out of 58 falling within a circle of the same size as that here shown, while 15 of them fall within the present circle.

Two explanations naturally suggest themselves, one or both of which may be correct, for the deviation between the results deduced from the stars and those deduced from the comets. One explanation is that the absorbing medium is not stationary with regard to the stars as a whole, and the other that the Sun is moving in a curve rather than a straight line, the comets indicating a region from which the Sun has formerly been moving. The result, based on the orbits of the earlier comets, can hardly be looked upon as confirmatory of this latter suggestion, since the change indicated by it is too rapid to be plausible, and, moreover, is due largely to three bright comets which appeared prior to the year 800 A.D., and whose aphelia are therefore rather uncertain.

On the other hand, the line of bright comets beginning with the great comet of 1882, $\lambda = 101^\circ\cdot6$, $\beta = -35^\circ\cdot2$, and extending along the ninetieth meridian to $\lambda = 89^\circ\cdot4$, $\beta = +46^\circ\cdot3$, with fainter comets on either side, radiating as it were from the southern end of the line, rather suggests a continuous change in the direction of the solar motion. This line, we may note, is inclined at an angle of about 20° to the direction of the Milky Way, crossing it in latitude +16°. It is true the head of the line is 10° east of the solar anti-apex with regard to the stars, but this we may perhaps assign to the other cause, the drift of the stars with regard to the absorbing medium. However, with two possible explanations of the deviation, we must clearly wait for further light before deciding upon their relative importance, and can merely conclude, therefore, that the solar motion relatively to the resisting medium differs somewhat in direction, but not very greatly, from its motion relatively to the stars.

On the Derivation of the Constants for the two Star-Streams. By J. C. Kapteyn, Assoc. R.A.S., with the co-operation of Dr. H. A. Weersma.

In two lectures, the one before the Congress at St. Louis in 1904, the other before the British Association in South Africa in 1905, I have communicated some of the elements of the two great star-streams without publishing at the same time the solution which led to these elements. I certainly intended to do so later, but as the method leads to very laborious calculations as soon as we wish to make a *detailed* comparison with the observations, I hoped at some future time to be able either to simplify it or replace the method by another which would be more commodious. Before I found leisure, however, to take up this task, Eddington published his solution. The elegance and convenience of his method leaving nothing to be desired, I provisionally gave up the plan of publishing my method at all. Afterwards Dyson published another method of exquisite simplicity, and Schwarzschild gave his well-known beautiful method, which probably is the most convenient of all. There thus would be no use in publishing a more cumbrous method, were it not for the two following facts:—

First.—Eddington, Dyson, and Schwarzschild all make the derivation of the stream elements exclusively dependent on the *number* of proper motions in different position angles, whereas in my method the amount of the proper motion plays a part.

Second.—The hypotheses from which Eddington and Schwarzschild start are different. The present method may help to settle the question which of the two must be nearest the truth.

These considerations have revived my plan to publish my results in the form in which they were arrived at some eight years ago. Other matters, however, have continually interfered with the execution of this project, and might still have prevented it had not Dr. Weersma, on the appearance of Eddington's recent work relating to the second of the two points just mentioned, generously offered to collect and arrange my notes on the subject, and to prepare the whole for the press. I have gladly availed myself of this offer, and wish to express my most cordial thanks for his kind and skilful co-operation.

1. *Definitions and notation.*—First consider a number of stars in the point S of space, and suppose the solar system to be at rest. Let (fig. 1)

O be the position of the observer,

Ω a plane through S,

S Z the line O S prolonged,

S X the projection on Ω of the line S-antapex,

S Y a line at right angle to the former in the plane Ω,

S V, S W the line towards the true vertex and its projection on Ω,

S P $= s$ and S Q the linear peculiar motion in space of a star and its projection.

Δx, Δy the projection of S P on the x- and y-axes,

$\sum\limits_{0}^{\pi}\Delta x$, etc., the sum of the Δx's making with the x-axis angles between 0 and π, etc.,

$\left[\Delta x\right]_{0}^{\pi}$, etc., the average values of these same quantities.

Let further (see fig. 1)

$$\phi = \text{V S P} \qquad \gamma = \text{W S X}$$
$$\sigma = \text{Z S P} \qquad \psi = \text{Q S W}$$
$$\lambda' = \text{V S Z} \qquad \rho = \text{O S}$$

while α, δ, A, D, A′, D′ represent the co-ordinates on the sphere of the star at S, the apex, and the true vertex.

