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J. B. CLARK'S FORMULÆ OF WAGES AND INTEREST.

I.

THE recent volume by Professor J. B. Clark on *The Distribution of Wealth* is an important contribution to the literature bearing on the interest question. The author, in no uncertain manner, takes sides with those who find the explanation of interest in the productive character of capital. "*The power of capital to create the product is, then, the basis of interest.*"¹ What may be called the instinctive or intuitive view of mankind is in line with this doctrine; since the inducement to create capital in the beginning was its productive efficiency. This same productive efficiency has been the only economic inducement to employ capital. Indeed, the surplus product due to capital is the only economic justification of the existence of capital; and surely the basis of the remuneration of capital must lie in the circumstances that afford its justification. But Professor Clark has not been content to find merely the *basis* of interest in productivity. He has industriously sought to establish a formula that will quantitatively identify the *rate* of interest with the

¹P. 135.

specific part of the product due to one unit of capital. He also presents a correlate formula for wages.

In order to place the theory fairly before the reader, we give the author's own formulation of it, as follows:

Let the number of units of labor be measured, in the following figure (Fig. 1),¹ along the line A D. Let them be set working in a series, in connection with a fixed amount of capital. The product of the first unit of

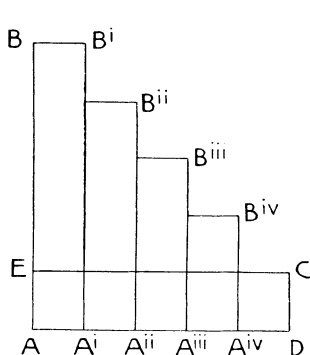


FIG. 1.
(Labor Diagram.)

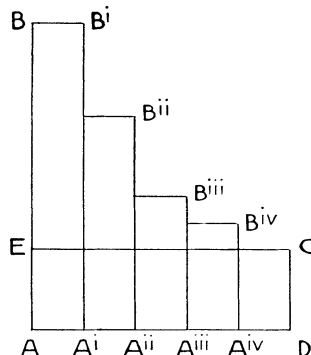


FIG. 2.
(Capital Diagram.)

labor, as aided by all the capital, is measured by the rectangle A Bⁱ. What the second unit of labor adds to this product is the amount expressed by Aⁱ Bⁱⁱ, the next by Aⁱⁱ Bⁱⁱⁱ, the next by Aⁱⁱⁱ B^{iv}, and the last by A^{iv} C. A^{iv} C measures the effective productivity of any unit of labor in the series, and fixes the general rate of pay. If the first unit of labor claims more than the amount A^{iv} C, employers will let it withdraw, and will substitute for it the last unit. What they lose by the withdrawal of any one unit in the entire force is the amount A^{iv} C.

A fact of great importance now appears. We may reverse the application of this law, and by so doing get a law of interest. Let the labor be the

¹ We have taken the liberty to change the curve of Professor Clark's diagrams to a broken line, in order the more accurately to represent the phenomena. As consistent with this change, we have replaced the letters of Professor Clark's text which indicate straight lines by others which indicate areas. This does no injustice to the author's exposition. He himself, in a note at page 329, explains the accuracy of the broken line representation, adhering to the curve simply for convenience in connection with lettering. "We should have a somewhat cumbersome mode of description to contend with."

For our purpose, that of analyzing the diagrams, it is preferable to use the more accurate form.

element that is unchanged in amount, and let capital be the one that is supplied in a succession of increments.

Aⁱ Bⁱ (Fig. 2) is now the product gained by using one investment of capital in connection with the whole working force. Aⁱ Bⁱⁱ is the additional product that is created by a second increment of capital. Aⁱⁱ Bⁱⁱⁱ is the product of the third increment, and A^{iv} C is the amount produced by the last. This amount, A^{iv} C, fixes the rate of interest. No one of the series of units of capital can secure for its owner more than the last one produces. If the owner of the first increment asks more than this for the use of it, the *entrepreneur* will relinquish this bit of capital and will put the last unit in its place. What he will lose, in the way of product, is measured by the amount A^{iv} C, the direct product of the final increment of capital. This expresses the *effective* product of every increment, since it is the amount that would be lost if any one of the series were withdrawn.¹

Professor Clark maintains that the final areas, A^{iv} C, of the figures will measure or express the actual wages and interest in a "static" condition of society, and that these are the standards toward which the rates are ever tending in a dynamic condition.

As real as gravitation is the force that draws the actual pay of men *toward* a standard that is set by the final productivity law. This law is universal and permanent; everywhere it will outlive the local and changeful influences that modify its operation. We are to get what we produce—such is the dominant rule of life; and what we are able to produce by means of labor, is determined by what a final unit of mere labor can add to the product that can be created without its aid. *Final productivity governs wages. . . .* The standard of wages thus attained is a static one. So long as the labor and the capital continue unchanged in amount, and produce the same things, by the same processes, and under an unchanging form of organization, wages will continue at the rate that this test establishes. . . . Every unit of capital can secure for its owner what the last unit produces, and it can secure no more. The principle of final productivity, in short, acts in two ways, affording a theory of wages and of interest.²

Profits are entirely excluded in a static state, according to Professor Clark, and the total product is divided between interest and wages, following the law of his formulæ. The question naturally suggests itself, how is provision made for the replacement of the capital consumed in the industrial process? While Professor Clark establishes a distinction between concrete and abstract capital, claiming that it is only the former that is

¹ Pp. 181-183.

² Pp. 180, 181, 187.

consumed, it must be remembered that industry is carried on by concrete agents, and that it is by the very re-creation of these concrete agents that the fund of abstract capital is maintained; so that the productive process must provide for the rehabilitation of the concrete capital. The exposition, to be complete, should show that the total product covers this element of consumed concrete capital. However, the investigation of Professor Clark's theory is in no wise affected by this consideration. It may be taken for granted that the capital must be maintained, and that the total area of the diagrams represents the residuum after this rehabilitation is provided for. We shall speak of the total product, then, as if this element were eliminated, as if the net product, so to speak, obtained by deducting the portion necessary to replace the capital, were the total product.

The position of Professor Clark, then, is that the marginal product A^ivC (Fig. 1) determines, in a static state, the wages of labor; that if, upon the employment of the last unit of labor, this product, A^ivC , appears, then indeed is A^ivC the static rate of wages, and the rectangle AC is the total share of wages. This rectangle is evidently equal to as many times the marginal product as there are units of labor employed. This area, AC , represents what Professor Clark calls the *virtual product*¹ of labor; and in a static state it measures the wages of labor, and in a dynamic state it is the standard toward which the wages are ever tending.

