

On the Construction of a Colour Map

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1890 Proc. Phys. Soc. London 11 323

(<http://iopscience.iop.org/1478-7814/11/1/336>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 131.91.169.193

This content was downloaded on 09/09/2015 at 02:37

Please note that [terms and conditions apply](#).

current coil which would replace all the eddy-current circuits is a coil of 2 turns whose resistance is about 1.9 ohms, short-circuited on itself.

$$e = 0.38, \text{ if } k = 600.$$

It is obvious that e is proportional to k and to the square of the radius of the iron wire.

Assuming constant permeability and no eddy currents,

$$C = .074 \sin (kt - 90^\circ).$$

With some saturation but no hysteresis,

$$C = .079 \sin (kt - 69^\circ.2) - .0148 \cos 3kt - .0037 \cos 5kt,$$

if $b = 0.2$, $m = .05$.

These values of b and m are usually employed by me for such magnetizations as are common in transformers. When I assume the existence of hysteresis, I take f about 20 degrees.

XXXV. *On the Construction of a Colour Map.*

By WALTER BAILY, M.A.*

[Plate XI.]

By the term Colour Map I mean a diagram each point of which defines by its position some particular colour. Such a colour map was designed by Clerk Maxwell in the form of a triangle, the angles of which were occupied by certain colours, and all other colours were treated as mixtures of these three primary colours, the composition of the mixture for the colour which occupied any particular point in the triangle being indicated by the length of the perpendiculars from that point on the sides of the triangle.

Now trilinear coordinates, although they afford very elegant methods for the solution of certain problems, are by no means so generally useful or so intelligible as the ordinary rectangular coordinates; and the fact that every colour can be defined by means of a spectrum colour and white light suggested to me the construction of a colour map with rectangular coordinates, in which measurement in one direction should indicate the wave-length of the spectrum colour employed,

* Read April 8, 1892.

and measurement at right angles to it should indicate the quantity of white light employed in defining the colour.

Let us take a vertical line to represent the spectrum, the lower end giving the red of the spectrum and the upper the violet. The spectrum is supposed to be formed so that equal differences of length measured along the spectrum represent equal differences in the wave-length; and when the quantity of colour at any point of the spectrum is mentioned, it is intended that a definite small part of the spectrum about that part is to be taken. Now all colours, except the purples, can be formed by adding white light to a spectrum colour. Let the amount of white light required be indicated by a line measured horizontally to *the right* from the proper point in the spectrum. Then the given colour is indicated by the point at the extremity of that line. Again, every colour except the greens has the following property: viz. that if it is added in the proper quantity to some spectrum colour, white is produced. Let the quantity of white produced be indicated by a line drawn from the proper point horizontally to the left. The point at the extremity of this line indicates the given colour. In this way a map is obtained in which every colour has its appropriate position. The greens occur only on the right hand, and the purples only on the left hand, but all other colours, as they can be indicated in both ways, occur on both sides of the spectrum line.

In using the term quantity of white light, I mean that a beam of white light is to be obtained in some definite manner from a definite source of light which forms the spectrum, and that the map is to show how much of this beam is used. Captain Abney finds that the positive pole of the electric arc is a source of light of constant quality, and uses it in his measurements; and he indicates the quantity of white light used by the ratio between its luminosity and that of the spectrum colour. It is a more complicated matter to express such a ratio than to express the amount of white light only, and I failed to work into a map Captain Abney's method of defining the quantity of white light.

The principle on which this map is founded will come out more clearly by the consideration of fig. 1, which may be considered as a sort of colour staff, to borrow a term from music.

The three horizontal lines represent the three colour sensations—Red, Green, and Violet, with such luminosity that the mixture represented by equal lengths of the three lines represents white light. Thus the vertical lines A, A', wherever they may be placed, will include between them white light, which will be the more intense the farther they are apart. Any colour whatever may be represented by taking the line A as a base and measuring off the quantities of the sensations to points, R, G, and V. The distances included between A'

Fig. 1.

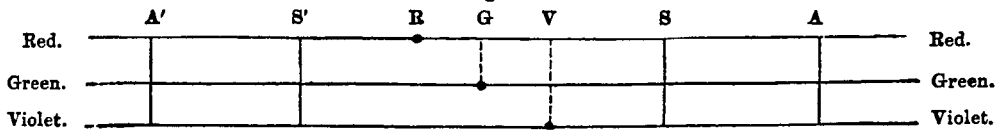


Fig. 2.

