

ART. XXVI.—*The Mathematical and Philosophical State of the Physical Sciences*; by Prof. JOSEPH LOVERING. From the Presidential Address of Prof. Lovering before the American Association at Hartford, August, 1874.

THE luminiferous ether and the undulatory theory of light have always troubled what is supposed to be the imperturbable character of the mathematics. The proof of a theory is indisputable when it can predict consequences, and call successfully upon the observer to fulfill its prophecies. It is the boast of astronomers that the law of gravitation thus vindicates itself. The undulatory theory of light has shown a wonderful facility of adaptation to each new exigency in optics, and has opened the eye of observation to see what might never have been discovered without the promptings of theory. But this doctrine, and that of gravitation also, have more than once been arrested in their swift march and obliged to show their credentials. After Fresnel and Young had secured a firm foothold for Huyghens' theory of light in mechanics and experiment, questions arose which have perplexed, if not baffled, the best mathematical skill. How is the ether affected by the gross matter which it invests and permeates? Does it move when they move? If not, does the relative motion between the ether and other matter change the length of the undulation or the time of oscillation? These queries cannot be satisfactorily answered by analogy, for analogy is in some respects wanting between the ether and any other substance. Astronomy says that aberration cannot be explained unless the ether is at rest. Optics replies that refraction cannot be explained unless the ether moves. Fresnel produced a reconciliation by a compromise. The ether moves with a *fractional* velocity large enough to satisfy refraction, but too small to disturb sensibly the astronomer's aberration. In 1814, Arago reported to Fresnel that he found no sensible difference in the prismatic refraction of light, whether the earth was moving with full speed toward a star or in the opposite direction, and asked for an explanation. Fresnel submitted the question to mathematical analysis, and demonstrated, that whatever change was produced by the motion of the prism in the relative velocity of light, the wave-length through the prism, and the refraction, was compensated by the physiological aberration when the rays emerged. Very recently, Ketteler of Bonn has gone over the whole ground again with great care, studying not only Arago's case but the general one, in which the direction of the light made any angle with the motion of the earth: and he proves that the light will always enter the eye in the same apparent direction as it would have done if the earth were at rest. The mathematical and physical view taken of this subject by Fresnel has been under discussion for sixty years, and forty eminent physicists and mathematicians might be enumerated who have taken part in it. Fresnel's explanation has encountered

difficulties and objections. Still, it is consistent not only with Arago's negative result, but with the experiments on diffraction by Fizeau and Babinet, and the preponderance of mathematical evidence is on that side. Mr. Huggins runs counter to the general drift of physical and algebraical testimony (although he appears to be sustained by the high authority of Maxwell), when he attributes some displacement of the spectrum lines to the motion of the earth, and qualifies the observed displacement on that account. The number of stars which Huggins has observed is insufficient for any sweeping generalization. And yet he seems inclined to explain the revelations of his spectroscope, not by the motion of the stars, but by that of the solar system: because those stars which are in the neighborhood of the place in which astronomers have put the solar apex are moving, apparently, toward the earth, while those in the opposite part of the sky recede. If it be true that the earth's annual motion produces no displacement in the spectrum, then the motion of the solar system produces none. Or, waiving this objection, if the correct explanation has been given by Huggins, astronomers have failed, by their geometrical method, of rising to the full magnitude of the sun's motion. The discrepancy appears to awaken no distrust in Mr. Huggins' mind as to the delicacy of the spectrum analysis or the mathematical basis of his reasoning. On the contrary, he would remove the discrepancy by throwing discredit on the estimate of star-distances made independently by Struve and Argelander from different lines of thought.

Next we ask, if it is certain that even the motion of the luminary will change the true wave-length, the period of oscillation, and the refrangibility of the light which issues from it? The commonly received opinion on this subject has not been allowed to pass unchallenged. It is fortified by more than one analogy: but it is said that comparison is not always a reason. It is not denied that, when the sonorous body is approaching, the sound waves are shortened, the number of impulses on the ear by the condensed air is increased, and the pitch of the sound is raised. Possibly, the color of light would follow the same law; but there is no experiment to prove it, and very little analogy exists between the eye and the ear. There is no analogy, whatever, between the subjective sensation by either organ and the physical action of the prism. The questions at issue are these: Does refraction depend upon the absolute or the relative velocity of light; are the time of oscillation of the particles of ether and the normal wave-length, corresponding to it, changed by any motion of translation in the origin; or is the conservation of these elements an essential attribute of the luminiferous medium. It has been said that Doppler reasoned as if the corpuscular theory of light were true, and then expressed himself in the language of undulations. Evidently, there is an obscurity in the minds of many physicists, and an uncertainty in all, when they reason upon the mechanical constitution of the ether, and the fundamental laws of light. The

