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DIFFRACTION PATTERNS IN THE PRESENCE OF SPHERICAL ABERRATION

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INTRODUCTION

A VERY well known test for spherical aberration in telescope objectives consists in observation of the image of a real or artificial star of which the angular diameter is too small for resolution by the telescope. In an objective free from aberration the focussed image shows clearly the well-known central disc surrounded by bright and dark rings.

More sensitive than the focussed image, however, as a test for spherical aberration is the appearance of the expanded image in the neighbourhood of the focus. Immediately inside and outside the focus, that is towards and away from the objective, are diffraction ring patterns, the number of rings increasing with the distance from the focus. The symmetry of these ring patterns inside and outside the focus is a valuable guide to the quality of the objective.

With an objective free from aberration the inside and outside focus appearances are almost the same. With an over-corrected objective the ring system outside the focus will be clearer and better defined than that inside the focus. With an under-corrected objective the reverse will be the case. The greater the aberration the greater will be the lack of symmetry.

OBJECT OF THE RESEARCH

The object of this research was to photograph and record the appearance of the images and of the expanded images on each side of the focus of a telescope objective, when there was a known difference of optical path between the marginal and paraxial rays.

METHOD AND APPARATUS USED

The lens used in the following experiments was a telescope objective consisting of two components—a plano-concave flint glass component and a biconvex crown glass component, the two components being separated by a narrow air-space.

The aperture of the objective was approximately 7.6 cms., and its focal length about 120 cms. The object used was an artificial star at a distance of 14 metres.

The difference of optical path at the focus between the marginal and the

paraxial rays was calculated by the method of trigonometrical ray tracing through the lens, following the formula and methods of Prof. Conrady. Used in the ordinary way with the crown component towards the star the difference of optical path was found to be $.15\lambda$. This aberration was regarded as negligible and when used in this way the lens is described as having zero aberration. A calculation was then made on the assumption that the lens was turned round so that the light first entered the flint component. The difference of optical path between the marginal and paraxial rays in this case was found to be $+3.43$ wave lengths.

All these calculations were made for the brightest light—the yellow-green. The calculations were made, using five figure logarithms and check calculations were performed in every case to ensure accuracy. The 4th power law was assumed and calculations were made as to the aperture which would give $.5$, 1 , 1.5 , and 2 wave lengths of aberration. Stops were then made to these sizes and the image observed and photographed.

DETAILS OF THE APPARATUS USED

Artificial Star. The artificial star consisted of a diminished image of an electric carbon arc contained in a box. The light from it fell on the eyepiece of a microscope, which gave a diminished image of the crater of the positive carbon. The light then passed through a minus-blue Wratten filter and stops were put at the back lens of the objective and at the point of focus of the image to cut out any stray light.

Telescope Objective. The telescope objective was held in a stand on an optical bench, the adjustments of the stand permitting the lens to be raised or lowered, moved sideways and rotated about a vertical or about an horizontal axis in order to secure correct centering.

Observing Microscope. The image was observed by a microscope having a 1 inch objective and a projection eyepiece. The microscope gave an enlargement of the image on the ground glass screen of a camera box. The magnification of the image on the camera plate was 26.5 .

In photographing the image Ilford special rapid panchromatic plates were used. The time of exposure was 3 minutes and the plates were all developed under the same conditions, and with developer from the same stock, for 4 minutes. Photographs were made for the focussed position and for the 2, 3, and 5 ring positions inside and outside the focus for 0 , $.5$, 1 , 1.5 , and 2 wave lengths difference of path.

The original negatives show more detail, especially in the case of the focussed images, than is exhibited by the prints. The negatives and prints have not been touched up or altered in any way but are the genuine photographic record as taken during the experiments.

Measurements of Diameters of Rings on Photographic Plate. The diameters of the rings on the photographic plates were measured by means of a microscope, with an eyepiece scale when possible. When the contrast of the rings was too poor to permit of the microscope being used a micrometer eyepiece was employed.

