips at any time. Since, however, this temperature is a function not only of the temperature of the room, but also of the strength of the main current and of the radiating power of the surface of the strip, a direct determination of it is impracticable. A value for this correction combined with the correction for the change in sensitiveness due to change in absolute temperature of the strip, could have been ascertained by comparing deflections given by a source of constant radiation, under different conditions of room temperature and current strength; but for the present work such refinements were deemed unnecessary, since the error introduced by not applying these corrections would form only a small proportional part of the quantities to be measured.

For observations requiring only a low degree of sensitiveness, the galvanometer coils were connected in the multiple-series arrangement described above. For higher degrees of sensitiveness all the coils were put in multiple, the increase in sensitiveness produced by this change being about twofold. With either connection the fine adjustment of sensitiveness was obtained by changing the distance of a controlling magnet. In nearly all cases the sensitiveness of the bolometer was such as to permit the controlling magnet to be placed so as to strengthen the earth's field, and the period of the galvanometer needle was correspondingly short.

The constants of the galvanometer and of the bolometer were as follows: With the galvanometer coils in the multiple-series connection and the sensitiveness adjusted to give a throw of the needle of 16.2 scale divisions on putting the 5000 ohms in the shunt (this being the standard sensitiveness for candles), the period

of the needles was 6 seconds for a complete vibration. A deflection of one scale division corresponded to a current of 68×10^{-10} amperes.

This deflection of one scale-division corresponded to a temperature rise in the bolometer strips of $0^{\circ}.00657$ C. If we reduce this to millimeter divisions on a scale placed at a distance of 1 m. from the mirror, we see that one millimeter deflection corresponded to a current of 68×10^{-11} amperes, and the corresponding rise in temperature of the strip was $0^{\circ}.00066$ C.

This temperature sensitiveness is much smaller than has usually been employed in bolometer work, but it was amply sufficient for the purpose. That the bolometer itself was one of high sensitiveness is evident from the fact that this degree of sensitiveness was attained with galvanometer needles swinging in a strengthened field and with a galvanometer of 190 ohms resistance. The conditions for maximum sensitiveness of the bridge would have required a galvanometer resistance of only 0.5 ohms.

The reason for the great sensitiveness lay in the nature of the strips employed. Their area was considerable, the temperature coefficient of the iron was high, and the current through it was large, ranging from 0.15 to 0.20 amperes, the size of the strips permitting the use of a large current without undue heating. As a result of the strong field in which the galvanometer needles swung, the drift due to magnetic changes was usually imperceptible.

The bolometer, compensating resistance, and battery were all placed in an interior room, with thick brick walls, and having communication with the outer room only by a door, *D*, Fig. 3.

The temperature of this room changed very slowly, and it was quite free from draughts. The bolometer box, *B*, was placed upon a shelf fastened to the door of the room, and looked out, through a hole in the door, upon the outer room. A double screen, *S*, of tin, arranged to slide up and down on the outside of this door, covered up the bolometer strips when desired. The box, *B'*, containing the compensating resistance, was also fastened to the inside of the door. The end of the box through which the screw projected, fitted into a hole in the door, so that the screw

could be turned and the bridge balanced from the outer room. The wires, g, g , led to the galvanometer, and s, s led to the box which was in multiple with the exposed strip. For further protection from draughts the bolometer box and connecting wires were covered with cotton waste.

Under this arrangement it was necessary to enter the inner room only occasionally to attend to the batteries, and, therefore, the bolometric resistances were subjected to no sudden changes

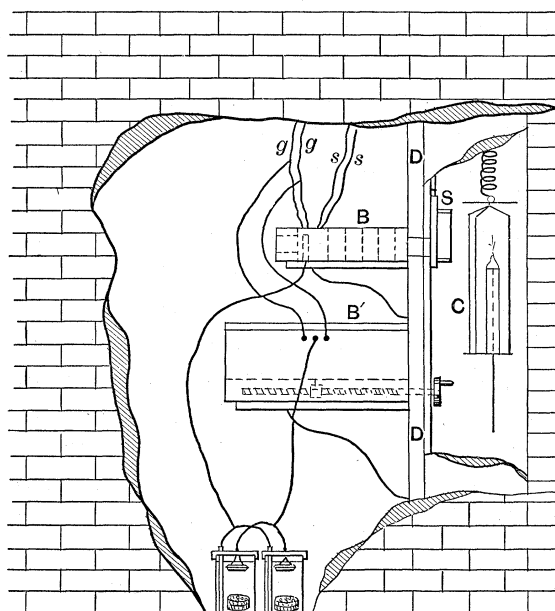


Fig. 3.

of temperature. As a consequence, drift due to changes of temperature of these resistances was very small and quite regular. Unfortunately, however, it was necessary to place the box containing the resistance which was in multiple with the exposed strip in the outer room, near the galvanometer telescope. The resistance of this box, and of the wires leading to it, changed slightly with temperature changes in the outer room. As a consequence a slight and regular drift would always ensue upon lighting the gas in the outer room preparatory to beginning work.

Since only a very small proportion of the current flowed through this outer circuit, this drift was never large, and would gradually cease as the temperature of the room became constant. Various expedients were adopted for minimizing this temperature change; such as opening a window slightly to admit enough cold air to counterbalance the heating due to the gaslights; closing the valves in the radiator pipes, etc. In every case correction was made for the drift and for any change of sensitiveness which might take place, on the assumption that the change had been a perfectly regular one. Whenever it would seem that this assumption could result in any considerable error, the observations were rejected.

In getting the variations of candles, a special device was required to keep the top of the burning candle at constant height in front of the bolometer strip. A suggestion of Mr. C. H. Bierbaum, M.E., resulted in the construction of the following simple and effective arrangement. A spiral spring, about 60 cm. long and 4 cm. in diameter, was attached to a small scale-pan, and the spring was cut off to such a length that when a candle was put on the scale-pan, the elongation was just equal to the length of the candle. The spring would then take up as fast as the top of the candle was lowered by burning. A small piece of sheet metal served to protect the spring from the heat of the candle. In order to keep the scale-pan from swinging side-wise and from oscillating up and down, a couple of wires were passed vertically through holes on opposite sides of it, and served as loosely fitting guides, which, without interfering with the take-up of the spring, effectually damped any vibrations. The adjustment of a spring to any candle could be made in a few minutes with sufficient accuracy so that the height of the top of a candle burning on the scale-pan would not vary over 1 mm. in an hour. *C*, Fig. 3, shows this arrangement in place.

III.

Method of Taking Observations.

All determinations of the variations of standards were made at times when the laboratory and its surroundings were very quiet. Most of them were made between the hours of seven and twelve in the evening; a few were made on holidays, when the laboratory was closed for general work. Before beginning a set of observations the sensitiveness was adjusted, and the galvanometer was carefully watched for a considerable time to get the amount of its swings due to currents of air about the bolometer strips or to changes in the earth's field. The sensitiveness was also tested from time to time in order to find whether this was variable or not. If swings of the galvanometer greater in amount than two-tenths of a scale division, or any irregular changes in sensitiveness were detected, no run was attempted. Since two-tenths of a scale division amounted to only about 0.5 per cent of the total deflection in most cases, and since the variation in the light sources themselves were many times two-tenths of a scale division in the case of all the standards except the Carcel lamp, this amount of error was considered entirely negligible.

