THE FALL OF MERCURY DROPLETS IN A VISCOUS MEDIUM.

BY O. W. SILVEY.

INTRODUCTION.

IN a recent comparison[1] of the fall of a mercury droplet in air and in xylol, it was found that if the simple Stokes's[2] equation were assumed to hold for the fall in the liquid a correction of the formula could be obtained for the fall in air, which was in fairly good agreement with the value found by Professor Millikan[3] by another method. Brownian movement measurements by Nordlund[4] on mercury particles, and a consequent determination of the Avagadro constant which agreed well with the value given by Professor Millikan showed that the simple Stokes's law was applicable to these minute particles. Nordlund[5] to verify his results, carried out an investigation of the fall of mercury drops in a mixture of glycerin and water and in 105 experiments found a variation from Stokes's law of only .076 per cent.

Although the study of the fall of solid spheres both in air and in liquids has been very carefully studied experimentally by Allen,[6] Ladenburg,[7] Zeleny,[8] Arnold[9] and others, the fall of liquid spheres has been neglected presumably because of difficulty met in measuring the radius of the drop.

Stokes[10] in his Memoirs discusses the case of liquid spheres falling in a gas and concludes that although the relative motion of the particles within the globule should be taken into account, if the globule is assumed to be preserved in a strictly spherical shape by capillary attraction, the motion will be the same as that of a solid sphere in the same medium.

[1] Loc. cit.

[2] Stokes, Mathematical and Physical Papers, Vol. III., p. 55–61.

[3] Millikan, PHYS. REV., 4, Apr., 1911, p. 349; also 2, Aug., 1913, p. 109.

[4] Nordlund, Zeitschrift fur Physikalische Chemie, 87, 1914, p. 40.

[5] Nordlund, Ark. für Mat. Astron. Och. Fysik, Stockholm, 9, No. 13, p. 1–18, 1913. Science Abstracts, 180, Vol. 17, 1914.

[6] Allen, Phil. Mag., 1900, p. 323.

[7] Ladenburg, Ann. der Physik, 32, p. 287 (1907) and 33, p. 447, 1907.

[8] Zeleny and McKeehan, PHYS. REV., Vol. XXX., 5, May, 1910, p. 535.

[9] Arnold, Phil. Mag., 22, 6, p. 755.

[10] Stokes, Mathematical and Physical Papers, Vol. III., p. 55–61.

The first work on the fall of liquid spheres was that of O. Jones,[1] who attempted to make use of mercury globules to determine the viscosity of viscous liquids. He determined the radius of his drops by weighing a rather large drop, then dividing it into smaller parts, and assumed that the droplets thereby produced were aliquot parts of the whole. His verification of the Stokes's formula was no better than one should expect from such a method of measurement.

Arnold[2] made a few measurements of the motion of alcohol globules in olive oil, and because of continual dragging away of the drop obtained too large a value for the viscosity of the oil. He recorded no results on mercury droplets because of uncertainty in the measurement of the radii. Arnold's chief interest was not in the liquid spheres but in the behavior of the solid ones.

In addition to their work on spores and wax spheres Zeleny and McKeehan studied the fall of mercury spheres in air. For this work, extremely small spheres were necessary and they too were uncertain of their measurement of the radii.

In 1911, Rybczynski[3] and quite independently Hadamard[4] derived from theoretical consideration, a modified form of Stokes's equation for liquid drops falling in a viscous medium. They assumed a vortex motion of the particles within the sphere, due to the drag at the surface, and a consequently greater speed of fall. Their correction term involves the viscosity of the liquid within the drop and that of the medium through which it falls. The modified Stokes's equation is of the form:

$$(\sigma - \rho)g = \frac{9}{2} u \frac{v}{a^2} \left(\frac{u_1 + \frac{2}{3}u}{u_1 + u} \right),$$

where a, σ, v and u_1 are the radius, density, velocity and viscosity of the falling sphere, ρ and u the density and viscosity of the surrounding medium.