2. *Formulæ.* — According to the theory of star-streaming, the true star-motions (freed from the Sun's motion) favour two opposite directions, the directions of the true vertices. In order

to express the predilection analytically, we assume that the frequency of a motion making the angle ϕ with the direction of one of the vertices is proportional to $1 + \epsilon \cos 2\phi$. This, of course, implies the assumption of equality in number of the two "star-streams." For the rest we take the frequency of the amounts of motion for any particular value of ϕ in accordance with Maxwell's law. The easiest way of representing the distribution of the

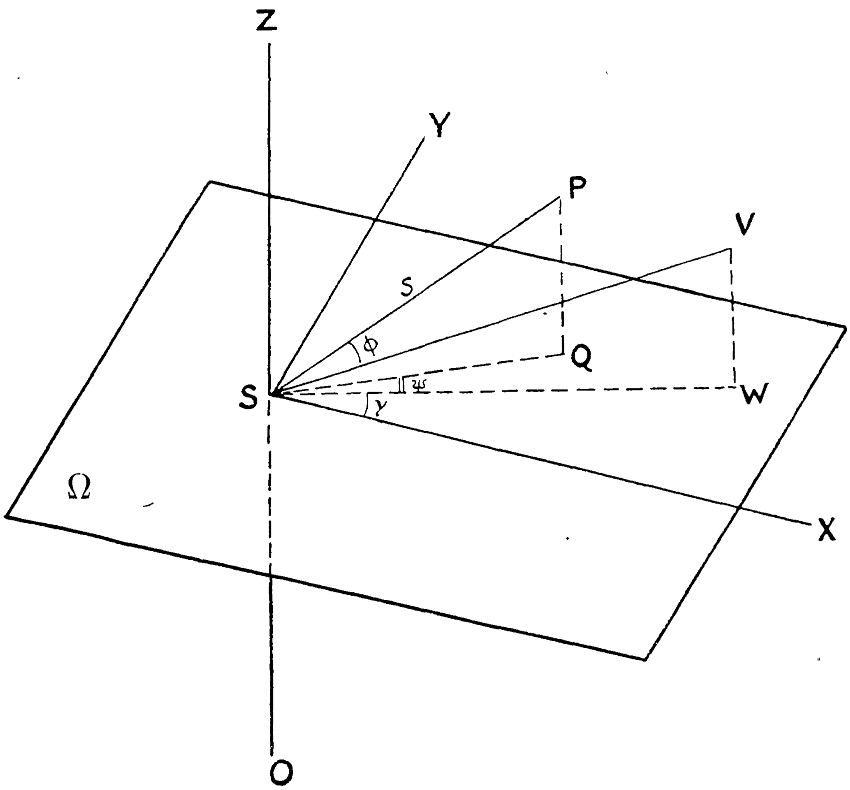

Fig. 1.—O, observer ; V, true vertex ; X, antapex.

direction is perhaps by considering the distribution of the positions of the *tops* P of the proper motions.

Let

Θ = the number of *tops* per unit of volume ;

then our assumption comes to putting

$$(1) \quad . \quad . \quad . \quad . \quad \Theta = Ce^{-h^2s^2}(1 + \epsilon \cos 2\phi)$$

where C, h, and ϵ are constants to be determined from the observations. The quantity ϵ is a measure of the degree of preference for the direction of the vertices. $\epsilon = 0$ means that there is *no* preference; the distribution will then, on our assumption, be wholly in agreement with Maxwell's law. ϵ *negative* expresses *averseness* to the direction of the vertices. For ϵ *positive* there is predilection. The distribution is best shown if in the different

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directions we draw radii vectores proportional to the frequencies in these directions. The tops of these radii vectores will form a surface. For $\epsilon < 1$ there will be one continuous surface, which, for the smaller values of ϵ, does not diverge too much from an ellipsoid of revolution (Schwarzschild's theory). For increased values of ϵ the surface becomes more and more constricted in the neighbourhood of $\phi = 90°$. For $\epsilon > 1$ it will break up into two separate surfaces. We thus come nearer to Eddington's assumption.

The form adopted has some advantages therefore in regard to the question which of the two assumptions, those made by Schwarzschild and by Eddington, must be nearer to the truth.

The three constants C, h, and ϵ are not independent. The total number of *tops* is, of course, equal to the total number of stars at S. If this number is n, then

$$2\pi C\int_0^\infty s^2 e^{-h^2s^2}ds\int_0^\pi(1+\epsilon\cos 2\phi)\sin\phi d\phi = n,$$

from which

$$(2) \quad . \quad . \quad . \quad C = \frac{h^3}{\pi\sqrt{\pi}}\,\frac{3}{3-\epsilon}\,n\,;$$

therefore

$$(3) \quad . \quad . \quad . \quad \Theta = \frac{nh^3}{\pi\sqrt{\pi}}\,\frac{3}{3-\epsilon}\,e^{-h^2s^2}(1+\epsilon\cos 2\phi).$$

The determination, from the observations, of the remaining constants h, ϵ and the co-ordinates A′ and D′ of the vertex, was made in the way which was suggested by the results communicated in the British Association lecture, p. 5. It was there found that the quantities $x_R - x_L$ $\left(\text{which below will be designated by } \Big[v\Big]_0^\pi - \Big[v\Big]_\pi^{2\pi}\right)$, which in the theory of random-distribution of the directions should all be zero, show on the contrary an astonishing amount of systematic variation. It was just the investigation of the law of this variation that finally led to the recognition of the phenomenon of star-streaming. It seems natural to use these same observed quantities as the basis for the derivation of the stream-constants. This was accordingly done; only, in order, if possible, to strengthen the determination, the perpendicular components τ of the proper motion were also considered.

We thus have to derive the values of $\Big[v\Big]_0^\pi, \Big[v\Big]_\pi^{2\pi}, \Big[\tau\Big]_0^\pi, \Big[\tau\Big]_\pi^{2\pi}$ in terms of the constants.