From Fig. 2, the capital diagram, we find in like manner that the rectangle AC is the virtual product of capital, and is likewise the standard toward which interest is ever tending, which standard is realized in a static state. It is plain that the total areas of the two diagrams are equal, since each one represents the product of the total forces of both capital and labor. For the validity of professor Clark's formulæ, it is necessary, moreover, that the two rectangles, AC of the labor diagram and AC of the capital diagram, have together the total area of either

¹ P. 55. "The theory of catallactics has to prove that the income of the one class that labors and that of the other which furnishes capital is, in each case, its virtual product."

diagram ; that is, that AC of the labor diagram, be equal to the broken area lying above EC of the capital diagram, and likewise that AC of the capital diagram be equal to the broken area lying above EC of the labor diagram. It was incumbent on Professor Clark, in substantiation of his theory, to show that these equalities hold. He seems to take it for granted that this is the case, as he nowhere undertakes a proof.¹ There is certainly no *a priori* consideration to establish the equality of these areas.

A^{iv}C (Fig. 1) is an *excess* product; it is not even the *actual* product of the last unit of labor working in connection with its quota of capital. This last unit creates a product greater than A^{iv}C, as Professor Clark himself distinctly states.² The point is that while the final unit of labor is just as efficient as any other unit, the distribution of the capital over a larger working force reduces the efficiency of every unit of labor. Instead of AⁱBⁱ now representing the efficiency of the first unit of labor, and AⁱBⁱⁱ the second unit, and so on, the entire figure must be divided by five, and the quotient will represent the efficiency of any of the five units of labor, each working with one fifth of the entire capital. A^{iv}C simply represents the *increase* of efficiency shown by five units of labor working with five units of capital beyond that shown by four units of labor working with the same five units of capital. It is, indeed, the excess product realized upon employing the final unit of labor, and this is its sole principle of association with that final unit. The same is true, of course, in the case of the marginal product of capital, represented in Fig. 2. But what is the *a priori* evidence, from these circumstances, that these two "virtual products" are together equal to the total product? Professor Clark does not vouchsafe to reply to this question beyond what is presently to be spoken of.

It is our task at present to show that there is a disparity between the areas which Professor Clark assumes to be equal. In

¹ At page 330 Professor Clark makes a statement on this point which he seems to regard as a proof. We shall have to notice this statement in another connection.

² P. 323. "The essential fact is that the new working force and the old one share alike in the use of the whole capital, and with its aid they *now* create equal amounts of product." See also p. 176.

setting out upon this task, it is desirable to have the conditions of the problem clearly before us. We start with a definite quantity of capital and a definite force of labor, which, for convenience of illustration, we divide into five units in each case. First, we take the case of the entire capital working with one unit of labor, and a certain product is the result, which is represented by the rectangle $A B^i$ (Fig. 3).

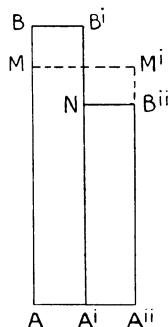


FIG. 3.
(Labor Diagram.)

Now, two units of labor working with this same capital will create a product not twice as great as this first rectangle, generally speaking, but a product somewhat less than a double one, say the rectangle $A M^i$. How much greater is this rectangle than $A B^i$? Evidently, if the small rectangle $N M^i$ is equal to $M B^i$, we have the difference between the two products represented by $A^i B^{ii}$. That is, the second unit of labor working with one half the capital creates not the product $A^i B^{ii}$, but the product $A^i M^i$; and the first unit of labor creates no more, since its efficiency is reduced by sharing the capital with the second unit. The two units together create a product in excess of $A B^i$, that is, in excess of the original product of one unit of labor working with the entire supply of capital, by the rectangle $A^i B^{ii}$. By continuing this process we build up Fig. 1 (page 162) as a complete labor diagram, the final rectangle $A^{iv} C$, representing the excess product created by $5C$ and $5L^1$; that is, representing the difference between the total product of $5C$ and $5L$, or the total area of the diagram, and what had been the total product of $5C$ and $4L$ before the last unit of labor entered the combination.

Likewise we may build up the capital diagram. That is, we may put the whole labor force to work with one unit of capital, and then add successively the remaining units of capital. At this point the crucial part of the theory presents itself. Let us ask, before setting down a single division of this capital diagram,

¹ $5C$ meaning five units of capital; $5L$, five units of labor.

what we know of it, *a priori*. Evidently we cannot say beforehand that the first division will equal the first division of the labor diagram, as we have an entirely new combination of operating forces to deal with. In the case of the labor diagram we started with $5C$ and $1L$. Now we start with $5L$ and $1C$. What will be the size of this first division? We do not know beforehand — that is to say, we cannot infer anything from any of the circumstances of the labor diagram. We can know only by actual experiment. If this is true in the case of the working combination $5C + 1L$ it is also true of $5C + 2L$, and $5C + 3L$, and $5C + 4L$. Do we know what will be the result with $5C + 5L$? Yes, we do know this result, for we had the same combination in constructing the labor diagram. This combination represents the total force of both capital and labor, and the total area of the capital diagram will equal the total area of the labor diagram. And this is all the definite knowledge we have, *a priori*, of this capital diagram. We know the area of the complete diagram. This area may be represented by the rectangle AC^i (Fig. 4). Of course, the figure must be reconstructed into a diagram representing diminishing returns, as in the case of the labor diagram, since the total labor force working with two units of capital will, generally speaking, accomplish somewhat less than twice the result achieved by this same labor force working with one unit of capital; and so on, down to the last unit of capital. We must then reconstruct this figure.

But what shall be the size of the various divisions? Actual experiment, of course, will tell us; but we know beforehand only that the total area must be the same as that of the large rectangle AC^i .

In how many ways can we construct this figure so as to satisfy this single condition? In an infinite number. For instance, we may extend the first division above the line MC^i , as far as the last division falls short of the same line. If we

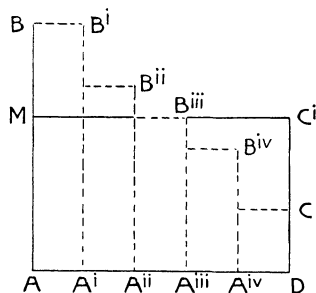


FIG. 4.
(Capital Diagram.)

construct divisions two and four likewise, we shall have a figure, bounded on the upper side by a broken dotted line, with the same area as the large rectangle AC^i . This is one of an infinite number of forms that the capital diagram may assume, and satisfy the only *a priori* condition, namely, that the entire area shall equal that of the labor diagram. The final division, $A^{iv}C$, may be varied at will (of course, within certain limits); but to be available for Professor Clark's theory, this final division must be a definite magnitude—that is, it must be such a magnitude that, when multiplied by the number of capital units, the area thus obtained shall, with the labor rectangle AC equal the entire area of either diagram. Professor Clark nowhere undertakes to show that this final division of the capital diagram will be such a magnitude, and as the only *a priori* condition is satisfied by any one of an infinite number of magnitudes, this neglect is fatal to the validity of his formulæ.

