

## COEFFICIENTS OF LINEAR EXPANSION AT LOW TEMPERATURES.

BY H. D. AYRES.

IN passing from ordinary temperatures upward it is a well-known fact that there is a slight increase in the coefficient of expansion of many substances. It has been thought probable that the curve thus obtained will hold for lower temperatures. But little is known as to the behavior of bodies at extremely low temperatures. Almost the only work that has been done at temperatures below  $-50^{\circ}$  C. is that of Dewar,<sup>1</sup> and of a Russian physicist, Zakrzewski.<sup>2</sup> In the work of the former the specific-gravity method was used in studying the behavior of bodies such as ice, some hydrated salts, solid carbonic acid, etc. Metal balls were used in the determination of the specific gravity, and the work was based on the assumption that these metals contract according to the Fizeau formula for coefficient of expansion, it being assumed that the parabolic formula might be legitimately extended to low temperatures. Zakrzewski examined the behavior of glass, iron and copper to  $-100^{\circ}$  C., finding the coefficients to decrease somewhat rapidly.

The ordinary methods for the determination of the expansion of bodies become difficult of application at low temperatures, for the reason that mirrors, lenses, etc., either attached directly to, or anywhere near a body having a temperature nearly two hundred degrees below zero centigrade, very quickly become frosted. Another fact to consider is that the body must be quite small if it is to be immersed in liquid air and kept at nearly a constant temperature for any length of time: that is to say, the liquid air must be kept in a receptacle which will prevent its rapid evaporation. Another necessary condition is that the body itself be well insulated, else the heat from the outside is conducted to it so rapidly

<sup>1</sup> "Expansion of Solids at Low Temperatures," Royal Soc. Proc., 70, July 8, 1902, pp. 237-246; *Nature*, Vol. 66, May 22, 1902, p. 88.

<sup>2</sup> "Über die Ausdehnung einiger fester Körper bei niederen Temperaturen," *Beibl.*, 1890, p. 491.

that it does not readily come to a temperature of equilibrium, and the parts expand and contract unequally. Until the behavior of at least one substance is known it is certain that no method of compensation can be used.

The principle used in the work now to be described is that first used by Fizeau<sup>1</sup> in the determination of the coefficient of dilatation of crystalline substances. The method devised by him depends essentially upon the determination of the difference of expansion between the screws of a small metallic tripod and the object under investigation, which is supported by it. The method has been employed with some modifications by Pulfrich,<sup>2</sup> and with still other modifications, including a method of compensation, by Tutton.<sup>3</sup>

The apparatus used in the present work, however, is on an entirely different plan, and is much simpler. The dilatometer as described by Pulfrich and Tutton, has been replaced by a single piece of metal<sup>4</sup>—the metal under consideration—used between two glass plates, thus very much simplifying the work in some ways, though it must be acknowledged, complicating it in others. It is believed, however, that the sources of errors are at least considerably decreased, since the expansion of the tripod screws and the need of compensation are eliminated. The metal piece is cut into the form of a cylindrical shell with its faces nearly parallel, so that the interference phenomena take place between the light reflected from the upper surface of the lower glass plate and the lower surface of the cover plate. The lower plate has one plane surface and is of black astronomical glass so that all light is absorbed except that reflected from the plane surface. The cover plate has both surfaces plane and inclined at a small angle, preferably about 30', though one or two were used in the course of this work with an angle of three or four degrees. By having the faces inclined the reflection from the upper surface is thrown out of the field of vision and does not illuminate the dark spaces between the bands.

As a mark by which to count the bands as they pass, a small

<sup>1</sup> *Compt. Rendus*, Vol. 58, p. 923, and Vol. 62, p. 1133. *Ann. Chim. Phys.* (4), Vol. 2, p. 143; (4), Vol. 8, p. 333.

<sup>2</sup> *Zeitschrift für Instrumentenkunde*, 1893, p. 365.

<sup>3</sup> *Phil. Trans.*, Vol. 191, A, 1899, p. 313.

