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ART. I.—*On the Pressure Coefficient of Mercury Resistance*;  
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DURING the last fifty years many physicists have investigated the specific resistance of mercury and its variations under different conditions, yet the only determinations of the pressure coefficient, previously published, are those of Barus,\* who found  $-0.00003$  by subjecting commercial mercury to pressures up to 400 atmospheres, and Lenz,† who found  $-0.0002$  for pure mercury between one and sixty atmospheres. The discordance of these results is far too great to be explained by impurities in the mercury and invites further study.

The long range of pressure desired for the present investigation was easily obtained with Professor Barus's "Screw compressor." This instrument, together with the vertical piezometer employed in my work, has been so fully described elsewhere‡ that only a cursory mention is necessary here. The piezometer proper consists of a cold drawn seamless steel tube, about 6<sup>mm</sup> internal and 13<sup>mm</sup> external diameter and 60<sup>cm</sup> long, connected to the compressor in such a manner that very perfect electrical insulation exists between the two without rendering the joint appreciably leaky. The whole apparatus was filled with heavy mineral oil which, though more viscous than water, attained uniformity of pressure with sufficient rapidity and possessed the advantage of being a very good insulator.

\* Barus, this Journal, III, xl, p. 219, 1890.

† Lenz, Stuttgart, 1882; Wied. Beibl., vi, p. 802, 1882.

‡ Barus, Phil. Mag., Oct., 1890, p. 340; Proc. Amer. Acad., xxv, p. 93, 1890; Bull. U. S. Geological Survey, No. 96.

Moreover it is much easier to prevent leakage of oil than water and it has no detrimental effect on the steel parts with which it comes in contact. An Amagat "manomètre à pistons libres"\* was used to determine the pressures attained and gave results within one-tenth per cent throughout the entire range. The ratio of its pistons is such that one millimeter difference in height of the mercury column supported by the larger corresponds to a difference of pressure of .647 atmospheres on the smaller. It is supplied with an open glass manometer three and one-half meters high and is therefore capable of indicating pressures somewhat greater than two thousand atmospheres, but above this limit a large element of uncertainty is introduced by leakage of oil around the pistons.

Commercial mercury was digested for about forty-eight hours in a solution of sulphuric acid and bichromate of potash in water and after being carefully washed, dried, and filtered, was distilled directly into the tubes in which its resistance was measured. An ordinary glass thermometer tube, about 18<sup>cm</sup> long and 0.1<sup>mm</sup> bore, had 10<sup>cm</sup> of 2<sup>mm</sup> bore tubing welded to its upper end in such a manner that a cavity, about 1<sup>cm</sup> long and 4<sup>mm</sup> in diameter, was formed between the two parts. This cavity and the elongated bulb at the lower end of the fine capillary had platinum electrodes melted through their walls and, when filled, formed the terminals of the mercury thread under investigation. The open end of the large tube was welded to a small glass mercury still connected, through a drying chamber, with a Geissler-Toepler air pump and the whole apparatus exhausted until the pressure fell below one millimeter. When the inside walls had become perfectly dry, heat was applied and mercury slowly distilled over and condensed in the experimental tube. As soon as this became full it was emptied and the operation repeated until the mercury thread, when examined with a magnifying glass, appeared perfectly bright and uniform throughout its entire length. Air was then admitted and the large capillary cut off about two centimeters from the point where it joined the still. After soldering silk insulated copper wires to the electrodes the tube was placed inside the steel piezometer and the upper wire connected directly to it, while the lower one, after passing down through a narrow glass tube, to insure good insulation, was soldered to the inside of the compressor. Oil was forced up into the piezometer and when its appearance at the top showed that all air had been expelled the opening was closed by a tinned steel screw.