mathematical theory is not so clear as to be able to dispense with the illumination of experiment. Within the present year, Van der Willigen has published a long and well considered memoir on the theoretical fallacies which vitiate the whole of Huggins' argument for the motion of the stars and nebulae. His analysis proves that the motion of the luminary will not interfere with the time of oscillation and the wave-length provided that the origin of the disturbance is not a mathematical point but a vibrating molecule, and that the sphere of action of this molecule upon surrounding molecules is large enough to keep them under its influence during ten or a hundred vibrations, before it is withdrawn by the motion of translation. If this theoretical exposition of the subject should be generally adopted by mathematicians, the spectroscopic observations on the supposed motion of the stars must receive another interpretation. On the other hand, if a luminary is selected which is known to move, independently of spectroscopic observations, and the displacement of the spectrum lines accords with this motion, it will be time to reconsider the mathematical theory, and make our conceptions of the ether conform to the experiment. The spectroscopic observation of Angström on an oblique electric spark does not favor Huggins' views. Secchi testifies to opposite displacements when he examined, with a direct vision spectroscope, the two edges of the sun's equator, one of which was rotating toward him and the other from him, and Vogel has repeated the observation with a reversion-spectroscope. This would have the force of a crucial experiment were it not that an equal displacement was seen on other parallels of latitude, and that the bright bands of the chromosphere were moved, but not the dark lines of the solar atmosphere.

When Voltaire visited England in 1727 he saw at the universities the effect of Newton's revolutionary ideas in astronomy. The mechanism of gravitation had exiled the fanciful vortices of Descartes, which were still circulating on the continent. So he wrote: "A Frenchman who comes to London finds many changes in philosophy as in other things: he left the world full, he finds it empty." The same comparison might be made now, not so much between nationalities as between successive stages of scientific development. At the beginning of this century the universe was as empty as an exhausted receiver: now it has filled up again. Nature's abhorrence of a vacuum has been resuscitated, though for other reasons than these which satisfied the Aristotelians. It is the mathematicians and not the metaphysicians who are now discussing the relative merits of the *plenum* and the *vacuum*. Newton, in his third letter to Bentley, wrote in this wise: "That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance, through a *vacuum*, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall

into it." Roger Cotes, who was Newton's successor in the chair of mathematics and natural philosophy at Cambridge, was only four years old when the first edition of the "*Principia*" was issued, and Newton outlived him by ten years. The venerable teacher pronounced upon the young mathematician, his pupil, these few but comprehensive words of eulogy: "If Cotes had lived, we should have known something." The view taken of gravitation by Cotes was not the same as that held by his master. He advocated the proposition that action at a distance must be accepted as one of the primary qualities of matter, admitting of no farther analysis. It was objected by Hobbes and other metaphysicians, that it was inconceivable that a body should act where it was not. All our knowledge of mechanical forces is derived from the conscious effort we ourselves make in producing motion. As this motion employs the machinery of contact, the force of gravitation is wholly outside of all our experience. The advocates of action at a distance reply, that there is no real contact in any case, that the difficulty is the same with the distance of molecules as that of planets, that the mathematics are neither long-sighted nor short-sighted, and that an explanation which suits other forces is good enough for gravitation.

Comte extricated himself from this embarrassment by excluding causes altogether from his positive philosophy. He rejects the word attraction as implying a false analogy, inconsistent with Newton's law of distance. He substitutes the word gravitation, but only as a blind expression by which the facts are generalized. According to Comte's philosophy, the laws of Newton are on an equality with the laws of Kepler, only they are more comprehensive, and the glory of Kepler has the same stamp as that of Newton. Hegel, the eminent German metaphysician, must have looked at the subject in the same light when he wrote these words: "Kepler discovered the laws of free motion; a discovery of immortal glory. It has since been the fashion to say that Newton first found out the truth of these rules. It has seldom happened that the honor of the first discoverer has been more unjustly transferred to another." Shelling goes farther in the same direction: he degrades the Newtonian law of attraction into an empirical fact, and exalts the laws of Kepler into necessary results of our ideas.