We have

$$(4) \quad . \quad . \quad . \quad . \quad \begin{aligned}\Delta x &= s\sin\sigma\cos(\psi+\gamma)\\ \Delta y &= s\sin\sigma\sin(\psi+\gamma).\end{aligned}$$

On the other hand, the number of *tops* P in an element of space is easily found to be

$$(5) \quad . \quad . \quad . \quad Cs^2e^{-h^2s^2}(1+\epsilon\cos 2\phi)\sin\sigma ds d\sigma d\psi\,;$$

or, writing

(6) . . . $\cos\phi = \cos\lambda'\cos\sigma + \sin\lambda'\sin\sigma\cos\psi$

(7) . . . $Cs^2e^{-h^2s^2}\{1 - \epsilon + 2\epsilon\cos^2\lambda'\cos^2\sigma$

$+ 4\epsilon\cos\lambda'\sin\lambda'\cos\sigma\sin\sigma\cos\psi + 2\epsilon\sin^2\lambda'\sin^2\sigma\cos^2\psi\}\sin\sigma ds d\sigma d\psi$

Therefore

$$\sum_0^\pi \Delta x = C(1-\epsilon)\int_0^\infty s^3e^{-h^2s^2}ds\int_0^\pi \sin^2\sigma d\sigma\int_{-\gamma}^{\pi-\gamma}\cos(\psi+\gamma)d\psi$$

$$+ 2C\epsilon\cos^2\lambda'\int_0^\infty s^3e^{-h^2s^2}ds\int_0^\pi \cos^2\sigma\sin^2\sigma d\sigma\int_{-\gamma}^{\pi-\gamma}\cos(\psi+\gamma)d\psi.$$

$$(8)\quad + 4C\epsilon\cos\lambda'\sin\chi\int_0^\infty s^3e^{-h^2s^2}ds\int_0^\pi \cos\sigma\sin^3\sigma d\sigma\int_{-\gamma}^{\pi-\gamma}\cos\psi\cos(\psi+\gamma)d\psi$$

$$+ 2C\epsilon\sin^2\lambda'\int_0^\infty s^3e^{-h^2s^2}ds\int_0^\pi \sin^4\sigma d\sigma\int_{-\gamma}^{\pi-\gamma}\cos^2\psi\cos(\psi+\gamma)d\psi.$$

The corresponding expression for $\sum_0^\pi \Delta y$ is obtained by substituting $\sin(\psi+\gamma)$ for $\cos(\psi+\gamma)$.

Working out the integrals and taking account of (2),

$$\sum_0^\pi \Delta x = \frac{3n}{4}\frac{1}{h\sqrt{\pi}}\frac{\epsilon}{3-\epsilon}\sin^2\lambda'\sin 2\gamma,$$

$$\sum_0^\pi \Delta y = \frac{n}{2}\frac{1}{h\sqrt{\pi}} + \frac{3}{4}\frac{n}{h\sqrt{\pi}}\frac{\epsilon}{3-\epsilon}\left(\sin^2\lambda'\sin^2\gamma - \frac{1}{3}\right);$$

or, if we introduce the mean values, indicated by square brackets,

$$(9)\qquad \Big[\Delta x\Big]_0^\pi = \frac{3}{2}\frac{1}{h\sqrt{\pi}}\frac{\epsilon}{3-\epsilon}\sin^2\lambda'\sin 2\gamma.$$

$$(10)\qquad \Big[\Delta y\Big]_0^\pi = \frac{1}{h\sqrt{\pi}} + \frac{3}{2}\frac{1}{h\sqrt{\pi}}\frac{\epsilon}{3-\epsilon}\left(\sin^2\lambda'\sin^2\gamma - \frac{1}{3}\right).$$

In order to be able to substitute the observed proper motions for the $\Sigma\Delta x$ and $\Sigma\Delta y$, we have now to introduce the consideration of the Sun's motion through space. We will take the velocity of this motion as our unit of velocity. Let v, τ be the components of the total observed angular proper motion in the directions towards the antapex and at right angles to it, λ the distance on the sphere of S from the apex. We find at once

$$(11)\quad . \quad . \quad . \quad \begin{aligned}\rho v &= \sin\lambda + \Delta x\\ \rho\tau &= \pm\Delta y.\end{aligned}$$

The sign of Δy will be taken such that $\rho\tau$ becomes alway positive. Considering that the angle which the total prope motion on the sphere makes with the great circle towards the apex varies between 0° and 180°, respectively, 180° and 360° together with the angle X S Q (fig. 1) and that ρ is constant for the stars in S, we have

$$(12) \quad \ldots \quad \rho\Big[v\Big]_0^{\pi} = \sin\lambda + \Big[\Delta x\Big]_0^{\pi} \qquad \rho\Big[\tau\Big]_0^{\pi} = \Big[\Delta y\Big]_0^{\pi}$$

and

$$(13) \quad \ldots \quad \rho\Big[v\Big]_{\pi}^{2\pi} = \sin\lambda + \Big[\Delta x\Big]_{\pi}^{2\pi} \qquad \rho\Big[\tau\Big]_{\pi}^{2\pi} = -\Big[\Delta y\Big]_{\pi}^{2\pi},$$