Great stress is laid in Professor Clark's volume on the independence of the two shares, interest and wages. For instance:

This point is of much consequence. The question at issue is nothing less than whether any static income is determined residually. Clearly it is never so determined. No static income is what it is merely because the deducting of another income from the social product leaves a remainder.¹

The two diagrams are, indeed, independent expedients that yield certain results. If the formulæ were valid, that is, if the the two actual products thus ascertained would tally with the entire product, the methods would be both adequate and independent. The two processes of arriving at the two virtual products are surely independent; and, if the results thus obtained, if the areas of the two rectangles AC were together equal to the total, the adequacy and the independence would both be established.

As a matter of fact, the second share *can* be ascertained residually. We have the entire product, and if we determine one share by the diagram method, for instance, if we determine the share of labor to be AC (Fig. 1), then, as the entire figure

¹ P. 202.

represents the sum of the two shares, the difference between the total area and the area of A C must be the share of capital. This share is thus *ascertained* residually. This residual share may be divided by the number of units of capital, and a result obtained which may be called the virtual product of the final unit of capital. Thus determined, however, it is not an independent share. To be independent, we must be able to find it quite apart from the labor diagram. A certain magnitude *is* found by means of Professor Clark's capital diagram, but he fails to show that this magnitude is the same as the one determined residually, and this omission constitutes the fatal hiatus in his argument.

Grant for a moment that it is possible to establish the equality of these two magnitudes. Let us see where such a concession leads. It means, first of all, that the marginal product of capital is given by the labor diagram. Now, if we deduct this marginal product from the entire product, there remains the product that represents the efficiency of $5L + 4C$. This is a remarkable result. The labor diagram, in each of its sections and combinations of sections, represents the efficiency of $5C$, working with a varying labor schedule. From such combinations of labor and capital, always involving $5C$, we can deduce the efficiency of a combination involving $4C$. That is, from the product of the combination $5C + 5L$ we may compute the product of $4C + 5L$. This latter product will be represented by that part of the capital diagram (Fig. 2) which remains after cutting off $A^{iv}C$, the final division.

If we can deduce the result of the combination $4C + 5L$ from that of $5C + 5L$, we may readily drop the units of capital, one by one, and get successively the results of the combinations $3C + 5L$, $2C + 5L$, $1C + 5L$; and thus we shall be enabled to construct the capital diagram entire, only in the reverse order, if we have given the labor diagram.¹

¹ Some curious results follow from a legitimate manipulation of the assumption that the marginal division of one diagram must tally with that determined residually. For instance, if we start with the labor diagram which represents the product $5C + 5L$, our assumption gives us the product of $4C + 5L$. Now alternating from capital

According to Professor Clark's exposition, we may have distinct static states without number, the varying circumstances having reference to diverse rates of interest and wages, and to differences in population or capital. Also the unit of capital, and that of labor are not restricted to any specific magnitudes.¹ It is quite possible that among these infinite sets of combinations, there may be one that will fit into the requirements of the formulæ—that is, a case in which the final division of the capital diagram will have by actual experiment the magnitude which is its *a priori* requirement. But to establish Professor Clark's theory, this must be the case in every set of circumstances.

Let us apply the test of a varying labor force. Suppose with an additional unit of labor we have an excess product, DD^i (Fig. 5). Then, if Professor Clark's theory is to hold, the final increment of the correlate capital diagram must, of course, conform to this change in the labor diagram. The virtual product of labor is now six times DD^i . That of capital must be the remainder of the total. The portion of the figure above the dotted line, $E^i D^i$, will represent this remainder. The marginal product of capital must be the quotient obtained by dividing this remainder by the number of units of capital. But, as before, the capital diagram, constructed independently, has but one *a priori* condition, and that concerns the total area. The marginal division may be any one of an infinite number of

to labor diagrams, we can get, successively, the products of $4C+4L$, $3C+4L$, $3C+3L$, $2C+3L$, $2C+2L$, $1C+2L$, $1C+1L$. At this point we have a single division wholly attributable to capital, or wholly attributable to labor, according to the diagram that represents it. If it be represented by a labor diagram, the correlate capital diagram vanishes, which would mean that if labor should come on the scene first, in which case the combination would appropriately be represented by $1L+1C$, no product would result. Likewise, if we represent the efficiency of the single units $1L+1C$ by a capital diagram, the correlate labor diagram (combination $1C+1L$) vanishes, and this combination would be barren.

¹P. 23. "At one time there is one standard of value, wages and interest set by static forces. . . . At a later time it will be found that the standards themselves have undergone a change." P. 174. "We will let a thousand workers constitute each increment of labor."

magnitudes, but Professor Clark's theory demands that the one possibility out of an infinite number must happen also in this case. Likewise this must happen each time we add a unit of labor. As the increments of product continually diminish, we may suppose that there is such a surplus of labor that eventually there will be no increment. This is a permissible supposition, theoretically, since the conditions of the organization of industry may be such that eventually the total product can not be increased by an additional unit of labor. As we are discussing a question of material product, and not one of wages, the latter question need not embarrass us at this point. Suppose, then, the final division of the labor diagram is zero in area. This means that the total virtual product of labor is zero, and the labor diagram terminates in a straight line, indicating no area.

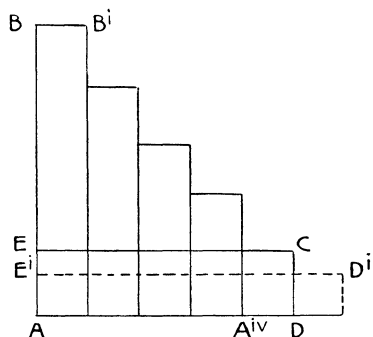


FIG. 5.
(Labor Diagram.)

What must be the character of the correlate capital diagram? Since no product is attributable to labor, the whole product must be imputed to capital, and the final division of the capital diagram must be as great as any other division, and the capital diagram will be a series of equal rectangles (Fig. 6).

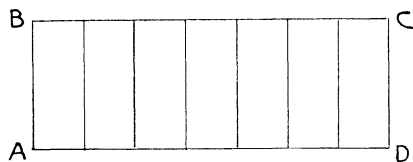


FIG. 6.
(Capital Diagram.)

