

**A HOMEMADE PLANIMETER FOR CLASS ROOM USE.**

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In the application of the integral calculus to actual problems arising in the field of applied science, especially in engineering practice, the evaluation of the definite integral is of fundamental importance. That the formulation of the definite integral, with its limits, from conditions existing where the calculus may be applied is one of the most difficult problems in the application of the calculus is quite generally recognized, especially by those instructors in the calculus who lead their students into the realm of the practical. The integrations involved, as such, do not begin to compare in difficulty with the formulation of the integral and its limits, and these in actual practice will be found to fall under a few heads, readily recognized and as readily executed. The difficulty generally lies in the fact that the student either does not have a clear notion of existing conditions which call for the use of a definite integral, or that he is not exactly certain as to what the calculus will do for him in the solution of his problem.

In order that the student may have the courage of his convictions and really believe that the calculus will help him it is essential that he have clearly in mind the notion that in its fundamental form the value of the definite integral may be represented by an area drawn to scale. This correspondence on the face of it may seem quite apparent from the general proofs given in most texts on the integral calculus—or in spite of these proofs; the fact remains that, in order to have the average student in the calculus appreciate the full significance of this correspondence, it is necessary to do much concrete and definite work in its application.

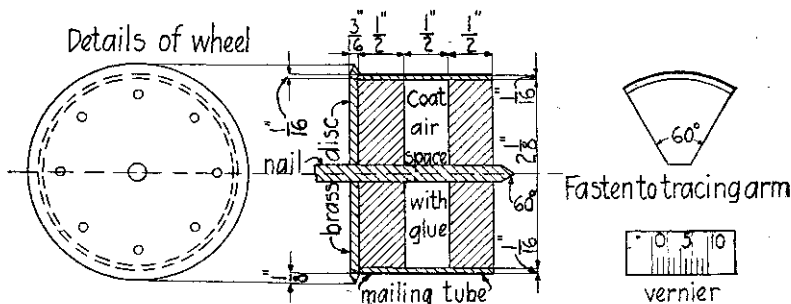
Several methods easily suggest themselves which, undoubtedly, ought to be incorporated early in a course on the integral calculus. Among the first of these methods to suggest itself is the practice quite common among engineers—why not in the class room?—of plotting accurately and to scale  $f(x)$  between  $x = a$  and  $x = b$  when the integral  $\int_a^b f(x)dx$  is desired, and then actually counting the units of area representing approximately the value of the definite integral so formulated. This gives only an approximate result? Suppose it does only that. The engineer using such a scheme generally has a fair estimate of what his result should be, and a definite notion of the limit of allowable error.

And very often he cannot get an algebraic expression for  $f(x)$  for the simple reason that no one can. An approximation is obtained which answers the purpose well, and which is sufficient for the problem in hand. The method is excellent and should be included in the fundamental principles of the teaching of the integral calculus.

A second method easily suggesting itself is found in the use of the formulas for approximate integration generally included in an appendix, if given at all. There is no valid reason why the Trapezoidal, Simpson's, or Durand's Rules for approximate integration should not be used at this stage, partly for their own sake in problems where real approximations only exist, and partly for the sake of checking up on the more accurate value of