Aberrations in wave-lengths

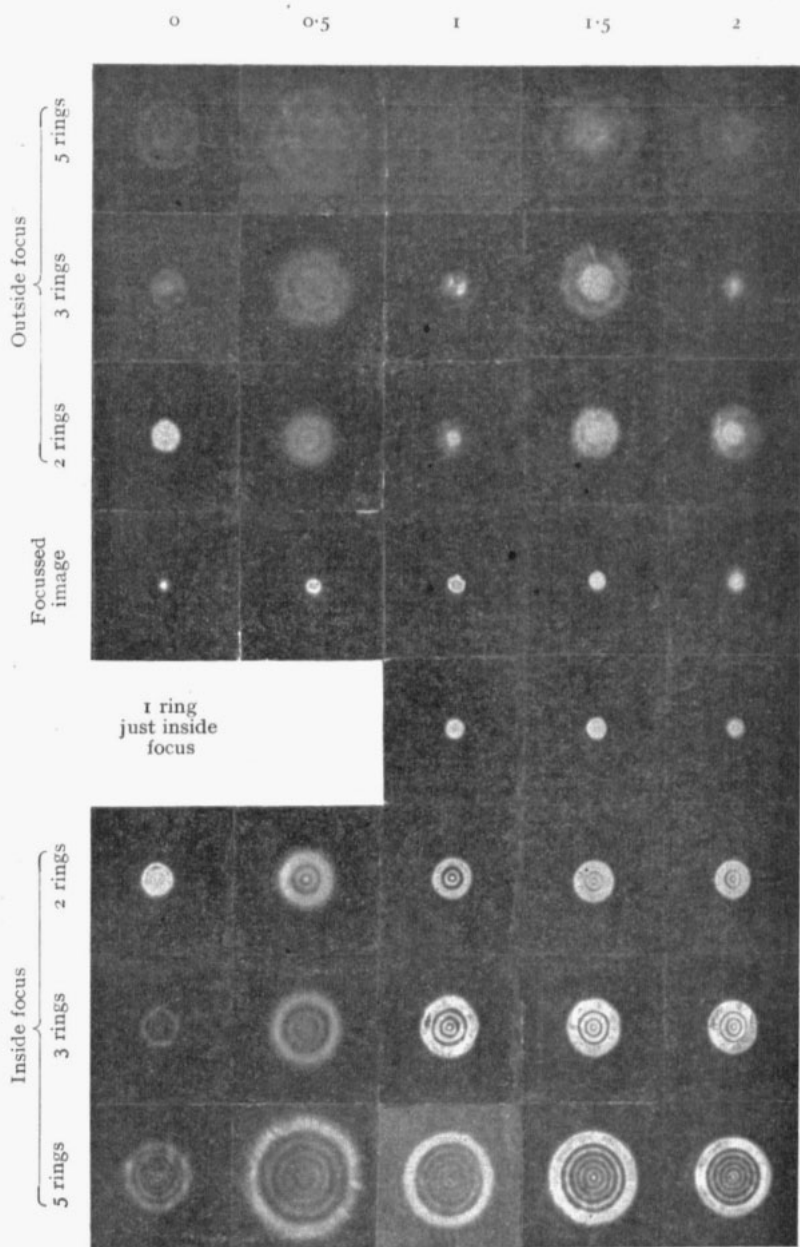


Fig 1.

RESULTS.

Focussed image. Zero aberration.

No. of Ring	Experimental measurements		Theoretical values for comparison	
	Dark rings	Bright rings	Dark rings	Bright rings
	cms.	cms.	cms.	cms.
1	·0568	—	·0570	—
2	·1	·0773	·105	·0768
3	·145	·118	·152	·126
4	·195	·171	·199	·174

The theoretical values are calculated from the figures given in Chap. 7 of Schuster's *Optics*.

The following tables give the measurements of the out of focus images.

Ring Diameters (2 rings)

Aberration		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring	Total diameter
Zero	Inside focus }	17	25	42	64	113	204
	Outside focus }	29	45	57	79	95	164
·5λ	Inside focus }	23	47	87	136	204	332
	Outside focus }	12	38	83	128	211	354
1λ	Inside focus }	16	30	61	95	170	226
	Outside focus }	11	30	72	91	166	207
1·5λ	Inside focus }	19	26	57	87	136	223
	Outside focus }	11	76	117	134	189	340
2λ	Inside focus }	19	23	53	87	134	223
	Outside focus }	23	38	64	143	234	340

Unit = 10^{-4} cm.