Therefore, as it is easily seen that

$$\Big[\Delta x\Big]_{\pi}^{2\pi} = -\Big[\Delta x\Big]_0^{\pi}, \quad \Big[\Delta y\Big]_{\pi}^{2\pi} = -\Big[\Delta y\Big]_0^{\pi}$$

we have by (9) and (10), τ being always taken positively,

$$(14) \quad \rho\Big[v\Big]_0^{\pi} - \rho\Big[v\Big]_{\pi}^{2\pi} = \frac{1}{h\sqrt{\pi}}\,\frac{3\epsilon}{3-\epsilon}\sin^2\lambda'\sin 2\gamma$$

$$\rho\Big[\tau\Big]_0^{2\pi} = \frac{1}{h\sqrt{\pi}} + \frac{3}{2}\,\frac{1}{h\sqrt{\pi}}\,\frac{\epsilon}{3-\epsilon}\Big(\sin^2\lambda'\sin^2\gamma - \frac{1}{3}\Big)$$

$$(15) \quad \ldots \quad \rho\Big[v\Big]_0^{2\pi} = \sin\lambda,$$

and therefore by division

$$(16) \quad \ldots \quad \frac{\Big[v\Big]_0^{\pi} - \Big[v\Big]_{\pi}^{2\pi}}{\Big[v\Big]_0^{2\pi}} = \frac{\frac{1}{h\sqrt{\pi}}\,\frac{3\epsilon}{3-\epsilon}\sin^2\lambda'\sin 2\gamma}{\sin\lambda}$$

and

$$(17) \quad \ldots \quad \frac{\Big[\tau\Big]_0^{2\pi}}{\Big[v\Big]_0^{2\pi}} = \frac{\frac{1}{h\sqrt{\pi}} + \frac{3}{2}\,\frac{1}{h\sqrt{\pi}}\,\frac{\epsilon}{3-\epsilon}\Big(\sin^2\lambda'\sin^2\gamma - \frac{1}{3}\Big)}{\sin\lambda}.$$

These formulæ have been derived on the supposition that we have to do with stars at one and the same point of space. If now, however, we assume that the quantities A′, D′, h and ϵ are the same for the whole of space, the second members of (16) and (17) become independent of the position of the star. These formulæ, therefore, will hold for the whole of the stars visible in any one region of the sky. If for the sake of brevity we put

$$(18) \quad \left\{ \begin{aligned} \frac{\Big[v\Big]_0^{\pi} - \Big[v\Big]_{\pi}^{2\pi}}{\Big[v\Big]_0^{2\pi}} &= u \\ \frac{\Big[\tau\Big]_0^{2\pi}}{\Big[v\Big]_0^{2\pi}} &= t \\ \frac{1}{h\sqrt{\pi}} \frac{3\epsilon}{3-\epsilon} &= \alpha \\ \frac{1}{h\sqrt{\pi}} &= \beta \\ \sin^2\lambda' \sin 2\gamma &= \mathrm{H} \\ \tfrac{1}{2}\sin^2\lambda' \sin^2\gamma &= \mathrm{G}, \end{aligned} \right.$$

we have simply

$$(19) \quad \alpha\mathrm{H} = u \sin \lambda.$$

$$(20) \quad \beta + \alpha\left(\mathrm{G} - \frac{1}{6}\right) = t \sin \lambda.$$

We will further suppose that the co-ordinates A and D of the apex have been determined by means of a method that does not involve suppositions inconsistent with the equation (1), for example, by means of the method of Bravais. The quantities $u \sin \lambda$ and $t \sin \lambda$ may then be derived from the observed proper motions. H and G are functions of the unknown quantities A′ and D′. Formulæ for expressing H and G in A′ and D′ will be given presently. By the aid of these the determination of the quantities A′, D′, α and β may be carried out in two steps, viz. by deriving, I., a set of approximations A_0', D_0', α_0 and β_0 for these quantities; and II., a set of corrections $\Delta A'$, $\Delta D'$, $\Delta\alpha$ and $\Delta\beta$ to be added to the former in order to obtain the values for A′, D′, α and β best satisfying the equations of condition.

As to I., approximate values for A_0' and D_0' may be obtained in several ways, for instance in the way indicated in "Star-streaming" (*Report of the British Association for the advancement of Science*, 1905, p. 257, etc.). With these approximations for A and D′ we may then derive approximations H_0 and G_0 for H and G. Introducing all these approximate values in the equations of condition (19) and (20), these will yield by their solution approximate values for the constants α and β.*

* A first approximation for β is not really necessary As a matter of fact, however, we have first derived such a value.