Meanwhile, the Newtonian theory of attraction, under the skillful generalship of the geometers, went forth on its triumphal march through space, conquering great and small, far and near, until its empire became as universal as its name. The whirlpools of Descartes offered but a feeble resistance, and were finally dashed to pieces by the artillery of the parabolic comets; and the rubbish of this fanciful mechanism was cleaned out as completely as the cumbrous epicycles of Ptolemy had been dismantled by Copernicus and Kepler. The mathematicians certified that the solar system was protected against the inroads of comets, and the border warfare of one planet upon another, and that its stability

was secure in the hands of gravitation, if only space should be kept open, and the dust and cobwebs which Newton had swept from the skies should not reappear. Prophetic eyes contemplated the possibility of an untimely end to the revolution of planets, if their ever-expanding atmospheres should rush in to fill the room vacated by the maelstroms of Descartes. When it was stated that the absence of infinite divisibility in matter, or the coldness of space, would place a limit upon expansion, and, at the worst, that the medium would be too attenuated to produce a sensible check in the headway of planets, and when, in more recent times, even Encke's comet showed but the slightest symptoms of mechanical decay, it was believed that the motion was, in a practical, if not in a mathematical sense, perpetual. Thus it was that the splendors of analysis dimmed the eyes of science to the intrinsic difficulties of Newton's theory, and familiarity with the language of attraction concealed the mystery that was lurking beneath it. A long experience in the treatment of gravitation has supplied mathematicians with a fund of methods and formulas suited to similar cases. As soon as electricity, magnetism and electromagnetism took form, they also were fitted out with a garment of attractive and repulsive forces acting at a distance: and the theories of Cavendish, Poisson, Aepinus and Ampère, endorsed as they were by such names as Laplace, Plana, Liouville and Green, met with general acceptance.

The seeds, which were destined to take root in a later generation, and disturb, if not dislodge, the prevalent interpretation of the force of gravitation, were sown by a contemporary of Newton. They found no congenial soil in which they could germinate and fructify until the early part of this century. At the present moment, we find the luminiferous ether in quiet and undivided possession of the field from which the grosser material of ancient systems had been banished. The *plenum* reigns everywhere; the vacuum is nowhere. Even the corpuscular theory of light, as it came from the hands of its founder, required the reinforcement of an ether. Electricity and magnetism, on a smaller scale, applied similar machinery. If there was a fundamental objection to the conception of forces acting at a distance, certainly the bridge was already built by which the difficulty could be surmounted. The turning-point between the old physics and the new physics was reached in 1837, when Faraday published his experiments on the specific inductive capacity of substances. This discovery was revolutionary in its character, but it made no great stir in science at the time. The world did not awake to its full significance until the perplexing problem of ocean telegraphs converted it from a theoretical proposition into a practical reality, and forced it on the attention of electricians. The eminent scientific advisers of the cable companies were the first to do justice to Faraday. This is one of the many returns made to theoretical electricity for the support it gave to the most magnificent commercial enterprise.