Ring Diameters (3 rings)

Aberration		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring	3rd dark ring	3rd light ring	Total diameter of image
Zero	Inside focus	15	23	42	64	87	113	166	204
	Outside focus	11	23	42	64	87	113	162	200
.5λ	Inside focus	11	38	79	113	158	219	302	400
	Outside focus	11	38	83	113	158	211	279	415
1λ	Inside focus	15	30	60	87	132	170	234	317
	Outside focus	11	45	68	91	143	181	249	302
1.5λ	Inside focus	15	26	57	83	113	155	204	283
	Outside focus	11	26	57	79	113	143	226	294
2λ	Inside focus	19	26	53	79	120	151	208	287
	Outside focus	19	38	57	75	91	121	151	302

Unit = 10^{-4} cm.

Ring Diameters (5 rings)

Aberration		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring	3rd dark ring	3rd light ring	4th dark ring	4th light ring	5th dark ring	5th light ring	Outside diameter of image
Zero	Inside focus	11	23	42	64	91	113	143	173	222	245	298	415
	Outside focus	11	23	53	75	90	128	166	—	230	—	317	378
.5λ	Inside focus	11	19	26	90	143	188	234	279	354	430	528	640
	Outside focus	15	23	45	91	120	135	188	226	264	347	453	545
1λ	Inside focus	15	30	53	94	120	166	196	245	294	340	378	491
	Outside focus	8	—	—	38	91	120	181	226	302	347	438	468
1.5λ	Inside focus	15	32	57	94	128	166	188	238	260	276	366	445
	Outside focus	9	19	23	45	83	113	151	230	302	378	423	528
2λ	Inside focus	11	30	53	79	109	139	166	208	238	282	302	408
	Outside focus	8	19	75	—	170	—	264	—	358	—	528	585

Unit = 10^{-4} cm.

INTENSITY OF LIGHT IN RINGS

In order to make some measurements of the intensity of the light in the rings as recorded by the photographic plate a special comparator apparatus was constructed. This consisted of a microscope having two objective tubes arranged at right angles to each other. By means of a right-angled prism in the focal plane of the positive eyepiece both fields of view could be viewed simultaneously. Half the field as seen through the eyepiece was the field seen through one objective. The other half of the field was that seen through the other objective.

In front of the one objective was placed a photograph of diffraction rings and the particular ring under consideration was focussed. In front of the other objective was placed a standard plate divided into squares, forming a graded series of densities. The objectives used were of $\frac{3}{8}$ in. focal length. A square of the standard plate was found that just matched the central portion of the ring under consideration. The measurements were made for the middle portion, that is the lightest or darkest of the light or dark rings.

STANDARD PLATE

The plate was exposed on an apparatus consisting of a box with, 4 cms. in front of the plate, a sheet of metal pierced by 30 holes of graded sizes. The part of the plate illuminated by one hole was divided from that illuminated by the next hole by a partition of blackened metal. In front of the holes was a stretched sheet of carefully selected grainless paper to diffuse the light. The apparatus was exposed to the sky and rotated during exposure. In this way the plate on development was obtained divided into squares, each square having been evenly illuminated and being of a density which is a function of the area of the hole. The diameters of the holes were carefully measured and the areas were in such a series that, approximately hole 4 had twice the area of hole 1, hole 5 had twice the area of hole 2, etc.

In calculating the results for the intensity curves the area of the hole whose corresponding square gave a match with the central disc of the focussed image was found and the intensity of an area of a ring was taken as proportional to the area of the hole whose corresponding square gave a match with the ring as seen through the comparator—the areas being expressed so that the area of the hole whose corresponding square matched the central disc of the focussed image is taken as 100.

The plate used in making the standard plate and the conditions of its development were similar to those employed in making the series of diffraction ring-pictures with which the standard plate was to be compared. In preparation for some visual observations described in the last paragraph the standard plate was also calibrated against an adjustable sector and the percentage of light transmitted by each square was found.