<sup>4</sup> *Annalen der Physik.* (9), Vol. 314, 1902, p. 857.

silver disc about a millimeter in diameter on the lower surface of the cover glass is used. This may be easily obtained by silvering the whole surface and then fastening the plate to a wooden block and revolving it in a small lathe, cleaning the surface, except this spot, by means of a small piece of soft wood. This is wet so as to avoid scratching the glass surface. In the case of one of the plates used a small hole was drilled through its center. The plates are circular in shape, 30 mm. in diameter and about 2 mm. in thickness. They are held firmly in contact with the metal by means of a small cylindrical lead weight having an opening through

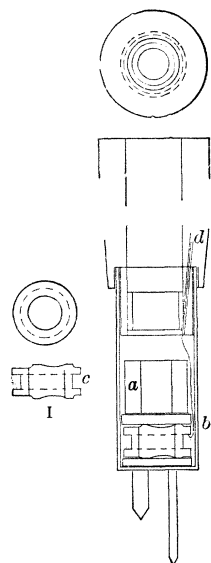


Fig. 1.

its center and resting on the upper cover plate, (Fig. 1, *a*). In order to decrease the likelihood of disturbances due to particles of dust or dirt between the faces of the metal and the glass surfaces, the faces of the metal are cut so as to leave three equidistant projections upon which it may rest (Fig. 1, *I*). The length of the metal cylinder used depends altogether upon the light available. If sodium light is used, the thickness must be very small, something like one or two millimeters, and the piece so ground that this thickness gives a maximum visibility for the interference bands. If the green mercury line is employed an air space of over a centimeter may be used.

The metal piece, plates, and lead weight are placed in position in a small cylindrical brass cup, something over 3 cm. in diameter and about 8 cm. in length, Fig. 1, *b*. This is easily accomplished by adjusting the pieces properly in an inverted position upon a support which can enter then brass cup, then placing the cup in position and inverting the whole. The brass cup fits closely over the end of a wooden tube, and is held by friction. The wooden tube is so turned that it slips into the brass cup for some distance and has in addition a flange, which extends for a short distance over the outside of the cup. This joint is liquid-tight and its object is two-fold: First to facilitate the adjustment of the different parts and of

the lead-wires to the coil used for temperature measurement, and second to act as a non-conductor of heat when the whole end is immersed in liquid air. The metal cup has soldered to its bottom and projecting downwards two metal pieces, one about 2 cm. in length and quite thick, the other one about twice as long but smaller in diameter. Both of these have their ends pointed, and their purpose is to extract the heat from the interference apparatus and brass cup slowly, which could not be done if the liquid air came in contact with the bottom of the cup before it was well cooled. The wooden tube is about 30 cm. long and its upper end fits into a wooden support which clamps it firmly, thus leaving the lower end free, so that it can extend down into a vessel containing water or a flask of liquid air, etc. To alter the temperature the receptacle containing the liquid is raised, the liquid rising around the brass cup and wooden tube. The movement must be very slow and under control so that the bands may be made to shift at such a rate that they may be counted. A piece of apparatus was made for this purpose consisting of a large, heavy tripod stand and to its vertical rod another smaller rod was attached at a distance of about 15 cm. Then a stage was made to move up or down these rods, being supported from above by a leather strap which passed to a wheel and axle. The axle worked with a friction grip, thus permitting the stage to move as slowly or rapidly as desired. For extremely slow movement a wooden arm can be attached to the wheel.

#### ILLUMINATING AND OBSERVING APPARATUS.

The illumination is that from a Cooper-Hewitt mercury arc lamp, the green line being used. This is obtained by separating the colors by a prism, *P*, Fig. 2, the light first being passed through the lens *A* to render it parallel. Beyond the prism is another lens, *B*, whose principal focus is at *p*. A small total reflecting prism is situated at *p*, which turns the light at right angles in a horizontal plane. It is again turned at right angles, downward, by the mirror *M*, which is silvered on its reflecting face. The mirror was found to serve much better in this position than a total reflecting prism, being much more easily adjusted. At *D* is placed an achromatic lens of 25 cm. focal length, and so adjusted with respect to *p* as

to act as a collimator, thus rendering the rays passing to the interference apparatus parallel. From the interference apparatus the light is reflected back along the same path and brought to a focus at  $p$ , forming a real image of the rectangular aperture of the total reflecting prism. Adjustment is so made that this is formed just to the right and touching the edge of the prism. The interference pattern would be observed at the conjugate focus, from the interference apparatus, of the lens  $D$ . In order to be nearer the apparatus for adjustment, etc., and also to gain in intensity of light,

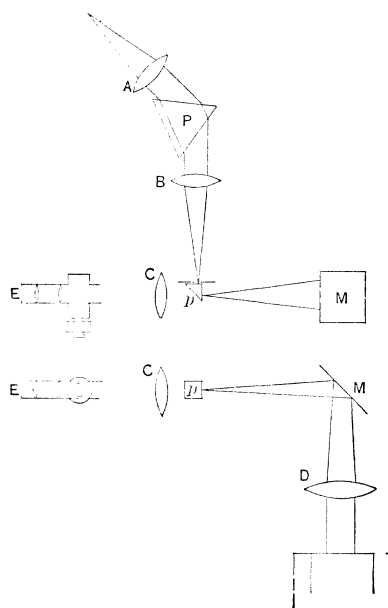


Fig 2.