As to II., approximate values A_0', D_0', a_0 and β_0 having been obtained, the corrections $\Delta A'$, $\Delta D'$, Δa and $\Delta\beta$ must be found by means of the differential equations

$$(21)\qquad a_0\left(\frac{dH}{dA'}\right)_0 \Delta A' + a_0\left(\frac{dH}{dD'}\right)_0 \Delta D' + H_0 \Delta a = u \sin\lambda - a_0 H_0.$$

$$(22)\qquad a_0\left(\frac{dG}{dA'}\right)_0 \Delta A' + a_0\left(\frac{dG}{dD'}\right)_0 \Delta D' + \left(G_0 - \frac{1}{6}\right)\Delta a + \Delta\beta$$
$$= t \sin\lambda - a_0 G_0 + \frac{1}{6} a_0 - \beta_0.$$

In order to obtain the necessary values of H and G we first, in the spherical triangle North Pole—star—antapex, compute the angle χ at the star by the formulæ

$$(23)\qquad \begin{aligned} \sin\lambda \sin\chi &= -\cos D \sin(A - a) \\ \sin\lambda \cos\chi &= -\cos\delta \sin D + \sin\delta \cos D \cos(A - a) \\ &(\lambda \leq 180^\circ) \end{aligned}$$

where $180^\circ - \lambda =$ the arc star-antapex.

We then find the quantities γ and λ' entering into the expressions (18) of H and G by the spherical triangle North Pole—star—true vertex, in which the sides are $90 - \delta$, $90 - D'$, λ', and the angles opposite to the two last $\chi - \gamma$ and $A' - a$. Therefore

$$(24)\qquad \begin{aligned} \sin\lambda' \sin(\chi - \gamma) &= \cos D' \sin(A' - a) \\ \sin\lambda' \cos(\chi - \gamma) &= \cos\delta \sin D' - \sin\delta \cos D' \cos(A' - a'), \end{aligned}$$

after which we find H and G by the expressions (18).

The differential quotients were computed by differential formulæ and checked by computing H and G also: (*a*) with somewhat different value of A′, and (*b*) with somewhat different value of D′.

3. *Application of the method.*—The materials used in the application of the present method are the corrected proper motions of the Bradley stars, as published in *Groningen Publ.* 9. The stars have been divided into 28 groups according to the galactic longitude and latitude (Table I.), and for each of these groups the quantities $u \sin\lambda$ and $t \sin\lambda$ have been calculated. In *Publ.* 9 the fainter components of double and multiple stars, the Hyades and Pleiades, were already excluded. We have here further excluded the stars of the Ursa group, β, γ, δ, ϵ, and ζ Ursæ majoris. Finally, 1 star of the first type and 28 of the second, all with excessive proper motions, have been omitted. Care was taken to omit approximately an equal percentage of the proper motions making the angles 0° to ±30°, ±30° to ±60°, etc., with the parallactic motion. It is expected that in this way the exclusion cannot have affected the results systematically. In order to lessen still further the influence of the very large proper motions, the stars of Spectrum-type I. have been taken with double weight. Table I. contains the data, obtained in this way, and

the weights adopted for the different groups; the latter have been taken generally proportional to the numbers of stars of the groups and to the values of $\sin^2 \lambda$ corresponding to the centres of the areas. From these data approximate values for α and β have been calculated by means of the equations of condition (19) and (20), adopting for the co-ordinates of the apex *

(25) . . $A = 273^\circ \qquad D = +29^\circ.5,$

and as approximate values for the co-ordinates of the true vertex

(26) . . $A_0' = 90^\circ \qquad D_0' = +15^\circ.$

The values found are

(27) . . . $\alpha_0 = 1.17 \qquad \beta_0 = 0.84.$

With the aid of these approximate values of A_0', D_0', α_0 and β_0, the equations of condition (21) and (22) have been formed for each of the 28 areas. They are shown in Tables II. and III. The solution by least squares of the equations of Table II. led to the values

(28) . $$A' = 92^\circ.0 \pm 2^\circ.0\dagger \qquad D' = +18^\circ.7 \pm 5^\circ.2$$ $$\alpha = 1.234 \pm 0.055.$$

In the equations of Table III. the unknown quantities are not well separable; moreover, the quantities $\Delta A'$ and $\Delta\alpha$ have small coefficients. These quantities were, therefore, taken from the solution of the equations of Table II. After that the solution yielded the values

(29) . $D' = +15^\circ.3 \pm 6.6 \qquad \beta = 0.839 \pm 0.043.$

Finally, the equations of condition of Table II. and Table III. have been combined into one solution. From the residuals obtained by substituting the results (28) and (29) into the equations of condition the following quantities were found for the probable errors corresponding with the unit of weight:

(30) . . . $$r_0 = \pm 0.152 \text{ for Table II.}$$ $$r_0 = \pm 0.108 \text{ for Table III.}$$

The squares of these quantities are nearly proportional to 2 and 1; for this reason the equations of condition from Table II. and those from Table III. have been combined with weights 1 and 2. The normal equations thus obtained are given in Table IV. Their solution leads to the values

(31) . $$A' = 91^\circ.4 \pm 1^\circ.8 \qquad D' = +17^\circ.9 \pm 4^\circ.1$$ $$\alpha = 1.232 \pm 0.054 \qquad \beta = 0.825 \pm 0.033.$$

* The values for the co-ordinates of the apex used here are those finally found in *Astron. Nachr.*, No. 3722, p. 17, and differ only little from those found in *Gron. Publ.*, No. 21, by means of the method of Bravais.

