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156. To Prove by Pascal's Theorem That the Straight Lines Meeting Three Non-Intersecting Straight Lines Generate a Conicoid, i.e. a Surface Every Plane Section of Which Is a Conic  
Author(s): W. H. Blythe

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part. If only he could have established the idea of the anharmonic ratio of imaginary elements, a formula which is to be found in the *Géométrie Supérieure* (p. 118, new edition) would have immediately enabled him to give the beautiful definition of an angle as the logarithm of an anharmonic ratio—a definition which enabled Laguerre, my lamented colleague, to find the long-sought, complete solution of the problem of the transformation of those relations which contain both angles and homographic and correlated segments.

## IV.

While Chasles, Steiner, and later, von Staudt, were endeavouring to build up a theory that would be a rival to Analysis, and were, as it were, erecting altar against altar, Gergonne, Bobillier, Sturm, and Plücker in particular, were perfecting the Cartesian Geometry, and were constructing an analytical system in some measure adequate to the discoveries of the geometers. To Bobillier and Plücker we owe what is known as the method of *Abridged Notation*. In the last volumes of Gergonne's *Annales* are to be found a few really original pages from the pen of the former. Plücker had begun to develop the method in his first volume, very soon to be followed by a series of works in which the foundations of modern geometry are deliberately established. To the same investigator we owe tangential and trilinear coordinates, used in homogeneous equations, and also the canonical forms, the validity of which is recognised in the fruitful but sometimes deceptive method known as the *enumeration of constants*. All these timely discoveries were to infuse new blood into the Cartesian Analysis; they enabled it to give their full significance to those conceptions which the so-called *synthetic* geometry had not completely grasped. Plücker—and with his name it is only fair to couple that of Bobillier, carried off by an untimely death—must be regarded as the first to familiarise us with those methods of modern Analysis, in which the use of homogeneous coordinates enables us to treat simultaneously, without the reader's knowledge, as it were, a figure, and all the figures that are deduced from it by homography and correlation.

(To be continued.)

## MATHEMATICAL NOTES.

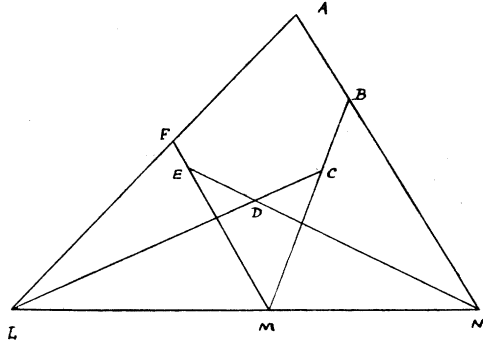
156. [L<sup>2</sup>. 7; 14. a.] To prove by Pascal's theorem that the straight lines meeting three non-intersecting straight lines generate a conicoid, i.e. a surface every plane section of which is a conic.

Let  $P_1, P_2, P_3; p_1, p_2, p_3$  be two sets of non-intersecting straight lines in space, each line of one set meeting all of the other set. Let them meet

any plane, not containing one of them, in the points  $A, C, E, B, D, F$ . Let  $AF, CD; FE, BC; ED, AB$  meet respectively in  $L, M, N$ . Denote the intersections  $P_1p_2, P_2p_3, P_3p_1$  by  $X, Y, Z$ . Now  $X, Y, L$  lie in one straight line for they will all be found in both the planes  $P_1p_3$  and  $P_2p_2$ , and therefore lie on their intersection.

Similarly  $Y, Z, M$  lie in the planes  $P_3p_3$  and  $P_2p_1$ , and  $X, Z, N$  lie in the planes  $P_3p_3$  and  $P_1p_1$ . Therefore  $L, M, N$  are the points in which the sides of the triangle  $XYZ$  meet the plane of  $ABCDEF$ , and are therefore in one straight line, so that by Pascal's theorem  $A, B, C, D, E, F$  lie in a conic.

Since five points determine the conic, we may consider  $F$ , and therefore  $p_3$ , as variable, and any straight line meeting  $P_1, P_2, P_3$  traces out the conic  $ABCDE$ .



W. H. BLYTHE.

**157. [P. 3. b. a.]** To find the relation between two maps of the same contour on the gnomonic projection.

Let  $l$  be any line touching at  $A$  a sphere whose centre is  $O$ , and let  $m$  be the parallel line through  $O$ . Let  $\xi$  be the projection from  $O$  on the sphere of any curve  $a$  on the tangent plane  $p$  at  $A$ . Let  $\zeta$  be the reflection of  $\xi$  in a plane through  $m$  making an angle  $\theta$  with the plane  $mA$  and cutting  $p$  in the line  $n$ . Let  $\eta$  be the reflection of  $\zeta$  in the plane  $mA$ . Then  $\eta$  is obtained from  $\xi$  by rotating  $\xi$  about  $m$  through an angle  $2\theta$ . Let  $\beta, \gamma$  be the projections of  $\eta, \zeta$  on  $p$ . Then  $\alpha, \beta$  are two different gnomonic projections of the same spherical curve. Draw  $VAW$  perpendicular to  $l$  in the plane  $p$  cutting  $n$  in  $W$ , such that  $VOW$  is a right angle. Then the line joining corresponding points of  $\alpha$  and  $\gamma$  evidently passes through  $V$  and is divided harmonically by  $V$  and its intersection with  $n$ . Hence  $\beta$  is the reflexion in  $l$  of the harmonic homologue of  $\alpha, V$  being the centre, and  $n$  the axis of homology.

Since the theorems given in the *Math. Gazette*, Jan. 1904, p. 383, and March, 1904, p. 7, on successive inversion with respect to coaxial circles can be proved by aid of the "relation between two maps of the same contour on the stereographic projection" (*Math. Gazette*, May 1904, p. 33; Oct. 1904, p. 88), we see that in the theorems referred to we may replace "inversion with respect to one of a system of coaxial circles" by "the operation of taking the harmonic homologue of a figure," "the axis of homology being one of a system of parallel lines and the centre its pole with respect to a fixed circle." An independent proof may be readily given.

HAROLD HILTON.

**158. [P. 3. b.]** On a Theorem in Inversion.

Since two points  $P, P'$  on a sphere which are reflexions of each other in the plane of a great circle  $s$  are projected stereographically into two points inverse with respect to the projection of  $s$ , the theorem 153 (p. 88) becomes: the stereographic projections of two figures on the sphere which are reflexions