another lens,  $C$ , of about 20 cm. focal length, is placed just back of  $p$  so that the rays are focused about six or eight centimeters to the left of  $p$ . For the same reason the mirror  $M$  is brought between  $D$  and  $p$  instead of the light passing through the lens  $D$  before reflection. To observe the interference pattern a Ramsden double eyepiece,  $E$ , is employed, which focuses the double cross hairs as in Tutton's apparatus, the lines being moved, and not the eyepiece itself, with respect to the interference bands and the silver disc. The eyepiece also contains fixed cross hairs. It is mounted upon

one of the posts of an optical bench, and thus has a rack and pinion vertical adjustment and a micrometer screw horizontal adjustment. The lens *C* is also mounted so as to be adjustable in any direction, as are also the mirror *M* and the lens *D*. The prism *p* is mounted on another post of the bench, and is fitted in a brass case, having on the side towards *B* an adjustable rectangular opening which can be varied in size from the size of the face of the prism to as small an opening as is desired. A large opening is necessary in making adjustments, but to obtain clear bands with a path difference of several millimeters the aperture must be cut down very much; one or two square millimeters give the best results in most cases.

#### TEMPERATURE MEASUREMENT.

The measurement of temperature is made by means of the change in resistance of a coil of very fine copper wire wound closely round the metal cylinder (see Fig. 1, *c*.) The ends of this fine wire are connected to larger wires which serve as lead-wires, and these latter pass out through small holes bored in the wooden tube, *d*. A calibration curve was obtained by finding the resistance of the coil at four points, namely, about  $100^{\circ}$ ,  $0^{\circ}$ ,  $-80^{\circ}$ , the latter by immersing the coil in solid carbon dioxide mixed with ether, and at about  $-190^{\circ}$  by immersing in liquid air. The temperature of the air was obtained from Baly's<sup>1</sup> curve. The air given off by the liquid while the coil was immersed is analyzed and the percentage of oxygen found.

A Wheatstone's bridge was used in determining the resistances of the coil at the different temperatures. The deflections of the galvanometer were observed by means of a spot of light reflected on a ground-glass scale, which was set directly in front of the expansion apparatus and directly in line with *EM*, Fig. 2, so that the attention could be readily and quickly turned from the spot of light to the eye-piece, or *vice versa*.

#### ADJUSTMENT AND USE OF THE APPARATUS.

In adjusting the apparatus the eye-piece *E*, Fig. 2, and the lenses *C* and *D* are removed and the tube is adjusted vertically so that the light passes through it parallel to its length. This is most easily

<sup>1</sup> Phil. Mag. (5), 49, 1900, p. 517.

accomplished by using daylight and removing the metal cup from the end of the tube. The lens  $D$  is placed in position and the distance  $CMD$  made about 25 cm. The metal cup, with a single reflecting surface in it, is then attached. This reflecting surface should be at about the same position as the lower surface of the cover plate when it is finally placed. The arc is now started and the tube so adjusted, by means of the set-screws against its support, that the light is reflected back to  $p$ . An ordinary eye-piece is adjusted so that it focuses a point just to the right of the edge of the prism. The lens  $D$  is then moved until an image of the rectangular opening at  $p$ , which must be quite small, is clearly in focus. Then replacing the eye-piece by the lens  $C$  and the Ramsden eye-piece  $E$ , and placing in position the two plates and metal instead of the single reflecting surface, the interference bands should appear in the field. The width of the bands is now measured by the movable double lines, and the initial position of the band nearest the reference mark found. This distance, divided by the width of the bands, gives a fraction to be added to the whole number of bands counted, if the bands move so that the band nearest the reference mark moves towards the mark, or one minus the fraction added if the bands move in the opposite direction.

Owing to the condensation and freezing of moisture on the plates some drying reagent must be introduced in the wooden tube after all adjustments are made, and allowed to remain until the air is completely dry. Upon removing this the tube is closed by means of a plane cover glass, the end of the tube being cut so that the cover glass is inclined at a small angle to a line at right angles to the length of the tube. It is then made air-tight by sealing with shellac. Through the wall of the wooden tube near its upper end is a small metal tube, by means of which connection is made to a drying tube, and as the air inclosed in the brass cup and wooden tube cools and decreases in volume, dry air enters.