After taking these preliminary observations, which usually required about an hour's time, the bolometer was exposed to the source of light. After waiting a few minutes for the strip and the surrounding parts to be thoroughly heated, the galvanometer deflections were read rapidly by one person, and were plotted by another as fast as they were read, the time being taken from a watch. In this way curves were traced which represent very truly all the changes in the radiation of the light source. This method of recording observations was found to be much preferable to the ordinary one of noting galvanometer readings and plotting them afterwards, both in point of saving of time and in accuracy attainable. It made possible, not only more numerous readings, but also the giving of oral instructions to the recorder, and the consequent plotting of curves which, in correctness of their minor as well as of their major details, would be but little surpassed by a photographic record.

An unfortunate feature in these curves is that the scale of abscissas, representing times, is so small. As a result of this it has been difficult to represent with the greatest accuracy the true slope of those portions of the curves which correspond to very rapid changes in radiation. In examining the curve it should be borne in mind that a very steep line may cover several seconds of time, and that it is quite possible that a photometric setting might be made during the time that a candle is executing just such a variation as is represented by the steepest parts of one of the candle curves. The number of galvanometer readings plotted during the space of five minutes was usually from 50 to 100.

At the end of a run the strips and surrounding parts were given time completely to lose the heat imparted to them, after which readings for zero and sensitiveness were taken. To correct the curves for changes in zero and in sensitiveness, a plan was followed which was justified only by the fact that the error introduced by employing it was entirely insignificant. Change in sensitiveness was reduced to ordinates of the curve by means of the equation :—

$$\frac{\text{change in sensitiveness}}{\text{initial sensitiveness}} \times \text{approximate mean ordinate} \\ = \text{correction to ordinate.}$$

This was combined with the drift, and a point was placed on the cross-section paper having its ordinate such as to represent the combined effect of drift and change of sensitiveness, its abscissa being the time of taking observations. This point was joined by a straight line with a point on the axis of *X* at the time of taking the initial zero readings, thus assuming that the drift and change of sensitiveness had been regular during the run. Then, if the initial sensitiveness happened to be unequal to the sensitiveness which was taken as standard, by an equation similar to the one given above, a correction to all the ordinates was computed to reduce the curve to the standard sensitiveness. If the distance of the light source from the strip was other than that taken as standard, a similar correction was computed for this. These two reductions were then combined, and a line was drawn parallel to the first correction line and distant from it by an amount equal to

these combined corrections. The mean ordinates of the curves, reckoned from the new base line so constituted, represent the relative quantities of radiation of the sources of light. The curves were then traced on bond paper, the axis of abscissas on which was put so as to fall on the correction line on the original cross-section paper. Since the corrections which have been spoken of above were all small (except in the case of curves XVI. and XVI. (a)), neither the proportionality of the variations to the total deflection, nor the slope of the lines of the curves, has been materially affected.

Before setting out on the work it was feared that the inertia of the moving parts of the galvanometer might have a decided effect on the accuracy of the results obtained, in that it would tend to exaggerate the rapid changes. This fear was found to be groundless. In the first place, the galvanometer was, as has been said, very nearly dead beat. Although the bolometer strip followed almost instantly every fluctuation of the source of radiation, the fluctuations themselves were not usually so rapid that the galvanometer needles were driven with any considerable velocity. Consequently the tendency of the needles to oscillate in their natural period was very slight. That the inertia of the moving parts had very little, if any, influence on the shape of the curves was shown by the fact that at the end of most swings the needle would pause for some seconds before beginning its return; while it rarely happened that the angular velocity did not change during a swing.

It has been impossible to draw the curves so as to show clearly these variations which involved only a few seconds of time, since, as has been said, the scale to which the time abscissas are plotted is very small compared with the scale of ordinates. These important peculiarities of the movements of the galvanometer needle were clearly perceptible, however, to one who observed the galvanometer deflections for only a little time; while their character made it perfectly evident that the swings of the galvanometer needles in their natural period exercised only an inappreciable influence on the slopes of the curves and the magnitudes of the variations recorded.

On the plates the smallest divisions of abscissas represent five minutes of time. The corresponding divisions of ordinates are five divisions of the galvanometer scale. In order to economize space, no attention has been given to the height of the curves relative to the base line on the plates. The total ordinates for each curve are indicated by such a symbol as this $\left. \begin{array}{c} \text{I} \\ \text{II} \end{array} \right\} 40$, indicating that the true X -axis for curves I. and II. should be placed 10 divisions below the base line on the plate. Consequently on the various plates curves having the same ordinates are plotted one above the other, the true ordinate being indicated in the way mentioned above.

The data by which the curves have been corrected are given in Table I.

Table II. gives, in its first column, the approximate abscissas of the portion of the curve in question, and in its second column the mean ordinates of these portions. The third column contains the deviations of the mean ordinates of the portions from the mean ordinate of the whole curve. The fourth column contains these deviations reduced to percentages of the whole. The fifth contains similar deviations from the mean ordinate of all the curves of the particular standard under consideration, and in the sixth these also are reduced to percentages.

The standard distance and sensitiveness for the candle and Hefner lamp were 26 cm. and 16.2, respectively; for the Carcel lamp they were 56.7 cm. and 15, and for the Methven screen they were 50 cm. and 40, respectively.

TABLE I.

| Number of curve. | Times of taking zero and sensitiveness readings. | | Time of lighting. | Time of taking the curve. | | Hourly rate of consumption. | Distance from Bolo-meter. | Initial sensitiveness. | Correction for | | Reduction to standard | | |
|------------------|--|-------|-------------------|---------------------------|-------|-----------------------------|---------------------------|------------------------|----------------|--------------------------|-----------------------|------------|-------|
| | h. m. | h. m. | | h. m. | h. m. | | | | Drift. | Change of sensitiveness. | Sensitiveness. | Dis-tance. | Rate. |
| I. | 8 47 | 10 01 | 8 50 | 8 50 | 9 35 | 7.910 | 25.3 | 16.2 | +0.9 | +0.2 | 0 | -2.3 | -0.70 |
| II. | 9 41 | 10 50 | 9 33 | 9 50 | 10 35 | 7.767 | 26.0 | 16.1 | -0.4 | 0 | +0.3 | 0 | +0.05 |
| III. | 8 00 | 9 15 | 8 00 | 8 00 | 9 00 | 7.842 | 26.0 | 16.1 | -2.9 | +0.8 | +0.3 | 0 | -0.34 |
| III. (a) | 9 38 | 10 30 | 8 00 | 9 45 | 10 15 | 7.842 | 26.0 | 16.08 | +0.3 | 0 | +0.3 | 0 | -0.34 |
| IV. | 7 50 | 9 05 | 7 50 | 7 50 | 8 50 | 7.692 | 25.3 | 16.15 | +1.0 | -0.1 | +0.1 | -2.1 | +0.45 |
| IV. (a) | 9 35 | 10 30 | 7 50 | 9 45 | 10 15 | 7.692 | 25.3 | 16.4 | -0.1 | 0 | -0.5 | -2.3 | +0.45 |
| V. | 7 55 | 9 10 | 7 55 | 7 55 | 8 55 | 8.60 | 25.3 | 16.27 | +1.1 | -1.6 | -0.3 | -2.8 | — |
| V. (a) | 9 50 | 10 40 | 7 55 | 9 55 | 10 25 | 8.60 | 25.3 | 16.3 | -1.7 | +1.0 | -0.3 | -2.9 | — |
| V. (b) | 8 21 | 9 25 | 8 10 | 8 25 | 9 10 | — | 26.0 | 16.2 | -5.7 | +4.7 | 0 | 0 | — |
| VI. | 8 05 | 9 20 | 8 05 | 8 05 | 9 05 | — | 26.0 | 16.15 | +1.3 | -0.3 | +0.15 | 0 | — |
| VI. (a) | 9 30 | 10 31 | 8 05 | 9 46 | 10 16 | — | 26.0 | 16.2 | -0.3 | -0.15 | 0 | 0 | — |

ENGLISH CANDLE.

