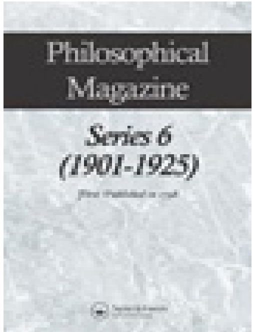

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XXXV. *On the Principle of Relativity and the Electromagnetic Mass of the Electron. A Reply to* Dr. A. H. Bucherer. *By* E. Cunningham, *Lecturer in Applied Mathematics, University College, London* *.

IN the March number of this Magazine (p. 316), Dr. A. H. Bucherer objects to the statement made by me in a paper published also in this Magazine (Oct. 1907) that his Principle of Relativity † was identical with the Lorentz-Einstein principle. Without at all wishing to depreciate the ingenious method which Dr. Bucherer has adopted to avoid the difficulties which cluster round this part of electromagnetic theory, I should like to consider his objections and to go more fully into the question as to whether the statement which I made was correct.

The following paragraph is quoted from my paper:—"It is required, among other things, to explain how a light-wave travelling outwards in all directions with velocity c relative to an observer A, may at the same time be travelling outwards in all directions with the same velocity relative to an observer B moving relative to A with velocity v." May I explain that I did not wish to assert that it was required by any known fact of observation, but that I took it to be involved in the statement of the principle. I may have read into it more than was intended, but if the Maxwell equations are assumed to hold when referred, as occasion requires, to various frames of reference moving relatively to one another, the deduction cannot be escaped that the velocity of propagation of a spherical wave will be found to be exactly the same, whatever the frame of reference. Thus what was proved in my former paper was that *if I have not read too much into Dr. Bucherer's principle in supposing that he assumes the Maxwell equations to hold, whatever particular point is considered to be at rest*, then that principle cannot be applied without taking into consideration a possible difference between the space and time measures of two observers moving relatively to one another, and that in fact this transformation between the space and time measures must be that associated with the names of Lorentz and Einstein.

Passing to another point raised by Dr. Bucherer, I feel myself on firmer ground, inasmuch as I am free from the fear that I may still be misconstruing his principle. He asks

* Communicated by the Author.

† Phil. Mag. April 1907.

me to carefully compare it with that of Lorentz. May I say that I should scarcely have ventured to approach the subject in this Magazine if I had not already done so, and that on exactly the point to which my attention is again called, viz., the expression given for the forces on a moving electron. An inspection of these instead of showing the impossibity of obtaining them by the Lorentz-Einstein transformation, shows that they may actually be derived from the ordinary Maxwellian expressions by means of that process. It may perhaps be worth while carrying out the calculation.

Consider first two electrons, A, B moving relatively to each other, the notation being that of Dr. Bucherer's paper. Taking the axis of x in the direction of the velocity of B relative to A, let the coordinates of B relative to A at a certain instant be x', y', z' to an observer moving with A. Then taking A to be at rest in the æther, the electric intensity at B due to it has components $\frac{qv^2}{r'^3}(x', y', z')$. Now apply the Lorentz-Einstein transformation. Then at the same instant to an observer moving with B the electric intensity is $\frac{qv^2}{r'^3}(x', \beta y', \beta z')$. But the coordinates of B relative to A to an observer moving with B will be $x = \frac{x'}{\beta}$, y, z; so that the intensity may be expressed as

$$\frac{\beta q v^2}{r'^3}(x, y, z) \quad \text{where} \quad \beta = \left(1 - \frac{u^2}{v^2}\right)^{-1/2}.$$

If γ is the angle between the line AB and the direction of u as seen by the observer moving with B, $\sin\gamma = \frac{\sqrt{y^2+z^2}}{r}$ and $r'^2 = \beta^2 r^2\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)$.

Thus finally the intensity to an observer moving with B is

$$\frac{qv^2(x, y, z)}{\beta^2 r^3\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}},$$

or in Dr. Bucherer's notation

$$\frac{\mathbf{r}_1 q v^2 s}{r^2\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}},$$

and therefore the force upon the electron B supposed at rest in the æther is

$$\frac{\mathbf{r}_1 q^2 v^2 s}{r^2\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}.$$

Similarly, suppose B to be a unit magnetic pole instead of an electron of charge q. We require now to know the magnetic intensity at B to an observer moving with it. Starting from the electrostatic force due to A as before, the Lorentz-Einstein expression for the magnetic intensity referred to axes moving with B is

$$\frac{\beta q u}{r'^3}(0,\ z',\ -y')$$

$$=\frac{qu}{\beta^2 r^3\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}(0,\ z,\ -y),$$

or in Dr. Bucherer's notation

$$\frac{qs}{r^3\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}\,\mathrm{V}u\mathbf{r}_1,$$

which is his expression (3). Similarly expressions (2) and (4) may be derived.

Having shown that these expressions may be obtained by means of the Lorentz transformation, there is hardly need to go further and obtain the expressions for the force acting on an electron moving in a uniform magnetic or electric field, since these are obtained by Dr. Bucherer by integration of the simpler expressions. But as a further verification of the equivalence of the two principles the work will be carried out for the case of the electric field, which gives the more complicated result.

