

MEASURING INSTRUMENTS OF LONG AGO.—II.*

THE MECHANICAL APPLICATION OF GEOMETRICAL PRINCIPLES

BY WILLIAM E. STARK.

Continued from Supplement No. 1795, Page 346.

THE GEOMETRIC SQUARE.

THE instrument most commonly referred to in the sixteenth century authors and which appears to have been in use for at least five hundred years is the geometric square (*quadratum geometricum*). In its simplest form, it consisted of a square frame, two sides of which were marked with scales of equal parts, the

a quadrant (Fig. XXII). Instead of an alidade, the quadrant had sights attached to one of the radial sides and the reading was taken under a plumb line which hung from the center.

An interesting modification of the quadrant form of the geometric square (Fig. XXIII) was devised for the purpose of reducing a distance measured on a

ured. The result was then calculated by the following rule: If both readings fall on the *umbra recta*, multiply the distance between the two stations by twelve and divide the product by the difference between the two readings. If both readings fall on the *umbra versa*, divide twelve by each reading in turn

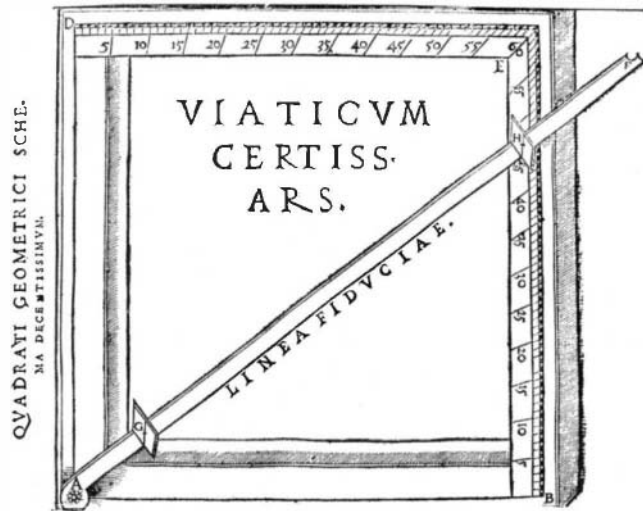


FIG. XVIII.—GEOMETRIC SQUARE.

From Finaeus. See note under Fig. XI.

number of divisions being usually twelve or some multiple of twelve. These scales, referring apparently to the measurement of altitudes by shadows, were called "*umbra recta*" and "*umbra versa*." At the corner of the square opposite the intersection of the scales was pivoted a sighting arm or alidade (Fig. XVIII). In the simplest problem, that of measuring the height of a tower, one edge of the square was held vertically and the alidade was aimed at the top of the tower. The reading where the edge of the alidade cut across the scale was then taken. If the reading was on the "*umbra recta*," the height of the tower was less than its distance from the observer; if on the "*umbra versa*," it was greater.

In the first case, assuming the reading to be five and the number of divisions on the scale twelve, the height of the tower was five-twelfths of its distance; in the other, with the same reading, it was twelve-fifths of the distance.

A quadrant was sometimes placed within the square for measuring angular altitudes, and a plumb line was often attached to one edge. Some forms had supporting staffs to which they could be fastened in either a horizontal or vertical position. The early instruments were made of wood and were three or four feet square, but the later ones were of metal. The one belonging to Prof. Smith (Fig. XIX), dating from about 1600, is made of brass and is about twelve inches square. "*Umbra versa*" and "*umbra recta*" scales were often engraved on astrolabes. Chaucer, in his treatise on the Astrolabe, 1390, describes this feature, and explains how to solve several problems by means of it.