GERMAN CANDLE.

| | 8 05 | 8 57 | 7 15 | 8 10 | 8 40 | — | 25.0 | 16.33 | -3.1 | +1.8 | -0.4 | -2.2 | — |
|-----------|-------|-------|------|-------|-------|-------|------|-------|------|------|-------|------|------|
| VII. | 9 05 | 10 00 | 7 15 | 9 10 | 9 45 | — | 25.0 | 16.1 | -1.3 | -1.9 | +0.3 | -2.3 | — |
| VII. (a) | 10 17 | 11 00 | 9 57 | 10 25 | 10 45 | — | 26.0 | 16.2 | -3.2 | +0.5 | 0 | 0 | — |
| VIII. | 8 08 | 9 05 | 7 13 | 8 20 | 8 50 | — | 26.0 | 16.2 | -0.3 | +0.1 | 0 | 0 | — |
| IX. | 8 42 | 9 20 | — | 8 50 | 9 00 | — | 36.8 | 32.5 | -0.3 | -0.3 | -0.1 | 0 | — |
| X. | 9 08 | 10 26 | 8 55 | 9 14 | 9 34 | — | 26.0 | 16.2 | -3.0 | +0.8 | 0 | 0 | — |
| XI. | 9 08 | 10 26 | 8 55 | 9 38 | 9 40 | — | 26.0 | 16.2 | -3.0 | +0.8 | 0 | 0 | — |
| XI. (a) | 9 08 | 10 26 | 8 55 | 9 46½ | 9 49½ | — | 26.0 | 16.2 | -3.0 | +0.8 | 0 | 0 | — |
| XI. (b) | 9 08 | 10 26 | 8 55 | 9 55 | 9 58 | — | 26.0 | 16.2 | -3.0 | +0.8 | 0 | 0 | — |
| XI. (c) | 7 53 | 8 35 | 7 40 | 8 00 | 8 20 | — | 26.0 | 16.2 | -2.3 | +0.4 | 0 | 0 | — |
| XII. | 8 40 | 9 25 | 7 40 | 8 50 | 9 10 | — | 26.0 | 16.2 | -0.7 | 0 | 0 | 0 | — |
| XII. (a) | 10 11 | 10 50 | 9 38 | 10 15 | 10 35 | — | 26.0 | 16.2 | -0.5 | +1.3 | 0 | 0 | — |
| XIII. | 10 00 | 10 35 | 9 53 | 10 08 | 10 18 | — | 26.0 | 16.2 | +0.7 | 0 | 0 | 0 | — |
| XIV. | 10 14 | 11 03 | 9 40 | 10 18 | 10 48 | 47.05 | 56.7 | 15.0 | +0.7 | -0.4 | 0 | 0 | -4.5 |
| XV. | 8 38 | 9 38 | 8 10 | 8 45 | 9 25 | 43.05 | 56.7 | 28.2 | +2.8 | -0.4 | -33.0 | 0 | -0.7 |
| XVI. | 10 00 | 10 38 | 8 10 | 10 03 | 10 23 | 44.14 | 56.7 | 28.4 | 0 | +0.2 | -35.6 | 0 | -1.5 |
| XVI. (a) | 8 13 | 8 55 | 7 59 | 8 20 | 8 40 | 46.85 | 56.7 | 15.03 | +0.6 | 0 | -0.1 | 0 | -3.0 |
| XVII. | 10 54 | 11 20 | 7 59 | 10 59 | 11 09 | 40.00 | 56.7 | 15.0 | 0 | 0 | 0 | 0 | +1.3 |
| XVII. (a) | 9 15 | 10 06 | — | 9 21 | 9 51 | — | 49.7 | 39.8 | +4.7 | -0.2 | +0.1 | -0.3 | — |
| XVIII. | 9 22 | 10 00 | — | 9 30 | 9 50 | — | 50.0 | 39.6 | +2.5 | -1.0 | +0.2 | 0 | — |
| XIX. | 9 00 | 9 51 | — | 9 06 | 9 36 | — | 50.0 | 40.2 | -0.5 | +0.2 | -0.2 | 0 | — |
| XX. | 8 17 | 8 59 | — | 8 24 | 8 44 | — | 50.0 | 40.1 | +0.6 | +0.1 | -0.1 | 0 | — |
| XXI. | 9 01 | 9 37 | — | 9 09 | 9 19 | — | 50.0 | 40.0 | +0.6 | 0 | 0 | 0 | — |
| XXII. | | | | | | | | | | | | | |

HEPNER LAMP.

CARCEL LAMP.

METHVEN SCREEN.

TABLE II.

| | Time: Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the English candle curves. | Deviations from general mean reduced to percentages. |
|-----------------|-------------------------------|---|--|--|---|--|
| CURVE I. | 15-20 | 41.68 | -1.04 | -2.43 | +0.59 | +1.44 |
| | 20-25 | 44.68 | +1.96 | +4.58 | +3.59 | +8.72 |
| | 25-30 | 40.84 | -1.88 | -4.40 | -0.25 | -0.61 |
| | 30-35 | 43.39 | +0.67 | +1.57 | +2.30 | +5.59 |
| | 35-40 | 43.71 | +0.99 | +2.32 | +2.62 | +6.37 |
| | 40-45 | 42.04 | -0.68 | -1.59 | +0.95 | +2.31 |
| | | 42.72 | Correction for rate of burning = - 0.70. | | | |
| CURVE II. | 45-50 | 39.24 | -1.77 | -4.32 | -1.85 | -4.50 |
| | 50-55 | 37.85 | -3.16 | -7.70 | -3.24 | -7.87 |
| | 55-60 | 39.12 | -1.89 | -4.61 | -1.97 | -4.79 |
| | 60-65 | 40.44 | -0.57 | -1.39 | -0.65 | -1.58 |
| | 65-70 | 40.76 | -0.25 | -0.61 | -0.33 | -0.80 |
| | 70-75 | 42.85 | +1.84 | +4.48 | +1.76 | +4.28 |
| | 75-80 | 43.52 | +2.51 | +6.12 | +2.43 | +5.92 |
| | 80-85 | 43.20 | +2.19 | +5.34 | +2.11 | +5.13 |
| | 85-90 | 42.11 | +1.10 | +2.68 | +1.02 | +2.48 |
| | | 41.01 | Correction for rate of burning = +0.05. | | | |
| CURVE III. | 15-20 | 43.04 | -0.54 | -1.25 | +1.95 | +4.74 |
| | 20-25 | 42.88 | -0.70 | -1.62 | +1.79 | +4.35 |
| | 25-30 | 42.82 | -0.76 | -1.76 | +1.73 | +4.21 |
| | 30-35 | 42.18 | -1.40 | -3.23 | +1.09 | +2.65 |
| | 35-40 | 43.23 | -0.35 | -0.80 | +2.14 | +5.20 |
| | 40-45 | 44.80 | +1.22 | +2.80 | +3.71 | +9.02 |
| | 45-50 | 43.58 | 0 | 0 | +2.49 | +6.06 |
| | 50-55 | 44.30 | +0.72 | +1.65 | +3.21 | +7.80 |
| | 55-60 | 45.38 | +1.80 | +4.13 | +4.29 | +10.45 |
| | | | 43.58 | Correction for rate of burning = -0.34. | | |
| CURVE III. (a). | 60-65 | 43.25 | +0.23 | +0.53 | +2.16 | +5.25 |
| | 65-70 | 43.52 | +0.50 | +1.16 | +2.43 | +5.91 |
| | 70-75 | 42.82 | -0.20 | -0.47 | +1.73 | +4.21 |
| | 75-80 | 43.68 | +0.66 | +1.54 | +2.59 | +6.30 |
| | 80-85 | 42.82 | -0.20 | -0.47 | +1.73 | +4.21 |
| | 85-90 | 42.04 | -0.98 | -2.28 | +0.95 | +2.31 |
| | | 43.02 | Correction for rate of burning = -0.34. | | | |