The significance of this diagram is that the doctrine of diminishing returns is eliminated. The excess product of the final unit of capital is the same as that of any prior unit. Will actual experiment ratify this showing? If we set the entire labor force to work, and apply the capital, unit by unit, our diagram will show varying returns. It is quite possible that the diagram will show increasing returns

in the initial stages, but, as unit after unit of capital is added, eventually diminishing returns will be the law, and we will build up a diagram like Fig. 7.¹

The only relation that this diagram has to the correlate labor

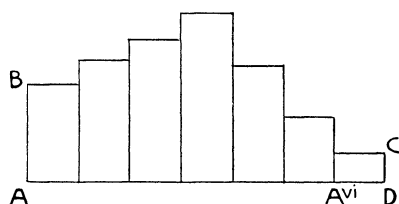


FIG. 7.
(Capital Diagram.)

when the whole force of capital and labor is at work again, the result achieved in the former case, by the same aggregate force, will be duplicated.² That is, the total area of the complete capital

diagram is equal to the total area of the complete labor diagram.

Let us see if Fig. 7, with its varying returns, may be manipulated so as to furnish the virtual product required by Professor Clark's formula. Perhaps the marginal area $A^vi C$ is the average area of the various divisions, and consequently the final product, taken as many times as there are units of capital, would tally with the total product. We will discuss, presently, this admissible case of increasing, followed by diminishing, returns from another standpoint. It need only be said at present that to postulate the final division of the capital diagram as an average division is to postulate one out of an infinite number of possibilities, any

¹ This law of increasing, followed by diminishing, returns is approved by Professor Clark, p. 163: "It is, of course, true that, if two men can combine their labors so as to assist each other in essential ways, such a diminution of their specific productivity may not appear. Two men may make possible a rudimentary organization of labor; and this is a new influence, of which a full study must take account. . . . A third, a fourth, and a fifth man might contribute to the perfection of the organization, and so hold somewhat in abeyance the law of diminishing returns that we have cited; but in the end the law would assert itself."

² It is true that diverse organizations of the same aggregate labor and capital would achieve diverse aggregate results, but, in this discussion, we assume the most efficient organization in every case, so that organization does not change unless the factors of industry, labor and capital, change in amounts, one or both. A new combination of labor and capital, such as $4C + 5L$, varying from any combination involving $5C$, would call for a special organization, and its efficiency would be known from trial only.

one of which is compatible with the only *a priori* condition of the problem, namely, that the total area shall be a specific magnitude.

If the objection be made that the efficiency of a marginal unit of labor can never vanish, however numerous the units may be, the reply is always that the marginal product does not represent the actual product of the marginal unit. The marginal unit actually produces as much as any other unit. The marginal product expresses the difference between the efficiency of the total capital working with the total labor force, on the one hand, and the efficiency of these same forces diminished by one unit of labor. A specific quantity of capital may easily be conceived to have reached its maximum efficiency with a certain quantity of labor, the efficiency diminishing with either a decrease or an increase of labor.

But let the objection stand for a moment. Let it be conceded that with a definite quantity of capital, the efficiency will continually increase with additional labor, so that the final division of the labor diagram shall remain a finite magnitude. Instead of increasing the labor force, we shall diminish it and shall show that we have the same predicament of a zero marginal product to face. Let us reduce the labor by one unit. The product of $5C + 4L$, as shown by Fig. 1, is evidently the sum of the first four rectangles. One by one, dispense with all but one unit of labor, and we have the product of $5C + 1L$ represented by the rectangle AB^1 , the first division of the diagram. According to the formula, this product must be imputed wholly to labor.¹ This first division, AB^1 , may be taken as a complete labor diagram, representing the product of five units of capital

¹ The whole product must be imputed to labor, because the single unit of labor is the final as well as the initial unit. The product, then, appears in its entirety upon the employment of this final unit. Professor Clark notices this case at page 195: "When there was available only a piece of land, with no labor to till it, the product was *nil*. When one unit of labor combined itself with the land, the product was AB^1 ; and in this form of statement we impute the whole product to the labor. . . . In the case of the first increment of labor, we might, by different dialectics, attribute the whole product to the land. Labor by itself creates nothing, and the addition of the land brings the whole product into existence."

and one unit of labor, the five units in the one case, and the one unit in the other, being the total force. AB^i , being the marginal product as well as the total product, must be attributed wholly to labor. Now, if the whole product is the virtual product of labor, it follows necessarily that the virtual product of capital is zero, and the correlate capital diagram must show this result by terminating in a division of no magnitude.

How may this predicament be met? It may be said that the one unit of labor which creates the initial product is an arbitrary unit, say, of 1000 men, that the initial division of the labor diagram represents indeed the total product of this body of men; but if analyzed, this product will show diminishing returns, so that the whole product is not strictly attributable to labor. Convenience of statement precludes the analysis of the labor force into minute units, and we must start in our illustration with a total product apparently attributable wholly to labor, but which, by further analysis, would prove partially imputable to capital. Grant that this is so. Then let us analyze this first unit. Decompose it into its individual constituents. However small we take the unit, we shall begin with this unit, and let it work with the total supply of capital. By the diagram method, the product of this combination is attributable wholly to labor—is labor's virtual product, and consequently the virtual product of capital is *nil*; so that the correlate capital diagram, in this case, must show a final division of no area.

Attention has been called to the circumstances that for a time occasion increasing returns.¹ Let us revert to this case.

We start with a large supply of capital and draft the labor force so that the second unit will show a marginal product superior to the first unit. A still larger return comes with the third unit. Let us say that this increase of marginal product will be the law up to the fifth unit. Our labor diagram will build up just the reverse of Fig. 1, say like Fig. 8.

If we terminate the diagram with this fifth division, we have the result of the combination of capital with five units of labor.

¹ Pp. 171, 172.

Suppose the five units of labor are the total force. Then we have a complete labor diagram, and we have to determine the virtual product by Professor Clark's formula. Evidently, in this case, the virtual product of the five units of labor is represented by the rectangle, AC, and is greater than the total actual product by an amount represented by the area above the broken line. What becomes of the virtual product of capital in this case? If we determine it independently, we shall begin with the total labor force, adding one unit of capital at a time, and obtain a marginal product, which will be at least as great as zero. If actually zero, the disparity between the sum of the two virtual products thus determined and the total actual product is the area above the broken line of the figure. If the marginal product of the capital be found greater than zero the disparity between the areas that should be equal is still greater.