Intensities (2 rings)

Aberration		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring
Zero	Inside focus	27.7	8.8	17.3	6.8	11.3
	Outside focus	22.6	8.8	11.3	8.8	11.3
.5λ	Inside focus	16	3.1	7.7	3.1	7.7
	Outside focus	7.76	3.9	5.1	3.88	5.1
1λ	Inside focus	56.1	.25	14.1	.25	6.8
	Outside focus	6.8	2.16	2.66	.81	.93
1.5λ	Inside focus	43.2	1.28	21.6	2.16	21.6
	Outside focus	17.3	8.5	11.0	6.8	8.5
2λ	Inside focus	50.3	1.15	25.3	3.13	12.6
	Outside focus	25.3	6.1	7.6	6.1	7.6

Intensities (3 rings)

Aberration		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring	3rd dark ring	3rd light ring
Zero	Inside focus	3.46	.20	.65	.20	.652	.20	1.02
	Outside focus	5.45	.74	1.36	1.02	1.36	.32	1.02
.5λ	Inside focus	7.74	5.01	6.29	3.85	7.74	3.85	9.92
	Outside focus	7.74	5.01	7.74	5.01	6.29	3.85	7.74
1λ	Inside focus	21.6	.81	8.47	.33	4.32	.33	8.47
	Outside focus	5.31	2.64	3.44	2.64	3.44	.25	.93
1.5λ	Inside focus	17.3	2.64	11	.93	6.8	1.28	6.8
	Outside focus	8.47	5.31	6.8	5.31	5.31	2.16	4.32
2λ	Inside focus	31	1.52	15.5	1.52	7.6	.73	13
	Outside focus	4.8	2.4	2.4	.22	.22	.36	.36

Intensities (5 rings)

Aberra- tion		Central disc	1st dark ring	1st light ring	2nd dark ring	2nd light ring	3rd dark ring	3rd light ring	4th dark ring	4th light ring	5th dark ring	5th light ring
Zero	Inside focus	2.11	.47	1.73	.20	.74	.266	1.02	.266	1.02	.266	1.36
	Outside focus	2.11	.47	.74	.47	.74	.47	.74	.47	.47	.47	.74
$.5\lambda$	Inside focus	3.85	1.36	3.85	1.36	3.85	1.36	3.85	1.36	3.85	1.36	3.85
	Outside focus	2.47	1.19	2.47	1.36	2.47	.84	2.47	.605	2.47	.605	2.47
1λ	Inside focus	1.69	.25	.81	.25	.81	.25	.58	.25	.93	.25	2.16
	Outside focus	4.15	—	—	—	.415	.40	.415	.33	.40	.25	.33
1.5λ	Inside focus	6.8	.33	4.32	.33	4.32	.33	2.64	.33	2.16	.33	6.8
	Outside focus	2.16	.93	1.69	1.28	1.69	1.28	1.69	1.28	1.69	.398	2.16
2λ	Inside focus	4.8	.83	2.4	.83	2.4	.83	2.4	.37	2.4	.52	4.8
	Outside focus	.73	.52	.83	.73	.73	.37	.37	.37	.37	.36	.37