† The errors given are all probable errors.

Finally, adopting these we get by (18)

(32) . . $h = 0{\cdot}684 \pm 0{\cdot}027$ $\epsilon = 0{\cdot}997 \pm 0{\cdot}039$.

The value found for ϵ is practically $= 1{\cdot}000$. Therefore, according to what has been said above, our solution is decidedly in favour of the *two*-stream theory as opposed to Schwarzschild's ellipsoidal theory.

Meanwhile, the uncertainty of the declination of the vertex is still undesirably large. It diverges considerably from an anterior, totally different, solution of the problem, of which it seems unnecessary to give details. The results there arrived at were

(33) . . $A' = 89^\circ{\cdot}5 \pm 2^\circ{\cdot}6$ $D' = +7^\circ{\cdot}5 \pm 1^\circ{\cdot}8$.

Careful considerations of the circumstances finally led us to adopt

(34) . . . $A' = 90^\circ 5$ $D' = +13^\circ{\cdot}5$,

which values were first published in 1904, in a lecture before the congress at St. Louis. The theoretical values of $u \sin \lambda$ and $t \sin \lambda$ used in deriving the O – C of our tables have been computed with the constants (31). It seems hardly worth while to recompute them for the co-ordinates (33). As they stand, they show plainly how strongly the two-stream hypothesis has contributed to represent the observations. In the old theory of random-distribution of motions, $u \sin \lambda$ ought in each case to be zero. The observed values would therefore have to be considered as accidental deviations. On the same theory $t \sin \lambda$ ought to be constant, and the deviations from a mean value would represent the accidental deviations. The observed values have therefore been inserted in the table in a form that shows these deviations from a mean value.

In order to see more clearly what has been gained by the two-stream theory, we have, in Table I., to compare the values in the columns 6 and 7 with those of O – C in the columns 10 and 11. Calling Δ the residuals, p the weights, we find for the sum of the $p\Delta^2$

	Random-Distribution.	Two Star-Streams.
$u \sin \lambda$	18·66	1·25
$t \sin \lambda$	1·90	0·90[5]

In the case of the residuals for $u \sin \lambda$ the new theory has thus reduced the sum of the squares of the errors to one-fifteenth of its amount.

4. *Test of the theory by radial velocities.*—If the stars all moved exactly in the direction of the two great stream-lines, the peculiar radial velocity (*i.e.* the observed radial velocity freed from the Sun's motion through space) would be *zero* in the regions of the sky which lie at a distance of 90° from the true vertices. On the other hand, in the regions near the vertices, the peculiar motions of the stars would be wholly radial, and would be *positive* for the

stars belonging to the one stream, *negative* for those of the other. For regions further and further away from the vertices, therefore, observation would show a regular diminution in the average radial velocity (all the velocities being taken positive) from the average value of the total star-velocity at the vertices, down to *zero* for regions 90° distant from those points.

Since the stars do *not* all move exactly in the direction of the stream-lines, such extreme variation will not obtain. Still, however, there will remain a regular change, the law of which can be easily derived from the preceding theory. The comparison of the theoretical change with that found by observation, must furnish a perfectly independent, and consequently extremely valuable, test of the theory.

We thus have to find, in the first place, how, according to our theory, the average peculiar radial velocity $\Delta\rho$ changes with λ' (distance on the sphere from vertex).

With the preceding notation we have

$$(35) \quad . \quad . \quad . \quad . \quad \Delta\rho = s\cos\sigma.$$

Let $[\Delta\rho]$ represent the average value of the $\Delta\rho$, these quantities being all taken *positive*. Since evidently the absolute value of the average *positive* and average *negative* $\Delta\rho$ is the same, we have, taking in the first place only stars at one and the same distance from the sun

$$\begin{aligned} n[\Delta\rho] = 2\mathrm{C}(1-\epsilon)\int_0^{\infty} s^3 e^{-h^2s^2}ds\int_0^{\frac{1}{2}\pi}\sin\sigma\cos\sigma d\sigma\int_0^{2\pi}d\psi \\ + 4\mathrm{C}\epsilon\cos^2\lambda'\int_0^{\infty} s^3 e^{-h^2s^2}ds\int_0^{\frac{1}{2}\pi}\sin\sigma\cos^3\sigma d\sigma\int_0^{2\pi}d\psi \\ + 8\mathrm{C}\epsilon\sin\lambda'\cos\lambda'\int_0^{\infty} s^3 e^{-h^2s^2}ds\int_0^{\frac{1}{2}\pi}\sin^2\sigma\cos^2\sigma d\sigma\int_0^{2\pi}\cos\psi d\psi \\ + 4\mathrm{C}\epsilon\sin^2\lambda'\int_0^{\infty} s^3 e^{-h^2s^2}ds\int_0^{\frac{1}{2}\pi}\sin^3\sigma\cos\sigma d\sigma\int_0^{2\pi}\cos^2\psi d\psi. \end{aligned}$$

The integrals offer no difficulty. We get

$$(36) \quad . \quad . \quad . \quad n[\Delta\rho] = \mathrm{C}\frac{\pi}{2h^4}(2-\epsilon\sin^2\lambda').$$