The piezometer was surrounded by a long brass cylinder,

\* Amagat, C. R., ciii, p. 429. 1886. Professor Tait has described a similar apparatus in the "Challenger Reports," 1873-76, Physics and Chemistry, vol. ii.

closed at the ends by rubber stoppers, through which water from the city mains was allowed to flow continuously. A thermometer with its bulb inside of this cooler showed that the temperature never varied more than one degree from  $9^{\circ}\text{C}$ . throughout the entire series of experiments and that the variations during the same day were very much less than this. For the measurements at the boiling point of water a tin can about  $30^{\text{cm}}$  long and  $13^{\text{cm}}$  in diameter, having short brass tubes soldered to apertures in the center of its ends, was placed on the piezometer and fastened, by short pieces of rubber tubing, in such a position that it entirely covered the experimental tube within. Two openings in the top were provided, one for the reception of a thermometer and the other for a vertical water condenser. The latter, being open at the top, kept the steam at atmospheric pressure and at the same time obviated the necessity of frequently renewing the supply of water. The whole arrangement with the exception of the bottom was covered with asbestos to prevent radiation and heat was applied by means of a ring burner surrounding the piezometer below the can. Small water coolers were placed above and below to prevent the conduction of heat through the steel tube to the joints where it might cause leaks.

Various methods for the measurement of resistance were tried with more or less success, but that due to Carey Foster was found to give the best results owing to its sensitiveness to small variations. The general arrangement of the apparatus for this method is too well known to need description here. The transposition of the standard and unknown resistances was accomplished by means of an eight pole mercury commutator similar to those put on the market by Nalder Bros. A series of platinoid coils, by Queen & Co., so arranged that their combined resistance could be varied by tenths from zero to ten thousand ohms, without altering the number of plugs in the circuit, was used as a standard with which to compare the mercury thread under investigation. As noted above, the electrodes of the thread were connected respectively to the piezometer and compressor, and since these parts were otherwise very perfectly insulated from one another they served admirably as poles from which to make connection with the commutator. A very uniform german silver wire, about No. 17 B. & S. gauge, was wound in ten uniform spirals about a vulcanite cylinder  $10^{\text{cm}}$  in diameter and  $3.6^{\text{cm}}$  long. Its ends were fastened, in the same generating line of the cylinder, to two thick brass plates that formed the ends of the drum and were rigidly fastened to two stout brass pillars which were connected with the poles of the commutator. An insulated frame work was arranged to revolve about this drum in such a man-

ner that a spring contact, which served as one terminal of the galvanometer, could be readily placed on any point of the wire and its position accurately determined by a large micrometer head divided into one hundred equal parts. This arrangement presents a great advantage over the ordinary form of drum bridge since the only friction connections are in the galvanometer circuit, where the worst effect they can produce is small variations in sensitiveness, and not, as they are usually placed, at the terminals of the wire, where changes in their resistance produce the maximum effect on the result. Current was supplied to the bridge by a single Daniell's cell and was so regulated by a small rheostat in series with the battery that its intensity was never sufficient to appreciably alter the temperature of the resistances in circuit. The attainment of balance was judged by a very sensitive and dead beat D'Arsonval galvanometer of the horizontal magnet type and its indications were observed by the telescope and scale method.

When the Queen resistance box was bought, some years ago, it was accompanied by a certificate from Professor Anthony to the effect that its readings were correct to one-fiftieth of one per cent at  $17.5^{\circ}$  C. and that its temperature coefficient was  $\cdot 00023$ . The coils used in the present investigation have nevertheless been very carefully calibrated and the values thus found used in all the calculations, for, though many of them came quite up to the guarantee, several showed deviations somewhat larger than the probable errors of observation. The resistance of the bridge wire was determined in the following manner. Let the reading of the bridge micrometer, when balance has been obtained, with two nearly equal resistances  $R$  and  $R'$  in circuit be  $x$ , and when  $R$  and  $R'$  are interchanged  $x'$ . Then if  $z$  and  $z'$  are the corresponding readings when  $R$  has been increased by a known increment  $dR$  it is easy to prove that

$$r = \frac{dR}{(x-z) + (z'-x')}$$

where  $r$  is the resistance of a length of the wire corresponding to one division of the micrometer. About one hundred determinations of this quantity, involving various lengths and different portions of the wire, gave the mean value

$$r = \cdot 000898 \text{ ohms}$$

the greatest difference between a single observation and the mean being less than  $3 \times 10^{-6}$  ohms. These measurements also showed that the error of a single setting of the micrometer was about one-tenth of one division and hence that the mean error of a single determination of a resistance, due to this cause alone, was less than  $\cdot 0001$  ohms. Throughout the entire

investigation the resistances in the various arms of the bridge were so proportioned as to give a maximum of sensitiveness; and a movement of the spring contact on the wire equal to one-tenth of a division was always sufficient to reverse the direction of the galvanometer deflection when balance was obtained. During the measurements at 9° C. no difficulty was experienced from thermoelectro-motive forces, since the water cooler was long enough to keep all the joints at the same temperature, but when the steam jacket was employed they caused so much trouble that it became necessary to replace the copper connections inside of the piezometer by iron wires. Disturbances of this nature were thus reduced to a minimum and trustworthy results could be obtained by closing first the galvanometer and then the battery circuit. The temperature of the room and of the standard resistances, determined by a small mercurial thermometer placed between the coils, remained nearly constant during the actual time of observation but varied considerably from day to day.

