



535. Geometrical Construction for the Trisection of an Angle to Any Required Degree of Accuracy

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the point F will describe a straight line, which can easily be shown to pass through G .

The particular case of parallelism referred to is of course when E is at infinity, and therefore AB parallel to CD .

The modern form in which this, the only surviving one of Euclid's Porisms, usually appears is: "coaxial triangles are copolar," a proposition generally supposed to be of quite recent discovery.

JOHN J. MILNE.

535. [K¹. 21. b.] *Geometrical Construction for the Trisection of an Angle to any required Degree of Accuracy.*

[For the main idea and for a considerable share in the calculation involved, I am indebted to Cadets J. Y. G. Torlesse and M. H. C. Young, R.N.]

Let AB be an arc of a circle subtending an angle θ at the centre O .

Let C be the mid-point of the arc AB , D the mid-point of the arc BC , E the mid-point of the arc CD , F the mid-point of the arc DE , etc.

1st approximation. Let the straight line through C parallel to OA meet the straight line through D parallel to OB in Q_1 .

Then $\hat{B}OQ_1 = \phi_1 = \text{approximately } \frac{\theta}{3}$.

2nd approximation. Let the straight line through D parallel to OB meet the straight line through E parallel to OC in Q_2 .

Let $\hat{B}OQ_2 = \phi_2$.

3rd approximation. Let the straight line through E parallel to OC meet the straight line through F parallel to OD in Q_3 .

Let $\hat{B}OQ_3 = \phi_3$, etc.

Then if ϕ_n be the angle obtained in the n th approximation (i.e. $\hat{B}OQ_n$), it is easy to show that

$$\tan \phi_n = \frac{\sum \sin \alpha}{\sum \cos \alpha},$$

where $\alpha = \frac{\theta}{3}[1 + 8(-\frac{1}{2})^{n+1}]$, $\beta = \frac{\theta}{3}[1 - 7(-\frac{1}{2})^{n+1}]$, $\gamma = \frac{\theta}{3}[1 - (-\frac{1}{2})^{n+1}]$,

or $\tan\left(\phi_n - \frac{\theta}{3}\right) = \frac{\sum \sin u}{\sum \cos u}$,

where $u = \frac{8\theta}{3}(-\frac{1}{2})^{n+1}$, $v = -\frac{7\theta}{3}(-\frac{1}{2})^{n+1}$, $w = -\frac{\theta}{3}(-\frac{1}{2})^{n+1}$,

(so that $\sum u = 0$),

$$\begin{aligned} \text{i.e. } \phi_n &= \frac{\theta}{3} + \tan^{-1} \frac{\sum \sin u}{\sum \cos u} \\ &= \frac{\theta}{3} + f(\theta), \end{aligned}$$

where $f(\theta) = \frac{28}{3}\left(\frac{\theta}{6}\right)^3 \times (-\frac{1}{2})^{3n} + 133\left(\frac{\theta}{6}\right)^5 \times (-\frac{1}{2})^{5n} + \frac{(133)^2}{10}\left(\frac{\theta}{6}\right)^7 \times (-\frac{1}{2})^{7n} + \dots$

Thus, if the n th approximation gives an error of any given size in the trisection of a given angle θ , the $(n+1)$ th approximation will give this same error (with the sign changed) for an angle 2θ , and approximately (rather less than) $\frac{1}{8}$ th of this error for the same angle θ .

In particular, an error of less than 1 minute is given by the n th approximation if the angle θ is less than $2^n \times 10\frac{1}{2}^\circ$, and an error of less than 10 minutes is given by the n th approximation if the angle θ is less than $2^n \times 22\frac{1}{2}^\circ$.

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