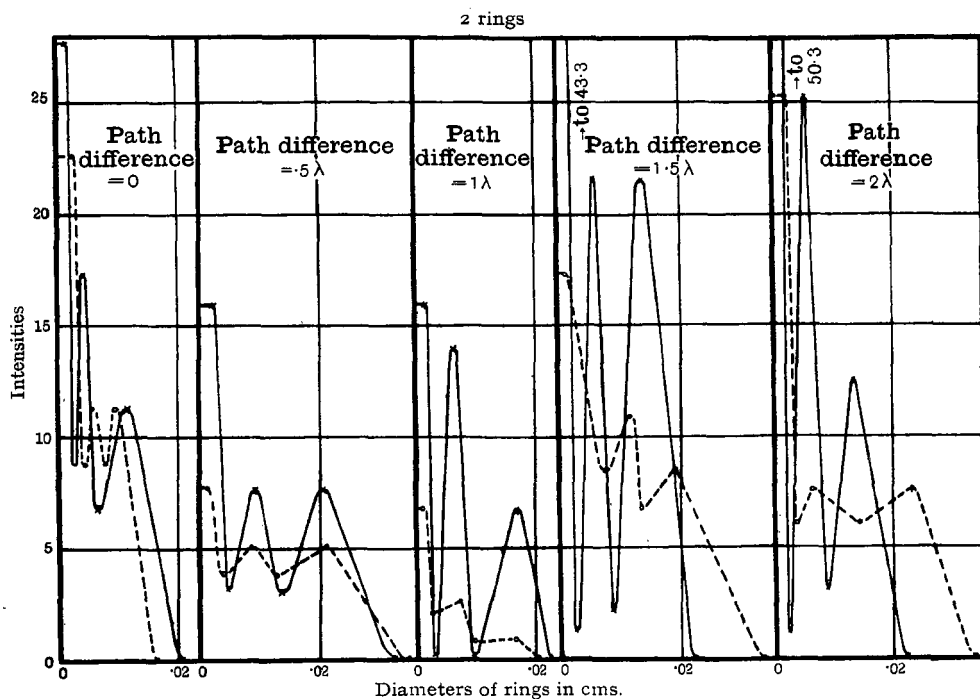


Fig. 2.

In drawing the curves shown in Figs. 2, 3, and 4, the intensity of the light referred to the central disc of the focussed image as 100 is plotted as the ordinate, and the diameter of the ring in cms. is plotted as the abscissa. The continuous

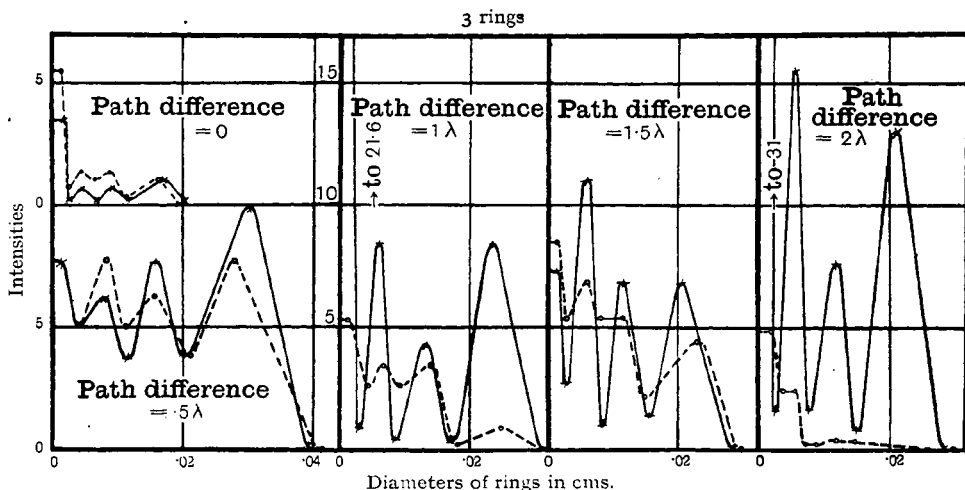


Fig. 3.

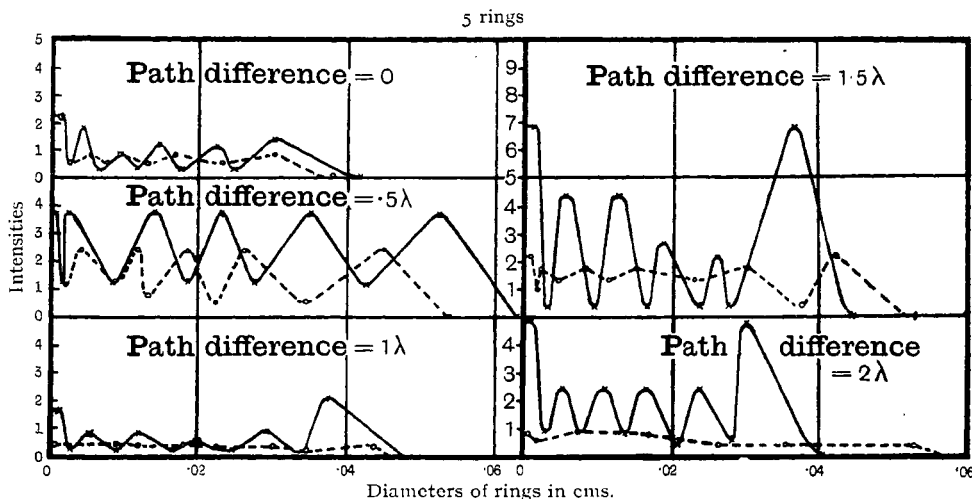


Fig. 4.