Let the field be of intensity E_0, and let an electron of charge q move with velocity u at an angle α with the direction of E_0 as seen by an observer at rest with the field. Then to an observer moving with the electron, the direction of the normal to the condenser plates will be slewed round to an angle α' with the direction of u where $\tan\alpha' = \tan\alpha\sqrt{1-\frac{u^2}{v^2}}$ so that

$$\cos\alpha' = \frac{\cos\alpha}{\sqrt{1-\frac{u^2}{v^2}\sin^2\alpha}} \quad \text{and} \quad \sin\alpha' = \frac{\sin\alpha\sqrt{1-\frac{u^2}{v^2}}}{\sqrt{1-\frac{u^2}{v^2}\sin^2\alpha}}.$$

Now referred to the original frame of reference the electric field was made up of $E_0 \cos\alpha$ parallel to u and $E_0 \sin\alpha$ perpendicular to u. Hence, according to the Lorentz-Einstein transformation, referred to axes moving with the electron it is made up of $E_0 \cos\alpha$ parallel to u and $\beta E_0 \sin\alpha$ perpendicular to u.

Thus the total component in the direction of the normal to the condenser plates is

$$E_0(\cos\alpha\cos\alpha' + \beta\sin\alpha\sin\alpha')$$

$$= \frac{E_0}{\sqrt{1-\frac{u^2}{v^2}\sin^2\alpha}} = \frac{E_0\sqrt{1-\frac{u^2}{v^2}\cos^2\alpha'}}{\sqrt{1-\frac{u^2}{v^2}}}.$$

The component parallel to the plate is similarly

$$E_0(\cos\alpha\sin\alpha' - \beta\sin\alpha\cos\alpha')$$

$$= \frac{E_0 \sin\alpha\cos\alpha}{\sqrt{1-\frac{u^2}{v^2}\sin^2\alpha}}\left(\frac{1}{\beta} - \beta\right)$$

$$= -\frac{E_0 \sin\alpha\cos\alpha\frac{u^2}{v^2}}{\sqrt{1-\frac{u^2}{v^2}}\sqrt{1-\frac{u^2}{v^2}\sin^2\alpha}}$$

$$= -\frac{E_0\frac{u^2}{v^2}\sin\alpha'\cos\alpha'}{\sqrt{1-\frac{u^2}{v^2}}\sqrt{1-\frac{u^2}{v^2}\cos^2\alpha'}}$$

The two components can be replaced by two in the directions of u and of the normal to the plate respectively of magnitudes

$$-\frac{E_0\frac{u^2}{v^2}\cos\alpha'}{\sqrt{1-\frac{u^2}{v^2}}\sqrt{1-\frac{u^2}{v^2}\cos^2\alpha'}}$$

and

$$\frac{E_0}{\sqrt{1-\frac{u^2}{v^2}}\sqrt{1-\frac{u^2}{v^2}\cos^2\alpha'}}.$$

Now the ratio of the areas of a given portion of the plate as seen by an observer at rest relative to them and to the electron respectively, is $\sec\alpha : \sec\alpha'$, so that if σ is the true density of the electricity on the plate, the apparent density to an observer moving with the electron is

$$\sigma' = \frac{\sigma\cos\alpha'}{\cos\alpha} = \frac{\sigma\sqrt{1-\frac{u^2}{v^2}\cos^2\alpha'}}{\sqrt{1-\frac{u^2}{v^2}}},$$

and $\quad E_0 = 4\pi\sigma v^2.$

Hence the electric intensity obtained above is made up of two components

$$-\frac{4\pi\sigma' v^2\cdot\frac{u^2}{v^2}\cos\alpha'}{\left(1-\frac{u^2}{v^2}\cos^2\alpha'\right)} \quad\text{and}\quad \frac{4\pi\sigma' v^2}{\left(1-\frac{u^2}{v^2}\cos^2\alpha'\right)}$$

in the directions of u and the normal to the plate respectively to an observer moving with the electron; and this agrees exactly with expression (8) for the force on the electron.

With regard to the evaluation of the mass of the electron, I must admit that I did not fully understand Dr. Bucherer's process, but I cannot rid myself of the feeling that he has somehow supposed the electron to be moving and at rest simultaneously in its different aspects as *active* and *passive* respectively.

In this connexion there is another difficulty that appears, if the transformation of space and time measures between the two observers be neglected, which may most simply be illustrated by considering the case of two electrons which at a certain instant have the same velocity u through the æther. Then, since the relative velocity is zero, the force acting on either will according to Dr. Bucherer's principle be the electrostatic force $\frac{q^2}{r^2}$ in the direction of the line joining the electrons. But the longitudinal and transverse masses being different, the acceleration of the electron will not in general be in the direction of the force, so that instead of the relative motion of the electrons due to their mutual action being in the line joining them, this line will begin to rotate; in fact, it will tend to set itself at right angles to the direction of motion, a conclusion scarcely consonant with a principle of relativity.

If we revert to the ordinary theory as applied to the same illustration, the field due to one electron in the neighbourhood of the other consists of an electric intensity

$$\frac{q\left(1-\frac{u^2}{v^2}\right)}{r^2\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

in the direction of r, and a magnetic intensity

$$\frac{qu\sin\theta\left(1-\frac{u^2}{v^2}\right)}{vr^2\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

at right angles to r and to u.

Combining these, the mechanical force on the second electron is made up of a component

$$\frac{q^2\cos\theta\left(1-\frac{u^2}{v^2}\right)}{r^2\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

in the direction of motion, and a component

$$\frac{q^2\sin\theta\left(1-\frac{u^2}{v^2}\right)^2}{r^2\left(1-\frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

perpendicular to this direction.

Thus, the only theory which would give an acceleration in the direction of r is one in which the ratio of the transverse to the longitudinal mass is $\left(1-\frac{u^2}{v^2}\right)$ as in the Lorentz theory. Taking the Abraham values of the masses the line joining the electrons will rotate as before.