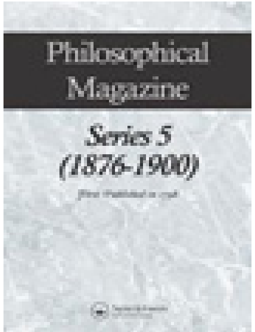

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V. *On Silk* v. *Wire Suspensions in Galvanometers, and on the Rigidity of Silk Fibre.* *By* Thomas Gray, *B.Sc., F.R.S.E.**

IN the last Number of the Philosophical Magazine there is a short article by R. H. M. Bosanquet drawing attention to some eccentricities of a galvanometer used by him. A determination of the rigidity of the suspending "fibre" of the galvanometer-needle would have been interesting, as it would have thrown considerable light on the probability or improbability of the explanation offered. It must have caused no little surprise to many of the readers of the Philosophical Magazine to find that Mr. Bosanquet based his condemnation of silk-fibre suspensions on the trouble he experienced with an instrument the suspending fibre in which was "left just stout enough to carry the weight," and which was of such a nature that it could possibly *twist* or *untwist* with stretching or with hygrometric changes in the atmosphere. Surely Mr. Bosanquet is scarcely in earnest when he writes about suspending the needles of a sensitive galvanometer with a *twisted* silk thread, or when he proposes to go back something like half a century in the history of this subject†, and adopt galvanometers with needles seven inches long made of stout knitting needles and suspended by a wire five feet long.

A galvanometer-needle should never be so heavy that it cannot be suspended by a *single* fibre of silk (that is, *half an ordinary cocoon fibre*), because such a fibre will bear easily, leaving a good margin of safety, two grammes; and it is an easy matter to so arrange such a mass that the period of vibration will be not only so much as thirty seconds but even several minutes. With an astatic arrangement, especially if it be only "nearly astatic," there will be changes of zero certainly, but I can hardly see any thing comparable to a "ghost" in what could occur.

About a year ago I made, in the Physical Laboratory of Glasgow University, a number of experiments on silk fibres, which included among other things some determinations of their rigidity. Mr. Bosanquet's paper has suggested to me that possibly a few of the results may be worth publication. Some of the results of these experiments are in type in vol. iii. of the Reprint of Sir W. Thomson's Mathematical and Physical Papers now in the press.

Two methods were used for the determination of the rigidity.

* Communicated by the Author.

† Some interesting experiments "On the Suspension of the Magnetic Needle by Spiders' Fibre" are described by the Rev. A. Bennet, F.R.S., in the R. S. Trans. vol. lxxxii. 1792.

The first method was almost identical with that introduced in this laboratory thirty-five years ago by Sir W. Thomson, and now commonly adopted for the determination of the rigidity of metallic wires. It consisted in suspending from a fixed support, by means of a measured length of the fibre, a thin circular rim of non-magnetic material and of easily calculated moment of inertia, and observing the period of the torsional vibrations. From this the torsional rigidity of the fibre can be readily calculated by a well-known formula. The second method consisted in suspending a small mirror, to which was rigidly fixed a small magnetic needle of known magnetic moment by means of a measured length of the fibre, and observing the deflection of the mirror produced by twisting the top of the fibre through a measured angle. This gives a ready means of calculating the rigidity of the fibre in terms of the magnetic moment of the suspended needle, and the strength of the magnetic field in which it is suspended.

The fibres were of Japanese floss-silk, which had been thoroughly washed in hot water to remove the gum which is always found in considerable quantity on cocoon fibres. The fibres were in all cases single fibres; and it will be seen, both from the direct measurements by the microscope and from the rigidity, that they vary considerably in thickness. Even a rough estimate of the rigidity per square centimetre section of the substance is impossible, as the fibre is not even approximately circular in section, and its diameter not nearly regular along its length. The results of the experiments are given in the following Table, the headings of the different columns being sufficiently explanatory of the numbers.