If  $R$  represents the resistance of the mercury thread and  $W$  that of the standard of comparison and if  $x$  and  $x'$  are the readings of the bridge-wire micrometer for the position of balance before and after they are interchanged, we have

$$R = W + r(x - x')$$

where  $r$  has the meaning and value assigned to it above and all the connection resistances are eliminated except those between  $R$  and  $W$  and the commutator. To determine these the mercury tube was replaced by a thick copper wire soldered to the same connecting wires and measurements were then made under as nearly as possible the same conditions of pressure and temperature that were used with the mercury. The mean of a large number of observations gave .0632 ohms with the copper connections used at low temperature and .5095 ohms with the iron ones used at the boiling point, and no variation with the pressure could be detected. In reducing the resistances to the standard temperature of the Queen box the bridge wire was assumed to have the same temperature coefficient and to be always at the same temperature as the standard coils. This assumption could introduce no appreciable error in the results, since the factor  $r(x - x')$  was always less than 0.1 ohm and the temperature of the room never varied much from that of the box, but it greatly simplified the calculation of the corrections. It was further found that the slight variations in the temperature of the mercury thread, from 9° C. in one case and from 100° C. in the other, introduced errors that could not be neglected and corrections were introduced using .0009 as the temperature coefficient of mercury. Finally the effect of changes in the volume of the glass tube, due to compression,

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were eliminated on the assumption that the coefficient of cubical compressibility of the glass used was  $\cdot 0000025$ . Calculations were instituted to determine the effect on the measurements of the slight changes in pressure, and hence in the resistances of the mercury thread, due to leakage of the compressor and gauge between the direct and reverse settings of the bridge wire contact, and it was found that the errors so introduced were generally so small and their calculation so uncertain that no appreciable benefit could be obtained by attempting their correction.

TABLE I.

P	R	R'	R-R'	P	R	R'	R-R'
1	12·470	12·451	·019	1199	11·963	11·956	·007
57	12·414	12·428	—·014	1400	11·871	11·872	—·001
149	12·382	12·390	—·008	1651	11·749	11·768	—·019
241	12·344	12·352	—·008	1890	11·678	11·669	·009
301	12·308	12·327	—·019	1984	11·637	11·630	·007
386	12·294	12·292	·002	113	12·406	12·405	·001
459	12·265	12·262	·003	186	12·374	12·375	—·001
515	12·250	12·239	·011	282	12·333	12·335	—·002
544	12·228	12·227	·001	378	12·302	12·295	·007
581	12·205	12·211	—·006	461	12·255	12·261	—·006
1	12·456	12·451	·005	537	12·236	12·230	·006
1	12·452	12·451	·001	623	12·195	12·194	·001
375	12·261	12·297	—·036	692	12·168	12·165	·003
552	12·215	12·225	—·010	777	12·142	12·130	·012
581	12·216	12·211	·005	867	12·071	12·100	—·029
605	12·200	12·201	—·001	133	12·396	12·396	·000
649	12·192	12·183	·009	881	12·105	12·087	·018
729	12·163	12·150	·013	918	12·094	12·072	·022
1441	11·837	11·855	—·018	990	12·050	12·042	·008
1504	11·809	11·829	—·020	1045	12·035	12·019	·016
1574	11·785	11·800	—·015	1173	11·976	11·966	·010
1619	11·765	11·782	—·017	1230	11·942	11·943	—·001
1686	11·740	11·754	—·014	1297	11·919	11·915	·004
1755	11·719	11·725	—·006	1369	11·878	11·885	—·007
1831	11·702	11·694	·008	1425	11·858	11·862	—·004
1923	11·666	11·656	·010	1479	11·838	11·839	—·001
177	12·378	12·378	·000	1542	11·815	11·812	·003
359	12·306	12·303	·003	1571	11·828	11·801	·027
532	12·230	12·231	—·001	154	12·388	12·388	·000
684	12·180	12·169	·011	154	12·388	12·388	·000
851	12·116	12·100	·016	106	12·406	12·408	—·002
1047	12·020	12·018	·002	106	12·403	12·408	—·005