Besides the work of Nordlund, an attempt was made by Roux[5] to test the Hadamard equation by allowing mercury droplets to fall in castor-oil. Since the viscosity of castor-oil is about 300 times as great as that of mercury, this provided conditions such that the Hadamard correction should have very nearly its maximum value, unless other physical conditions are introduced which are not included in Hadamard's hypothesis. Roux used comparatively large droplets, 618 μ to 905 μ,

[1] Jones, Phil. Mag., 37, p. 451, 1894.

[2] See note 4 above.

[3] Rybczynski, Ac. R. des Sc. Cracovie, 9, 1911.

[4] Hadamard, Comptes Rendus, March, 1911.

[5] Roux, Annales de Chimie et De Physique, May, 1913, p. 69.

and determined their radius by weighing the tube of oil before and after introducing the droplet. His results indicate a greater velocity than obtained by the Stokes equation, ranging from 6 per cent. greater for the smaller to 10 per cent. greater for the larger ones, although in no case did the speed attain a value necessary to agree with that calculated by the Hadamard equation.

In view of the fact that Nordlund obtained such good agreement with Stokes's law, while Roux's results all show a divergence in the direction of the Hadamard values in spite of the fact that he does not mention having used the Ladenburg correction for the diameter and length of the fall tube, it seemed worth while to repeat his work. It was hoped that by using a sufficiently wide range of droplets that an experimental curve could be obtained, which would show the transition from the Stokes to the Hadamard curve.

The Apparatus.

The apparatus was very simple. It consisted of a glass tube 40 cm. long and 5.5 cm. in diameter, drawn at the two ends to join to short tubes about 2 cm. long and .8 cm. in diameter. On each of these end tubes was placed a short piece of rubber tubing, which was closed by a clamp. Near each end of the tube and at about one third its length, etched lines encircling the tube permitted one to read without parallax the initial and final instant of the period of fall. The tube was supported by a clamp at its middle, such that it could be inverted when the droplet had fallen the desired distance. Behind the tube at a distance of about 50 cm., was placed a white paper screen, which was illuminated from behind by three incandescent lamps. These lamps were sufficiently near the tube to change its temperature, if allowed to glow continuously, and were, therefore, not used more than two or three seconds at a time, when it was necessary to ascertain the position of the falling drop. Owing to the changeable summer temperature, the whole apparatus was set up in a basement room, where a thermostat and an electrical heating device maintained the temperature constant to 0.4° C.

Measurement of the Diameter of the Drops.

The diameters of the drops were measured by means of a Zeiss micrometer ocular on a Leitz Wetzlar microscope, which had a magnification of about 60 diameters. Each turn of the screw corresponded to a shift of the cross hair of .00758 cm. The screw head bore fifty scale divisions and a setting of the cross hair tangent to the sphere could be duplicated with no greater variation than one half scale division. Allowing one half division for setting on each side, the measurement of the diameter of the smallest sphere used was correct to 1.4 per cent. The microscope was calibrated by comparison with a grating made for such measurements by Zeiss.

A preliminary study showed that the small mercury droplets could be readily picked upon the point of a steel needle and retained there if the needle were kept horizontal. If the droplet did not at first adhere to the needle, it was found to do so after dipping the needle into the oil to be used in the investigation, and the oil then removed by means of a dry cloth. Sufficient oil remained to cause the drop to adhere to the needle, yet not enough to cause any error in the measurement. To pick up the smaller drops, a very fine needle was used, while for the larger ones, it was necessary to use a larger needle. The microscope was turned to a horizontal position, and the needle (which for convenience in handling was held in the end of a small soft wooden cylinder about 10 cm. long), was placed in a suitable support which was held in position on the telescope table by the clamps designed for holding the slide. The microscope field was illuminated by an incandescent lamp behind a white paper screen about 30 cm. away.

When the microscope was focused on the drop, it was seen as a spherical object clinging to the lower side of the needle. Measurements along different diameters showed it to be spherical within the accuracy of the measuring device. If the droplet had recently been blown by an atomizer from clean mercury on to a clean piece of paper it was seen to be free from dust particles.

Observations.