In another common form of the instrument, for use in measuring altitudes the square was inscribed in

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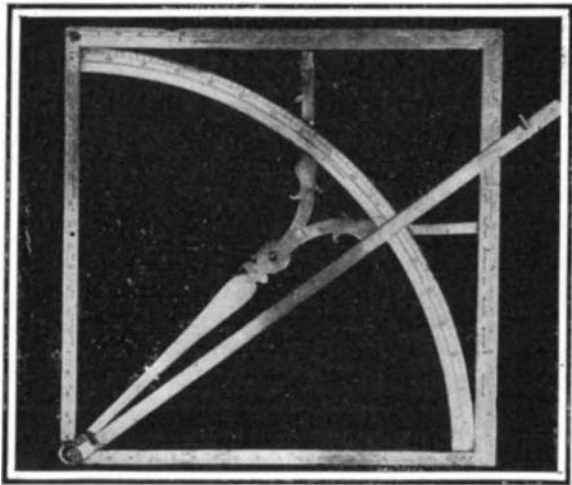


FIG. XIX.—GEOMETRIC SQUARE OF THE SEVENTEENTH CENTURY.

Belonging to Prof. David Eugene Smith of Teachers' College, New York.

slope to a horizontal measurement. Near the center of the quadrant, *BAC*, which is made of wood or metal a graduated straight edge is pivoted. The line *EM*, which represents the position of the straight edge when *AB* is vertical, is graduated in the same way as the straight edge, and a series of semicircles is drawn through the points of division as shown in the



FIG. XX.—USE OF GEOMETRIC SQUARE.

From Finaeus.

diagram. In use (Fig. XXIV) the side *AC* is held parallel to the slope and the straight edge is allowed to take its natural position. Suppose the distance on the slope to be 70 feet. Then the reading of the straight edge where it cuts the semicircle which passes through 70 on scale *EM* gives the horizontal distance. If the slope distance is too long to be applied directly to the scale *EM*, it is divided by two or three for example, and the reading is multiplied by the same number.

A general lack of skill in computation is implied in the proud claims of many authors that with their instruments all calculation is avoided. Belli's square (Fig. XXV) was solid and ruled like a checker board. Holes were bored at the graduation marks along two sides of the square, into which the pivot of the alidade could be thrust. With this device it was possible to make one side of the triangle formed on the instrument correspond in length to the base line, and so to read the required distance directly.

The most interesting problem solved by means of the square was that of finding the length of a line neither end of which is accessible. (Fig. XXVI.) Supposing the required distance to be the height of a tower, two observations were taken from points on the same level and in line with the tower, and the distance between the points of observation meas-

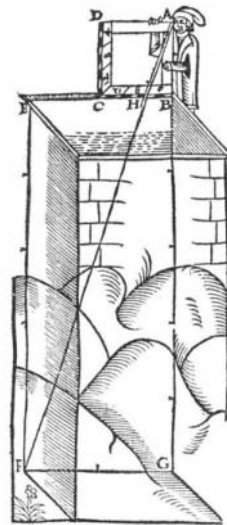


FIG. XXI.—USE OF GEOMETRIC SQUARE IN MEASURING DEPTH OF WELL.

From Finaeus.

and take the difference of the quotients. Divide the distance between stations by this difference. If one reading fell on the *umbra recta* and the other on the *umbra versa*, it was necessary to reduce one reading to its equivalent on the other scale before making the calculation according to the rules. Here is an opportunity for a good algebraic problem. Pupils may verify the correctness of the foregoing rules by stating a pair of simultaneous equations and getting an expression for the height of the tower in terms of the three measurements.

A few of the additional problems mentioned by Finaeus in connection with the geometric square are: (1) To measure a vertical distance from a higher elevation; (2) to measure a distance on a slope; (3) to measure the depth of a well.

THE CROSS-STAFF.

The cross-staff *baculus*, called also the Jacob staff, in its simplest form was a rod of rectangular cross section about four feet long with a short cross-piece having a square hole in the middle. This hole fitted the rod snugly enough to keep the crosspiece at right angles to the rod, but not too tightly to prevent its position being changed.

The staff was marked off into sections equal in length to the crosspiece.