The second member of (36) being independent of the distance of S from O, we may extend in the first member the Σ over all stars on the line O Z without changing the second member; only the number n will change. We thus have, if now $[\Delta\rho]$ represents the average value of *all* the $\Delta\rho$'s,

$$(37) \quad . \quad . \quad [\Delta\rho] = \frac{1}{h\sqrt{\pi}}\,\frac{3}{3-\epsilon}\left(1-\frac{1}{2}\epsilon\sin^2\lambda'\right).$$

With the values (32) of the constants this becomes

$$(38) \quad . \quad [\Delta\rho] = 1{\cdot}24 - 0{\cdot}62 \sin^2 \lambda' = 0{\cdot}62(1 + \cos^2 \lambda'),$$

and assuming 20·0 kil. for the sun's velocity,

$$(39) \quad . \quad . \quad [\Delta\rho] = 12{\cdot}4(1 + \cos^2 \lambda') \text{ kil. per second.}$$

The constants are very slightly different from those adopted in the *Report of the British Association* for 1905, p. 9. How observations are represented may be seen in that paper.

TABLE I.

Nr. Reg.	Limits of gal. long.	Limits of gal. lat.	Mean α.	Mean δ.	$u \sin \lambda$.	$t \sin \lambda$ = +0·79+.	Number of Stars.	Weight.	O − C. $u \sin \lambda$.	O − C. $t \sin \lambda$.
	° °		h m	°						
1	325–4	−40° to −90°	21 43	−23·5	+0·29	−0·05	33	0·3	−0·44	−0·08
2	5–44		22 34	−10·0	+0·96	+ ·28	92	1·4	− ·04	+ ·16
3	45–84		23 44	+ 6·0	+1·58	+ ·36	72	1·1	+ ·35	+ ·21
4	85–124		1 12	+ 6·0	+1·12	+ ·41	83	1·4	− ·01	+ ·28
5	125–164		2 39	− 2·5	+0·73	+ ·19	55	0·7	− ·03	+ ·08
6	325–4	+41° to +90°	15 5	+14·0	−0·62	+ ·42	53	0·4	+ ·07	+ ·28
7	5–44		15 24	+33·0	−0·96	+ ·32	44	0·3	+ ·02	+ ·19
8	45–84		13 41	+48·7	−0·36	− ·40	29	0·3	+ ·64	− ·43
9	85–124		11 30	+57·7	−1·31	− ·02	37	0·6	− ·39	+ ·04
10	125–164		10 5	+40·5	−0·64	+ ·01	66	1·1	+ ·21	+ ·04
11	165–204		9 55	+17·7	−0·51	− ·06	83	1·1	+ ·29	− ·12
12	205–244		11 12	+ 4·8	−1·35	+ ·02	74	1·1	− ·20	− ·12
13	245–284		12 37	− 3·3	−1·04	+ ·06	55	1·0	+ ·18	− ·08
14	285–324		13 52	− 2·1	−0·49	− ·11	46	0·6	+ ·50	− ·23
15	325–4	+1° to +40°	16 56	− 2·5	−0·22	− ·05	99	0·6	− ·23	+ ·03
16	5–44		18 17	+30·0	0·00	− ·76	102	0·0	− ·59	− ·84
17	45–84		19 56	+65·5	+0·09	+ ·18	103	0·7	− ·39	+ ·31
18	85–124		4 42	+76·0	−0·31	− ·14	106	1·8	− ·43	+ ·02
19	125–164		6 58	+33·3	+0·10	− ·13	132	2·1	+ ·20	+ ·02
20	165–204		8 4	+ 9·0	−0·12	− ·06	116	1·0	+ ·01	− ·03
21	205–244		9 40	−15·5	−0·66	− ·03	52	0·4	+ ·24	− ·18
22	285–324		15 17	−22·3	−0·92	− ·31	69	1·3	− ·41	− ·29
23	325–4	0° to −39°	19 19	−17·0	+0·16	+ ·08	107	1·1	+ ·12	+ ·19
24	5–44		20 36	+10·7	+0·26	+ ·06	156	1·1	− ·12	− ·06
25	45–84		22 32	+39·0	+1·14	− ·07	117	1·3	− ·07	− ·14
26	85–124		2 5	+43·8	+1·07	− ·13	144	3·1	+ ·27	− ·10
27	125–164		4 32	+20·1	+0·07	− ·24	160	2·5	− ·08	− ·14
28	165–204		5 35	− 8·5	−0·58	+ ·58	119	0·4	− ·34	+ ·70

Table II.

Equations of Condition derived from the quantities $u \sin \lambda$.