After the measurement of the diameter, the needle was lifted by the wooden cylinder from the support and the droplet carried to the fall tube. If the droplet was not too small it fell into the oil when the needle was placed vertically. To free the smaller ones, it was necessary to dip the point of the needle into the oil and move it slightly. The released droplet fell along the axis of the tube, the interval of fall between two selected lines being measured by means of a stop watch. As soon as the droplet had crossed the lower line the tube was inverted and another fall measured. Six to ten falls were recorded for each drop. In no case was the droplet allowed to touch the walls of the tube until all measurements were made. After the last measurement, the droplet was allowed to fall to the end of the tube, the tube was inverted, quickly opened, and the droplet removed by means of a medicine dropper, and another put in.

The following data were obtained:

Radius in Cm.	Velocity in Cm. per Second.	Velocity Calculated from the Stokes-Ladenburg Equation.	Velocity Calculated from the Stokes-Hadamard Equation.
.00577	.0139	.0139	.0202
.00829	.0281	.0286	.0416
.00863	.0305	.0301	.0451
.00974	.0398	.0395	.0575
.01198	.0589	.0596	.0868
.01599	.1045	.1057	.1538
.01696	.1217	.1189	.1728
.01928	.1511	.1533	.2229
.02090	.1800	.1798	.2615
.02118	.1842	.1846	.2685
.02149	.1903	.1900	.2763
.02194	.2014	.1979	.2878
.02251	.2113	.2083	.3029
.02506	.2572	.2575	.3744
.02659	.2893	.2893	.4208
.02789	.3226	.3214	.4675
.02997	.3775	.3668	.5335
.03779	.5982	.6211	.9033

The first and second columns are respectively, the observed radii and corresponding velocities. The third column contains the velocity calculated by the Stokes-Ladenburg equation:

$$v = \frac{2}{9} \frac{a^2(\sigma - \rho)}{u\left(1 + 2.4\frac{a}{R}\right)\left(1 + 3.1\frac{a}{L}\right)},$$

where R and L are the respective radius and length of the tube. The values given in the fourth column were obtained by multiplying those of the third column by the Hadamard correction.

The density of the castor oil was found by the pyknometer method to be .95594 at 24.°8 C. The viscosity determined by a modified Ostwald viscosimeter was 6.5256. The viscosity of mercury was taken from the tables. These values of the viscosity gave for the Hadamard correction the value 1.4544.

Discussion of Results.

It is seen that the observed values agree with those obtained by the Stokes-Ladenburg equation within the limit of experimental error for all drops observed. The agreement is shown more clearly by the curves, Fig. 1. The full line curves are drawn from values determined by the equations. The points are the actual observations. Curve S

represents the Stokes-Ladenburg equation, and Curve H the Hadamard values. The Hadamard correction, therefore, does not apply to the

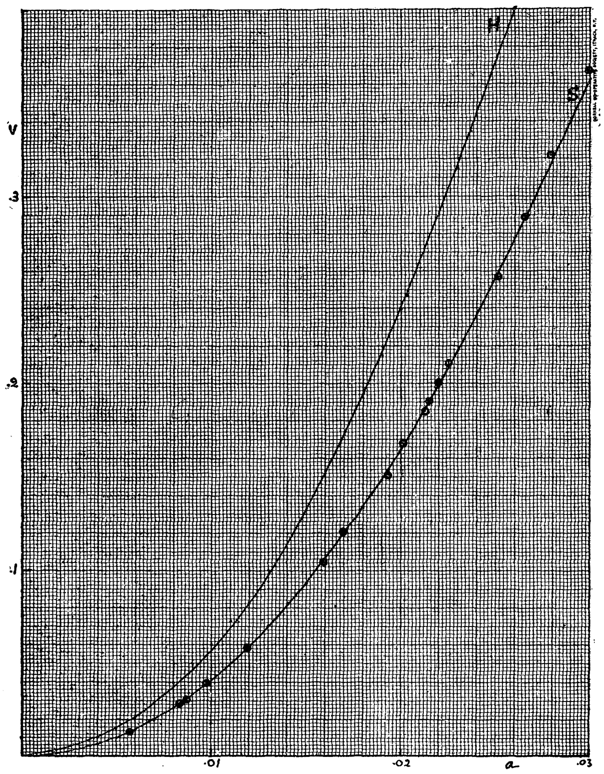

Fig. 1.

case of mercury droplets in castor-oil of this viscosity. The failure is probably due to some surface effect not included in the Hadamard hypothesis.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO.