The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables

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XXIII. *The Use of the Wien Bridge for the Measurement of the Losses in Dielectrics at High Voltages, with Special Reference to Electric Cables.* By A. Rosen, *A.C.G.I., B.Sc. (Engineering), A.M.I.E.E.*

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ABSTRACT.

In the preliminary section, the loss angle of an imperfect condenser is defined, the equations for the Wien bridge are derived, and the effects of variations of frequency and voltage on the balance are discussed.

One of the difficulties in the application of large potential differences to a bridge is the effect on the arm which has to withstand the high voltage. In the arrangements due to Monasch and Schering, this arm is the known condenser; in the bridge as used by the author, the voltage is applied to the ratio coils. The errors introduced by earth impedance are eliminated by using the Wagner auxiliary bridge.

The applications to measurements on cables are considered, and the use of the double bridge in determining the "wire-to-wire" and "wire-to-sheath" losses in a multi-core cable is described.

In Appendix I. the corrections due to imperfections of the bridge arms are discussed, and in Appendix II. a simple quantitative theory of the double bridge is given.

Preliminary.

Loss Angle of a Condenser.

Consider a periodic E.M.F. of pure sine form and frequency $\frac{\omega}{2\pi}$, represented by $v=\hat{V}\sin\omega t$, applied to the terminals of a condenser; the resulting current, which may possibly be complex, can be represented in the general case by

$$i=\hat{I}\sin(\omega t-\varphi)+\hat{I}_3\sin(3\omega t-\varphi_3)+\hat{I}_5\sin(5\omega t-\varphi_5)+\ldots$$

The mean power dissipated will be $W=\frac{1}{T}\int_0^T vi dt$, T being taken over a whole number of periods, $=\frac{1}{2}\hat{V}\hat{I}\cos\varphi$, so that, in measuring the power, we may disregard the harmonics in the current wave, and consider only the fundamental, $\hat{I}\sin(\omega t-\varphi)$. For any particular values of $\hat{V}$ and ω we can imagine the condenser replaced by a combination of a perfect, unvarying capacity C, shunted by a resistance which obeys Ohm's law, of conductance G, such that the total current is $i=\hat{I}\sin(\omega t-\varphi)$; C will be the effective capacity of our condenser, and the loss at a root-mean-square voltage V will be V^2G. The current leads on the P.D. by an angle $\varphi=\frac{\pi}{2}-\theta$, such that $\tan\theta=\frac{G}{\omega C}$, and the power may be expressed by $W=V^2\omega C\tan\theta$; θ is therefore a measure of the loss in the condenser, and is referred to as the "loss angle." The "power factor" as defined by $\int_0^T vi dt\Big/\left(\int_0^T i^2dt\,.\int_0^T v^2dt\right)^{\frac{1}{2}}$ equals $\cos\varphi\,\frac{\hat{I}}{(\hat{I}^2+\hat{I}_3^2+\hat{I}_5^2+\ldots)^{\frac{1}{2}}}$, so that only when the current wave is free from harmonics can $\cos\varphi$ be strictly termed the power factor of the condenser; the difference can, however, usually be neglected.

The Wien Bridge.

The Wien Bridge,* a particular application of which is described in this Paper, measures the values of C and G, and thus θ directly. Diagrammatically it is represented in Fig. 1. The arms AB, AD consist of pure resistances R_1, R_2 respectively.

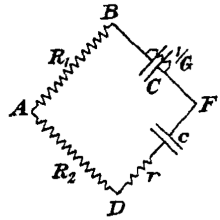

FIG. 1.—WIEN BRIDGE.

The unknown condenser to be measured is placed in the arm BF, and is represented by the capacity C shunted by the resistance $\frac{1}{G}$. The fourth arm is occupied by a perfect condenser c in series with a pure resistance r. An alternating P.D. of frequency $\frac{\omega}{2\pi}$ periods per second, and a suitable detector are placed across the conjugate points BD, AF. The impedance of the arm BF is $Z_3=\frac{1}{G+j\omega C}$, and of DF is $Z_4=r-\frac{j}{\omega c}$. The condition for no current in the detector is

$$\frac{R_1}{R_2}=\frac{Z_3}{Z_4}=\frac{1}{(G+j\omega C)\left(r-\frac{j}{\omega c}\right)}.$$

Equating the real and imaginary terms, we obtain, after transformation,

$$G=\frac{R_2}{R_1}\cdot\frac{\omega^2c^2r}{1+\omega^2c^2r^2}$$

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2c^2r^2}$$

$$\tan\,\theta=\frac{G}{\omega C}=\omega cr.$$

In the majority of cases, $\tan\theta$ is a small quantity, and its square can be neglected in comparison with unity. Writing $\frac{R_1}{R_2}=m$, the expressions reduce to

$$G=\frac{\omega^2c^2r}{m}\text{ mhos}$$

$$C=\frac{c}{m}\text{ farads}$$

$$\theta=\omega cr\text{ radians,}$$

c and r being in farads and ohms respectively.

* M. Wien, Wiedemann's Annalen, Vol. 44, p. 689 (1891).

For a given set of conditions, G and C are fixed, and to satisfy these equations we must vary any two of c, r and m. In the arrangement described in detail below m is kept fixed, and c and r are varied for a balance.

Variation of Balance with Frequency.