The instrument was used for measuring dimensions of inaccessible objects such as a fort (Fig. XXVII) The crosspiece was first placed at one of the division marks of the staff. The observer then took a position opposite the middle of the line, which he wished to measure, and moved forward or backward until when one end of the staff was placed at the eye, the crosspiece just covered the object. The crosspiece was then moved to the next division of the staff, and the observer took such a position that the object was again exactly covered. The required distance was

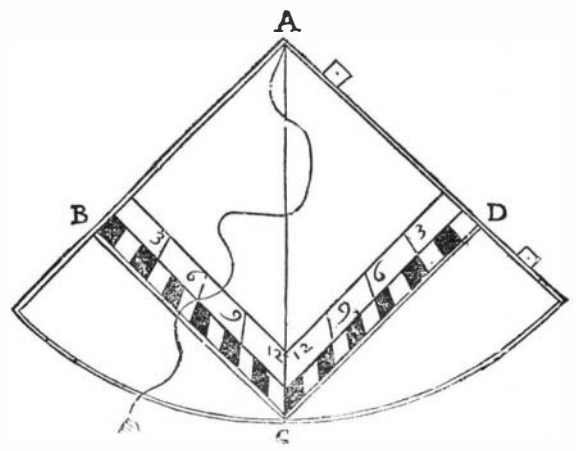


FIG. XXII.—QUADRANT FORM OF GEOMETRIC SQUARE

From *Del Modo di Misurare* by Cosmo Bartoli, Venice, 1689.

obtained by measuring from one point of observation to the other.

Fiammelli* says that Finaeus and Bartoli, who speak highly of the *baculus*, were mathematical theorists and not practical workmen and that their claims

about six inches long, joined by a hinge like the common two-foot rule. (Fig. XXVIII.) From the center of the pivot six pairs of lines radiated, three on each face of the instrument. The lines were graduated, the two members of each pair, one on each rule, being in

a straight line of the same length as the circumference of a given circle. To do this take the given diameter in a pair of dividers and set the compasses so that the dividers span from 50 to 50. The distance from 157 to 157 is then the length of the required line.

The line of squares is simply a scale of square roots, that is, the distance from the pivot to any point on the scale is proportional to the square root of the number standing opposite the point. To find the side of a polygon, whose area is three-fifths of that of a

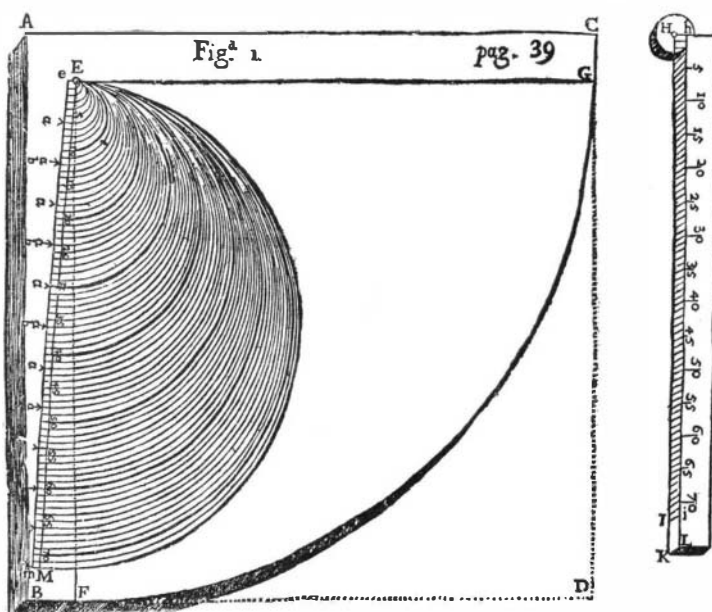


FIG. XXIII.—INSTRUMENT FOR REDUCING MEASUREMENTS ON A SLOPE TO HORIZONTAL DISTANCES.

From *Trattato della Pratica di Geometria* by Ludovico Perini. Third Edition, Verona, 1751.

are not justified. He says that it is often impossible to get observations from points as far apart as the length of the required line and gives a general rule for calculating the desired dimension from two observations and the distance between them.

His rule is to multiply the distance between stations by the fractional part of a cross-bar length between the two settings of the cross-bar. To obtain the latter

symmetrical positions and marked with identical divisions (Fig. XXIX.)