Nr. Reg.					O–C.
1	$-1{\cdot}027\Delta A'$	$-0{\cdot}363\Delta D'$	$+0{\cdot}639\Delta\alpha$	$= -0{\cdot}46$	$-0{\cdot}44$
2	−0·797	−0·199	+ ·846	= − ·03	− ·04
3	−0·086	+0·064	+ ·998	= + ·41	+ ·35
4	+0·688	−0·034	+ ·896	= + ·07	− ·01
5	+1·147	−0·335	+ ·597	= + ·03	− ·03
6	−1·274	+0·259	− ·542	= + ·01	+ ·07
7	−0·936	−0·480	− ·740	= − ·09	+ ·02
8	+0·559	−0·645	− ·926	= + ·72	+ ·64
9	+1·350	−0·308	− ·739	= − ·45	− ·39
10	+1·148	+0·087	− ·685	= + ·16	+ ·21
11	+0·993	+0·359	− ·691	= + ·30	+ ·29
12	+0·486	+0·056	− ·951	= − ·24	− ·20
13	−0·376	−0·033	− ·972	= + ·10	+ ·18
14	−0·935	+0·156	− ·784	= + ·43	+ ·50
15	−0·216	+0·765	+ ·043	= − ·27	− ·23
16	−0·801	+0·978	+ ·451	= − ·53	− ·59
17	+2·080	+0·528	+ ·298	= − ·26	− ·39
18	+1·990	+0·152	+ ·032	= − ·35	− ·43
19	+0·768	+0·301	− ·122	= + ·24	+ ·20
20	+0·746	+0·832	− ·168	= + ·08	+ ·01
21	+1·034	+0·054	− ·758	= + ·23	+ ·24
22	−1·055	+0·362	− ·404	= − ·45	− ·41
23	−0·417	−0·611	+ ·078	= + ·07	+ ·12
24	−1·286	−0·590	+ ·376	= − ·18	− ·12
25	+0·262	+0·675	+ ·943	= + ·04	− ·07
26	+1·326	+0·158	+ ·601	= + ·37	+ ·27
27	+0·649	−0·449	+ ·132	= − ·08	− ·08
28	−0·489	−0·691	− ·147	= − ·41	− ·34

Normal Equations.

$+22{\cdot}067\Delta A'$	$+2{\cdot}443\Delta D'$	$+0{\cdot}975\Delta\alpha$	$= +0{\cdot}991$
+ 2·443	+3·126	−0·077	= +0·283
+ 0·975	−0·077	+7·708	= +0·523

Table III.

Equations of Condition derived from the quantities t *sin* λ.

Nr. Reg.					O − C.
1	−0·401 $\Delta A'$	+0·355 $\Delta D'$	−0·005 $\Delta\alpha + \Delta\beta$	= −0·09	−0·08
2	−0·260	+ ·516	+ ·058	= + ·16	+ ·16
3	+ ·044	+ ·584	+ ·078	= + ·22	+ ·21
4	+ ·185	+ ·555	+ ·060	= + ·29	+ ·28
5	+ ·241	+ ·526	+ ·046	= + ·09	+ ·08
6	+ ·143	+ ·570	+ ·067	= + ·29	+ ·28
7	− ·202	+ ·546	+ ·069	= + ·19	+ ·19
8	− ·402	+ ·352	− ·008	= − ·44	− ·43
9	− ·398	+ ·196	− ·073	= + ·02	+ ·04
10	− ·413	+ ·260	− ·046	= + ·02	+ ·04
11	− ·369	+ ·421	+ ·020	= − ·13	− ·12
12	− ·140	+ ·571	+ ·076	= − ·12	− ·12
13	+ ·098	+ ·581	+ ·073	= − ·07	− ·08
14	+ ·205	+ ·546	+ ·056	= − ·22	− ·23
15	+ ·326	+ ·158	− ·109	= + ·03	+ ·03
16	+ ·322	+ ·458	+ ·016	= − ·83	− ·84
17	+ ·166	+ ·039	− ·155	= + ·31	+ ·31
18	+ ·021	+ ·001	− ·166	= + ·01	+ ·02
19	− ·199	+ ·025	− ·150	= ·00	+ ·02
20	− ·413	+ ·250	− ·050	= − ·05	− ·03
21	− ·042	+ ·589	+ ·080	= − ·17	− ·18
22	+ ·376	+ ·271	− ·063	= − ·28	− ·29
23	− ·316	+ ·087	− ·120	= + ·17	+ ·19
24	− ·261	+ ·516	+ ·059	= − ·06	− ·06
25	+ ·335	+ ·442	+ ·008	= − ·13	− ·14
26	+ ·362	+ ·224	− ·083	= − ·08	− ·10
27	+ ·295	+ ·123	− ·123	= − ·14	− ·14
28	+ ·241	+ ·079	− ·140	= + ·70	+ ·70

Normal Equations.

+1·622 $\Delta A'$	+0·060 $\Delta D'$	−0·089 $\Delta\alpha$	+ 0·395 $\Delta\beta$	= −0·108
+0·060	+2·780	+0·118	+ 6·264	= +0·019
−0·089	+0·118	+0·183	− 0·801	= +0·016
+0·395	+6·264	−0·801	+20·5	= −0·023

Table IV.

Normal Equations derived by combining the Equations of Table II. and those of Table III.

25·311 $\Delta A'$	+ 2·563 $\Delta D'$	+0·797 $\Delta\alpha$	+ 0·790 $\Delta\beta$	= +0·775
2·563	+ 8·686	+0·159	+12·528	= +0·321
0·797	+ 0·159	+8·074	− 1·602	= +0·555
0·790	+12·528	− 1·602	+40·100	= −0·046