This will depend on the properties of the dielectric being tested. It has been found experimentally* that for practically all the materials used as dielectrics for condensers the variation of the conductance G with frequency may be represented approximately by an equation of the form $G=a+b\omega$, where a and b are constants which are determined by the temperature ; also the capacity C is nearly independent of frequency. Substituting in the above equations, we obtain

$$r \propto \frac{a}{\omega^2}+\frac{b}{\omega}$$

$$c \doteqdot \text{constant.}$$

We may say that r varies inversely as some power of the frequency between the first and second, the precise number depending on the relative values of a and b. (For

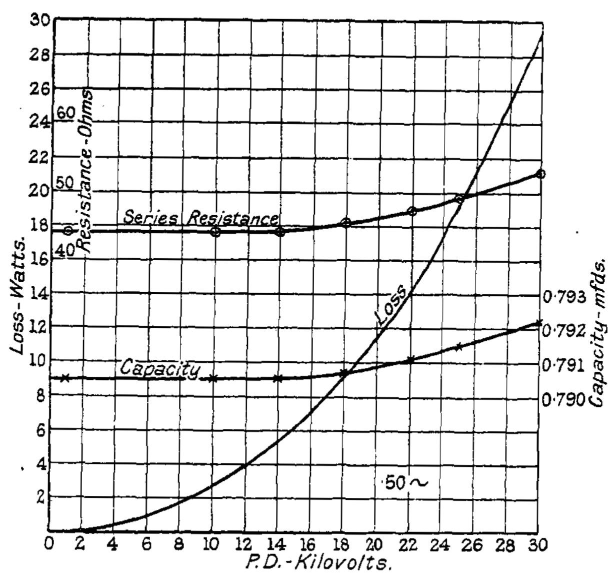

FIG. 2.—VARIATION OF BALANCE WITH VOLTAGE.

electric cables with impregnated paper insulation, at normal temperatures, a is relatively small, and r varies very nearly inversely as the frequency.)

It follows that if the current in the condenser tested is complex—i.e., contains terms of more than one frequency, the current in the detector can never be zero. However, by using an instrument that will respond to only one periodicity—e.g.,

* Fleming and Dyke, Journal I.E.E., Vol. 49, p. 323 (1912).

a tuned vibration galvanometer—a balance can be obtained for the fundamental, and the effect of the harmonics is eliminated. As no loss is caused by the harmonics, the result still gives the power spent in the dielectric. Further, if the P.D. applied to the bridge is not of pure sine form, the balance obtained will be in terms of the fundamental components of the voltage and current—i.e., the bridge will measure only the loss caused by the fundamental; in other words, the readings are independent of the wave-form of the applied voltage; this has been experimentally verified by Monasch.*

Variation of Balance with Voltage.

The change of capacity and loss angle of a condenser with voltage, if any, is generally not rapid, and, in any case, if the loss varies as V^n, to balance the bridge, r will vary approximately as V^{n-2}. This is a considerable advantage compared with the wattmeter method of measuring power, in which the reading varies as V^n, as firstly, the balance is not much disturbed by fluctuations of voltage, and, secondly, errors in the voltmeter which measures the P.D. are of less importance. In Fig. 2 are plotted some figures relating to a paper-insulated cable, and show the slow variation of r and c with voltage and the comparatively rapid change in the power loss.

Previous Work.

Monasch's Method.†

The application of high voltages to a bridge presents certain difficulties which were only partially overcome by Monasch's arrangement. Referring to Fig. 3(*a*),

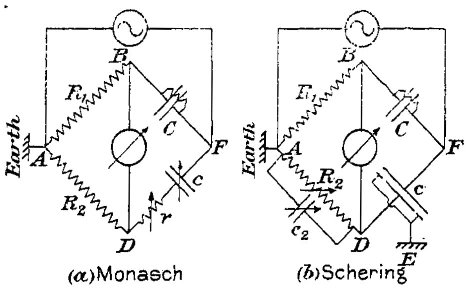

Fig. 3.—Bridges used by Monasch and Schering.

the P.D. was applied to the points AF, A being earthed, and a tuned optical telephone was placed as detector across BD. It will be seen that the full voltage on the condenser tested was applied to the adjustable capacity c, and this introduced serious limitations. After many efforts, a variable air condenser was constructed capable of withstanding 11 kilovolts without appreciable loss due to ionisation of the air, its maximum capacity being 230×10^{-6} mfd.; this set a definite upper limit to the testing pressure. The large surface area of this condenser introduced considerable earth capacity effect, so that the readings obtained were only

* B. Monasch, Electrician, Vol. 59, p. 416 (1907).
† Electrician, loc. cit.

comparative and not suitable for calculating the true loss angle. For his quantitative measurements, Monasch used a much lower P.D. and larger capacities, as he had concluded that the balance was independent of the voltage. As regards cable, this is only true for the comparatively low dielectric stresses used by Monasch and for the higher values it is necessary to test at the full voltage.

Schering's Method.

A more successful arrangement due to Prof. Schering, and described by Alfred Semm,* is shown in Fig. 3(*b*). It differs from Monasch's scheme in that the leakance in condenser C is compensated by a capacity c_2 shunted across R_2. The equations for balance are of the same form as previously, viz.,

$$C=\frac{R_2}{R_1}\cdot\frac{c}{1+\omega^2 c_2{}^2 R_2{}^2}$$

$$\tan\theta=\omega c_2 R_2$$

and balance was obtained by varying R_2 and c_2. The air condenser c, which was of fixed capacity, had a "guard plate" connected to earth; this placed the "edge-effect" with the possibility of ionisation outside the bridge arm. The effect of the earth capacity of the L.T. plate was negligible, as it was in parallel with c_2, which was of the order of 1 mfd. By making the distance between the plates large, c was made capable of withstanding 100 KV., but the capacity was small, being only 50 cm., i.e., $55{\cdot}6\times10^{-6}$ mfd. A defect is that the impedances of the bridge arms were very unequal, causing loss of sensitivity; e.g., R_1 is given as usually 200 ohms, which is $\frac{1}{300{,}000}$ of the impedance of c at 50 cycles per second. Further, a serious practical disadvantage when testing cables is that the low-tension side of C has to be well insulated from earth, as any earth impedance shunts the arm R_1.