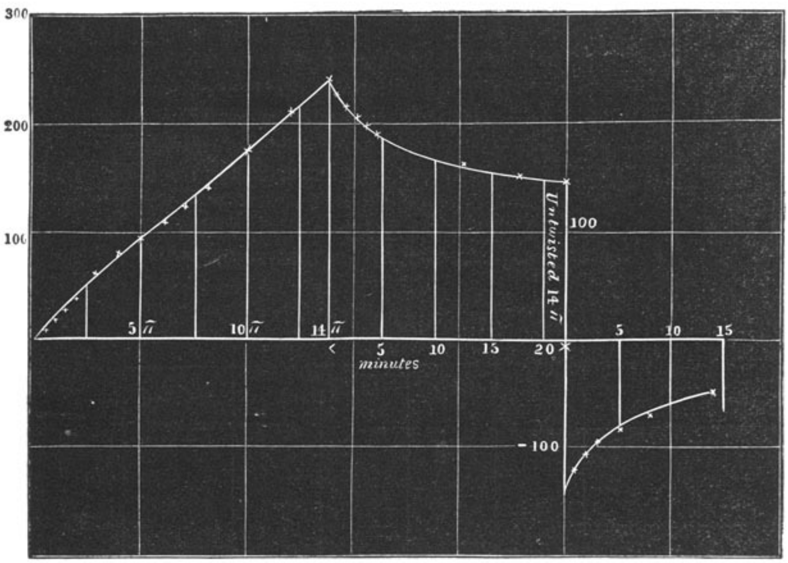
Vibrator Method.

Mass of vibrator, in grammes.	Radius of gyration of vibrator, in centimetres.	Length of fibre, in centimetres.	Diameter of fibre, in centimetres.	Period of vibrator, in seconds.	Torsional rigidity in C.G.S. units of a length of 1 centim. of the fibre.
0·0274	0·20	8·60	0·0008	29	0·00096
0·0114	0·29	8·60	0·0010	16	0·00132

Magnetic Method.

Length of fibre, in centimetres.	Diameter of fibre, in centimetres.	Deflection per turn of torsion-head on a scale of millimetres. Magnetic field =·153.	Distance of the scale from the mirror, in centimetres.	Torsional rigidity, in C.G.S. units, of a length of 1 centim.	Young's modulus for the fibre, weight in grammes*.
9·05	0·0010	8·0	117·0	·00143	75·4
9·20	0·0009	8·0	117·0	·00090	61·3
8·45	0·00145	21·0	117·0	·00216	65·6
9·55	0·0015	21·5	117·0	·00250	73·5

The following curve illustrates an experiment, and shows how nearly proportional the first deflection is to the torsion even after the elastic limit of the fibre has been far exceeded. In



the first part of the curve the ordinates are the scale-readings, the abscissæ the angle turned through by the torsion-head,

* This is the ratio of the product of the pull applied to the fibre, and the length of the fibre, to the elongation produced by the pull, or, if E be the modulus, $E = \frac{\text{weight applied} \times \text{length of fibre}}{\text{elongation}}$.

which we may, without appreciable error, assume to be the torsion of the fibre, as the angle turned through by the mirror is so small as to be negligible; in the last part the ordinates have the same meaning, but the abscissæ indicate time. This second part of the curve shows the rate at which the fibre takes a set under the torsional stress; the part of this curve below the zero-line shows the working out of the set after the fibre was untwisted. The length of the fibre in this experiment was 8·5 centim. and the average thickness about 0·0015 centim.

When a galvanometer is made sufficiently sensitive for the fibre to play an important part in directing the needle, the set of the fibre due to continued deflection always produces an apparent change of zero which, in exact measurements, it is somewhat difficult to properly allow for. Except, however, in very special cases, as, for instance, in taking deflections with a Thomson's "dead-beat" galvanometer in a weak magnetic field, the error is small, and it is not in any way capricious. It is important to bear in mind, however, that for very sensitive galvanometers to be used as deflectional instruments the suspension should be of considerable length, such, for example, as is provided in the Thomson's astatic galvanometer.