TABLE II. (continued).

| | Time: Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the English candle curves. | Deviations from general mean reduced to percentages. |
|-----------|-------------------------------|---|--|--|---|--|
| CURVE IV. | 15-20 | 38.65 | +1.42 | +3.82 | -2.44 | -5.94 |
| | 20-25 | 39.84 | +2.61 | +7.01 | -1.25 | -3.04 |
| | 25-30 | 38.07 | +0.84 | +2.26 | -3.02 | -7.35 |
| | 30-35 | 35.76 | -1.47 | -3.87 | -5.33 | -12.95 |
| | 35-40 | 35.72 | -1.51 | -4.05 | -5.37 | -13.08 |
| | 40-45 | 37.04 | -0.19 | -0.51 | -4.05 | -9.84 |
| | 45-50 | 35.84 | -1.39 | -3.75 | -5.25 | -12.78 |
| | 50-55 | 37.16 | -0.07 | -0.02 | -3.93 | -9.55 |
| | 55-60 | 36.96 | -0.27 | -0.72 | -4.13 | -10.05 |
| | 37.23 | Correction for rate of burning = +0.45. | | | | |

| | | | | | | |
|----------------|-------|-------|---|-------|-------|-------|
| CURVE IV. (a). | 60-65 | 38.26 | -1.47 | -3.70 | -2.83 | -6.88 |
| | 65-70 | 39.04 | -0.69 | -1.74 | -2.05 | -4.99 |
| | 70-75 | 38.26 | -1.47 | -3.70 | -2.83 | -6.88 |
| | 75-80 | 40.84 | +1.11 | +2.80 | -0.25 | -0.62 |
| | 80-85 | 40.84 | +1.11 | +2.80 | -0.25 | -0.62 |
| | 85-90 | 41.16 | +1.43 | +3.60 | +0.07 | +0.17 |
| | | 39.73 | Correction for rate of burning = +0.45. | | | |

Mean ordinate of all the English candle curves is 41.09.

Mean ordinate of all the English candle curves corrected for rate of burning is 41.05.

Mean ordinate of all the English candle curves corrected for rate and reduced to true deflections is 41.06.

| | | | | German candle curves. | | |
|----------|-------|-------|-------|--------------------------|-------|-------|
| CURVE V. | 15-20 | 49.48 | -0.18 | -0.36 | -1.03 | -2.05 |
| | 20-25 | 48.62 | -1.04 | -2.10 | -1.89 | -3.77 |
| | 25-30 | 49.20 | -0.46 | -0.92 | -1.31 | -2.60 |
| | 30-35 | 49.44 | -0.22 | -0.44 | -1.07 | -2.12 |
| | 35-40 | 49.40 | -0.26 | -0.52 | -1.11 | -2.20 |
| | 40-45 | 49.20 | -0.46 | -0.92 | -1.31 | -2.60 |
| | 45-50 | 49.52 | -0.14 | -0.28 | -0.99 | -1.96 |
| | 50-55 | 51.96 | +2.30 | +4.64 | +1.45 | +2.88 |
| | 55-60 | 50.12 | +0.46 | +0.92 | -0.39 | -0.77 |
| | 49.66 | | | | | |

TABLE II. (continued).

| | Time : Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the German candle curves. | Deviations from general mean reduced to percentages. |
|----------------|--------------------------------|---|--|--|--|--|
| CURVE V. (a). | 60-65 | 52.80 | +1.43 | +2.78 | +2.29 | +4.53 |
| | 65-70 | 51.00 | -0.37 | -0.72 | +0.49 | +0.97 |
| | 70-75 | 49.44 | -1.93 | -3.76 | -1.07 | -2.12 |
| | 75-80 | 50.24 | -1.13 | -2.20 | -0.27 | -0.53 |
| | 80-85 | 50.68 | -0.69 | -1.35 | +0.17 | +0.34 |
| | 85-90 | 54.08 | +2.71 | +5.27 | +3.57 | +7.07 |
| | | 51.37 | | | | |
| CURVE V. (b). | 50-55 | 53.14 | -0.78 | -1.45 | +2.63 | +5.21 |
| | 55-60 | 55.41 | +1.49 | +2.77 | +4.90 | +9.71 |
| | 60-65 | 52.53 | -1.39 | -2.58 | +2.02 | +4.00 |
| | 65-70 | 52.82 | -1.10 | -1.88 | +2.31 | +4.58 |
| | 70-75 | 52.62 | -1.30 | -2.40 | +2.11 | +4.18 |
| | 75-80 | 53.30 | -0.62 | -1.15 | +2.79 | +5.53 |
| | 80-85 | 55.22 | +1.30 | +2.42 | +4.71 | +9.32 |
| | 85-90 | 56.33 | +2.41 | +4.47 | +5.82 | +11.52 |
| | | 53.92 | | | | |
| CURVE VI. | 15-20 | 45.24 | -3.67 | -7.35 | -5.27 | -10.43 |
| | 20-25 | 46.63 | -2.28 | -4.62 | -3.88 | -7.69 |
| | 25-30 | 48.17 | -0.74 | -1.51 | -2.34 | -4.63 |
| | 30-35 | 49.20 | +0.29 | +0.59 | -1.31 | -2.60 |
| | 35-40 | 49.84 | +0.93 | +1.90 | -0.67 | -1.33 |
| | 40-45 | 49.52 | +0.61 | +1.25 | -0.99 | -1.96 |
| | 45-50 | 49.88 | +0.97 | +1.99 | -0.63 | -1.25 |
| | 50-55 | 50.20 | +1.29 | +2.64 | -0.31 | -0.61 |
| | 55-60 | 57.48 | +2.57 | +5.16 | +0.97 | +1.92 |
| | | 48.91 | | | | |
| CURVE VI. (a). | 60-65 | 48.93 | +0.14 | +0.28 | -1.58 | -3.13 |
| | 65-70 | 49.20 | +0.41 | +0.84 | -1.31 | -2.60 |
| | 70-75 | 47.78 | -1.01 | -2.07 | -2.73 | -5.40 |
| | 75-80 | 48.52 | -0.27 | -0.55 | -1.99 | -3.94 |
| | 80-85 | 48.97 | +0.18 | +0.37 | -1.54 | -3.05 |
| | 85-90 | 49.36 | +0.57 | +1.17 | -1.15 | -2.28 |
| | | 48.79 | | | | |