Five or ten minutes were always allowed to elapse after each increment to the pressure before the resistance measurements

were made in order that the irregularities in temperature, due to compression, might become equalized and the distribution of pressure throughout the whole apparatus become uniform. It was also possible by this method to determine whether the rate of leakage was sufficient to seriously affect the results and when this was found to be the case to adopt means to prevent it. The observations at 9° C. are given, in the order in which they were taken, in table I, and those at 100° C. in table II, where the columns P and R contain respectively the pressure in atmospheres and the corresponding corrected resistances in ohms. The chart, fig. 1, shows the same data graphically, the horizontal scale being one hundred atmospheres and the vertical one-tenth ohm per division. The base line corresponds to 11.6 ohms and the temperature at which each series was made is marked above it.

TABLE II.

P	R	R'	R-R'	P	R	R'	R-R'
88	13.388	13.360	.028	1521	12.711	12.714	-.003
161	13.348	13.327	.021	1600	12.685	12.678	.007
239	13.310	13.292	.018	1690	12.644	12.638	.006
325	13.257	13.253	.004	1788	12.603	12.594	.009
407	13.227	13.216	.011	1895	12.573	12.545	.028
471	13.204	13.188	.016	1969	12.518	12.512	.006
531	13.177	13.160	.017	2139	12.459	12.435	.024
572	13.156	13.142	.014	2214	12.430	12.401	.029
647	13.125	13.108	.017	59	13.378	13.373	.005
697	13.103	13.086	.017	210	13.299	13.305	-.006
739	13.069	13.067	.002	316	13.246	13.257	-.011
801	13.030	13.039	-.009	420	13.207	13.211	-.004
862	13.014	13.011	.003	513	13.148	13.168	-.020
904	12.982	12.992	-.010	596	13.114	13.131	-.017
947	12.974	12.973	.001	713	13.047	13.078	-.031
1008	12.931	12.945	-.014	834	13.004	13.024	-.020
1053	12.920	12.925	-.005	955	12.939	12.969	-.030
1156	12.865	12.878	-.013	1067	12.898	12.919	-.021
1235	12.830	12.843	-.013	1199	12.845	12.859	-.014
1293	12.824	12.817	.007	1324	12.800	12.803	-.003
1306	12.797	12.811	-.014	1441	12.721	12.750	-.029
1400	12.762	12.768	-.006	1599	12.684	12.679	.005
1468	12.743	12.738	.005	1741	12.616	12.615	.001
1509	12.724	12.719	.005	1877	12.574	12.553	.021
1580	12.693	12.687	.006	2029	12.476	12.485	-.009
1652	12.668	12.655	.013	2113	12.481	12.447	.034
1705	12.647	12.631	.016	2236	12.339	12.392	-.053
1719	12.625	12.625	.000	1838	12.567	12.571	-.004
9	13.394	13.396	-.002	1409	12.746	12.764	-.018
1292	12.834	12.817	.017	1023	12.908	12.938	-.030
1368	12.763	12.783	-.020	587	13.107	13.135	-.028
1452	12.733	12.745	-.012	103	13.401	13.354	.047

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Combining these observations by the method of "Least Squares" we have

$$\begin{array}{ll} \text{at } 9^{\circ} \text{ C.} & R = 12.4518 - .000414P \\ \text{at } 100^{\circ} \text{ C.} & R = 13.3999 - .000451P \end{array}$$

The lines on the chart have been drawn in accordance with these equations, and it will be seen that the plotted points are very nearly in coincidence with them. The values of  $R$  computed by these formulæ have been entered in the tables under  $R'$  and the relative errors, from which the probable error of a single observation has been found to be .008 ohms at  $9^{\circ} \text{ C.}$  and .012 ohms at  $100^{\circ} \text{ C.}$ , under  $R - R'$ . Hence the resistance measurements are accurate to less than one-tenth of one per cent and are as good as could be expected when it is remembered that the uncertainty in determining the pressure is about the same in magnitude and that it is impossible to entirely prevent leakage when very high pressures are employed. Furthermore small errors were probably introduced by the lag in the indications of the mercurial thermometers behind the actual temperature variations. Putting the above equations in the form

$$R = R_0(1 + \beta P)$$

where  $\beta$  is the increment to unit resistance caused by one atmosphere increase in pressure, we have, after calculating the probable error in the usual way from the sum of the squares of the errors,

$$\begin{array}{ll} \text{at } 9^{\circ} \text{ C.} & \beta = -.00003324 \pm .00000014 \\ \text{at } 100^{\circ} \text{ C.} & \beta = -.00003367 \pm .00000019 \end{array}$$