From the data given above we may very easily form an estimate as to when the rigidity of a silk fibre comes to be an important factor, affecting the sensibility of a galvanometer. If C be the current flowing through the galvanometer, K a constant depending on the coils, I and I′ the field at the upper and lower needles respectively, m and m' the magnetic moments of these needles, τ the torsional rigidity of the fibre, and θ the deflection, we have

$$C=K\left\{\frac{Im-I'm'}{m+m'}\tan\theta+\frac{\tau\theta}{(m+m')\cos\theta}\right\}.$$

When the needle system is perfectly astatic, $m=m'$, and this reduces to

$$C=K\left\{\frac{I-I'}{2}\tan\theta+\frac{\tau\theta}{2m\cos\theta}\right\};$$

and for small deflections this may, without great error, be written

$$C=K\theta\left\{\frac{I-I'}{2}+\frac{\tau}{2m}\right\}.$$

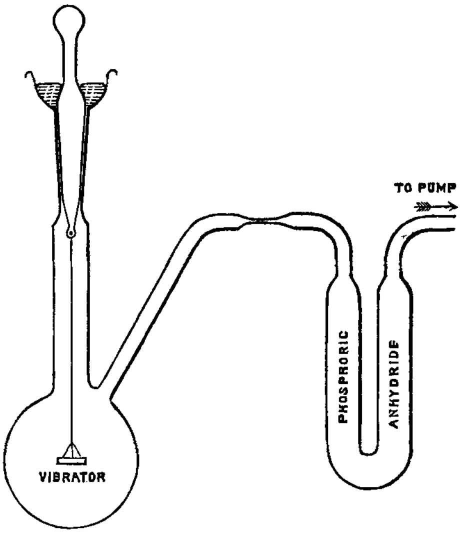
From this equation we see that the fibre becomes important when $\frac{\tau}{m}$ is not small compared with $I-I'$. Now in a very sensitive instrument it is not unusual for $I-I'$ to be reduced to about ·001, and m in such an instrument as we are considering will not differ much from unity. Hence τ must be much less than ·001; and we find, from the tables given above, that, for a fibre of about the usual length, say 5 centim., τ will be about ·0003; or about one fourth of the total force is, in this case, due to the fibre. This, then, may be taken as about the limit of sensibility beyond which we cannot easily pass with an ordinary Thomson's astatic galvanometer with small needles; to get beyond it, attention must be directed to an increase of m*. The limit here indicated is, however, far beyond anything that can be reached with wire suspension, the smallest current which can be measured being about 10^{-9} ampere for a galvanometer of 1 ohm resistance, and about $0{\cdot}2 \times 10^{-10}$ ampere for one of 10,000 ohms resistance. When $I-I'$ is as much as 0·01, or between a tenth and a twentieth of the earth's horizontal force in this country, the effect of the set of the suspending fibre is extremely small. With such a value of $I-I'$, however, a properly constructed galvanometer, the resistance of which is as low as 1 ohm, will measure a current of 10^{-8} ampere. When very high sensibility is absolutely necessary, it may be to some extent obtained by increasing the length of the fibre; but if this prove insufficient, some alteration or other arrangement of the parts becomes necessary. Such an arrangement is described in the paper referred to in the footnote; but it may be remarked that, in so far as this arrangement is intended to increase m, it is only important when $I-I'$ is made practically zero. So long as $I-I'$ is considerably greater than τ, a high value of m is of no importance; and the Thomson form is, because of the small inertia of its needle system, decidedly the best.

NOTE.—Since the above was written Mr. J. T. Bottomley has suggested to me that some interesting results might be obtained if the vibrational method, above referred to, were carried out with the fibre and vibrator in a good Sprengel vacuum; and in conjunction with him I have made some preliminary experiments, the results of which seem worth quoting.