Mean ordinates of all German candle curves is 50.51

Reduced to true deflections = 50.40

TABLE II. (continued).

| | Time : Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the Hefner lamp curves. | Deviations from general mean reduced to percentages. |
|-----------------|--------------------------------|---|--|--|--|--|
| CURVE VII. | 0-5 | 38.81 | -0.50 | -1.27 | +0.17 | +0.44 |
| | 5-10 | 40.02 | +0.71 | +1.81 | +1.38 | +3.57 |
| | 10-15 | 39.64 | +0.33 | +0.84 | +1.00 | +2.59 |
| | 15-20 | 39.19 | -0.12 | -0.31 | +0.55 | +1.42 |
| | 20-25 | 39.03 | -0.28 | -0.71 | +0.39 | +1.01 |
| | 25-30 | 39.19 | -0.12 | -0.31 | +0.55 | +1.42 |
| | | 39.31 | | | | |
| CURVE VII. (a). | 30-35 | 39.96 | +0.44 | +1.11 | +1.32 | +3.42 |
| | 35-40 | 40.27 | +0.75 | +1.90 | +1.63 | +4.22 |
| | 40-45 | 39.83 | +0.31 | +0.79 | +1.19 | +3.08 |
| | 45-50 | 39.54 | +0.02 | +0.05 | +0.90 | +2.33 |
| | 50-55 | 39.09 | -0.43 | -1.09 | +0.45 | +1.16 |
| | 55-60 | 39.03 | -0.49 | -1.24 | +0.39 | +1.01 |
| | 60-65 | 39.19 | -0.33 | -0.84 | +0.55 | +1.42 |
| | 65-70 | 39.27 | -0.25 | -0.63 | +0.63 | +1.63 |
| | 39.52 | | | | | |
| CURVE VIII. | 70-75 | 38.91 | -1.12 | -2.80 | +0.27 | +0.70 |
| | 75-80 | 39.83 | -0.20 | -0.50 | +1.19 | +3.08 |
| | 80-85 | 40.54 | +0.51 | +1.28 | +1.90 | +4.92 |
| | 85-90 | 40.86 | +0.83 | +2.08 | +2.22 | +5.74 |
| | | 40.03 | | | | |
| CURVE IX. | 0-5 | 38.26 | -0.54 | -1.39 | -0.38 | -0.98 |
| | 5-10 | 38.62 | -0.18 | -0.46 | -0.02 | -0.05 |
| | 10-15 | 38.97 | +0.17 | +0.44 | +0.33 | +0.85 |
| | 15-20 | 38.87 | +0.07 | +0.18 | +0.23 | +0.59 |
| | 20-25 | 39.04 | +0.24 | +0.62 | +0.40 | +1.03 |
| | 25-30 | 39.03 | +0.23 | +0.59 | +0.39 | +1.01 |
| | | 38.80 | | | | |
| CURVE X. | 30-35 | 38.89 | 0 | 0 | +0.25 | +0.65 |
| | 35-40 | 39.16 | +0.27 | +0.70 | +0.52 | +1.34 |
| | 40-45 | 38.62 | -0.27 | -0.70 | -0.02 | -0.05 |
| | | 38.89 | | | | |

TABLE II. (*continued*).

| | Time : Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the Hefner lamp curves. | Deviations from general mean reduced to percentages. |
|----------------------------------|--------------------------------|---|--|--|--|--|
| CURVE XI. | 45-50 | 39.42 | -0.83 | -2.06 | +0.78 | +2.02 |
| | 50-55 | 40.50 | -0.10 | -0.25 | +1.51 | +3.91 |
| | 55-60 | 40.50 | +0.25 | +0.62 | +1.86 | +4.81 |
| | 60-65 | 40.92 | +0.67 | +1.67 | +2.28 | +5.90 |
| | | 40.25 | | | | |
| CURVES XI. (a), (b), and (c). | 70 | 39.37 | -0.24 | -0.61 | +0.73 | +1.89 |
| | 75-80 | 40.00 | +0.39 | +0.98 | +1.36 | +3.52 |
| | 85-90 | 39.46 | -0.15 | -0.38 | +0.82 | +2.12 |
| | | 39.61 | | | | |
| CURVE XII. | 0-5 | 36.47 | -0.90 | -2.27 | -2.17 | -5.62 |
| | 5-10 | 37.21 | -0.16 | -0.43 | -1.43 | -3.70 |
| | 10-15 | 37.85 | +0.48 | +1.28 | -0.79 | -2.05 |
| | 15-20 | 37.94 | +0.57 | +1.52 | -0.70 | -1.81 |
| | | 37.37 | | | | |
| CURVE XII. (d). | 25-30 | 35.53 | -0.67 | -1.85 | -3.11 | -8.03 |
| | 30-35 | 35.73 | -0.47 | -1.30 | -2.91 | -7.52 |
| | 35-40 | 36.38 | +0.18 | +0.50 | -2.26 | -5.84 |
| | 40-45 | 37.18 | +0.98 | +2.71 | -1.46 | -3.78 |
| | | 36.20 | | | | |
| CURVE XIII. | 50-55 | 38.84 | +0.15 | +0.39 | +0.20 | +0.52 |
| | 55-60 | 38.74 | +0.05 | +0.13 | +0.10 | +0.26 |
| | 60-65 | 38.20 | -0.49 | -1.27 | -0.44 | -1.14 |
| | 65-70 | 39.00 | +0.31 | +0.80 | +0.36 | +0.93 |
| | | 38.69 | | | | |
| CURVE XIV. | 75-80 | 38.46 | -0.16 | -0.41 | -0.18 | -0.47 |
| | 80-85 | 38.78 | +0.16 | +0.41 | +0.14 | +0.36 |
| | | 38.62 | | | | |

Mean ordinate of first five minutes of Hefner lamp curves = 38.645.
Reduced to true deflections = 38.66.