curves give the maxima and minima of light inside the position of the focus, that is towards the object glass and the broken curves give the maxima and minima outside the position of the focus. Only the maxima and minima are plotted from measurements. They are joined by a curve to make the diagram clearer, but the

precise slope of the curve between a maximum and the adjoining minimum is without meaning. The curves show clearly the increasing lack of symmetry with increasing aberration and show also how outside the focus with increasing aberration the light is scattered from the ring system into the surrounding field.

VISUAL OBSERVATIONS ON FOCUSED IMAGE

Visual comparisons were also made of the intensities of the light in the focussed image where $\cdot 5$, 1, 1.5, and 2 wave lengths of path difference were present with the intensity of the central disc of the focussed image with zero aberration.

The lens was set up in the stand and the pin of the stand provided with a collar so that the lens and holder could be rotated through 180° without any other adjustments being disturbed.

A stop was arranged on a separate stand 6 cms. away and between the lens and the observing microscope. The diameter of the stop was calculated and was so constructed that the required aberration would be obtained when the lens was in the reversed position, that is, with the flint glass component towards the object. On rotating the lens through 180° to place the crown glass component towards the object and slightly adjusting the position of the stand forwards or backwards along the optical bench the image could again be obtained in focus with zero aberration and, as the observing microscope, the stop and consequently the cone of light producing the image were kept unaltered the intensities in the two cases, the one with and the other without aberration, were comparable.

The image was observed through the eyepiece—an extra density being inserted at the star end to adjust the light to a suitable amount. The intensity of the central disc of the focussed image was then cut down by interposing between the eye and the eyepiece a square of the standard plate and choosing such a square that the central disc just became invisible. Knowing the percentage of light transmitted by that particular square a comparison of the intensity of the central disc with some certain amount of aberration and with zero aberration could be made. Measurements were also made for the 1st and 2nd rings of the focussed images.

Aberration	Central disc	1st ring	2nd ring
Zero	1.0	.174	.046
$\cdot 5\lambda$.81	.17	—
1λ	.78	.13	.054
1.5λ	.78	.165	.033
2λ	.69	.217	.027

In conclusion I wish to express my indebtedness to Professor Conrady for initiating the research and my thanks to Professors Cheshire and Conrady for kind interest and valuable assistance throughout the work.

DISCUSSION

Mr P. F. Everitt thought that in order to measure the spherical aberration in an objective Hartmann's well-known method would give less trouble and yield more accurate results. He did not quite follow the purpose of Mrs Griffiths' work, as it did not seem to be a case of forming an estimate of the spherical aberration of an objective and then checking the results by other methods.

Mr T. Smith asked in what way the position of focus was determined, and what was the meaning attached to certain errors of phase; did the phase difference refer to the position where the marginal rays met the axial rays?

Mrs Griffiths replied that the position of focus was determined chiefly by appearance; it was thus found separately for each of the stops used. The phase differences were calculated for the marginal focus.

Mr T. Smith, continuing, said it was unfortunate that the positions of focus differed in the various series of photographs. A number of theories were held by different authorities regarding the best position of focus, and each theory involved a different result for the best method of compensating aberrations of different orders. The classical view, in accordance with which the great bulk of calculations were carried out at the present time, considered the paths of the rays only, and correction was directed to bringing all rays within the smallest area on the image plane. According to another view aberrations must be corrected so as to bring the variations of phase in the wave front within the closest limits at the paraxial focus. Professor Conrady had gone further and suggested that the eye would choose as the position of best focus that point where the errors of phase were least. Other theories, as plausible as any of those mentioned, could be brought forward, and these would give still other results. The important fact to bear in mind, however, is that no one can say at present with any certainty what is the correct principle to follow. Any theoretical discrimination must be based on a study of diffraction patterns, and our knowledge of the way in which these should be calculated, though good enough to give the general distribution of light with fair accuracy, is not necessarily sufficiently precise for this much nicer problem. It will be possible to discuss this theoretically when exact solutions have been obtained for the propagation of waves through apertures, but this problem has so far defied solution. There remains the possibility of an experimental investigation, and Mrs Griffiths' photographs might have been of value in this connection had they been referred to a definite focus correlated to, say, the paraxial focus. By comparing photographs and visual impressions with the results of examination by Hartmann's method and with an interferometer test, and by extending such comparisons over a number of objectives of varying focal lengths and apertures and afflicted with differing amounts of aberration, much light should be thrown on this interesting problem.