The discovery of diamagnetism furnished another argument in favor of the new interpretation of physical action. What that new interpretation was, is well described by Maxwell. "Faraday, in his mind's eye, saw lines of force traversing all space, where the mathematicians saw centers of force attracting at a distance; Faraday saw a medium where they saw nothing but distance; Faraday sought the seat of the phenomena in real actions going on in the medium; they were satisfied that they had found it in a *power* of action at a distance impressed on the electric fluids." The physical statement waited only for the coming of the mathematicians who could translate it into the language of analysis, and prove that it had as precise a numerical consistency as the old view with all the facts of observation. A paper published by Sir William Thomson, when he was an undergraduate at the university of Cambridge, pointed the way. Prof. Maxwell, in his masterly work on electricity and magnetism, which appeared in 1873, has built a monument to Faraday, and unconsciously to himself also, out of the strongest mathematics. For forty years mathematicians and physicists had labored to associate the laws of electrostatics and electrodynamics under some more general expression. An early attempt was made by Gauss in 1835, but his process was published, for the first time, in the recent complete edition of his works. Maxwell objects to the formula of Gauss because it violates the law of the conservation of energy. Weber's method was made known in 1846; but it has not escaped the criticism of Helmholtz. It represents faithfully the laws of Ampère and the facts of induction, and led Weber to an absolute measurement of the electrostatic and electromagnetic units. The ratio of these units, according to the formulas, is a velocity; and experiment shows that this velocity is equal to the velocity of light. As Weber's theory starts with the conception of action at a distance, without any mediation, the effect would be instantaneous, and we are at a loss to discover the physical meaning which he attaches to his velocity. Gauss abandoned his researches in electromagnetism because he could not satisfy his mind in regard to the propagation of its influence in time. Other mathematicians have worked for a solution, but have lost themselves in a cloud of mathematical abstraction. The two theories of light have exhausted all imaginable ways in which force can be gradually transmitted without increase or loss of energy. Maxwell cut the Gordian knot when he selected the luminiferous ether itself as the arena on which to marshal the electromagnetic forces under the symbols of his mathematics, and made light a variety of electromagnetic action. His analysis gave a velocity essentially the same as that of Weber, with the advantage of being a physical reality and not a mere ratio. Of the two volumes of Mr. Maxwell, freighted with the richest and heaviest cargo, the reviewer says: "Their author has, as it were, flown at everything; and, with immense spread of wing and power of beak, he has hunted down his victims in all quarters, and from each has extracted

something new and interesting for the intellectual nourishment of his readers." Clear physical views must precede the application of mathematics to any subject. Maxwell and Thomson are liberal in their acknowledgments to Faraday. Mr. Thomson says: "Faraday, without mathematics, divined the result of the mathematical investigation; and, what has proved of infinite value to the mathematicians themselves, he has given them an articulate language in which to express their results. Indeed, the whole language of the *magnetic field* and *lines of force* is Faraday's. It must be said for the mathematicians that they greedily accepted it, and have ever since been most zealous in using it to the best advantage."

It is not expected that the new views of physics will be generally accepted without vigorous opposition. A large amount of intellectual capital has been honestly invested in the fortunes of the other side. The change is recommended by powerful physical arguments, and it disentralls the theories of science from many metaphysical difficulties which weigh heavily on some minds. On the other hand, the style of mathematics which the innovation introduces is novel and complex; and good mathematicians may find it necessary to go to school again before they can read and understand the strange analysis. It is feared, that with many, who are not easily deflected from the old ruts, the intricacies of the new mathematics will outweigh the superiority of the new physics.

The old question, in regard to the nature of gravitation, was never settled: it was simply dropped. Now it is revived with as much earnestness as ever, and with more intelligence. Astronomy cast in its own mould the original theories of electrical and magnetic action. The revolution in electricity and magnetism must necessarily react upon astronomy. It was proved by Laplace, from data which would now, probably, require a numerical correction, that the velocity of the force of gravitation could not be less than eight million times the velocity of light; in fact, that it was infinite. Those who believe in action at a distance cannot properly speak of the transmission of gravitation. Force can be transmitted only by matter; either with it or through it. According to their view, action at a distance *is* the force, and it admits of no other illustration, explanation or analysis. It is not surprising that Faraday and others, who had lost their faith in action at short distances, should have been completely staggered by the ordinary interpretation of the law of gravitation, and that they declared the clause which asserted that the force diminished with the square of the distance to be a violation of the principle of the conservation of energy.

Must we then content ourselves with the naked facts of gravitation, as Comte did, or is it possible to resolve them into a mode of action, in harmony with our general experience, and which does not shock our conceptions of matter and force? In 1798, Count Rumford wrote thus: "Nobody surely, in his sober senses, has ever pretended to understand the mechanism of gravitation."