TABLE II. (continued).

| | Time: Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the Carcel lamp curves. | Deviations from general mean reduced to percentages. |
|---------------------|-------------------------------|---|--|--|--|--|
| CURVE XV. | 0-5 | 27.11 | -0.49 | -1.78 | -1.16 | -4.11 |
| | 5-10 | 27.37 | -0.23 | -0.83 | -0.90 | -3.19 |
| | 10-15 | 27.62 | +0.02 | +0.07 | -0.65 | -2.30 |
| | 15-20 | 27.68 | +0.08 | +0.29 | -0.59 | -2.09 |
| | 20-25 | 27.85 | +0.25 | +0.91 | -0.42 | -1.49 |
| | 25-30 | 27.98 | +0.38 | +1.34 | -0.29 | -1.03 |
| | | 27.60 | | | | |
| CURVE XVI. | 0-5 | 29.00 | -0.37 | -1.26 | +0.73 | +2.58 |
| | 5-10 | 29.09 | -0.28 | -0.95 | +0.82 | +2.90 |
| | 10-15 | 29.19 | -0.18 | -0.61 | +0.92 | +3.26 |
| | 15-20 | 29.32 | -0.05 | -0.17 | +1.05 | +3.72 |
| | 20-25 | 29.35 | -0.02 | -0.07 | +1.08 | +3.83 |
| | 25-30 | 29.61 | +0.24 | +0.82 | +1.34 | +4.74 |
| | 30-35 | 29.61 | +0.24 | +0.82 | +1.34 | +4.74 |
| | 35-40 | 29.76 | +0.39 | +1.33 | +1.49 | +5.28 |
| | 29.37 | | | | | |
| CURVE XVI. (a). | 40-45 | 29.00 | +0.37 | +1.29 | +0.73 | +2.59 |
| | 45-50 | 28.75 | +0.12 | +0.42 | +0.48 | +1.70 |
| | 50-55 | 28.65 | +0.02 | +0.07 | +0.38 | +1.34 |
| | 55-60 | 28.14 | -0.49 | -1.72 | -0.13 | -0.46 |
| | | 28.63 | | | | |
| CURVE XVII. | 60-65 | 25.90 | -0.92 | -3.44 | -2.37 | -8.39 |
| | 65-70 | 26.86 | +0.04 | +0.15 | -1.41 | -4.97 |
| | 70-75 | 27.27 | +0.45 | +1.68 | -1.00 | -3.54 |
| | 75-80 | 27.27 | +0.45 | +1.68 | -1.00 | -3.54 |
| | | 26.82 | | | | |
| CURVE XVII. (a). | 80-85 | 27.82 | -0.24 | -0.86 | -0.45 | -1.59 |
| | 85-90 | 28.30 | +0.24 | +0.86 | +0.03 | +0.11 |
| | | 28.06 | | | | |

TABLE II. (continued).

| | Time: Minutes on curve. | Mean ordinates and their mean. | Deviations from the mean ordinate of the curve. | Deviations reduced to percentages. | Deviations from mean ordinate of all the Carcel lamp curves. | Deviations from general mean reduced to percentages. |
|---|-------------------------------|---|--|--|--|--|
| CURVE XVIII. | 0-5 | 25.95 | +0.09 | +0.35 | -0.66 | -2.48 |
| | 5-10 | 25.82 | -0.04 | -0.15 | -0.79 | -2.97 |
| | 10-15 | 25.57 | -0.29 | -1.12 | -1.04 | -3.91 |
| | 15-20 | 25.57 | -0.29 | -1.12 | -1.04 | -3.91 |
| | 20-25 | 25.95 | +0.09 | +0.35 | -0.66 | -2.48 |
| | 25-30 | 26.31 | +0.45 | +1.74 | -0.30 | -1.13 |
| | | | 25.86 | | | |
| Mean ordinate of all the Carcel lamp curves = 28.27. | | | | | | |
| | | | | | Methven screen curves. | |
| CURVE XIX. | 35-40 | 23.87 | -0.91 | -3.67 | -2.74 | -10.30 |
| | 40-45 | 24.89 | +0.11 | +0.44 | -1.72 | -6.47 |
| | 45-50 | 24.98 | +0.20 | +0.81 | -1.63 | -6.13 |
| | 50-55 | 25.38 | +0.60 | +2.42 | -1.23 | -4.63 |
| | | | 24.78 | | | |
| CURVE XX. | 60-65 | 25.31 | -1.07 | -4.06 | -1.30 | -4.88 |
| | 65-70 | 25.82 | -0.56 | -2.12 | -0.79 | -2.97 |
| | 70-75 | 26.24 | -0.14 | -0.53 | -0.37 | -1.39 |
| | 75-80 | 26.59 | +0.21 | +0.80 | -0.02 | -0.08 |
| | 80-85 | 26.82 | +0.44 | +1.67 | +0.21 | +0.79 |
| | 85-90 | 27.48 | +1.10 | +4.17 | +0.87 | +3.27 |
| | | 26.38 | | | | |
| CURVE XXI. | 0-5 | 28.10 | -0.63 | -2.19 | +1.49 | +5.60 |
| | 5-10 | 28.97 | +0.24 | +0.84 | +2.36 | +8.87 |
| | 10-15 | 28.81 | +0.08 | +0.28 | +2.20 | +8.27 |
| | 15-20 | 29.06 | +0.33 | +1.15 | +2.45 | +9.27 |
| | | | 28.73 | | | |
| CURVE XXII. | 25-30 | 28.87 | -0.16 | -0.55 | +2.26 | +8.51 |
| | 30-35 | 29.19 | +0.16 | +0.55 | +2.58 | +9.70 |
| | | | 29.03 | | | |
| Mean ordinate of all the Methven screen curves = 26.61. | | | | | | |

IV.

Discussion of the Results Obtained.

Plate I. exhibits the curves given by the English standard candles. In curves III. and IV. the candles were lighted at their tops. The wicks flared up and gave a high point on each curve, which in the case of curve IV. is not shown on the plate. The flames then increased gradually to their normal size, which was reached after about 15 minutes. Curves III. (a) and IV. (a) are continuations of III. and IV. The candles were allowed to burn during the interval between the curves, which in the case of III. and III. (a) was 45 minutes, and in the case of IV. and IV. (a) was 55 minutes. During III. the room was rather more draughty than during III. (a), and the effect of the draughts is seen in the much larger numbers of small irregularities in the former than in the latter curve. During the interval between IV. and IV. (a) the height of the flame of the candle was found to vary between 43 and 48 mm.

Curve I. was taken with the portion of candle left over from IV. and IV. (a). It was lighted, its wick being already charred and its crater formed, and readings were taken immediately. Curve II. was taken with the lower half of the candle used in getting III. and III. (a). The bottom, *i.e.* the larger end of this, was hollowed out to expose the wick and readings were taken after the candle had been burning long enough to come to its normal light-giving power. The agreement in the amount of radiation of the candle burned in this way, with the amount when burned from the smaller end, shows that the variation in the diameter of the candle has little if any influence on the intensity of the light emitted.

One marked peculiarity which characterizes, to a greater or less degree, all these curves, is the succession of sudden drops followed by gradual rises to a maximum. In the case of the drop in curve IV. at 55 minutes, the change amounted to 15 per cent of the total deflection, and in other instances the change was nearly or quite as large. The reason for these drops is to be looked for in the action of the wick, which, as the candle burns

down, projects farther above the spermaceti, causing a tall flame. Finally, by reason of charring and because of its own weight, it bends over, and the end burns off. The flame, following the wick, becomes shorter and, sometimes, broader. Since the wicks in the English standard candles are very uniform in construction, these drops succeeded each other after more or less regular time intervals, which were, in most cases, not far from three minutes. The curves show also a comparative freedom of these flames from minor fluctuations due to disturbing draughts of air. The ordinates of the English candle curves are not corrected for rate of burning. It will be seen from the tables that these corrections, while they have in each case the proper sign, are, in most cases, entirely insufficient to reduce the curves to a common mean ordinate.

Plate II. exhibits the curves of German standard paraffin candles as prescribed by the *Deutscher Verein von Gas- und Wasserfachmännern*, and called the "Vereinskerze."