AUTHOR'S ARRANGEMENT.

The method used is shown in Fig. 4; it consists essentially of the usual Wien

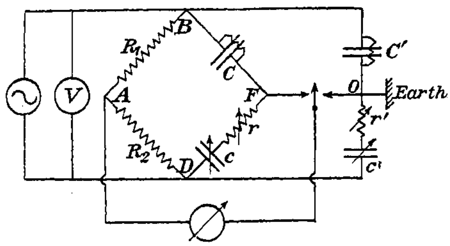

FIG. 4.—WAGNER DOUBLE BRIDGE.

bridge with the Wagner earthing arrangement.† The arms of the bridge $ABFD$ are as in Fig. 1, and in addition an auxiliary bridge, consisting of the condensers C', c' and a resistance r', is placed across the points BD, the junction O of C' and

* Archiv für Elektrotechnik, Band 9, p. 29 (1920).
† K. W. Wagner, E.T.Z. Vol. 40, p. 1001 (1911).

r' being put to earth. The detector is switched alternately between the points AF, AO, c, r and c', r' being varied respectively to obtain a balance. When finally there is no deflection in either position, A, F and O are at the same, i.e., earth potential, and c, r give the true readings required. Considering the currents to earth from the points $ABFD$, those from B and D will merely modify the impedance of the arms BO, DO, and since A, F and O are at the same potential, no earth currents can flow from A and F. The effect of the impedance to earth of the apparatus, including the source and the detector, is thus eliminated.* (*See* Appendix II.)

The following points are to be noticed:—

(1) By connecting the source across the ratio coils, the arm of the bridge which has to withstand the high voltage is a resistance and not a capacity; it is easier to construct a satisfactory resistance for this purpose than a condenser.

(2) By using a large value for the ratio $\frac{R_1}{R_2}$, e.g., 100 : 1, the P.D. across the measuring condenser c can be brought to within everyday values, so that ordinary condensers can be used for this purpose, and there is no limit as to their size.

(3) The galvanometer is brought to earth potential, and the P.D. between the adjustable arm FD and earth is small, so that the apparatus can be manipulated conveniently and in safety, which contributes to speed and accuracy in working.

(4) The voltage on the condenser tested is a fixed proportion of the P.D. of the source, which can be read directly on a voltmeter.

Practical Details.

Ratio Arms.

In obtaining the equations for the Wien bridge, it was assumed, for simplicity, that the phase angle of the resistances R_1, R_2 was zero. The equations still hold good if the resistances are not perfectly non-reactive, providing they have the same time-constant, in which case their ratio, m, has zero angle. This result can be largely attained by constructing both arms of identical units. R_1, R_2 consisted of 100 and one 5,000 ohm coils respectively, the total resistance being 505,000 ohms. They were wound on flat ebonite cards 0·036 in. thick with 0·002 in. d.s.c. Eureka wire in one layer, the direction of winding being reversed three times; this form of winding gives a coil with small inductance and self-capacity suitable for withstanding high voltages. The cards were mounted in supports in such a way that the voltage and capacity between adjacent ones was small, and the whole was immersed in oil in a wooden tank supported on insulators. This resistance was capable of withstanding 30 KV. without undue temperature rise, and up to 50 KV. for a short time. A smaller coil of 2,500 ohms resistance was provided for use above 30 KV., giving a ratio of 200 : 1, and thus a lower voltage on the adjustable condensers.

For higher potential differences resistances become too elaborate, besides absorbing considerable power. In this case it is better to use inductances of the

* S. Butterworth, Proc. Phys. Soc., Vol. 34, p. 8 (1921).

same phase angle for the ratio arms. A tapping from the H.T. winding of the transformer would serve, but, apart from the difficulty of adapting an existing transformer, this has the disadvantage that the time-constant ratio would probably alter with the voltage and load. A more suitable arrangement would be a specially designed iron-cored choke coil, the winding being built up of a number of similar units in series. Each one of these would then have the same phase angle under all conditions, and wherever the tapping is made the two parts would have equal time constants.

Adjustable Capacity Arms.

For general work, three condensers were used in parallel as the variable capacity c, (1) mica dielectric, adjustable by means of plugs up to 20 mfds. in steps of 1 mfd., (2) mica dielectric of 3-decade type with dial switches giving continuous variation up to 1 mfd. in steps of 0·001 mfd., (3) air dielectric, continuously variable up to 0·0018 mfd. For the "extra" bridge capacity, accuracy of calibration and small loss angle are not needed, and Mansbridge condensers with paper dielectric were used for the values from 1 to 10 mfds.; a 3-decade condenser in parallel gave continuous adjustment from 0·001 to 1 mfd. The balancing was much facilitated by using the condensers with rotating switches.

Source.

The source of current used was an alternator of 60 KW. capacity directly driven by a D.C. shunt motor. The frequency could be varied by altering the excitation of the motor field, and was measured by a frequency meter of the vibrating-reed type, actuated by a device mounted on the main shaft. The meter could be read to within $\frac{1}{2}$ cycle per second, and a variation of about $\frac{1}{10}$ cycle per second could be detected.

Leads.

The lay-out of the testing plant was such that all parts at a dangerous potential were completely protected, e.g., the transformer was housed in an enclosed chamber, and the voltmeter guarded by a large glass screen. It was desirable that the addition of the apparatus described should not alter this general rule. The ratio arms, which have to withstand the full voltage, were therefore placed in the chamber alongside the transformer, and the galvanometer, variable condensers and resistances, which are all at low working potential, were placed conveniently near the controls for the frequency and voltage. This necessitated somewhat long connecting leads; these were brass-taped over the insulation and then laid up in pairs, the screens being joined to earth. Thus, the possibility of inductive interference from the primary circuit was avoided; further, the impedance between the leads was eliminated, as it was taken account of by the Wagner auxiliary bridge. This impedance consists, not only of the capacity and leakance of the leads, but also leakage over the ends, and this last may, in certain cases, cause serious errors unless avoided by having the leads screened.

When tests were taken on drums of cable, the P.D. was conveyed through highly-insulated leads, about 20 yards in length, consisting of two single core cables with impregnated paper insulation, the lead sheathing being earthed. Here, again, the double bridge arrangement eliminated the impedance to earth of these leads, and no correction was needed to take account of them. The resistance of these H.T. leads was negligible, as the equivalent series resistance of the cable never amounted to less than several hundreds of ohms. However, the resistance of the leads to the adjustable arm *FD* could not in certain cases be neglected, and its value, 0·20 ohm., was added to the readings of the series resistance *r*.

Detector.

A vibration galvanometer made by H. Tinsley & Co. was used as detector. A step-down transformer was found greatly to increase the sensitivity. (Actually

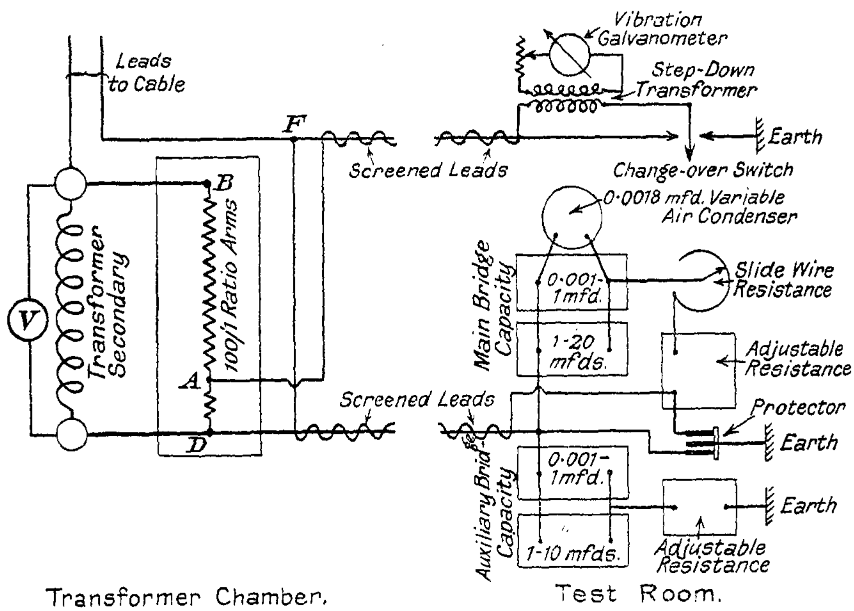

FIG. 5.—ARRANGEMENT FOR ROUTINE TESTING OF CABLES.

a voltmeter transformer 6,000 V./110 V. was used, because it happened to be available, and this gave an increase of about ten times in the deflection.) An adjustable resistance was put in the circuit to reduce the deflection to a suitable value when the bridge was out of balance. Theoretically best results are obtained when the galvanometer is exactly tuned to the supply frequency. It was found that this made the balance very sensitive to speed variation, and further, the response to adjustments of the bridge was sluggish. As there was generally no lack of sensitivity, it was found better to work with the galvanometer adjusted to about 3 cycles from the working value ; even so, for the higher voltages the deflections were very large, and it was usually advisable for rapid working to cut them down by means of the series resistance.

Protective Devices.

Under normal working conditions no part of the apparatus to be adjusted is at a higher potential than 300 volts (A.C.) from earth ; if, however, the dielectric of the test condenser breaks down, then the full voltage appears on the measuring condenser. This was guarded against by connecting the points F and D through spark gaps to earth. A protector designed by the Post Office for guarding their aerial lines against lightning discharges was used very successfully. It consisted of carbon blocks mounted on a porcelain base and separated by thin sheets of mica pierced with holes. This acted as a perfect insulator until the P.D. rose to about 450 volts, at which value the air broke down and a low resistance arc was formed ; this was capable of passing enough current to operate an overload release in the primary circuit of the transformer.

Determination of Corrections.

In Appendix I. the effect of the imperfections of the bridge arms is discussed, and it is shown that it is only necessary to take into account the loss angle of the variable condenser c, measured with the bridge ratio arms. This correction was determined for the various mica condensers by comparison with a known air condenser and a non-reactive series resistance placed in the arm BF, the rest of the circuit being as usual ; the capacity to earth of the leads served as the corresponding auxiliary bridge condenser. The P.D. of the transformer secondary was cut down to 200 V. (this being a safe figure for the air condenser) by using a 10 V. accumulator to excite the field of the alternator instead of the usual 110 V. mains. By employing exact tuning of the galvanometer, the power factor could be measured to within a few per cent. On the large 1-20 mfd. condenser the values were of the order 0·001, and on the 3 decade condenser about 0·0008. The correction is thus roughly 10% when measuring power factors of 0·01, and an error of as much as 10% in the determination of the loss angle of the mica condensers would produce an error of only 1% in the power measurement.

It will be seen that the assumption is made that the constants of the bridge arms do not alter as the voltage is raised. This was checked by direct comparison with a simple air condenser constructed as follows :—

The two plates were formed of thin sheets of tinned iron 6 ft. × 2 ft. 6 in. fixed on to flat wooden frames. The L.T. plate was cut into two portions by a continuous slot parallel to and about 4 in. from the sides, the sharp edges being turned down. The upper (H.T.) plate was supported by porcelain insulators standing on the outer ring, which served as a guard plate and was earthed ; thus there was only air as dielectric between the plates of the condenser proper.

To assist in obtaining a balance, a coil having an inductance of 16 millihenries and resistance of 20 ohms was placed in series with the 5,000 ohm ratio arm. With the plates about 2 in. apart, the capacity was 0·000197 mfd., and the bridge balance was constant up to 12 KV., beyond which an increase in the series resistance r was necessary. On altering the distance to $4\frac{1}{2}$ in. by using bigger insulators, the capacity was reduced to 0·0000828 mfd., and the balance was undisturbed up to 25 KV. It is evident that no change was occurring in the bridge arms, the alteration being due to ionisation of the air in the H.T. condenser. Similar results were obtained using both air and mica condensers as the adjustable capacity c, and it was concluded that the bridge constants do not alter with voltage.

From the results of these tests, the phase angle of the ratio arms (a) was calculated as $a=\frac{\omega L}{R_2}-\omega cr$, L being the inductance (16 mh.) in series with R_2. When the capacity of the known air condenser c was 0·00828 mfd., the series resistance at 50 ∿ was 400 ± 10 ohms; thus $a = -0{\cdot}00004$, which is less than the experimental error. This result shows that the unit method of constructing the ratio arms is very satisfactory.

Measurements on Cables.

In a single-core cable, the conductor forms one plate of a condenser and the outer lead sheath the other. The core is connected to the high potential point B on the bridge, and the sheath to the low potential point F. A second length of cable capable of withstanding the voltage, forms a suitable condenser C' for the auxiliary bridge, the sheath being put to earth. The sheath of the cable being tested must not be earthed, but the insulation need not be of a high order, e.g., satisfactory measurements were obtained on a coil wound on a dry wooden drum.

Multicore Cables.

Consider a cable containing n cores insulated from each other and from the

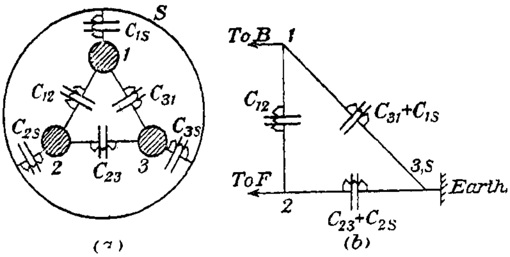

Fig. 6.—Capacities in 3-Core Cable.

metal sheath surrounding them. The "wire to wire" capacities between them may be represented by a series of $\frac{1}{2}n(n-1)$ leaky condensers connecting every pair of wires, and the "wire-to-sheath" capacities by a further n condensers connecting each core to the sheath.* A three-core cable is shown diagrammatically in Fig. 6(a). The Wagner double bridge enables us to determine the loss in, and capacity of any of these condensers directly.† To measure C_{12}, core 1 is connected to B in the bridge, and core 2 to F, the remaining core being joined to the sheath, which is earthed; the resulting system of condensers is shown in Fig. 6(b). The capacity of 1 to earth is $C_{31}+C_{1S}$, and this forms the condenser C' in the auxiliary bridge circuit. When final balance is obtained core 2 is brought to the same potential as S, and thus the capacity of 2 to earth, viz. $C_{23}+C_{2S}$ does not affect the main bridge; condenser C_{3S} is short-circuited, since 3 is joined directly to S. We have therefore eliminated all the capacities but the one to be measured, viz., C_{12}. Similarly to determine a

* A. Russell, Alternating Current Theory, Vol. 1, Chap. 4.
† K. W. Wagner, E.T.Z., Vol. 25, p. 635 (1912).

"wire-to-sheath" capacity, say C_{1S}, we connect 1 to the high potential point B, the sheath to F, and the other cores to earth. It is necessary for this test, as when taking a single core cable, that the sheath should be insulated from earth.

Cable Sheath Earthed.

If the cable sheath is already earthed, as when the coil is immersed in water, it is not possible to obtain any "wire-to-sheath" measurements in this way, and we have to fall back on the ordinary single bridge. We can allow for the earth impedance of the bridge arms and of the leads to the cable, if we first take a reading with the cable disconnected; this gives the value of the resultant earth impedance imagined all located in the arm BF. In what follows, the earth impedance of the bridge arms is understood to be included in the term "leads."

Let C_l, θ_l be the capacity and loss angle respectively of the leads.

C_c, θ_c be the capacity and loss angle respectively of the cable.

C_{c+l}, θ_{c+l} be the capacity and loss angle respectively of the leads and cable.

Then $C_c = C_{c+l} - C_l$,

and $\tan\theta_c = (C_{c+l}\tan\theta_{c+l} - C_l\tan\theta_l)/(C_{c+l} - C_l)$.

With the existing double bridge arrangement we can compensate for the leads and measure the cable directly as follows:—

With the cable disconnected and the arm FD (Fig. 7) open, the auxiliary bridge

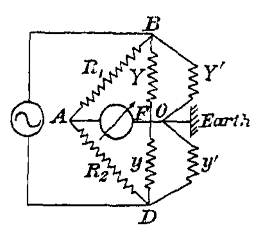

FIG. 7.—CABLE SHEATH EARTHED.

arm OD is adjusted to balance the leads. Then when the cable is on, the main bridge condenser and resistance are adjusted for balance, and the readings give the capacity and loss angle of the cable in the usual way.

For, let Y' represent the admittance of the leads, which is balanced by y'.

$$\therefore \frac{R_1}{R_2}\cdot Y' = y'.$$

Again Y and Y' in parallel are balanced by y and y' in parallel.

$$\therefore \frac{R_1}{R_2}(Y+Y') = y+y'.$$

Hence $\dfrac{R_1}{R_2}\cdot Y = y$, or y is the required balance for Y.

Loss in a 3-core Cable.

The dielectric loss in a 3-core cable is given by

$$W=3\omega(V_w^2 \tan \theta_w C_w + V_e^2 \tan \theta_e C_e)$$

the subscripts w and e referring to "wire-to-wire" and "wire-to-sheath" quantities respectively; when the voltages are balanced, $V_e=\frac{V_w}{\sqrt{3}}$, V_w being the line pressure.

Two tests were employed to determine W: (1) the three cores against sheath, at a voltage $\frac{V_w}{\sqrt{3}}$, giving a loss W_e; it the sheath were earthed, the single bridge method as described in the previous section, was used; (2) a "wire-to-wire" test with the double bridge, at the full voltage V_w, giving a loss W_w; then $W=W_e+3W_w$.

Routine Work.

From the apparently complicated series of adjustments required to obtain a balance using the double bridge, it might appear that this method is not so suitable for routine testing when time is a consideration, as the simpler wattmeter. This is not the case. It is shown in Appendix II. that, because of the high ratio of R_1 to R_2, a balance is quickly obtained, and only one cycle of adjustments, i.e., one of the auxiliary and then one of the main bridge, is needed for each subsequent step in voltage. Further, as was pointed out in the preliminary discussion of the Wien bridge, the balance is much less affected by fluctuations of speed and voltage than a wattmeter.

When testing cables, a reading was first taken at a low voltage (1 or 2 *KV.*). This indicated if the circuits were correct before the high tension was applied, and provided a datum for determining the change in the capacity and loss angle with voltage. As a rule, a series of readings at 4 or 5 different pressures was taken in addition to the working voltage, as not much extra time was occupied in obtaining the subsequent balances.

The following is an example of a "wire-to-wire" test on a 22,000 volt 3-core cable at 50 ~. Ratio=100 : 1.

(1) Voltage V kilovolts.	(2) Capacity, c mfds.	(3) Resistance reading ohms.	(4) Resistance, r ohms.	(5) ωcr.	(6) $\tan \theta$	(7) Loss $= V^2\omega C \tan \theta$ watts.
1	0·7905	40·5	40·7	0·0101	0·0109	0·027
10	0·7905	40·5	40·7	0·0101	0·0109	2·70
14	0·7905	40·5	40·7	0·0101	0·0109	5·30
18	0·7907	42·1	42·3	0·0105	0·0113	9·10
22	0·7911	44·1	44·3	0·0110	0·0118	14·2
25	0·7915	46·0	46·2	0·0115	0·0123	19·1
30	0·7922	49·7	49·9	0·0124	0·0132	29·6

Column (4) is obtained from (3) by adding 0·2 ohm, the resistance of the connecting leads to the variable arm. Column (6) is obtained from (5) by adding 0·0008, the loss angle of the measuring condenser. These results are plotted in Fig. 2, where, however, the resistance figures are adjusted to include the effective series resistance of the condenser.

Comparison with Wattmeter.

Comparative tests were carried out using a Duddell-Mather dynamometer wattmeter with the scheme of connections shown in Fig. 8. The effect of the capacity current flowing between the series resistance and its metal containing tank was eliminated by joining the case to the junction of the wattmeter coils. Correcting only for the loss in the current coils, the results, when measuring power factors

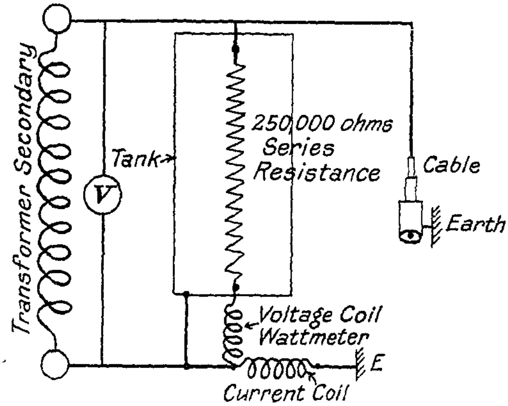

Fig. 8.—Wattmeter Connections.

of the order 0·01 agreed with the bridge figures to within 3 per cent., which was about the estimated experimental error due to unavoidable fluctuations in speed and voltage. An example is appended :—

Bridge. 50~. Ratio=100 : 1.

Voltage *KV.*	Capacity, *c* mfds.	Total Resistance, *r* ohms.	ωcr	$\tan\theta$	Loss watts.
12·7	8·103	3·40	0·0086	0·0096	39·5
22·0	8·117	4·17	0·0106	0·0116	143

Wattmeter 50~.

Voltage *KV.*	Constant, watts per division.	Reading of torsion head divisions.	Reading less lead correction divisions.	Loss watts.	Loss in current coils watts.	Loss in cable watts.
12·7	12·5	3·65	3·27	40·9	2·5	38·4
22·0	12·5	13·8	12·3	154	7·5	146·5

General Remarks.

The bridge arrangement described in this Paper is the result of an attempt to devise an alternative to the wattmeter for measuring the dielectric losses in cables using only such apparatus as was already available, or could be easily made. It has proved more successful than was originally anticipated, in particular regarding the accuracy and ease of balancing. Whilst it is most suitable for the purpose for which it was designed, it has shown a surprising degree of flexibility and has proved capable of measuring a large range of capacities of varying power factors, at low and high voltages, without alteration of the connections.

In the measurement of dielectric loss, the accuracy obtainable is limited by the closeness with which the frequency can be controlled and measured. For ordinary work, 1 per cent. was considered sufficient, and with special care, an accuracy of one-fifth per cent. has been obtained.

Arising from the use of the double bridge, the method is particularly adapted to the application of the "guard-wire" principle, e.g., the "wire-to-wire" tests of cables, the elimination of lead corrections, and the measurement of condensers with "guard-plates." This, combined with the ability to deal with small capacities, of the order 0·0001-0·001 mfd., has proved very useful in the testing of dielectric materials in small quantities.

It should be noticed that one pole of the condenser tested is brought to earth potential; this is a disadvantage when taking a "wire-to-wire" test on a cable, as the voltage is limited to the safe "wire-to-earth" pressure; on a three-phase system with earthed neutral, the W/E voltage is about $\frac{1}{\sqrt{3}}$ of the W/W voltage.