Probably Rumford had never seen the paper of Le Sage, published by the Berlin Academy in 1782, in which he expounded his mechanical theory of gravitation, to which he had devoted sixty-three years of his life. In a posthumous work, printed in 1818, Le Sage has developed his views more fully. He supposed that bodies were pressed toward one another by the everlasting pelting of ultramundane atoms, inward bound from the immensity of space beyond, the faces of the bodies which looked toward each other being mutually screened from this bombardment. It was objected to this hypothesis, which introduced Lucretius into the society of Newton and his followers, that the collision of atoms with atoms, and with planets, would cause a secular diminution in the force of gravity. Le Sage admitted the fact. But as no one knew that the solar system was eternal, the objection was not fatal. As the necessity for giving a mechanical account of gravitation was not generally felt at the time, the theory of Le Sage fell into oblivion. In 1873, Sir William Thomson resuscitated and republished it. He has fitted it out in a fashionable dress, made out of elastic molecules instead of hard atoms, and has satisfied himself that it is consistent with modern thermo-dynamics and a perennial gravitation.

Let us now look in a wholly different quarter for the mechanical origin of gravitation. In 1870, Prof. Guthrie gave an account of a novel experiment, viz: the attraction of a light body by a tuning-fork when it was set in vibration. Thomson repeated the experiment upon a suspended eggshell and attracted it by a simple wave of the hand. Thomson remarks "that what gave the great charm to these investigations, for Mr. Guthrie himself, and no doubt also for many of those who heard his expositions and saw his experiments, was, that the results belong to a class of phenomena to which we may hopefully look for discovering the mechanism of magnetic force, and possibly also the mechanism by which the forces of electricity and gravity are transmitted." By a delicate mathematical analysis, Thomson arrives at the theorem that the "average pressure at any point of an incompressible, frictionless fluid, originally at rest, but set in motion and kept in motion by solids, moving to and fro, or whirling round in any manner, through a finite space of it," would explain the attractions just described. Moreover, he is persuaded by other effects besides those of light, that, in the interplanetary spaces and in the best artificial vacuum, the medium which remains has "perfectly decided mechanical qualities, and, among others, that of being able to transmit mechanical energy, in enormous quantities;" and he cherishes the hope that his mathematical theorems on abstract hydrokinetics are of some interest in physics as illustrating the great question of the eighteenth and nineteenth centuries—Is action at a distance a reality, or is gravitation to be explained, as we now believe magnetic and electric forces must be, by action of intervening matter?



In 1869 and 1873, Prof. Challis of Cambridge, England, published two works on the Principles of Mathematical Physics. They embody the mature reflections of a mathematical physicist at the advanced age of three score years and ten. Challis believes that there is sufficient evidence for the existence of ether and atoms as physical realities. He then proceeds to say: "The fundamental and only admissible idea of *force* is that of *pressure*, exerted either actively by the ether against the surface of the atoms, or as reaction of the atoms on the ether by resistance to that pressure. The principle of deriving fundamental physical conceptions from the indications of the senses does not admit of regarding *gravity*, or any other force varying with distance, as an essential quality of matter, because, according to that principle, we must, in seeking for the simplest idea of physical force, have regard to the sense of *touch*. Now, by this sense, we obtain a perception of force as pressure, distinct and unique, and not involving the variable element of distance, which enters into the perception of force as derived from the sense of sight alone. Thus, on the ground of simplicity as well as of distinct perceptibility, the fundamental idea of force is pressure." As all other matter is passive except when acted upon by the ether, the ether itself, in its quiescent state, must have uniform density. It must be coextensive with the vast regions in which material force is displayed. Challis had prepared himself for the elucidation and defence of his dynamical theory by a profound study of the laws of motion in elastic fluids. From the mathematical forms in which he has expressed these laws he has attempted to derive the principal experimental results in light, heat, gravitation, electricity and magnetism. Some may think that Mr. Challis has done nothing but clothe his theory in the cast-off garments of an obsolete philosophy. If its dress is old, it walks upon new legs. The interplay between ether and atoms is now brought on to the stage, not as a speculation sustained by metaphysical and theological arguments, but as a physical reality with mathematical supports. I should do great injustice to this author if I left the impression that he himself claimed to have covered the whole ground of his system by proof. Mathematical difficulties prevented him from reaching a numerical value for the resultant action of a wave of ether upon the atom. What he has written is the guide-post, pointing the direction in which science is next to travel: but the end of the journey is yet a great way off. The repeated protests of Mr. Challis against the popular physics of the day, and his bold proclamation of the native, independent motion of the ether, have aroused criticism. What prevents the free ether, asks the late Sir John Herschel, from expanding into infinite space? Mr. Challis replies that we know nothing about infinite space or what happens there; but the existence of the ether, where our experience can follow it, is a physical reality. The source of the motion which the ether acquires is not the sun: for the most efficient cause of solar radiation is gravitation and condensation. Our