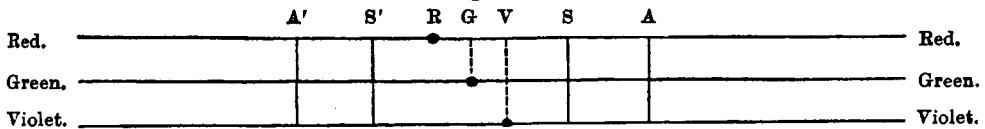
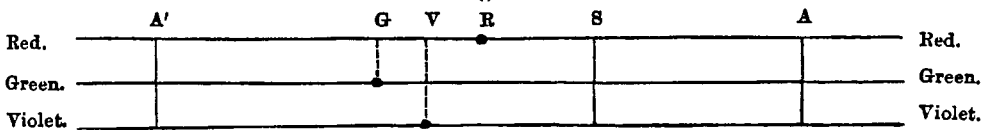


Fig. 3.



and R, G, V give every complementary colour to that represented by R, G, V, and A. The whole of the colours of this system are related together only by the position with respect to one another of R, G, V—that is, only by the differences R G and G V. But if we express the same colours at (say) half the luminosity, we must reduce all these distances to one half, as in fig. 2, and so with any other proportion. It is then not the differences R G and G V, but the ratio of these differences which is constant for all this system of colours. Hence, to determine to what system a colour belongs of which we know r , g , v , the quantities of red, green, and violet sensations respectively, we have only to obtain $\frac{r-g}{g-v}$.

In fig. 4, Pl. XI., the vertical line called “spectrum colours” is that along which the spectrum is thrown; and the lines

called "line of no Red," "line of no Green," and "line of no Violet" are lines to which distances are to be measured horizontally from any point to show the quantity of red, green, and violet sensations in the colour represented at that point. When these distances are measured from points on the spectrum line, they give the amount of such sensation for the corresponding spectrum colour. The curves which I have used are not intended to represent the true form of such curves, as it is sufficient for explaining the principles of the map that they should be curves having a maximum and shading off on each side. The numbers marked along the "spectrum line" give the value of the fraction $(r-g)/(g-v)$ at each point; and it will be seen that the value is large at the red end of the spectrum, probably beginning with infinity, and diminishes to zero, where the red and green are equal. It then changes sign and remains negative until g and v become equal, when the fraction becomes infinite and again changes sign. For the remainder of the spectrum the fraction continues positive and passes from infinity to zero. The fraction $(r-g)/(g-v)$, which may be called the "Colour Index," has therefore in the spectrum every value from plus to minus infinity, and has all the positive values twice over. Every positive colour index has two spectrum colours:—one in which the order of magnitude of the sensations is Red, Green, Violet, and the other in which the order is Violet, Green, Red. In fig. 1, where the order is that required, let the lines S, S' give the spectrum colours. Then it is clear that these two spectrum colours are complementary to one another. Also that the colour represented by A is equal to the spectrum colour S plus the colour included between S and A , which is white; and also that the colour at A plus the spectrum colour at S' form the white between A and S' .

Now suppose the colour index negative, then R, G, V must be arranged in the order $R V G$ or $V R G$ (see fig. 3). We have $A, A',$ and S , as before; but S' , the second spectrum colour, does not occur, inasmuch as there is no spectrum colour in which green is less than both the red and the violet. Hence the green, which is represented by A , can be defined only by the addition of white to a spectrum colour; and the purple, which is represented by A' , can be defined only by

the fact that when added to a spectrum colour they can form white.

To see how what precedes is represented in the Colour Map (Plate XI. fig. 4), take any line perpendicular to the spectrum-line, say the line in the orange for which the Colour Index is 1.0, and compare this line with fig. 1. S is the point on the spectrum-line, V is the point at which the "line of no violet" is crossed, and G and R the points in which the lines of no green and no red respectively are crossed, and S' represents the complementary spectrum colour, which is represented on the thick line at the point marked 1.0. This thick line, along which the figures are marked, represents the spectrum which is complementary to that from Red to Yellow, and itself extends from Violet, of which the colour index is zero, to Blue, of which the colour index is infinite. A similar line gives the complementary spectrum of the part from Blue to Violet, and itself extends from Red when the colour index is infinite to Yellow when it is zero. The region on the right outside *all* the lines gives all the colours to be obtained by adding white to a spectrum colour; and to ascertain the amount of each sensation, we have only to measure horizontally to the line giving the zero of that sensation. The region on the left outside *all* the lines gives all the colours capable of making white with spectrum colours; and here, again, to ascertain the amount of each sensation we have only to measure horizontally to the line giving the zero of that sensation. It will thus be seen that the whole map is really constructed on one single principle. It is obvious that if a series of colours are obtained by some definite law, their positions on the map will lie on some line straight or curved.