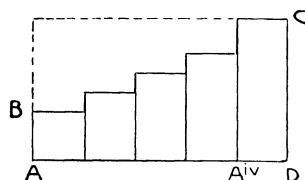


FIG. 8.
(Labor Diagram.)

Let us now determine the virtual product of capital by the residual method. The remainder of the area representing the total actual product, after deducting the rectangle AC, the virtual product of labor, is the area lying above the broken line considered negatively. This would give the marginal product of capital, by the residual method, a magnitude less than zero.¹

This paramount question of the equivalence of the two rectangles AC of Fig. 1 and Fig. 2, on the one hand, and the whole of either figure, on the other, has, as we have said, been practically ignored in Professor Clark's exposition. We have

¹ It may be argued, and justly, that such a result is not necessarily inconsistent, since one can conceive of such an adjustment of capital and labor that the reorganization, occasioned by the addition of a unit to the labor force, would result in a product less than the former product. This would involve a marginal product negative in character. While this is true, it would not establish the validity of Professor Clark's formulæ, unless the negative magnitude were just such as would appear by using the process of the residual method. Besides, the negative marginal products are within the bounds of possibility for both capital and labor at the same time. This will be made clear in the text, as we discuss the problem from a different standpoint.

alluded to a statement in his volume on this point which has the form of a demonstration. It runs as follows:

Now, in Fig. 1, EBC is all that is left of the entire product that is not produced by labor. If AECD, of the second figure, is as large as EBC, of the first, this amount, EBC, is the product of capital; since the rectangle AECD is certainly the product of capital. We know that, by our hypothesis of perfect competition and a complete static adjustment, there is no profit realized by the *entrepreneur*, as such; and the figure ABCD cannot contain more than wages and interest. The amount EBC is, therefore, not larger than is AECD of the second, and all of EBC is the product of capital.

This proof, of course, rests upon the assumption that the two rectangular areas AECD of the two diagrams, taken together, are equal to the total area of either diagram. Under this assumption the result is a case of simple subtraction as follows: AECD (Fig. 1) + AECD (Fig. 2) = AECD (Fig. 1) + EBC (Fig. 1). Subtracting AECD (Fig. 1) from both sides of this equation, we have AECD (Fig. 2) = EBC (Fig. 1), the equality sought to be established. However, the second equation depends upon the first, so that if we have succeeded in discrediting the one, the sole support of the second disappears.

In the foregoing discussion the attempt has been to show that there is no ground for assuming that the sum of the virtual products of capital and labor, as independently determined by the diagram method, tallies with the total product, which is represented by the total area of either diagram. It remains to show that these virtual products, as thus determined, will vary with the magnitudes taken to represent the unit of capital and the unit of labor. There is nothing in the circumstances of the problem that suggests units of any specific magnitudes, and certainly Professor Clark has not stipulated that the unit of capital, or that of labor, shall be of a special size. He distinctly states the contrary.

And whether we take a single man or a body of men as the unit of labor, *any unit can get, as pay, what the last one would produce, if the forces were set working in this way.*¹

¹ P. 181.

We may put this statement to a ready and simple test. Let us, for convenience, take an even number of units, say six, to represent the labor force. Our labor diagram then will take the form of Fig. 9.

Let us see what the result would be, if we halve the magnitude of the unit of labor, thus doubling the number, without changing the total force. In this case unit one becomes one and two, and so on down to unit six, which becomes eleven and twelve. With ten units of labor employed the product is the same evidently as when five of the double magnitude were employed—that is, we

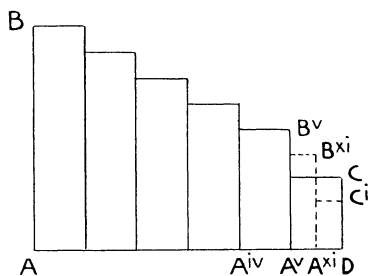


FIG. 9.
(Labor Diagram.)

would have the product $A B B^v A^v$. We will test the statement of Professor Clark with units eleven and twelve, although it could be tested as effectually at any other point. When units eleven and twelve were a single unit six, the marginal product was the rectangle $A^v C$; but with our new schedule of units the marginal product depends on just half of the former unit six, namely, the unit twelve. Unit eleven brings a marginal product, and unit twelve brings a distinct marginal product; and, coming into the combination later than number eleven, its marginal product, according to the law of diminishing returns, will be less than that of eleven. What will be the relation of these two marginal products to that of former unit six—that is, to the rectangle, $A^v C$? Evidently the sum of these two margins will be $A^v C$; but the marginal product of unit eleven being greater than that of unit twelve, the former will be greater than one half of $A^v C$, and the latter will be less by the same amount. That is, the marginal product of unit eleven will be something like the rectangle $A^v B^{xi}$, and that of unit twelve like $A^{xi} C^i$. The law of diminishing returns necessitates this discrimination in their magnitudes.

Now, what is the virtual product of labor, as determined by

our modified diagram? Evidently, the rectangle $A^x C^i$ multiplied by 12. But this gives a less result than $A^v C$ multiplied by 6, since $A^v C$ is more than twice $A^x C^i$, while 12 is just twice 6. It is plain, then, that diminishing the magnitude of the unit diminishes the virtual product of labor. The same thing is true in the case of capital. So we have by the law of Professor Clark's formulæ virtual products constantly diminishing with the size of the units taken.¹ Let us proceed with the division of the unit indefinitely. It is easily conceivable that we may arrive at a point in the halving process, where the second half of the terminal unit will give a negative marginal product, the first half giving a product greater than the entire unit, owing to circumstances of organization of capital and labor over the whole field. We thus have the case referred to in note 1, page 175.

Now, instead of diminishing, let us increase the magnitude of the unit, by combining two units into one. Referring to Fig. 9, let us suppose that units five and six are consolidated into a single unit. In this case the marginal product of units five and six will be the sum of the rectangles $A^{iv} B^v$ and $A^v C$. But this sum is greater than twice $A^v C$, the former marginal area. The number of the units being just half, the virtual product, under this schedule, will be greater than that under the original schedule. This marginal product may be increased at will, by proceeding with the consolidation of the units. The same will be true in the case of capital; and finally, if we consolidate all the units of labor into a single unit, and those of capital into a single unit, we shall have virtual products each of which is the entire product.²

¹ Professor Clark, in note at page 195, analyzes the initial unit of labor, and takes notice of this very contingency of a diminishing virtual product, but fails to realize the significance of it. He says: "Again, by subdividing the one unit of labor into a series of smaller units, we might attribute the product partly to the labor and partly to the land."

² It is evident that by a proper manipulation of the units, we can create a schedule of virtual products, the sums of the sets constituting a descending scale, the maximum sum being twice the area of either diagram, and the minimum at or below zero. This

II.