In conclusion, the author desires to express his thanks to the management of Messrs. Siemens Bros. & Co., Ltd., Woolwich, for permission to present this Paper; to Mr. B. R. Chaplin, for his help in the construction of apparatus, and in particular, to Mr. E. A. Beavis, A.C.G.I., B.Sc.(Eng.), A.M.I.E.E., for his co-operation both in that part and in the experimental portion of the work.

Appendix I.

Corrections for Imperfections of Bridge Arms.

The possible departures from the assumptions made in obtaining the equations for balance of the bridge are :—

(1) The time constants of the ratio arms may not be the same, in which case $\frac{R_1}{R_2}=m\angle\alpha$, where α is a small angle.

(2) The condenser used as the balancing capacity c may have a small loss angle β.

(3) The series resistance r may have a small phase angle γ.

Then $$Z_4=r\angle\gamma-\frac{j}{\omega c\angle\beta}=r+\frac{\beta}{\omega c}-j\left(\frac{1}{\omega c}-\gamma r\right).$$

Since r is small compared to $\frac{1}{\omega c}$ and γ is small compared to $r, \gamma r$ may be neglected in comparison with $\frac{1}{\omega c}$, i.e., the phase angle of the series resistance does not introduce any appreciable error.

Further, $$Z_3=m\angle\alpha \,.\, Z_4=m(1+j\alpha)\left(r+\frac{\beta}{\omega c}-\frac{j}{\omega c}\right)=m\left(r+\frac{\alpha+\beta}{\omega c}\right)-j\,.\,\frac{m}{\omega c},$$

neglecting αr and $\frac{\alpha\beta}{\omega c}$ in comparison with $\frac{1}{\omega c}$.

Thus the corrections amount to adding a small quantity $\frac{\alpha+\beta}{\omega c}$ to the value of the series resistance r. In the expression for the capacity C, r itself enters only as a small correcting factor, so that the effect of the angles α and β can be here safely ignored. For the loss angle, we have

$$\tan\theta=\omega c\left(r+\frac{\alpha+\beta}{\omega c}\right)=\omega cr+\alpha+\beta.$$

The correction consists simply in adding the constant $(\alpha+\beta)$ to the loss angle as determined in the usual way; $\alpha+\beta$ may be regarded as the loss angle of the measuring condenser in terms of the particular ratio arms used.

Appendix II.

Theory of Double Bridge.

The general network of admittances resulting from the double bridge is shown in heavy lines in Fig. 9(a). To obtain the effect of the parallel system BOD with

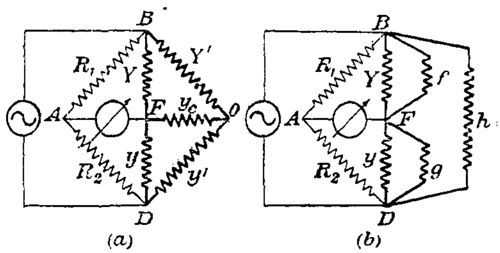

Fig. 9.—Transformation of Double to Single Bridge.

the "link" admittance OF when balancing BFD, we can transform the "star" OB, OF, OD, into its equivalent "mesh" as shown in Fig. 9(b).* The arrangement reduces to a simple single bridge; the admittance f is in parallel with the unknown arm Y, g is in parallel with the known arm y, while h is a shunt to the source and does not affect the balance.

$$f=\frac{y_e Y'}{Y'+y'+y_e}=pY' \text{ where } p=\frac{y_e}{Y'+y'+y_e}$$

$$g=\frac{y_e y'}{Y'+y'+y_e}=py'.$$

* S. Butterworth, Proc. Phys. Soc., Vol. 33, p. 315 (1921).

If the bridge ratio is m, the net effect is to add $g-mf$ to the known arm, i.e., $p(y'-mY')$. The conditions for no effect are (1) $y_e=0$, in which case the auxiliary circuit is not necessary, (2) $y'=mY'$, which is obtained when the bridge is finally balanced.

If all the admittances consist of capacities of fairly small phase angle, p becomes approximately $\frac{c_e}{C'+c'+c_e}$, where c_e is the "link" capacity; (when the bridge is balanced, $c'=mC'$ and $p=\frac{c_e}{(m+1)C'+c_e}\Big)$.

Now $y+py'=m(Y+pY')$.

As an approximation p is assumed unaffected by small changes in y'.

$$\therefore \qquad y+py'=\text{constant},$$

i.e., $$\omega^2c^2r+j\omega c+p(\omega^2c'^2r'+j\omega c')=\text{constant},$$

whence $$c+pc'=\text{constant},$$

$$r+p\left(\frac{c'}{c}\right)^2 r'=\text{constant}.$$

Thus small changes $\delta c'$, $\delta r'$ in c', r' respectively, produce the same effect on the balance of the main bridge as variations $p\delta c'$, $p\left(\frac{c'}{c}\right)^2\delta r'$ in c, r respectively. Conversely, when balancing the auxiliary bridge, to obtain the effect of small changes in c and r, multiply by p', $p'\left(\frac{c}{c'}\right)^2$ respectively, where $p'=\frac{c_e}{C+c+c_e}$.

A given "out-of-balance" in the main bridge capacity will give rise to $\frac{1}{p}$ of the error when adjusting the auxiliary arm, and this again, when we return to the main bridge, will cause a new "out-of-balance" $\frac{1}{pp'}$ of the original value. Thus, in one cycle, we have reduced the error by the over-all factor pp'; in the same way, the over-all reduction factor per cycle for the resistance is also pp'. This shows that, to facilitate working, the "link" capacity c_e should be kept as small as possible, and also the advantage of using a large value for the ratio m.