Before beginning a set of observations the resistances of the coil for the desired temperatures were obtained from a calibration curve, and the box resistance adjusted for the first temperature. The liquid air was now raised slowly and the bands began to move. A key in the galvanometer circuit was closed frequently to note

the deflection. A D'Arsonval galvanometer was used and two pins stuck in the woodwork back of the air vane, one at each end, prevented more than a very small deflection of the needle. When the deflections had become less violent the circuit was left closed, and soon the spot of light began to move towards the zero position. The cooling at this point must proceed very slowly, so that the metal and coil may come to the same temperature, and it is best that the cooling cease entirely just about the time the spot of light reaches its zero position. By proceeding slowly enough this can be obtained almost exactly. In any case, however, the spot of light can be shifted to the one side or the other of the zero mark by slightly raising or lowering the liquid, the bands meanwhile moving a small fraction of the distance between two bands, one way or the other, thus proving very conclusively that coil and metal are at the same temperature. The cross hairs may easily be set just as the spot of light is on the zero position. The fraction of the band is thus obtained, and together with the fraction found in starting, added to the whole number of bands counted. This gives the expansion of the body in terms of the wave-length of light used. This number of bands is different, however, from what it would have been if the refractive index of the air inclosed in the metal cylinder had not changed, due to change in temperature. The formula for making this correction has been worked out and explained by Pulfrich in the article referred to above. He gives in addition to this correction one for change in barometric pressure, but this is always small, and since in this work a temperature interval is covered in about one hour the small change in pressure likely to occur may be entirely ignored. We shall consider therefore only that part due to change of temperature. Since the change in length of the metal spool due to change in temperature is slight, we can assume the thickness of the air space as constant in the correction. Call this  $d$ . Then

$$d = \beta_1 \frac{\lambda_1}{2}$$

where  $\lambda_1$  is the wave-length of light and  $\beta_1$  the number of wave-lengths in the path difference of the light reflected from the two surfaces. Now suppose the above temperature change to have



taken place without affecting the length of the metal spool. A certain number of bands would have shifted, caused by the change in wave-length due to change in refractive index. Denoting this new wave-length by  $\lambda_2$  and the corresponding number of wave-lengths by  $\beta_2$ , again we have

$$d = \beta_2 \frac{\lambda_2}{2}.$$

Since the correction,  $k$ , is equal to  $\beta_1 - \beta_2$  (supposing the temperature to increase) we obtain from the above and the relation

$$\lambda_2 = \frac{\lambda_1 \mu_1}{\mu_2},$$

the expression,

$$k = \frac{2d}{\lambda_1} \cdot \frac{\mu_1 - \mu_2}{\mu_1}.$$

As  $\lambda_1$  differs a very little from  $\lambda$ , and  $\mu_1$  is very nearly equal to *one*, the expression may be written,

$$k = \frac{2d}{\lambda} (\mu_1 - \mu_2). \quad (1)$$

By Gladstone and Dale's law,

$$\frac{\mu - 1}{\rho} = \text{constant}$$

where  $\mu$  is the refractive index at any temperature and  $\rho$  the density. Then,

$$\mu_s - 1 = K \rho_s, \quad (2)$$

the subscript  $s$  denoting standard conditions. From this we have,

$$\mu_{t_1 b} - 1 = K \rho_s \frac{b}{760} \cdot \frac{1}{1 + at_1} \quad (3)$$

in which the temperature is  $t_1$  and the barometric pressure  $b$ , and  $a$  the coefficient of expansion of air. Replacing  $K \rho_s$  by its value from (2),

$$\mu_{t_1 b} - 1 = (\mu_s - 1) \frac{b}{760} \cdot \frac{1}{1 + at_1}. \quad (4)$$

For a temperature  $t_2$  this becomes,

$$\mu_{t_2 b} - 1 = (\mu_s - 1) \frac{b}{760} \cdot \frac{1}{1 + at_2}. \quad (5)$$

Subtracting (5) from (4), and substituting in (1) where the subscript 1 corresponds to  $t_1b$ , and 2 to  $t_2b$ , we obtain,

$$k = \frac{2d}{\lambda} (\mu - 1) \frac{b}{760} \cdot \frac{a(t_2 - t_1)}{(1 + at_1)(1 + at_2)},$$

the subscript  $s$  being dropped. Then if  $f$  denote the number of bands counted,  $f'$  the corrected number,  $f' = f + k$  or

$$f' = f + d(t_2 - t_1) \cdot \frac{b}{760} \cdot \frac{1}{1 + at_1} \cdot \frac{1}{1 + at_2} \cdot \frac{2(\mu - 1)\alpha}{\lambda}.$$

The logarithm of  $[2(\mu - 1)\alpha]/\lambda$ , as given by Pulfrich for the green mercury line,  $\lambda = 0.0000546$  cm. is  $7.59901 - 10$ .