The vibrator used was the lighter of the two referred to in connection with the former experiments, and consisted of a small ring of brass 0·295 centimetre radius and 0·012 gramme in weight. It was suspended, as shown in the diagram, inside a small spherical bottle provided with a long neck and a ground stopper, to the lower end of which the fibre was attached. A tube passed from the side of the bottle to one

* On this subject see a paper "On a New Reflecting Galvanometer of Great Sensibility," by T. and A. Gray, Proc. Roy. Soc. No. 230 (1884).

end of a U-tube, containing phosphoric acid and beads of glass, the other end of which was sealed to a tube leading to the Sprengel pump. The vibrator was attached to the fibre by means of three short single fibres, in the manner shown in



the sketch. The results are given in the following table, the meaning of the numbers in the different columns of which will be readily understood from the headings. In the column headed "numbers of vibrations observed" the figures represent roughly the number of periods which could be observed at the different degrees of exhaustion, shown in the preceding column, beginning in each case from an amplitude of about 60°, and observing directly the transits of a black spot on the ring over a fixed mark until the amplitude fell to about 10°.

The results are sufficient to show that the effect of the viscosity of the fibre in damping the vibrations is very small in comparison with the effect of the air friction; and it seems probable that a moderately heavy vibrator (say about 2 grammes in weight) with a small magnetic needle attached, and suspended by a single silk fibre, may prove a good arrangement for experiments such as have been carried out by Maxwell, Kundt and Warburg, Crookes and others on the friction and viscosity of gases. It certainly would have the advantage that the period would depend mainly on the strength of the magnetic field, and could be varied at pleasure. Should opportunity offer, Mr. Bottomley and the writer hope to continue these experiments.

Number of experiment.	Length of fibre, in centimetres.	Diameter of fibre, in centimetres.	Period of vibrator, in seconds.	Pressure of air at the vibrator, in atmospheres.	Numbers of vibrations observed.	Torsional rigidity of fibre per centimetre of its length in C. G. S. units.
1.	3·9	·00095	10·5	1·00	7	
,,	,,	,,	9·6	$1·46\times10^{-5}$	14	·00134
2.	3·7	·00120	7·78	1·00	7	
,,	,,	,,	7·64	0·066	7	
,,	,,	,,	7·42	$1·46\times10^{-6}$	(?)	
,,	,,	,,	7·42	$0·40\times10^{-6}$	60	·00261
3.	3·65	·00105	12·5	1·00	5	
,,	,,	,,	11·92	$7·45\times10^{-3}$	7	
,,	,,	,,	11·83	$8·35\times10^{-6}$	15	
,,	,,	,,	11·63	$0·53\times10^{-6}$	40	
,,	,,	,,	11·57	$0·13\times10^{-6}$	50	
,,	,,	,,	11·61	$0·13\times10^{-7}$	50	·00110

VI. *On Stationary Waves in Flowing Water.*—Part IV. *Stationary Waves on the Surface produced by Equidistant Ridges on the Bottom.* *By* Sir WILLIAM THOMSON, *F.R.S.**

THE most obvious way of solving this problem is by the use of periodic functions, which we have been so well taught by Fourier in his 'Mathematical Theory of Heat;' and in this way it was solved in Part III. (formulas 1 to 15); the solution being (15) Part III. with

$$\kappa=1, \quad m=2\pi/a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1);$$

where a denotes the distance from ridge to ridge. Thus, reproducing (15) Part III. with the notation modified to shorten it in form and to suit it for numerical computation, we have

$$\mathfrak{h}=\sum_{i=1}^{i=\infty}\frac{4A/a\,.\cos i\psi}{e^{i}+e^{-i}-\frac{M}{i}\left(e^{i}-e^{-i}\right)} \quad . \quad . \quad . \quad (2);$$

* Communicated by the Author.