In a particular 3-core cable, the "wire-to-wire" capacity was 0·007 mfd., and the "wire-to-earth" capacity 0·0275 mfd. per core. When taking a "wire-to-wire" test, the calculated figures allowing for the capacity of the leads, were as follows :—

$C=0{\cdot}00700$ mfd., $c=0{\cdot}700$ mfd., $C'=0{\cdot}0368$ mfd., $c'=3{\cdot}68$ mfd., $c_e=0{\cdot}0463$ mfd.
Main bridge balance,

capacity factor, $$p=\frac{0{\cdot}0463}{0{\cdot}037+3{\cdot}68+0{\cdot}046}=0{\cdot}0123(0{\cdot}016)$$

resistance factor, $$p\left(\frac{c'}{c}\right)^2=0{\cdot}0123\left(\frac{3{\cdot}68}{0{\cdot}70}\right)^2=0{\cdot}34\ (0{\cdot}33).$$

Auxiliary bridge balance,

capacity factor, $$p'=\frac{0{\cdot}0463}{0{\cdot}037+3{\cdot}68+0{\cdot}046}=0{\cdot}062(0{\cdot}05)$$

resistance factor, $$p'\left(\frac{c}{c'}\right)^2=0{\cdot}062\left(\frac{0{\cdot}70}{3{\cdot}68}\right)^2=0{\cdot}0022(0{\cdot}0033).$$

Over-all reduction factor, $pp'=0{\cdot}00076(0{\cdot}00080)$.

The figures in brackets are the measured results, and these agree closely enough with the calculated figures to prove the approximate theory given above. In this case, the capacity was read to the nearest 0·0001 mfd., and the resistance to the nearest 0·1 ohm; hence the permissible error in balancing the auxiliary circuit was $\frac{0{\cdot}0001}{0{\cdot}016}=0{\cdot}006$ mfd., and $\frac{0{\cdot}1}{0{\cdot}33}=0{\cdot}3$ ohm. The penultimate adjustments on the main bridge could be to within $\frac{0{\cdot}0001}{0{\cdot}0008}=0{\cdot}125$ mfd. and $\frac{0{\cdot}1}{0{\cdot}0008}=125$ ohms without causing error, i.e., 0·062 mfd. and 62 ohms from the final values. Actually, the maximum variation with the usual voltage steps never approached these figures, so that once the first balance was obtained, only one cycle (i.e., one adjustment of the auxiliary and then one on the main bridge) was needed for each step in voltage.

DISCUSSION.

Dr. E. H. RAYNER: This description of some of the methods available for what is an increasingly important branch of electrical measurements is very useful to have on record. I should like to point out one or two practical details more as a warning than a criticism. The author suggests a series of iron-cored inductances for a potential divider. It is to be remembered that the important region of phase angle in this kind of work is from about 2 deg. to zero, and any want of accuracy or similarity in subsidiary apparatus may become of first importance. I should not trust to any iron-cored inductances with joints in the iron. Our experience with nominally exactly similar potential transformers, which might make excellent potential dividers, is to the effect that their phase-angles may differ in the ratio of 3 to 2, and I am informed that the value can be made to vary by a hammer blow which may open or close the iron joints.

Another point to beware of is distributed capacity in the 500,500 ohm potential divider, either from the sections to earth or between sections. The same applies to the series resistance of the Duddell wattmeter. At 50 ~ it may be negligible; but it is one of the most difficult technical points to deal with, and the use of condensers, which are much more " pure " than resistances or inductances, is always a great point in their favour when phase angles of minutes or seconds are of importance.

Mr. L. HARTSHORN: Having made many measurements of dielectric losses with the Wien Bridge and the Schering Bridge, I was surprised to find that Mr. Rosen had used the Wien Bridge for high voltage tests, instead of the Schering one. Grebe and Zickner have shown that the Schering Bridge is capable of giving great precision under suitable conditions, and the saving of time in doing away with the necessity of an auxiliary balance is very considerable. It seems unlikely that it is more difficult to make a condenser to stand the high voltages than a resistance. Could not the condenser constructed out of sheet iron by the author be used for this purpose? As the resistance arms can be kept very small and yet the sensitivity remains quite adequate at such high voltages, the effect of the earth impedances on the low-tension side should not prove to be serious, provided it is made as small as possible by a suitable arrangement of the apparatus. An important advantage of the Schering Bridge is that it requires very little power. The resistance arms carry but small currents, so that they can be small and shielded, whereas the large resistances used by the author will, as Dr. Rayner pointed out, have considerable distributed capacity to earth, and as the Wagner earthing device only compensates for capacities to earth concentrated at the corners of the bridge, this bridge is not likely to be entirely free from error due to earth impedances. If comparative tests could be made, workers on dielectric losses would be glad to know how the two bridges compare as regards accuracy. In the matter of making the adjustments the Schering Bridge is unquestionably simpler.

AUTHOR'S reply (communicated): In reply in Dr. Rayner, I appreciate that it would be difficult to obtain two separate iron-cored inductances with the same phase-angle. However, what is suggested in the Paper is that the two ratio coils should have a common iron circuit; the effect of joints in the iron should then be the same for both. Further, the phase-angle of the ratio coils is easily measured and allowed for. This applies also to the resistances used as ratio arms. It is realised that the Wagner method of eliminating earth capacity is not perfect, but the resistances are connected in exactly the same way when their phase-angle ratio is measured, so that the correction takes account of any residual earth capacity.

In reply to Mr. Hartshorn, the objection to the Schering bridge is that the impedance between the L.T. pole of the condenser tested and earth must be of a high order. This is not always practicable in cable testing, whereas successful tests have been obtained with the arrangement described in which the resistance between the lead sheath and earth was only a few hundred ohms.