The marginal products represented by the final divisions of Figs. 1 and 2, are designated by Professor Clark as specific or virtual products. In each case it is the virtual product of one unit ; and in each case the total virtual product is this marginal product multiplied by the number of units at work. The rectangles AC of the two diagrams represent respectively the total virtual product of labor and the total virtual product of capital.

Let us inquire into the logic of this nomenclature, "virtual product." Why is this final area taken to represent the virtual technical product of every unit of labor ? In what way is this area associated with one labor unit ? It is, confessedly, not the *actual* product of the final unit of labor, as all units have equalized efficiency. It is not the actual product of the last unit employed, in conjunction with its quota of capital. As has already been pointed out, this final area represents an *excess* product, not an actual product due directly to any specific combination of labor and capital. It is the excess result of $5C + 5L$, over what had been accomplished by $5C + 4L$. Here is, indeed, an indirect association with one unit of labor, since $5C + 5L$ varies from $5C + 4L$ by $1L$, one unit of labor. Is, then, this marginal product, this excess product, the *virtual* product, because of this indirect association ? But, by a similar expedient, we can associate the *first* division of the labor diagram, or any given division, with one unit of labor. For instance, the last four divisions of the diagram represent the difference between the result of the combination $5C + 5L$, the total product, and the result of $5C + 1L$; the latter schedule will include a set that will give the sum demanded by Professor Clark's formulæ. Indeed, with any specific marginal product on the part of either capital or labor we can, by the proper manipulation of the units of the other factor, obtain the requisite marginal product. This would be a practical problem, and could be solved only by actual experiment. There would be a brace of difficulties that would probably discourage the attempt, however. (1) It would be necessary that society should tender the use of her capital and labor for the experiment ; and (2) if absolute mathematical accuracy were required, it would necessitate an infinite number of experiments. In the realm of theory, however, all such difficulties vanish.

combination being the initial one that created the product represented by $A B^i$. Since $5C + 1L$ differs from $5C + 5L$ by $4L$, we can associate $4L$ with the last four divisions of the diagram, thus leaving $1L$ to the first division. Or we may associate the $1L$ more immediately with the first rectangle of the figure by considering that this first rectangle is the initial marginal product. It is the product realized upon setting to work the first unit of labor. It is the result of the combination $5C + 1L$. Now $5C$, the capital alone, can create no product. $5C + 1L$ creates the product $A B^i$. But $5C + 1L$ differs from $5C$ by $1L$; hence we may associate $1L$ with the first division of the figure. Again, the marginal product $A^{iv}C$ is an *excess* product that appears when the last unit of labor is set working, as the author explains. The initial product $A B^i$ is a *residual* product that remains when all but one unit of labor is dispensed with. So we have an indirect association of one unit of labor with the first division of the diagram as well as with the final division. Similarly, we could associate a unit of labor with any division.

Of course it will be said that it is absurd to call the first division of the diagram the virtual product of one unit of labor, as that would make the total virtual product of labor exceed the entire product. So let us examine the diagram showing increasing returns, p. 175. Shall we call the final area here the virtual product of one unit of labor? It is the marginal product, in the orthodox sense, that is to say, the final marginal excess product. But that would give to labor a total virtual product greater than the total actual product. It would be less absurd in this case to let the initial area represent the virtual product of a unit of labor.

Why, again, should we take either division to represent the virtual product? Is there any direct association of either of these divisions with one unit of labor, disengaged from capital? Not if we confine our study to mechanical, or material, products. There *is* an association, a very direct association, if either of these areas represents the *wages* of one unit of labor. In this case we may, indeed, call this area, $A^{iv}C$ (Fig. 1), the

virtual product of one unit of labor. It measures, or rather it *is*, the wages of one unit of labor. Labor is entirely disengaged from capital in this area, if we regard the area as a value product. It is the virtual technical product of one unit of labor, because it is the remuneration of one unit of labor. It is the virtual product because it is the value product.

Perhaps this view will be more convincing if we embrace the correlate virtual product in the same statement. We have seen that we cannot take both marginal products simultaneously to represent the two virtual products. Their sum does not tally with the total product. If in any instance the marginal product of labor be taken as the virtual product of one unit, the virtual product of capital must, in that case, be determined *residually*; so that the virtual product of capital, from the mechanical side, would have no definite relation whatever to a unit of capital. We are absolutely obliged to seek this association from the value side. The residual area EBB^iC must be the virtual product of capital if the remainder of the area AC be taken as the virtual product of labor. But EBB^iC has no definite quantitative relation, from the mechanical side, to one unit of capital, direct or indirect; yet if AC is the wages of labor, then EBB^iC must be the remuneration of capital, and it is the virtual product of capital because, and only because, it is the value product of capital.

It is evident that the converse is true also. That is, if the rectangle AC (Fig. 2) be taken as the virtual product of capital, then must the residual area EBB^iC (Fig. 2), and not the area AC of Fig. 1, be taken as the virtual product of labor. The virtual products are so called because they are value products. This is the logic of the nomenclature, and this nomenclature may stand whatever the interest and wages may be and whether profits are an element of the division or not. Virtual products are value products and must be proportional to the wages, interest, etc. That is to say, if we identify virtual products with the products that represent the rewards of the various factors of industry, then this must be the principle of the identification.

III.

An important circumstance in the relation of capital and labor confers a permanent and significant advantage upon the latter. This is the circumstance that the unit of labor, the ultimate unit, is an individual man; while that of capital is impersonal. The advantage gained here is due to the fact that the laborer is personally interested in the rate of wages, that is, the pay to the individual, and he has no personal interest in the *gross* amount of wages paid to the whole body of laborers. On the other hand, the capitalist, who may own an indefinite number of units of capital, has a personal interest in the *gross* remuneration to capital, as well as in the rate, or the quantity paid per unit. So 5 per cent. of \$50,000 is \$2500, while 4 per cent. of \$100,000 is \$4000. The capitalist would prefer the investment of \$100,000 at the lower rate to a restricted investment at the higher rate. Hence he is personally interested in the expansion of his capital, even at the expense of a diminishing rate.

IV.

The theory of Professor Clark is that a static state sets the standard of interest and wages. Let us see whether a "static state" can effect any change in interest and wages. Of course, this will depend on definitions. What is a static state? Professor Clark expresses his conception of a static state perhaps as succinctly and clearly as anywhere in the following passage:

What would be the rate of wages if labor and capital were to remain fixed in quantity, if improvements in the mode of production were to stop, if the consolidating of capital were to cease, and if the wants of consumers were never to alter? The question assumes, of course, that industry shall go on, and that, notwithstanding a paralysis of the forces of progress, wealth shall continue to be created under the influence of a perfectly unobstructed competition. The values and the rates of wages and interest which, under such conditions, would prevail, are those to which, in spite of all disturbances that progress occasions, the rates in the actual market tend, at any one time, to conform.¹

¹ Preface, pp. vi, vii.