It remains to consider the spaces enclosed within the lines. On the right between the spectrum-line and the nearest sensation zero-lines lies a space which has a real meaning, as the points in it represent colours in which the sensations have certain positive ratios to one another; but these ratios give a more intense colouring than the spectrum colours themselves, and therefore such points cannot represent any colours which can be seen by a normal eye, because, as was known to Newton, every mixture of colours is more diluted than the spectrum colour which it most nearly resembles. This region may be

called an abnormal region. The colours it represents would be visible to eyes more or less colour-blind. There are two abnormal regions on the left of the figure between the complementary spectrum-lines and the red and violet zero-lines respectively.

The remaining portion of the map, viz. that lying *between* zero sensation lines, is of a different nature. At any point in this region the distances measured to the zero lines are not all in the same direction; so that one or two out of the three sensations must be considered to be negative. As no one possesses a negative colour sensation, the colours represented in this region are imaginary. This may be called the imaginary region. Though it has no physical meaning it will be found to have its value in connexion with the geometrical structure of the map. As an example of this, consider the complementary spectrum-lines. They end abruptly, leaving a gap opposite the green; but they may be continued across the gap in such a way as their general form seems to point, and this has been done in fig. 4, by continuing the complementary spectrum-lines until they meet in a cusp at the point on the right marked -1.0 . This extension lies wholly in the imaginary and abnormal region, and may represent the missing complementary spectrum of green.

The map affords convenient methods for calculating the effect of mixing colours. Let a colour which has the sensation red, green, and violet in the proportion r_1, g_1, v_1 be represented by $r_1 | g_1 | v_1$. Then, if we take two colours $r_1 | g_1 | v_1$ and $r_2 | g_2 | v_2$, the mixture of these colours in the proportions l_1 and l_2 will give the result $l_1 r_1 + l_2 r_2 | l_1 g_1 + l_2 g_2 | l_1 v_1 + l_2 v_2$. The index of this colour is

$$\frac{l_1(r_1 - g_1) + l_2(r_2 - g_2)}{l_1(g_1 - v_1) + l_2(g_2 - v_2)}.$$

Let the spectrum colour having the same index be $r | g | v$. In order to find the quantity of white which must be added to this spectrum colour to produce the required colour, it is necessary that the luminosity of the colour should be altered to that luminosity at which the colour is represented in the map. This can be done by multiplying the coefficient of each sensation by the fraction $(r - g) / \{l_1(r_1 - g_1) + l_2(r_2 - g_2)\}$ or one

of the equivalent fractions. The resulting sensation coefficients are

$$\begin{aligned}\text{Red} & . . . (l_1 r_1 + l_2 r_2)(g-v)/\{l_1(g_1-v_1) + l_2(g_2-v_2)\}. \\ \text{Green} & . . (l_1 g_1 + l_2 g_2)(v-r)/\{l_1(v_1-r_1) + l_2(v_2-r_2)\}. \\ \text{Violet} & . . (l_1 v_1 + l_2 v_2)(r-g)/\{l_1(r_1-g_1) + l_2(r_2-g_2)\}.\end{aligned}$$

The coefficient of the white to be added to the spectrum colour is obtained while the colour lies on the right hand by subtracting r from the red coefficient above obtained, or by subtracting g and v from the other coefficients respectively. When the colour lies on the left hand, the white is obtained by adding r to the above red coefficient, or g and v to the green and violet coefficients respectively.

I have applied the formulæ given above to obtain the curves showing the results of adding together in any proportion two spectrum colours so related to each other that if the first is $r \mid g \mid v$ the second is $v \mid g \mid r$. The index of the second colour being $\frac{v-g}{g-r}$ is the reciprocal of the index of the first.

The curves obtained are shown in fig. 5. Consider the curve numbered 2. This is the locus of mixtures of the blue whose index is 2.0, and the yellow whose index is 0.5. The curve passes through these two points of the spectrum, giving the cases in which a zero quantity of one of the colours is taken; and every other mixture is indicated by some point on the curve joining these two points and lying to the right of the spectrum-line. In this figure the dotted horizontal lines occupy the positions where the indices are zero and infinity respectively, so that the portion of any curve which lies outside of them must be repeated again on the left side of the complementary spectrum-line. In curve No. 2 two small parts do lie outside the dotted lines, and, accordingly, these two parts are repeated to the left of the complementary spectrum-line. We have then the curve No. 2 in three separate portions, which it is not possible to connect physically, as the missing part of the curve lies in the imaginary and abnormal regions. But what is not possible for physics is easy for geometry. We cannot subtract one spectrum colour from another, but we can subtract the lines representing the sensations in one

spectrum colour from the lines representing the sensations in the other spectrum colour ; and so by subtracting one spectrum colour from the other in any proportions we can complete the curve No. 2 through the imaginary and abnormal regions and so obtain the complete and continuous curve. Curves Nos. 1 and 0 have no portion on the complementary side, but curves Nos. 3 and 4 have a considerable portion on that side. A new feature is shown when we take the locus numbered 5. This is got by combining the spectrum indigo, having index 1.0, with spectrum orange, having the same index. These are complementary colours. When added together in the proper proportion they produce white, and when added in any other proportion they produce white plus whichever spectrum colour predominates. Hence the locus consists of horizontal straight lines through the two points in the spectrum-line, going off to infinity, where the colour indicated is white, considered as a spectrum colour infinitely diluted with white light. Next consider curve No. 7. The main portion of the curve lies to the left, and starts from points in the complementary spectrum-line which indicate the spectrum colour chosen. The parts of this portion which lie outside the horizontal dotted lines are repeated to the right of the spectrum-line ; the remainder, obtained by subtraction, lies wholly in the imaginary and abnormal regions. All these curves pass through a certain pair of points, as may be easily shown.

The first spectrum colour is . . . $r \mid g \mid v$.

The second is . . . $v \mid g \mid r$.

By subtraction of one from the other, we get a colour

$$r-v \mid \text{zero} \mid v-r.$$

The resulting colour has therefore no green, and has the red and violet equal in amount but opposite in sign. These conditions are satisfied at the two points shown in the figure.

In this figure the lines are drawn under the condition that the index of one spectrum colour is the reciprocal of the index of the other ; but any number of other systems of lines might be drawn showing combinations of two spectrum colours, so that it is evident that every colour can be resolved into two spectrum colours in an infinite number of ways.

There are three regions in fig. 5 which are shaded to show

that none of the curves pass through them. These regions might probably be filled up by curves drawn through points in the imaginary part of the complementary spectrum to which I have already alluded.

Now the complementary spectrum-line and the curves giving mixtures of two spectrum colours have been drawn by strict arithmetical methods from certain curves of hypothetical form which indicate the intensity of the sensations for each point of the spectrum ; but they can also be plotted out by direct experiment.

To plot out the complementary spectrum-line, add to a spectrum colour its complementary until white is produced, measure the quantity of white, and mark off a horizontal line to the *left* from the point in the spectrum of a length proportional to the quantity of white. The end of this line is a point in the complementary spectrum ; other points may be obtained in the same way, and the normal part of the complementary spectrum-line be drawn.

To plot out the curve giving the mixtures of two spectrum colours, take a third spectrum colour and make a colour patch of the first two colours, and another colour patch of the third colour and white. Keep the luminosity of the third colour constant, and vary that of the other colours and the white until both patches are of the same colour. Then measure the quantity of white used and mark off a line from the position of the third spectrum colour to the right proportional to the quantity of white. The end of this line gives a point in the curve. By taking other spectrum colours as the third colour other points may be obtained. If, however, it is found impossible to make the two patches of the same colour, then throw the three spectrum colours together, and keeping the luminosity of the third colour constant vary that of the other two until the three produce white ; measure the quantity of white, and mark off to the left from the position of the third spectrum colour a line proportional to the quantity of white obtained. The end of this line is a point in the curve. If both these methods fail the point on the curve corresponding to the third spectrum colour lies in the abnormal or imaginary regions, and cannot be determined by experiment.

When the derived curves have been plotted out by experi-

ment, it will be possible to modify the hypothetical forms of the curves of intensity of the sensation in the spectrum so as to make the curves derived from them accord more closely with the results of experiment, and so to arrive by gradual approximation to the true form of those curves.

XXXVI. *A Note on the Electromotive Forces of Gold and of Platinum Cells.* By E. F. HERROUN, *Professor of Natural Philosophy in Queen's College, London* *.

IN nearly all modern text-books of Physics the metal platinum is placed after gold in Volta's Electropositive Series. This no doubt is partly owing to the well-known fact that gold is attacked by chlorine or nitrohydrochloric acid more readily than platinum, and it might therefore be reasonably supposed that gold evolves more heat in the formation of its chloride than does platinum. On referring to the values for the heats of formation of the chlorides of these two metals, as given by Julius Thomsen †, one finds, however, that the heat attending the formation of auric chloride is, per equivalent, only about half as great as that in the case of platinic chloride.

Assuming that the voltaic constants of metals are deducible from the thermochemical values of their compounds, the above facts would compel us to regard gold as more negative than platinum, at least when immersed in chloride solutions. (The same observations would also apply if oxygen were the attacking medium, as Thomsen gives the heat of formation of platinic hydrate as a considerable positive number, while that of auric hydrate is a large *negative* quantity.)

It was, therefore, an interesting point to determine how far the actual electromotive forces obtained with gold and with platinum agreed with these conclusions, and I endeavoured to find records of the electromotive forces of cells in which these metals are immersed in solutions of their chlorides opposed

* Read March 25, 1892.

† *Thermochemische Untersuchungen*, iii. pp. 412 & 430.