The coefficient of expansion  $e$  is given by the expression,

$$e = f' \frac{\lambda(t_2 - t_1)}{2d}. \quad (6)$$

Following are some results for aluminium :

SERIES I. *Aluminium.*

$d = \text{length of metal spool} = 0.603$  cm.     $b = \text{barometric pressure} = 74.0$  cm.  
 $\lambda = 0.0000546$  cm.,  $\lambda/2 = 0.0000273$  cm.

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to - 40	17.7	1.1	18.8	213
- 40 " - 80	15.8	1.5	17.3	196
- 80 " -120	13.9	2.3	16.3	184
-120 " -160	9.5	4.0	13.5	152
-160 " -185	3.6	4.0	7.7	139
0 " 40	18.0	0.8	18.8	213
40 " 78	19.2	0.6	19.8	236

SERIES II. *Aluminium.*

$d = 0.603$  cm.     $b = 74.3$  cm.

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to - 40	17.7			
- 40 " - 80	16.0			
- 80 " -120	13.9			
-120 " -160	9.9			
-160 " -175	2.0			
-175 " -160	2.0	2.4	4.4	133
-160 " -120	10.0	4.1	14.1	159
-120 " - 80	14.0	2.3	16.3	185
- 80 " - 40	16.1	1.5	17.6	199
- 40 " 0	17.8	1.1	18.8	213
0 " 40	18.4	0.8	20.2	229
40 " 80	19.6	0.7	20.3	230

SERIES III. *Aluminium.* $d = 0.603$  cm.  $b = 74.3$  cm.

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to - 40	17.3			
- 40 " - 80	16.2			
- 80 " -120	12.6			
-120 " -160	10.3			
-160 " -187	2.7			
-187 " -160	2.5	5.4	8.0	134
-160 " -120	10.4	4.1	14.4	163
-120 " - 80	13.8	2.3	15.7	178
- 80 " - 40	16.0	1.5	17.6	199
- 40 " 0	17.8	1.1	18.7	212
0 " 40	18.4	0.8	19.2	217
40 " 99.3	29.7	0.4	30.6	234

In series II. and III. the total number is the sum of the correction and the mean value of the bands counted for that interval as the temperature was falling as it was rising.

The mean of the results of the three series are plotted in Fig. 4. Taking the value from the plot we find  $e$  for  $40^{\circ}$  C. to be about  $225 \cdot 10^{-7}$ . Fizeau gives  $2313 \cdot 10^{-8}$  for  $40^{\circ}$ ,  $2336 \cdot 10^{-8}$  for  $50^{\circ}$  and  $315 \cdot 10^{-7}$  for  $600^{\circ}$ . Tutton has determined  $e$  for aluminium, finding it to be  $220 \cdot 10^{-7}$ , the next figure varying, and an increment per degree rise in temperature of  $10 \cdot 10^{-10}$ . The values gotten from this and those taken from the curve are quite close for a range of temperature of a few degrees on either side of zero.

Below are given some results for silver.

$$d = 1.318 \text{ cm.}$$

$$b = 74.4 \text{ cm.}$$

SERIES IV. *Silver.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to - 50	43.0			
- 50 " -100	38.0			
-100 " -150	30.0	9.0	39.0	163
-100 " - 50	41.5	5.0	44.8	187
- 50 " 0	43.7	3.1	46.5	193
0 " 50	44.3	2.2	46.5	193
50 " 99.3	43.4	1.5	44.9	189

SERIES V. *Silver.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$\epsilon \times 10^{-7}$
0 to - 50	43.2			
- 50 " -100	38.8			
-100 " -150	31.8			
-150 " -184	12.8			
-184 " -150	11.8	12.2	24.5	151
-150 " -100	31.2	9.0	40.5	169
-100 " - 50	37.4	5.0	43.1	179
- 50 " 0	43.6	3.1	46.5	193
0 " 50	43.3	2.2	45.5	189
50 " 99.3	43.4	1.5	44.9	189