The essential elements of this conception of a static state are: (1) stability of labor; (2) stability of capital; (3) stability of modes of production; (4) stability of organization of capital; (5) stability of consumers' wants; (6) unobstructed competition. It is (6) of this schedule that invites controversy. Why should unobstructed competition, or competition under any qualification, characterize a static state?

If we examine the various passages in Professor Clark's text on this head, we find some confusion. For instance: "In that static condition in which competition would produce its full effects and bring wages to a natural standard, etc."¹ Elsewhere, the competition seems to keep the wages and interest at the proper standard, as well as to bring them there: "With an ideally complete and free competitive system, each unit of labor can get, etc."² But this competition that continues after the static standards are realized is such a "perfect competition" that it ceases to manifest itself. It is described as not being competition at all: "A natural price is a competitive price. It can be realized only where competition goes on in ideal perfection—and that is nowhere."³ That is to say, perfect competition ceases to be competition.⁴ But whether it is proper to identify the two conceptions, "perfect competition" and "no competition," is not specially important for our purpose. The important thing to note is that there are actually two types, or degrees, of static state, namely, one in which imperfect competition prevails, and a second blessed with perfect competition, the latter being the period after the standards of wages and interest have been attained; the former being the period that witnesses the struggle to reach those standards.

The two kinds of competition may perhaps be better described

¹ P. 95.

² P. 179.

³ P. 77.

⁴ The confusion arises from confounding liberty of competition with the actual exercise of the privilege. One would hardly commit this error in ordinary relations. The privilege to dissipate would hardly be considered equivalent to the practice. Professor Clark explains that the absence of a motive for competition would lead to its extinction. Possibly, but that would hardly justify designating this non-existent competition *perfect* competition.

as *active* and *latent*. The matter of terminology is not important. It is the distinction that has significance. This distinction establishes two degrees, or classes, of static state. Why is competition brought into the static state? It is necessary to have some agency that can level the rates of interest and wages to the proper standards. A dynamic state manifests a *tendency* to attain these standards, but it never succeeds in reaching them. At least, if it does succeed, it can be only momentarily, since the dynamic forces are continually active to cause divergences. When the dynamic changes are supposed to cease, we cannot be sure that the rates of interest and wages are "normal." Hence some agency must be enlisted to perform this service. Clark assigns this function to competition. But why should not one of Clark's dynamic forces discharge this office? Fluctuation, either on the part of capital or on the part of labor, would answer the purpose. If interest is too high in comparison with wages, increase of capital will lower it. *Vice versa*, decrease of capital will raise the rate. Fluctuations in population will serve the same end. If it be said that fluctuations in population involve changes in organization, let it be so; let there take place whatever dynamic changes are required to adjust the rates properly.

Besides, it is difficult to see how reorganization can be avoided under the rule of competition. Clark claims that profits are to be eliminated by competition. If so, reorganization will take place; and whether this elimination does or does not take place, if there is to be an adjustment of interest and wages, changes in organization will be part of the means by which this is accomplished.

If the objection is made that fluctuations in capital and labor are dynamic forces, while competition is not, then what is meant by a dynamic force? We are concerned with normal scales of interest and wages. It is dynamic forces that baffle the pursuit of these standards. Fluctuations in capital and labor cause constant fluctuations in wages and interest. Therefore they are dynamic forces. But competition of the active kind induces these same fluctuations. Consequently competition is a dynamic

force ranking with the others. To postulate static conditions involves the exclusion of competition.

It is easy to see that if we eliminate competition in addition to Clark's five dynamic forces, wages and interest will that moment attain fixity. No matter what rates are prevailing, upon the suspension of all dynamic forces those rates will assume a character of permanency. The point is that all changes of interest or wages are dynamic changes, and a static state is incapable of setting a standard because it is incapable of creating one. Rates are made and altered under the régime of dynamic conditions, and not under that of static ones.

V.

We have seen that the rate of interest and the rate of wages cannot be identified with the marginal products, for the reason that these marginal products do not tally with the total. It may be asked, further, have these rates any definite quantitative relation to specific products. Professor Clark's theory has been shown to be incompatible with the facts. However, since the basis of interest and wages lies in this productive capacity of capital and labor, it was perhaps natural to expect that the productive relation would afford not merely the basis, but a definite measure for the various rewards, of the industrial factors. It must be borne in mind, however, that the productive agents exercise joint efficiency rather than isolated efficiency. If such an expedient as Professor Clark's could really isolate the efficiency of the two factors, so that we could confidently say that this fraction of the product is the creation of labor alone, and that, of capital alone, we should have, at least, a tangible basis of division in the productive relation. It would, of course, still remain a question whether this principle would be available practically, as long as the co-operation of the factors continues.

To study this question of rates, let us make use of the labor diagram (Fig. 10). If A^ivC is the last excess product, the marginal product of labor, Professor Clark would say that the rectangle AC represents the standard of wages toward which actual

wages are tending — an upward tendency, because it is found that a part of the virtual product of labor goes to profits, which latter has a tendency to be eliminated.¹ It will be pertinent to ask how wages happen to be at or below the line EC. If we are to take this diagram to represent a case of real organiza-

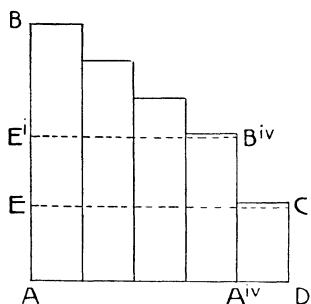


FIG. 10.
(Labor Diagram.)

tion, and not merely a device to ascertain a "marginal product," it is plain that, prior to the employment of the final unit of labor, Professor Clark's formula would fix the wages at $E^i B^{iv}$. What is left for interest is represented by the area above this dotted line. Now any reduction of wages below $E^i B^{iv}$ will inure to the benefit of capital. Evidently it will pay capital to hire the additional unit of labor and

give it even more than the marginal product $A^{iv} C$, provided the scale of wages for the entire force is reduced to the same level as that of the marginal unit. There is a margin of product represented by the rectangle $E B^{iv}$ which may be divided between interest and wages, any part of which will keep wages above the line $E C$, and therefore above the marginal product of labor. Of course, in time wages are apt to settle at or below this lower dotted line, since the last unit of labor may be alternately employed and discharged till the competition of laborers for employment brings down the rate. But in the process of change the wages line will drop from $E^i B^{iv}$ down through the space to $E C$.