On one face of the sector of the eighteenth century were the lines of equal parts, the lines of squares, and the lines of polygons. On the other, the lines of chords, the lines of cubes, and the lines of metals. On the first face also there was usually a scale giving the caliber of cannon corresponding to the weight of the ball, and in a corresponding position on the other

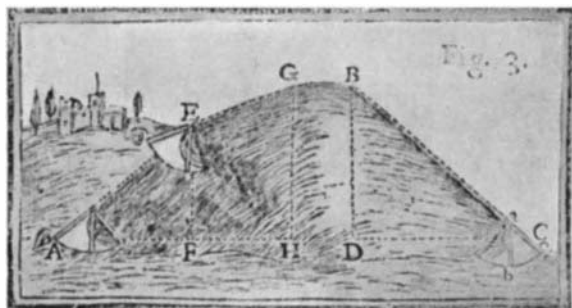


FIG. XXIV.

From Perini. See Fig. XXIII.

factor with accuracy, he proposes that the principal spaces on the staff be each divided into one hundred parts.

The verification of Fiammelli's rule and of the special relation referred to by Finaeus are valuable student's exercises.

SECTOR COMPASSES.

The sector compasses or sector (*compas de proportion*) combined in a single small instrument the means of solving a great variety of problems in arithmetical and geometrical proportion. It was used by architects, surveyors, military engineers, and practical mathematicians generally. It was made in various sizes and some examples were of beautiful workmanship. The earliest description of the sector which I have found is in Bettinus' *Apiaria*,† 1645, although it is said to have been invented nearly a century earlier.‡

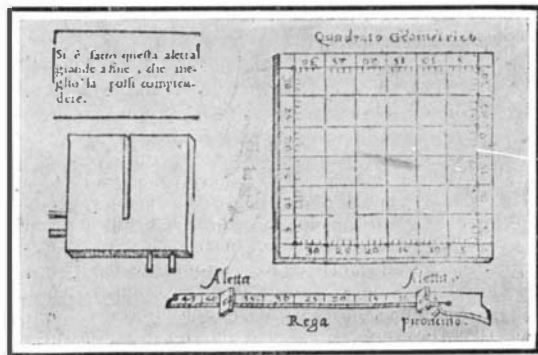


FIG. XXV.—GEOMETRIC SQUARE DESIGNED TO AVOID CALCULATION.

From Belli. See note under Fig. XII.

Though still listed in the catalogues of dealers, this device, which maintained its utility for two hundred years, appears to have been almost completely forgotten.

The instrument consisted of two metal rules, usually

* *La Riga Matematica di Gio. Francesco Fiammelli*, Fiorentino Matematico, Rome.

† *Apiaria Universae Philosophiae Mathematicae*, Marius Bettinus of Bologna and the Society of Jesus. Bologna, 1645.

‡ *International Encyclopedia*. Article on Sector.

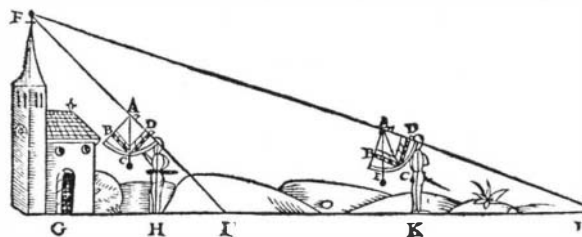


FIG. XXVI.—MEASURING THE HEIGHT OF A TOWER WHOSE BASE IS INACCESSIBLE.

From Finaeus.

face a scale of diameters of bullets of given weight. Lines of sines, of tangents, and of secants were added at a later period.