It was by no means evident that a photograph similar to one of Mrs Griffiths' by another lens would imply that the amount of aberration in the one case could be derived from that known to exist in the other.

Instructor-Commander T. Y. Baker pointed out that the appearance photographed was bound to consist of a combination of the ordinary diffraction patterns with an interference pattern due to the spherical aberration itself. He asked if any attempt had been made to disentangle the two effects.

Mr J. Rheinberg drew attention to the fact that precisely similar patterns can be obtained in a microscope by focussing on a piece of gold leaf having microscopic holes or on a piece of badly platinised glass. With a correctly adjusted microscope, the out of focus diffraction patterns are similar, whether slightly above or below the focus, and when not in correct adjustment the patterns will be dissimilar, corresponding to Mrs Griffiths' observations with the telescope objective. With the microscope, want of adjustment can be introduced at will and controlled by varying the tube length, or altering the correction collar of the objective. The study of these patterns in the microscope under known conditions might therefore be useful as an aid in interpreting the appearances presented by the patterns produced by the telescope objective.

Mr L. C. Martin thought that the scope of Mrs Griffiths' work had been misunderstood. The idea of the research was to give information as to what kind of diffraction patterns could be expected with given path differences. The results obtained, however, agreed with Professor Conrady's theoretical results on the light distribution in the image of a star. The remarkable result is confirmed that for a considerable region outside and inside the position of focus the size of the spurious disc altered very little. Also for considerable amounts of spherical aberration (two or three wave lengths) the size of the spurious disc is not greatly increased, if at all. These conclusions were of great importance in the consideration of the performances of instruments.

(Communicated remarks.) With regard to the position of best focus it seems clear that in this, as in any other type of wave motion, the maximum displacement must be found where the disagreement in phase of the disturbances from all contributing elements is a minimum. There is no reason to think that the eye could select any other region for the brightest light, and experiment leaves no room for doubt that the image appears brightest when the star image is smallest. Physicists must regard the ray conventions for best focus of the mathematician and trigonometrical computer alike as attempts to secure the above condition in a form convenient for calculation.

With regard to the desirability of correlating the chosen focus to that of the paraxial rays it would certainly be interesting to do this if the experimental difficulties of finding the paraxial focus can be overcome, but the position is impossible to calculate from the data of the system owing to constructional errors, and cannot easily be found experimentally owing to the necessity of restricting the aperture of the lens, whereby the depth of focus is greatly increased. Hartmann tests can give no information as to the paraxial focus except by extrapolation from many results, and give no information whatever as to the distribution of light at any point. Hartmann tests are also liable to lend undue weight to the zonal effects of figuring.

With regard to the interpretation of other diffraction patterns than the present

photographs I hope shortly to publish results which will tend to confirm this possibility. For general purposes it seems a common experience amongst those used to testing that aberration may be judged quite effectively from out of focus diffraction patterns and Mrs Griffiths' paper is a first attempt to put the estimation on a definite though approximate basis. It was not intended primarily to check theoretical results.

Prof. Cheshire considered that, even if all that Mrs Griffiths had done was to take the series of beautiful photographs, the paper was a success. Some years ago he had been very much interested in what he thought were direct photographs of star discs in Dennis Taylor's book, *The Adjustment and Testing of Telescopic Objectives*, but upon enquiry he was informed that they had been carefully drawn from observation of the discs.

Mrs Griffiths, in reply to Commander Baker, said she had not attempted to disentangle the two effects to which he had drawn attention.

She was very interested in the microscope analogy Mr Rheinberg had mentioned.