The fact that the wages line is at or below $E C$ does not mean that interest is at or above the corresponding line in the capital diagram, since, as we have seen,² the virtual products of labor and capital thus determined do not tally with the entire product.

¹ "There is a profit on labor as long as the men in a working force are paid less than the final one produces; but competition tends to annihilate that profit and to make the pay of labor equal to the final unit of it." — P. 179.

² Pp. 165 *et seq.*

If competition on the part of employers and laborers is equally keen, the chances that the line of wages will approximate the line *EC* are much better than we should find to be the case if we were studying the rate of interest from a capital diagram. There is a twofold economic force to induce this state of things. We have shown¹ that wages have the advantage of interest in the fact that laborers are personally interested in the *rate*, and not in the gross amount of wages, while capital is concerned with the gross amount of its remuneration. There is the additional consideration that a vague subsistence line bars the way to a reduction of wages below a minimum point, whereas no such subsistence consideration stands in the way of an unlimited reduction in the rate of interest. Labor enjoys this advantage in the competition for the excess products, and the history of industry shows that the advantage has not been a barren one. The history of industry shows that with the growth of capital and improvements in organization the rate of wages has manifested a tendency to advance, while that of interest has had the opposite tendency. Clark emphasizes this point, with a purpose different from ours. He says :

This process represents the actual condition of industry. Improvements are, in fact, occurring so rapidly as to tread upon one another's heels. They take place in all the different groups and subgroups of which society is composed, and every one of them does its minute part toward pushing upward the standard of pay for all labor.²

We have supposed that competition is equally strong among laborers and capitalists. Of what value is such a supposition in the study of actual rates ? We have seen³ that a static state is not the field in which to study rates, since a static state is simply an instantaneous photograph of a dynamic period at any moment. The only field in which to study rates with effect is the field that represents real life. In this field the actual circumstances of competition are the ones to consider in the study of rates. For this reason the capital diagram is preferable to the labor diagram as a basis of study, since it comes nearer to being a true graphic

¹ P. 182² Pp. 406, 407.³ Pp. 182-185.

representation of actual conditions. "Capital is the element that is outgrowing labor,"¹ says Professor Clark.

If we consult the capital diagram (Fig. 2), we see that the entire marginal product due to the last unit of capital may be transferred to labor, and capital will not suffer in the *gross* amount of its reward. We have seen that the personal importance of the *rate* of wages and the existence of a subsistence minimum give an advantage to labor in the contest for the marginal portion of the product. Indeed, it is conceivable that interest should suffer a decrease of *gross* amount while capital increases. There are two circumstances that give color to this hypothesis. The first grows out of the competition among capitalists. If capital should act as a unit, of course the contingency of an increase in its amount, with a diminution in the *gross* amount of interest, would not be likely, since capital could withdraw from the market and thus restore the gross interest. But under the régime of a spirited competition, the avidity of each capitalist to secure the lion's share of the decreasing total might be the means of putting on the market a larger aggregate of capital. The second circumstance leading to the possibility of a decreasing aggregate of interest lies in the fact that capital cannot escape destruction, whether used in production or not. If destroyed outside of the productive process the economic result is the same as to diminish the gross amount of interest.² It must be remembered that the abstract capital of Professor Clark's theory can be maintained only by the productive use of the concrete forms of it. Abstract capital is perpetuated by the re-creation of these concrete forms, and these can be re-created only by putting them into service. The great accumulation of the world's capital would soon perish by non-use. For this reason capital is under irresistible pressure to find investment. It would be better for

¹ P. 183.

² That is to say, the capitalist's total economic power is measured by the sum of the capital and the interest. This sum is an indivisible whole so far as concerns his economic welfare. If this aggregate is to be diminished, it is evidently immaterial, so far as the residual amount is concerned, which of the elements, principal or interest, shows the decrease.

it to work without remuneration than to lie idle. This circumstance counts in favor of labor. And whatever the offsets to this and other advantages may be, it is evident that the advantages exceed the disadvantages for the rate of wages is rising while that of interest is falling.

But what concerns us at present is that these rates are the creatures of intense dynamic forces. They cannot be forecast by an attempt to analyze the technical product. As a question of material product it is futile and valueless to inquire into the proportion of the two agencies. As an economic question it may be studied with some success and some significance. Historically, labor in the main has preceded capital, and, as Professor Clark says, it is capital that is now outgrowing labor. The record of industry shows that capital has come to re-enforce labor, and the result has been that labor has reaped a large reward. Economically, then, capital may be credited with the excess product beyond what was achieved by labor unaided. This, as a general statement, is not specially valuable, since the history of industry is the record of labor and capital re-enforcing each other alternately. But the study may with good effect be directed to specific cases. An improvement in industry due to a specific invention or change of organization may be investigated, and a comparison of results under the two conditions may be made. The difference of results may be approximately determined, and the proper credit assigned.

The rates of wages and interest at any time are the result of the play of dynamic forces. The gross amount of these elements have a maximum beyond which they cannot pass, barring extraordinary circumstances. For instance, interest and wages together cannot exceed the entire product, what we have called the *net product*¹ of both factors. It would be difficult to assign an upper limit to wages much below this maximum, since the *rate* of interest is ever tending downward, and even the *gross* amount of it is not inviolable. But the point here is that the *rates* of both wages and interest have a dynamic quality. That

¹ P. 164.

is, they enjoy no immunity from change. They are in the hands of tireless dynamic masters, the industrious artisans, or agents, of change. These agents are, according to Professor Clark's enumeration: (1) fluctuating population; (2) fluctuating capital; (3) fluctuating methods of industry; (4) fluctuating organization of capital; (5) fluctuating wants of consumers. This group should be extended so as to include what may be called "circumstances of competition," an important agent of a very dynamic character.

The determination of rates of interest and wages is a fascinating problem, for the reason that it is a *living*, ever-changing problem. The dynamic forces which we have enumerated both dominate the actual rates and exercise control over that real but elusive center of movement, the so-called standard, about which the daily rates are ever fluctuating; for the center and the seat of the external perturbations are co-ordinate parts of one system, which is sensitive as a whole to the action of the dynamic forces. A determination of the rates of remuneration for today is not a determination of them for tomorrow, for the circumstances that control the rates are ever shifting, ever modifying the elements that must be taken account of in the solution. The chess board of the market of interest and wages proposes no stereotyped problem. The powers of analysis must be addressed to the mastery of each novel combination.

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