In the instrument as described by Bion each line of equal parts was divided into two hundred segments and numbered beginning at the hinge. To divide a line into any number of equal parts, for example seven, take the length of the line on a pair of dividers and, placing one point of the dividers on the point marked 70 or 140 on one line of equal parts, open the compass until the other point of the dividers touches the same number on the other line of equal parts. Then the distance from one 10 (that is from the point marked 10 on one of the lines of equal parts) to the

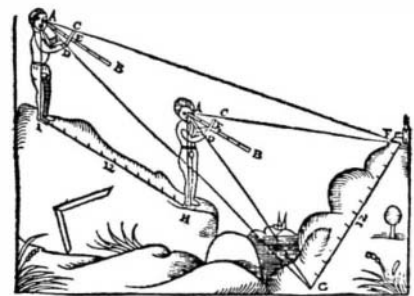


FIG. XXVII.—THE CROSS-STAFF IN USE.
From Finaeus.

given similar polygon, set the compasses so that the distance from 50 to 50 on the line of squares is equal to the side of the given polygon. Then the distance from 30 to 30 is the required line. Other problems to be solved by the line of squares are (1) to determine the ratio of areas of two given similar polygons, and (2) to construct a polygon similar to two given polygons and equal to their sum or their difference.

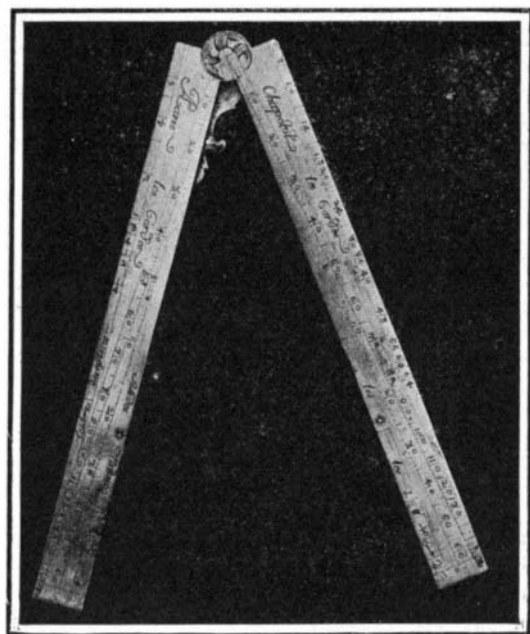


FIG. XXVIII.—SECTOR COMPASSES, 18TH CENTURY.

Belonging to Prof. David Eugene Smith of Teachers' College, New York.

The method of laying out the line of squares is a good application of the Pythagorean proposition. (Fig. XXIX.) A line *KL* is drawn equal in length to the proposed line of squares. This is divided into eight equal parts and the points of division numbered 1, 4, 9, 16, 25, 36, 49. The end of the line is numbered 64. At *K*, the zero end of the line, a perpendicular *KM*

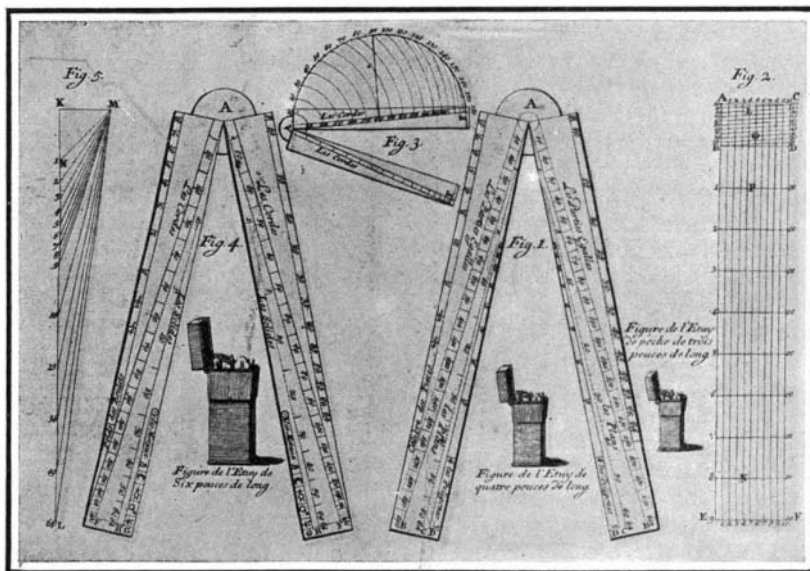


FIG. XXIX.—SECTOR COMPASSES.