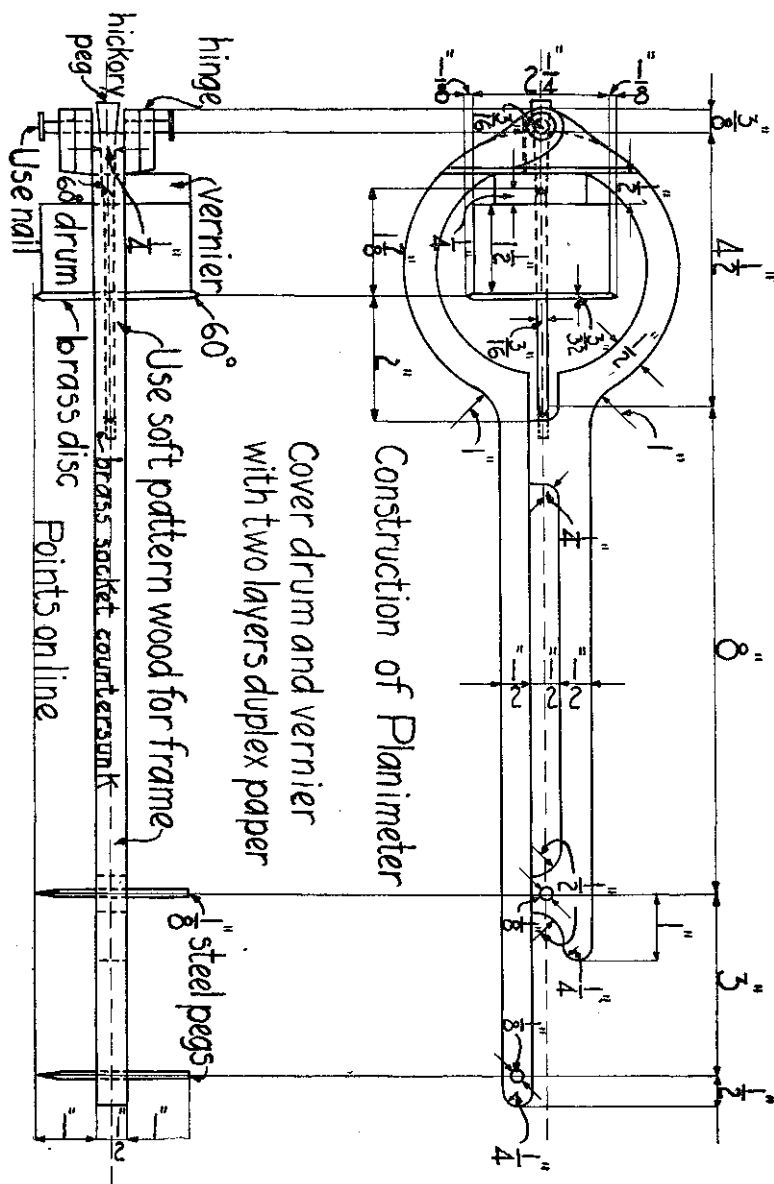
$\int_a^b f(x) dx$  whenever the latter can be evaluated. A judicious use of these formulas for such checking processes, and to emphasize the fundamental notion of the area correspondence is well worth while.

However, of all efficient methods for the emphasis of this notion of the correspondence of definite integral and area, the use of the planimeter in the class room—and by the students themselves—is the most effective. The writer has had in use in his class room for several years a homemade planimeter, which in its construction turned out to be quite accurate; and has insisted on its use by the students, not only for the purpose of checking results but also as an instrument on which they could depend for an approximate first solution. The first demonstration of its efficiency in measuring an area invariably has brought out exclamations of surprise, not at the accuracy of the result but at the idea that such a mechanical measurement is possible. The point is easily made and incidentally there is emphasized the importance of laying off figures accurately and to scale. The problem is concrete and the definite result before their eyes when the vernier



is read. This method of checking results is recommended as thoroughly efficient and should find a place in the teaching of the integral calculus.

The details of the construction of the planimeter used in my class room are given in the accompanying figures. Sufficient ex-



planation is given so that any student having a reasonable proficiency in the use of tools can construct one for himself, an exercise in manual training which might easily be recommended for its own sake. No strenuous endeavors were made to proportion the lengths of the arms; but when calibrated it was found that one revolution of the drum when the pointer on the tracing arm was moved around an area corresponded to about 90 square inches, a convenient size for use with coördinate paper in letter size sheets and on an ordinary drawing board. The drum was graduated by tracing in both directions the peripheries of accurately constructed cardboard rectangles containing 2, 4, 6, . . . 50 square inches respectively. It was found that equal additional parts of a revolution of the drum measured equal additions to the area. These divisions were first marked with pencil on the drum of the rolling wheel and later inked when the calibration was completed.

The construction of the roller wheel is shown in the figure and the brass disc used has given excellent results, though perhaps a piece of soft steel would give better service. This roller part including the drum with disc attached and nail for axis should be turned to shape in a metal lathe, and care should be taken to have the whole roll true.

The vernier, on a 60° sector of wood coated with mailing tube and paper in a manner similar to the drum, should be laid off so that its ten divisions subtend the same arc as nine divisions on the drum.

When graduated the whole instrument should be cleaned carefully and several coats of clear shellac applied, after which a bit of floor wax and plenty of polishing will give the whole a finished appearance. There is no reason why the same instrument could not be used in the teaching of geometry whenever congruent, equivalent, proportional, or irregular areas are under consideration.

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### GRADES OF COAL.

All the coal mined in Georgia is high-grade bituminous and makes a good steam fuel. As bunker coal it has no superior in the South Atlantic states. It also makes excellent coke, and about 30 per cent of the output is made into coke which is sold to the furnaces at Chattanooga and other points in Tennessee and Georgia.