Hence it follows at once that at any temperature

$$\beta = -.0000332 - 5 \times 10^{-9} t$$

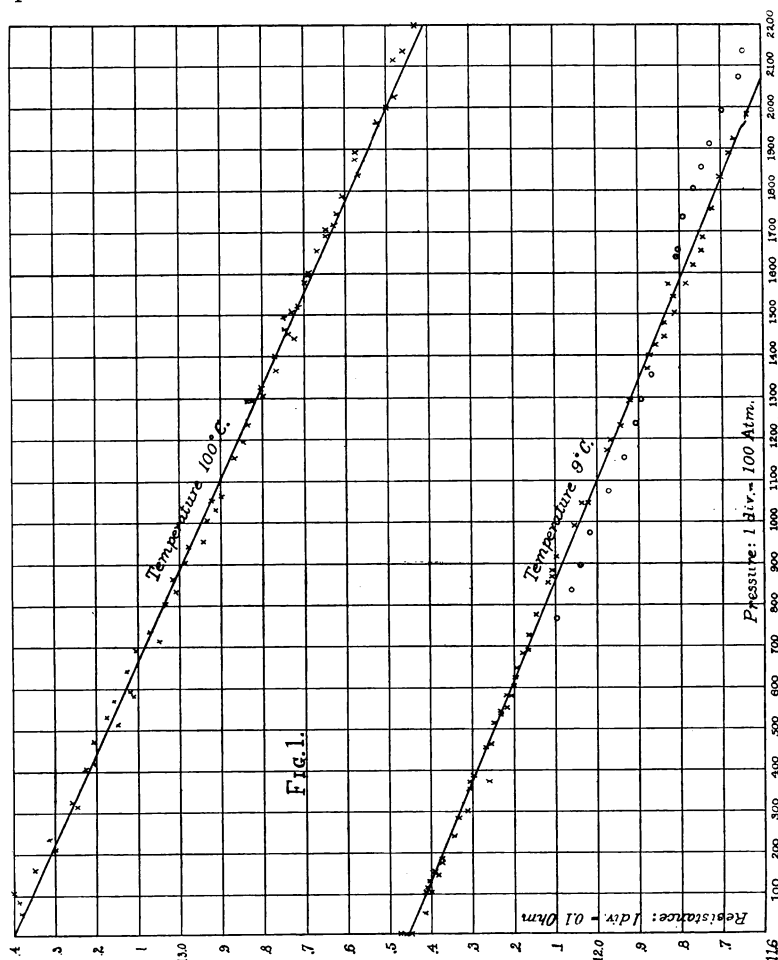
where the last term, owing to its extreme smallness, is probably only approximately accurate.

The difference between this result and that of Barns ( $-.00003$ ) is so small that it can be easily accounted for by the slight impurities in the commercial mercury used by him. Lenz's original paper is unfortunately inaccessible and the account of it in the *Beiblätter* is meager. He used a tube 1.2 meters long filled at atmospheric pressure, and it is probable that the very large coefficient ( $-.0002$ ) obtained was due to the imperfect removal of air bubbles from its inside walls, a source of error having its maximum effect at the low pressures employed by him.

The two series of observations marked by circles on the chart, fig. 1, are so obviously affected by consistent errors that they have been left out of the calculations. The first was



obtained after the apparatus had been left sustaining a pressure of about 750 atmospheres for two hours and lies below the line, while the second, obtained after rapidly increasing the pressure from one to 1640 atmospheres, lies above it. Similar



operations at another time failed to produce similar results and an entirely satisfactory explanation does not present itself, but it is probable that the first is due largely to imperfect freedom of the gauge pistons, caused by particles of dirt in the oil leaking past them, and the second to the heat produced by rapid compression.

Brown University, March 18, 1897.