Curves V. and V. (a) and VI. and VI. (a) were obtained from candles burned in exactly the same way as III. and III. (a) and IV. and IV. (a), the time interval between V. and V. (a) being one hour, and that between VI. and VI. (a) being 41 minutes. Curve V. (b) was taken with the remainder of the candle used for V., relighted on another night; so that it is entirely independent of V. and V. (a), excepting that the same candle was used.

The general level of these curves is rather closely adhered to. The numerous minor fluctuations show that the flame of this candle is quite subject to disturbances by draughts. Occasionally the flame increases to a size far beyond its normal, and smokes. These large fluctuations, extending over a considerable period of time, form a marked feature of the curves. It will be noticed that VI. did not reach its normal light-giving power until 15 minutes later than V.

When we come to compare the English with the German candle, we notice, first, that the variations of the English candle were much larger and that large variations were much more frequent. The German curves are free from the semi-periodic drops which

characterize the English. Curve I. at 24 minutes reaches a value which is 1.235 of the mean ordinate of the English candle curves. Curve IV. at 49 minutes drops to a point which is only 0.77 of the mean ordinate. The total variation is, consequently, 46.5 per cent. Both of these curves were taken with the same candle, but on different nights. The highest point of the German curves is on V. (a) at 86 minutes. The ordinate reaches a value which is 1.155 of the mean. The lowest point is on VI. (a) at 72 minutes, and is 0.915 of the mean. The total variation was consequently 24 per cent, or only about half that shown by the English candle.

Moreover, we see from the table that the percentage deviation of curve III., 55–60 minutes, from the mean of the English candles is + 10.45 per cent. Curve IV., 35–40 minutes, shows a deviation of – 13.08 per cent. The total deviation for a period of five minutes is 23.53 per cent for the English candles. In the German candles the maximum positive and negative deviations for five-minute periods are + 11.52 per cent and – 10.43 per cent respectively. These give a total of 21.95 per cent, and exhibit a performance but little better than that shown by the English candle.

These figures make perfectly evident the futility of any attempt to get concordant photometric results from freely burning candles, unless, indeed, a very long series of observations is taken. In favor of the English candle it can be said that its flame is more stable and is much less subject to smaller fluctuations than is the German candle. When it comes to measuring the dimensions of the flame and reducing readings to a standard size of flame, there is little doubt that the English candle would be found much superior to the German. In the early part of the work with candles a rather rude measuring device was set up and heights of flame of the English candle were determined and compared with the corresponding galvanometer readings. The accordance between definite flame heights and definite galvanometer readings was found to be very close.

Investigations along this line have not been prosecuted farther for lack of another observer. A device has been made, however, by which it is hoped that the height, or the height and the width, of a candle flame can be measured with great quickness and accu-

racy, and it is the purpose of one of the present observers to attempt to ascertain, by means of this measuring apparatus and the bolometer, whether the radiating power of a candle can be expressed conveniently in terms of its simpler dimensions or not. That the light-giving power of a candle must vary directly with the cubical contents of the luminous portion of its flame, provided the surrounding conditions are unaltered, seems obvious. That the relative luminous intensities of candles of different materials can be determined with a fair degree of accuracy by measuring the cubical contents of the light-giving portion of their flames has recently been shown by Glan.¹ For photometric work, however, it is essential that the luminous intensity at any time should be deducible from simple measurements of the flame — such measurements as can be made the instant before or after a photometer setting is made.

Plate III. shows the behavior of the Hepner-Alteneck amylicetate lamp. The lamp used is one by Hartmann and Braun, of the type having the optical projection device for fixing the height of flame at 40 mm.

Curve VII. was taken when a window in the farthest corner of the room was raised about 2 cm. Curve VII. (*a*) was begun 25 minutes after the end of VII., the lamp having burned during the interval, and the flame having been readjusted in height. The window was closed. The marked difference between the two curves is due to the stoppage of this slight draught.

In taking curve X., the sensitiveness was adjusted at 32.5, or about double the standard sensitiveness for the other tests of the candles and of the Hefner lamp. Then a computation was made, from the law of inverse squares, of the distance at which it would be necessary to place the lamp in order to produce the ordinary deflection. The fact that the ordinates of this curve agree with those of the curves taken with a sensitiveness of 16.2 shows that the method of getting the sensitiveness by taking only first throws of the galvanometer needles, was an allowable one.

Curves XI., XI. (*a*), XI. (*b*), XI. (*c*) show the results of successive attempts to adjust the height of flame to just 40 mm., the wick

¹ P. Glan, Ueber ein Gesetz der Kerzenflammen, Wied. Ann., Vol. 51, p. 584.

having been lowered between each one. No zero and sensitiveness readings were taken between the beginning of XI. and the end of XI. (c).

If the times that the lamp had been burning, when curves VIII., XI., XII., and XIV. were begun, are taken from Table I., it will be seen that, in each case, it was 20 minutes or less. Each of these curves shows a gradual increase in the amount of radiation. This emphasizes the advisability of lighting the Hefner lamp a considerable time before it is to be used in photometric work.

It is to be understood that, in the case of each of these runs except the one when curve VIII. was taken, the greatest care was exercised to have the room as quiet as possible. Curve VII. was the first of the Hefner lamp curves taken, and the need of extraordinary care in excluding draughts of air was not appreciated.

The Hefner light is a much more promising standard than the candles, although its curves show that it is subject to incessant fluctuations and that considerable variations may last for a minute or more. It is clear, however, that from a series of observations, extending over a comparatively short time, a very good average may be expected.

The highest point on the Hefner curves is 1.093 of the mean, and the lowest is 0.867, giving a total deviation of 22.6 per cent. This large deviation is not so significant, however, as a similar one in the case of candles, since in dealing with the Hefner lamp we have to do with an adjustable flame, and these fluctuations may indicate only that a slight readjustment was needed.

The most important question in regard to the Hefner lamp is the accuracy with which it can be adjusted to its normal light intensity. Some idea may be formed of this by noting in Table II. the percentage deviations of the first five minutes of each of the curves from the mean ordinate of the first five minutes of all the curves. The maximum deviation will be seen to be 8 per cent, while the mean deviation is 2.3 per cent. To show the accuracy with which the flame can be adjusted under given unchanging conditions, curves XI., XI. (a), (b), and (c) are of especial value.

In Table III. will be found the mean ordinates for the first five minutes of XI. and of all of XI. (a), (b), and (c), and the percentage deviation of each from the mean of all.

TABLE III.

| Curve. | Mean ordinate. | Deviation from mean. | Percentage deviation. |
|------------------------------|----------------|----------------------|-----------------------|
| XI. (<i>g^m</i>) | 39.42 | -0.14 | -0.35 |
| XI. (<i>a</i>) | 39.37 | -0.19 | -0.48 |
| XI. (<i>b</i>) | 40.00 | +0.44 | +1.11 |
| XI. (<i>c</i>) | 39.46 | -0.10 | -0.25 |
| Mean | 39.56 | Mean | 0.57 |

If, now, the assumption be made that the light-giving efficiencies of the English and German candles and of the Hefner lamp are equal, it is possible to get their relative intensities by a comparison of the mean ordinates of their curves. Reducing the deflections as read on the telescope scale to angular measure and taking double the tangents of these angles, we have for the true deflections the following values:—

$$\text{English candle} = 41.06$$

$$\text{German candle} = 50.40$$

$$\text{Hefner light} = 38.66$$

From these values we get the following ratios:—

$$\frac{\text{German candle}}{\text{English candle}} = 1.2275$$

$$\frac{\text{Hefner light}}{\text{English candle}} = 0.9415$$

Violle's values for these ratios are 1.13 and 0.98 respectively.