author avoids the vicious circle of making gravitation, first the reason, and afterward the consequence of the motion of the ether. He says: "It follows that the sun's heat, and the heat of masses in general, are stable quantities, oscillating, it may be, like the planetary motions, about *mean* values, but never permanently changing, so long as the Upholder of the universe conserves the force of the ether and the qualities of the atoms. There is no law of destructibility: but the same Will that conserves can in a moment destroy." The following remarks upon this theory deserve our attention: "The explanation of any action between distant bodies by means of a clearly conceivable process, going on in the intervening medium, is an achievement of the highest scientific value. Of all such actions that of gravitation is the most universal and the most mysterious. Whatever theory of the constitution of bodies holds out a prospect of the ultimate explanation of the process by which gravitation is effected, men of science will be found ready to devote the whole remainder of their lives to the development of that theory."

The hypotheses of Challis and Le Sage have one thing in common; the motion of the ether and the driving storm of atoms must come from outside the world of stars. "On either theory, the universe is not even temporarily automatic, but must be fed from moment to moment by an agency external to itself." Our science is not a finality. The material order which we are said to know makes heavy drafts upon an older or remoter one, and that again upon a third. The world, as science looks at it, is not self-sustaining. We may abandon the hope of explaining gravitation, and make attraction itself the primordial cause. Our refuge then is in the sun. When we qualify the conservation of energy by the dissipation of energy, the last of which is as much an induction of science as the first, the material fabric which we have constructed still demands outward support. Thomson calculates that, within the historical period, the sun has emitted hundreds of times as much mechanical energy as is contained in the united motions of all the planets. This energy, he says, is dissipated more and more widely through endless space, and never has been, probably never can be, restored to the sun, without acts as much beyond the scope of human intelligence as a creation or annihilation of energy, or of matter itself, would be.

From the earliest dawn of intellectual life, a general theory of the constitution of matter has been a fruitful subject of debate, and human science and philosophy have ever been dashing their heads against the intractable atoms. The eagerness of the discussion was the greater, the more hopeless the solution. For every man who set up an hypothesis upon the subject there were half a dozen others to knock it down; until at last speculation, which bore no fruit, was suspended. A lingering interest still hung around the question, whether matter was not infinitely divisible, and the atomic philosophers were not chasing a chimera. From every new decision on this single point there was an appeal, and

the foothold which the atoms had secured in chemistry was gradually subsiding. Of a sudden, the atomic theory has gained a new lease of life. But the hero of the new drama is not the atom but the molecule. In all the physical sciences, including astronomy, the war has been carried home to the molecules: and the intellectual victories of this and the next generation will be on this narrow field. From the outlying provinces of physics; from the sun, the stars and the nebulae; from the comets and meteors; from the zodiacal light and the aurora; from the exquisitely tempered and mysterious ether; the forces of nature have been moving in converging lines to this common battle-ground, and some shouts of victory have already been heard. In the long and memorable controversy between Newton and Leibnitz, and their adherents, as to the true measure of force, it was charged against the Newtonian rule that force was irrecoverably lost whenever a collision occurred between hard, inelastic bodies. The answer was, that nature had anticipated the objection and had avoided this kind of matter. Inelastic bodies were yielding bodies, and the force which had disappeared from the motion had done its work in changing the shape. But unless the body could recover its original figure by elasticity, there was no potential energy and force was annihilated. It is now believed, and to a large extent demonstrated, that the force, apparently lost, has been transformed into heat, electricity, or some other kind of molecular motion, of which the change of shape is only the outward sign. The establishment on a firm foundation of theory and experiment of the so-called conservation of energy, the child of the correlation of physical forces, is one of the first fruits of molecular mechanics.

It is no disparagement of this discovery, on which was concentrated the power of several minds, to call it an extension, though a vast one, of Newton's law of inertia, of Leibnitz's *vis viva*, and of Huyghens' and Bernouilli's conservation of living forces; these older axioms of mechanics having free range only in astronomy, where friction, resistance and collision do not interfere. The conservation of energy, in its extended signification, promises to be, like its forerunners, a valuable guide to discovery, especially in the dark places into which physical science has now penetrated. The caution which Lagrange has given in reference to similar mechanical principles, such as the conservation of the motion of the center of gravity, the conservation of moments of rotation, the preservation of areas, and the principle of least action, is not without its applicability to the new generalization. Lagrange accepts them all as results of the known laws of mechanics and not as the essence of the laws of nature. The most that physical science can assert is that it possesses no evidence of the destructibility of matter or force.