The results in the two sets of observations, IV. and V., check fairly well, and in both cases the value of  $\epsilon$  above zero falls somewhat below that between zero and  $-50^\circ$ . This seems to indicate that the point of inflection of the curve, Fig. 3, is considerably

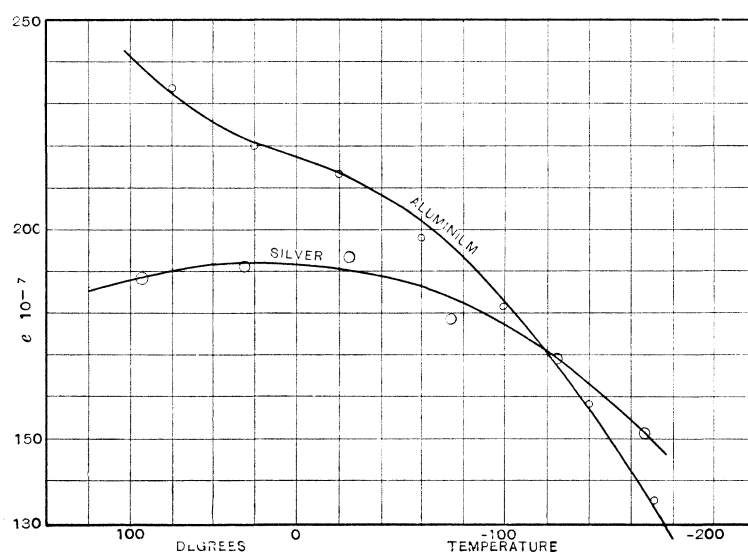


Fig. 3.

above zero. It may be due however to some inaccuracy in the thermo-coil, since a variation of a tenth of an ohm means a shift of almost a band.

The resistance in the case of both coils was not found to be a linear function of the temperature and this makes the determina-

tion of the temperature, especially the lower temperatures, somewhat uncertain.

The lengths of the metal spools have also been measured only roughly, to the third decimal place, by means of a micrometer eyepiece, and the change in length, that is  $d$  in formula (6), has been ignored except in the case of the silver spool at temperatures below  $-100^{\circ}$ .

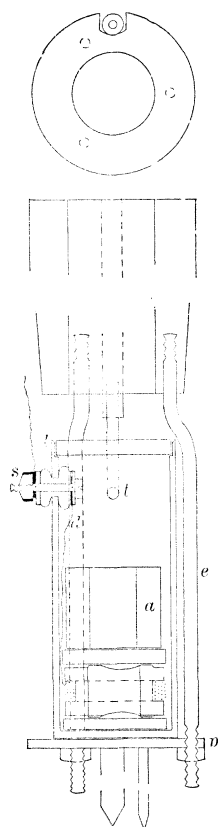


Fig. 4.

It appears from the above data that at low temperatures the refractive index of the air enters as a very important factor, making a correction of more than one hundred per cent. at temperatures somewhat below  $-150^{\circ}$ . It was desired to check the results obtained by eliminating the correction and to this end work was undertaken *in vacuo*. The only modification made was in the apparatus shown in Fig. 1. A brass cup similar to the one described was made and the top closed by means of a piece of plane plate glass. This was ground to fit in the mouth of the cup, the cup being cut out slightly so as to leave a ledge to prevent the glass being pressed in too far (Fig. 4,  $\lambda$ ). Fish glue was applied to the edge of the glass when it is inserted. Near the top of the cup was soldered a small copper tube,  $t$ , by means of which connection was made to the air pump. The lead-wires were very short, extending just above the lead weight. Connection was then made to wires outside by means of small bolts passing through rubber stoppers,  $s$ . The cup was attached to the wooden tube by means of three slender brass bolts, about 15 cm. long.

These screwed into the end of the wooden tube and had nuts on their lower ends.

Pulfrich has shown by means of the expression

$$k = 2 \frac{d}{\lambda_1} \cdot \frac{\mu_2 - \mu_1}{\mu_1},$$

after making two approximations, that the number of transited bands in the passage from the air-filled space to vacuum is equal to the number of millimeters which the air film is thick. This interesting statement was verified with the apparatus.

The work has not been carried far enough as yet to prove conclusively the correctness of the results obtained, using the correction, but the evidence so far seems to indicate that the correction holds for low temperatures.

I wish to extend my sincere thanks to Professor E. L. Nichols for his interest and suggestions, and to Professor J. S. Shearer, at whose suggestion and under whose direction this work was undertaken and pursued.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,  
June, 1904.