The curves given by the Carcel lamp are numbered XV., XVI., XVI. (*a*), XVII., and XVII. (*a*). They are all reduced to a common rate of burning of 42 grams per hour. XV. was begun when the lamp had been burning 38 minutes. It shows a slight upward tendency which decreased as time went on. The rate of consumption was high, — 48 grams per hour. XVI. was begun 35 minutes after the lighting of the lamp, and the rate of burning was 43 grams. XVI. (*a*) shows the radiation from the lamp after it had

been burning nearly two hours. During XVI. the radiation was increasing throughout the test. After the lamp had been burning during the interval between XVI. and XVI. (a), the radiation was falling off very rapidly as the wick became more and more charred. The rate for XVI. (a) was higher than for XVI., having been altered by a change in height of the chimney.

XVII. was begun 21 minutes after the lamp was lighted, while XVII. (a) was not begun until the lamp had been burning three hours. XVII. shows a rise as the lamp became warmed, the curve becoming parallel to the base line when the lamp had been burning half an hour. XVII. (a), however, shows that the radiation was again increasing. During XVII. the rate was 46.8 grams per hour, while the rate during XVII. (a) was only 40 grams per hour.

These curves show clearly the great gain in steadiness of a flame resulting from the use of a proper chimney. The deviations were, for the most part, no larger than the swings of the galvanometer when the bolometer was unexposed to radiation. The variations extending over a considerable period of time are, however, by no means inconsequential. In curve XVII. there was a total deviation in the mean ordinates of the five-minute periods of 5.1 per cent, taking place in 10 minutes. Curve XVI. shows a deviation of less than 0.8 per cent in 35 minutes, and this is perhaps a more typical illustration of what may be expected from this lamp when burned under the best conditions. The highest and lowest points on any of the Carcel curves are 29.9 and 25.4 respectively, corresponding to deviations from the mean of the curves of + 5.7 per cent and - 12.5 per cent, or a total of 18.2 per cent. Table II. shows the maximum deviations, for periods of five minutes, to be + 5.3 per cent and - 8.4, giving a total of 13.7 per cent.

Table IV. serves to show the reproducibility of the standard at different times.

TABLE IV.

| Curve. | Mean ordinate. | Deviation from mean. | Percentage deviation. |
|--------------|----------------|----------------------|-----------------------|
| XV. | 27.60 | -0.50 | +1.78 |
| XVI. | 29.37 | +1.27 | +4.52 |
| XVI. (a) | 28.63 | +0.53 | +1.89 |
| XVII. | 26.82 | -1.28 | -4.56 |
| XVII. (a) | 28.06 | -0.04 | -0.14 |
| Mean | 28.10 | Mean | 2.58 |

The mechanical parts of the lamp used were in good working order. Pure Colza oil was used for burning, and rates of combustion were determined by weighings made before and after the runs. The regulation wicks were employed, a fresh one being used for each run.

The curves given by the Methven screen are numbered XVIII., XIX., XX., XXI., and XXII. Curve XVIII. was taken with the Argand burner connected directly to the gas-pipes in the building. The pressure of the gas was controlled only by a large regulator on the main pipe leading into the building. The consumption of gas in the building was constant during the run. The curve shows many large, but quick, variations and certain decided waves. The other Methven curves were taken with a gas-holder of about 10 gallons' capacity interposed between the gas main and the Argand burner. The effect of this in smoothing the curves is very marked, and indicates that for photometric purposes the Methven screen is much improved by the interposition of a rather capacious reservoir. The use of such a reservoir tends to minimize the effect of changes of pressure and to absorb any waves in the gas due to water in the pipes or some similar cause.

The way in which these waves of variable pressure produce fluctuations in the amount of radiation is by causing changes in the quality of those portions of the flame which cover the slit. At times the top of the flame becomes forked, so that not all of the slit is covered by it. It happens perhaps more frequently

that some of the non-luminous portion of the flame rises so as partially to cover the slit. In either case the result is seen in a deviation of the curve. It must be true, also, that the amount of luminous radiation suffers a much larger proportional change than the total amount of radiation; hence the deviations recorded on the curves are too small to represent correctly the fluctuations in luminous intensity. There is an additional reason for these fluctuations, which is to be found in the slight protection against draughts of air which the wide open chimney used on the London Argand burner affords. In this respect its action is in marked contrast with that of the smaller close chimney of the Carcel lamp, which, of course, can be used only with much richer gases than ordinary illuminating gas.

These Methven screen curves cannot be trusted to give a true idea of the reproducibility of the standard. This is because the ordinates of the curve represent not only the radiation from the flame, but also that from the chimney and from the screens interposed. Since the burner was not always replaced in exactly similar positions before the bolometer, the amount of this radiation was variable. A more serious source of error, however, arose from the fact that after several of the curves had been taken it was found advisable to change the original arrangements for screening. The plan finally followed was to interpose between the Methven screen and the bolometer-box a heavy piece of asbestos paper, having a hole of proper size cut in it. This was placed so far from the burner that it did not become heated perceptibly.

The difficulty arising from the intermingling with the radiation from the flame of a variable amount of non-luminous radiation from chimney and screens could have been entirely obviated by determining, in a way which did not suggest itself until after most of the runs had been taken, the effect of this variable radiation. After the burner had been lighted long enough for the chimney and screens to become heated to their ultimate temperature, the flame was suddenly extinguished and the screen covering the bolometer strip was raised. Then a curve of cooling of the hot chimney and screens was plotted, using times as abscissas and deflections as ordinates. This curve carried back to the time of extinguishing the

light gave the amount of radiation received from all parts except the flame. The importance of this correction was not fully appreciated until the results of observations were worked over, after all the runs had been taken; consequently the method was not developed very completely. The records show that in the case of the Carcel lamp the amount of radiation from the chimney varied slightly. At the end of curve XV. the chimney radiation was such as to produce a deflection of 6.2 scale divisions, while at the end of XVII. (a) the chimney radiation was 5.8. For curve XVI. no observations of chimney radiation were made; but since the conditions under which the run was taken were very similar, the amount of such radiation could not have differed greatly from that found for the other curves. That the effect of applying this correction is not necessarily to tend to reduce the curves to a common mean ordinate is indicated by the fact that if we correct the end of XV. by subtracting from it 6.2, and correct the end of XVII. (a) by subtracting from it 5.8, the difference between these portions of the curves will be exaggerated.

The only determinations of the radiation from the chimney and the intervening screens in the case of the Methven standard were made at the end of curves XXI. and XXII. For curve XXI. this radiation caused a deflection of 2.8 divisions, and for XXII. the deflection was 2.7 divisions. These curves were taken on the same night, and only a short time intervened between them. The apparatus was not displaced in the meantime.

The above work was done during the fall and winter of 1893-4. We have, in conclusion, to express our earnest thanks to Professor E. L. Nichols, not only for having proposed the line of research, and having outlined the method to be employed, but also for many valuable suggestions given during the progress of the investigation.