It is not pretended that the existence of atoms has been or can be proved or disproved. Some chemists think that the atomic theory is the life of chemistry: others have abandoned it. Its importance is lost in that of the molecular theory. And what has this accomplished to justify its existence? If we define the mole-

cule of any substance as the smallest mass of that substance which retains all its chemical properties, we can start with the extensive generalization of Avogadro and Ampère, that the same volume of every kind of matter in the state of vapor, and under the same pressure and temperature, contains an equal number of such molecules. The conception of matter as consisting of parts, which are perpetually flying over their microscopic orbits and producing by their fortuitous concurrence all the observed qualities of bodies, is as old as Lucretius. He saw the magnified symbol of his hypothesis in the motes which chase one another in the sunbeam. One of the Bernouillis thought that the pressure of gases might be caused by the incessant impact of these little masses on the vessel which held them. The discovery that heat was a motion and not a substance, foreshadowed by Bacon, made probable by Rumford and Davy, and rigidly proved by Mayer and Joule when they obtained its exact mechanical equivalent, opened the way to the dynamical theory of gases. Joule calculated the velocity of this promiscuous artillery, rendered harmless by the minuteness of the missiles, and found that the boasted guns of modern warfare could not compete with it. Clausius consummated the kinetic theory of gases by his powerful mathematics, and derived from it the experimental laws of Mariotte, Gay-Lussac and Charles. By the assumption of data, more or less plausible, several mathematicians have succeeded in computing the sizes and the masses of the molecules and some of the elements of their motion. It should not be forgotten that mathematical analysis is only a rigid system of logic by which wrong premises conduct the more surely to an incorrect conclusion. To claim for all the conclusions which have been published in relation to the molecules the certainty which fairly belongs to some of them would prejudice the whole cause.

One of the most interesting investigations in molecular mechanics was published by Helmholtz in 1858. It is a mathematical discussion of what he calls ring-vortices in a perfect, frictionless fluid. Helmholtz has demonstrated that such vortices possess a perpetuity and an inviolability once thought to be realized only by the eternal atoms. The ring-vortices may hustle one another, and pass through endless transformations, but they cannot be broken or stopped. Thomson seized upon them as the impersonation of the indestructible but plastic molecule which he was looking for, to satisfy the present condition of physical science. The element of the new physics is not an atom or a congeries of atoms, but a whirling vapor. The molecules of the same substance have one invariable and unchangeable mass: they are all tuned to one standard pitch and, when incandescent, emit the same kind of light. The music of the spheres has left the heavens and condescended to the rhythmic molecules. There is here no birth or death or variation of species. If other masses than the precise one which represents the elements have been eliminated, where, asks Maxwell, have they gone? The spectroscope does not show them in the stars or nebulae. The hydrogen and sodium of remotest space are in unison with the hydrogen and sodium of earth.

In the phraseology of our mechanics we define matter and force as if they had an independent existence. But we have no conception of inert matter or of disembodied force. All we know of matter is its pressure and its motion. The old atom had only potential energy; the energy of its substitute, the molecule, is partly potential and partly kinetic. If it could be shown that all the phenomena displayed in the physical world were simply transmutations of the original energy existing in the molecules, physical science would be satisfied. Where physical science ends, natural philosophy, which is not wholly exploded from our vocabulary, begins. Natural philosophy can give no account of energy when disconnected with an ever present Intelligence and Will. In Herschel's beautiful dialogue on atoms, after one of the speakers had explained all the wonderful exhibition of nature as the work of natural forces, Hermione replies: "Wonderful, indeed! Anyhow, they must have not only good memories but astonishing presence of mind, to be always ready to act, and always to act, without mistake, according to the primary laws of their being, in every complication that occurs." And elsewhere, "Action, without will or effort, is to us, constituted as we are, unrealizable, unknowable, inconceivable." The monads of Leibnitz and the demons of Maxwell express in words the personality implied in every manifestation of force.