From Bion.

other is one-seventh of the line, from 20 to 20 two-sevenths, and so on.

The foregoing problem will suggest at once the utility of the instrument in drawing to a scale or in measuring distances taken from a map, whose scale is given. The only other problem for the line of equal parts which I shall mention is that of finding

is erected equal to one of the eight parts just referred to. Then the distance *M1* is taken in the dividers and laid off on the line from *K*. The point found is numbered 2. The distance *B2* is then laid off from *K* giving a point which is numbered 3. In general the distance from *K* to any point is equal to the distance from *M* to the preceding point.

The line of polygons gives the relative lengths of the sides of regular polygons inscribed in the same circle. As an example of the use of this line, let it be required to inscribe a regular polygon of seven sides in a given circle. Open the sector so that the distance from the points marked 6 on the lines of polygons is equal to the radius of the given circle. Then the distance from 7 to 7 is the length of the required side and may be stepped off on the circumference with the dividers. Other problems for the line of polygons are (1) to describe a given regular polygon on a given side, and (2) to divide a given line in extreme and mean ratio.

On the line of chords (Fig. XXIX, 3) the distance from the pivot to any point is the chord subtending a central angle measured by the number of degrees marked opposite the point, the whole length of the

line being the diameter or the chord of 180 deg. To open the compass so that the lines of chords make any desired angle, for example 57 deg., take in the dividers the distance from the pivot to point 57 on the line of chords, and open the compasses until this distance spans from 60 to 60. Other problems are (1) to lay out any desired angle; (2) to measure a given angle; and (3) to take on the circumference of a given circle an arc of any required number of degrees.

The line of cubes is a scale of cube roots corresponding to the scale of square roots on the other face of the compasses. To find the ratio of volume of two similar solids, set the compasses so that the distance from 10 to 10 on the lines of cubes is equal to some dimension of one of the solids. Then take the corresponding dimension of the other solid in the dividers and find the reading on the lines of cubes indicated by

this distance, spanned from one line to the other. The ratio of the reading to 10, of course, gives the desired ratio. The line of solids was used to make a scale showing the relation of the weight of a bullet to the caliber of the gun which would fire it; also to divide a gage for measuring the capacity of casks.

The distances from the pivot to the graduation marks on the line of metals are proportional to the dimensions of similar solids of equal weight made of gold, lead, silver, brass, iron and tin. Among the problems solved by use of the line of metals are: (1) to find the diameter of a ball of the same weight as a given ball, but of different metal; (2) to find the relative weight of the various metals; (3) to find the weight of a body of the same size and shape as a given body of another metal.

(To be concluded.)

REMARKABLE COMETS.—II.*

COMETS OTHER THAN HALLEY'S.

Concluded from Supplement No. 1795, page 342.

FEW comets excited greater sensation by their sudden appearance above the horizon than the great comet of 1861 (No. ii. of that year). It was discovered by J. Tebbutt, an amateur astronomer, at Windsor, N.S.W., on May 13th, previous to its perihelion passage, which took place on June 11th. Passing from the Southern Hemisphere into the Northern, it became visible in this country on June 29th, though it was not generally seen until the following evening. It is so rare for the inhabitants of the British Islands to have a big comet all their own, as it were, that in this case the multitude of observers and observations was so great that selection is difficult.

A good all-round description was that given by Sir John Herschel, who observed the comet at his house, "Collingwood," Hawkhurst, Kent. He says:

"The comet, which was first noticed here on Saturday night, June 29th, by a resident in the village of Hawkhurst (who informs me that his attention was drawn to it by its being taken by some of his family for the moon rising), became conspicuously visible on the 30th, when I first observed it. It then far exceeded in brightness any comet I have before observed, those of 1811 and the recent splendid one of 1858 not excepted. Its total light certainly far surpassed that of any fixed star or planet, except perhaps Venus at its maximum. The tail extended from its then position, about 8 deg. or 10 deg. above the horizon, to within 10 deg. or 12 deg. of the Pole star, and was therefore about 30 deg. in length. Its greatest breadth, which diminished rapidly in receding from the head, might be about 5 deg. Viewed through a good achromatic, by Peter Dollond, of 2¾ inches aperture and 4-foot focal length, it exhibited a very condensed central light, which might fairly be called a nucleus; but, in its low situation at that time, no other physical peculiarities could be observed. On the 1st instant it was seen early in the evening, but before I could bring a telescope to bear on it clouds intervened, and continued till morning twilight. On the 2nd (Tuesday), being now much better situated for observation, and the night being clear, its appearance at midnight was truly magnificent. The tail, considerably diminished in breadth, had shot out to an extravagant length, extending from the place of the head above α of the Great Bear at least to π and p Herculis; that is to say, about 72 deg., and perhaps somewhat further. It exhibited no bifurcation or lateral offsets, and no curvature like that of the comet of 1858, but appeared rather as a narrow prolongation of the northern side of the broader portion near the comet than as a thinning off of the latter along a central axis, thus imparting an unsymmetrical aspect to the whole phenomenon.

"Viewed through a 7-foot Newtonian reflector of 6 inches aperture the nucleus was uncommonly vivid, and was concentrated in a dense pellet of not more than 4 sec. or 5 sec. in diameter (about 315 miles). It was round, and so very little woolly that it might almost have been taken for a small planet seen through a dense fog; still so far from sharp definition as to preclude any idea of its being a solid body. No sparkling or star-light point could, however, be discerned in its center with the power used (96), nor any separation by a darker interval between the nucleus and the cometic envelope. The gradation of light, though rapid, was continuous. Neither on this occasion was there any unequivocal appearance of that sort of fan or sector of light which has been noticed on so many former ones.

"The appearance of the 3rd was nearly similar, but

on the 4th the fan, though feebly, was yet certainly perceived; and on the 5th was very distinctly visible. It consisted, however, not in any vividly radiating jet of light from the nucleus of any well-defined form, but in a crescent-shaped cap formed by a very delicately graduated condensation of the light on the side toward the sun, connected with the nucleus, and what may be termed the coma (or spherical haze immediately surrounding it), by an equally delicate gradation of light, very evidently superior in intensity to that on the opposite side. Having no micrometer attached, I could only estimate the distance of the brightest portion of this crescent from the nucleus at about 7 min. or 8 min., corresponding at the distance of the comet, at that time, to about 35,000 miles. On the 4th (Thursday) the tail (preserving all the characters already described on the 2nd) passed through α Draconis and r Herculis, nearly over π and ϵ Herculis, and was traceable, though with difficulty, almost up to α Ophiuchi, giving a total length of 80 deg. The northern edge of the tail from α Draconis onward, was perfectly straight—not in the least curved—which, of course, must be understood with reference to a great circle of the heavens.

"Viewed, on the 5th, through a doubly refracting prism well achromatized, no certain indication of polarization in the light of the nucleus and head of the comet could be perceived. The two images were distinctly separated, and revolved round each other with the rotation of the prism without at least any marked alternating difference of brightness. Calculating on Mr. Hind's data, the angle between the sun and the earth and the comet must then have been 104 deg., giving an angle of incidence equal to 52 deg., and obliquity 38 deg., for a ray supposed to reach the eye after a single reflection from the cometic matter. This is not an angle unfavorable to polarization, but the reverse. At 66 deg. of elongation from the sun (which was that of the comet on the occasion in question), the blue light of the sky is very considerably polarized. The constitution of the comet, therefore, is analogous to that of a cloud; the light reflected from which, as is well known, at that (or any other) angle of elongation from the sun, exhibits no signs of polarity."

A very interesting point was raised by Hind, and developed, so to speak, by E. J. Lowe, the well-known meteorologist. Hind stated that he thought it not only possible, but even probable, that in the course of Sunday, June 30th, the earth passed through the tail of the comet at a distance of perhaps two-thirds of its length from the nucleus. The head of the comet was in the Ecliptic at 6 P. M. on June 28th, at a distance from the earth's orbit of about 13,000,000 miles on the inside, its heliocentric longitude (its longitude seen from the center of the sun) being 279 deg. The earth at that moment was rather more than 2 deg. behind that point, but would arrive there soon after 10 P. M. on June 30th. The tail of a comet is seldom an exact prolongation of the radius vector, or imaginary line joining the nucleus with the sun; toward its extremity a tail is almost invariably curved; or, in other words, the matter composing it lags behind what would be its position if it traveled with the same speed as the nucleus. Now judging from the amount of curvature on June 30th, and the direction of the comet's motion, Hind thought that the earth very probably encountered the tail in the early part of that day; or, at any rate, that it was certainly in a region which had been swept over by cometary matter a short time previously. He added that on the evening of June 30th there was a peculiar phosphorescence or illumination of the sky which he attributed at the time to an

auroral glare. It was remarked by other persons as something unusual; and it seems scarcely open to doubt that the earth's proximity to the comet had something to do with it. Lowe confirmed Hind's statement of the sky having a peculiar appearance on the evening of June 30th. He says that the sky had a yellow, auroral, glare-like look, and that the sun, though shining, gave but a feeble light. The comet was plainly visible during sunshine at 7:45 P. M. In confirmation of the statement that there was something unusual and indescribable happening, Lowe adds that in his parish church the vicar had the pulpit candles lighted at 7 o'clock, which proves that some sensation of darkness was felt even while the sun was shining. Though unaware at the time that the comet's tail was enveloping the earth, he was so struck by the singularity of what he saw that he made the following entry in his day-book: "A singular yellow phosphorescent glare, very like diffused Aurora Borealis, yet, being daylight, such Aurora could scarcely be noticeable." The comet itself, he states, had a much more hazy appearance than on any subsequent evening.

De La Rue attempted to photograph the comet, but it left no impressions on two collodion plates, although neighboring stars did impress themselves on the plates.

No fewer than eleven envelopes were seen to spring from the head of this comet between July 2nd, when portions of three were in sight, and July 19th; a new one rising at regular intervals every second day. And their evolution and dispersion took place with much greater rapidity than was the case with Donati's comet in 1858; each envelope taking but two or three days to go through its various changes instead of two or three weeks.

On the question of the polarization of the light of the comet, Secchi said:

"The most interesting fact that I observed was this: the polarization of the light of the comet's tail and of the rays near the nucleus was very strong, and one could even distinguish it with the band polariscope; but the nucleus presented no trace of polarization, not even Arago's polariscope with double colored image. On the contrary, on the evenings of July 3rd, and following days, the nucleus presented decided indications, in spite of its extreme smallness, which, on the evening of July 7th, was found to be hardly 1 second.

"I think this a fact of great importance, for it seems that the nucleus on the former days shone by its own light, perhaps by reason of the incandescence to which it had been brought by its close proximity to the sun.

"During the following days the tail has been constantly diminishing, but it is remarkable that it has always passed near to α Herculis, and that it reached to the Milky Way up to July 6th. It would seem that the two tails were nearly independent, and that on July 5th the length and straightness had gone off from the large one, and that this bent itself to the southern side. Last night (July 7th) the long train was hardly perceptible. The light was polarized in the plane of the tail."

Observations on the polarization of the light of the comet were also made by Poey at Passy, near Paris. He found that "the plane of polarization seemed to pass sensibly perpendicular to the axis of the tail." Poey had in 1858 observed Donati's comet for polarization, and found that its light was polarized in a plane passing through the sun, the comet and the observer.

The comet of 1862 (iii.), though not one of first-class magnitude or brilliancy, was nevertheless a very interesting object on account of the fact that a jet of light, frequently altering in form, was observed

* "The Story of the Comets Simply Told for General Readers." By George F. Chambers, F.R.A.S. Oxford: at the Clarendon Press, 1909.