

## OUGHTRED'S IDEAS AND INFLUENCE ON THE TEACHING OF MATHEMATICS.\*

### GENERAL STATEMENT.

WILLIAM OUGHTRED has nowhere given a full and systematic exposition of his views on mathematical teaching. Nevertheless, he had very pronounced and clear cut ideas on the subject. That a man who was not a teacher by profession should have mature views on teaching is most interesting. We gather his ideas from the quality of the books he published, from his prefaces and from passages in his controversial writing against Delamain. As we proceed to give quotations unfolding Oughtred's views, we shall observe that three points receive special emphasis:

1. An appeal to the eye through suitable symbolism;
2. Emphasis upon rigorous thinking;
3. The postponement of the use of mathematical instruments until after the logical foundations of a subject have been thoroughly mastered.

The importance of these tenets is immensely reinforced by the conditions of the hour. This voice from the past speaks wisdom to specialists of to-day. Recent methods of determining educational values and the modern cult of utilitarianism have led some experts to extraordinary conclusions. Laboratory methods of testing, by the narrow-

\* For details of William Oughtred's life we refer to *The Open Court* of August, 1915, and for a description of his works to *The Monist* of July, 1915.

ness of their range, often mislead. Thus far they have been inferior to the word of a man of experience, insight and conviction.

MATHEMATICS, "A SCIENCE OF THE EYE."

Oughtred was a great admirer of the Greek mathematicians—Euclid, Archimedes, Apollonius of Perga, Diophantus. But in reading their words he experienced keenly what many modern readers have felt, namely, that the almost total absence of mathematical symbols renders their writings unnecessarily difficult to read. Statements that can be compressed into a few well-chosen symbols which the eye is able to survey as a whole are expressed in long drawn out sentences. A striking illustration of the importance of symbolism is afforded by the history of the formula

$$ix = \log (\cos x + i \sin x).$$

It was given in Roger Cotes's *Harmonia mensurarum*, 1722, not in symbols, but expressed in rhetorical form, destitute of special aids to the eye. The result was that the theorem remained in the book undetected for 185 years and was meanwhile re-discovered by others. Owing to the prominence of Cotes as a mathematician it is very improbable that such a thing could have happened, had the theorem been thrust into view by the aid of mathematical symbols.

In studying the ancient authors Oughtred is reported to have written down on the margin of the printed page some of the theorems and their proofs, expressed in the symbolic language of algebra.

In the preface of his *Clavis* of 1631 and of 1647 he says: "Wherefore, that I might more clearly behold the things themselves, I uncasing the Propositions and Demonstrations out of their covert of words, designed them in

notes and species appearing to the very eye. After that by comparing the divers affections of Theorems, inequality, proportion, affinity, and dependence, I tryed to educe new out of them."

It was this motive which led him to introduce many abbreviations in algebra and trigonometry. The pedagogical experience of recent centuries has endorsed Oughtred's view, provided of course that the pupil is carefully taught the exact meaning of the symbols. There have been and there still are those who oppose the intensive use of symbolism. In our day the new symbolism for all mathematics, suggested by the school of Peano in Italy, can hardly be said to be received with enthusiasm. In Oughtred's day symbolism was not yet the fashion. To be convinced of this fact one need only open a book of Edmund Gunter, with whom Oughtred came in contact in his youth, or consult the *Principia* of Sir Isaac Newton who flourished after Oughtred. The mathematical works of Gunter and Newton, particularly the former, are surprisingly destitute of mathematical symbols. The philosopher Hobbes, in a controversy with John Wallis, criticized the latter for that "Scab of Symbols," whereupon Wallis replied, "I wonder how you durst touch M. Oughtred for fear of catching the Scab. For, doubtlesse, his book is as much covered over with the Scab of Symbols, as any of mine. . . . As for my Treatise of Conick Sections, you say, it is covered over with the Scab of Symbols, that you had not the patience to examine whether it is well or ill demonstrated."<sup>1</sup>

Oughtred maintained his view of the importance of symbols on many different occasions. Thus, in his *Circles of Proportion*, 1632, p. 20:

"This manner of setting downe Theoremes, whether

<sup>1</sup> *Due Correction for Mr. Hobbes. Or Schoole Discipline, for not saying his Lessons right. In answer to his Six Lessons, directed to the Professors of Mathematicks.* By the Professor of Geometry. Oxford, 1656, pp. 7, 47, 50.

they be Proportions, or Equations, by Symbols or notes of words, is most excellent, artificiall, and doctrinall. Wherefore I earnestly exhort every one, that desireth though but to looke into the noble Sciences Mathematicall, to accustome themselves unto it: and indeede it is easie, being most agreeable to reason, yea even to sense. And out of this working may many singular consecretaries be drawne: which without this would, it may be, for ever lye hid."

#### RIGOROUS THINKING AND THE USE OF INSTRUMENTS.

The author's elevated concept of mathematical study as conducive to rigorous thinking shines through the following extract from his preface to the 1647 *Clavis*:

"...Which Treatise being not written in the usuall synthetical manner, nor with verbous expressions; but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newnesse of the delivery; and not any difficulty in the thing itselfe. For this specious and symbolically manner, neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation.

"Now my scope and intent in the first Edition of that my Key was, and in this New Filing, or rather forging of it, is, to reach out to the ingenious lovers of these Sciences, as it were Ariadnes thread, to guide them through the intricate Labyrinth of these studies, and to direct them for the more easie and full understanding of the best and antientest Authors;... That they may not only learn their propositions, which is the highest point of Art that most Students aime at; but also may perceive with what solertiousnesse, by what engines of aequations, Interpre-

tations, Comparations, Reductions, and Disquisitions, those antient Worthies have beautified, enlarged, and first found out this most excellent Science. . . . Lastly, by framing like questions problematically, and in a way of Analysis, as if they were already done, resolving them into their principles, I sought out reasons and means whereby they might be effected. And by this course of practice, not without long time, and much industry, I found out this way for the helpe and facilitation of Art."

Still greater emphasis upon rigorous thinking in mathematics is laid in the preface to the *Circles of Proportion* and in some parts of his *Apologeticall Epistle* against Delamain. In that preface William Forster quotes the reply of Oughtred to the question how he (Oughtred) had for so many years concealed his invention of the slide rules from himself (Forster) whom he had taught so many other things. The reply was:

"That the true way of Art is not by Instruments, but by Demonstration: and that it is a preposterous course of vulgar Teachers, to begin with Instruments, and not with the Sciences, and so instead of Artists, to make their Scholars only doers of tricks, and as it were Iuglers: to the despite of Art, losse of previous time, and betraying of willing and industrious wits, unto ignorance, and idlenesse. That the vse of Instruments is indeed excellent, if a man be an Artist: but contemptible, being set and opposed to Art. And lastly, that he meant to commend to me, the skill of Instruments, but first he would have me well instructed in the Sciences."

Delamain took a different view, arguing that instruments might very well be placed in the hands of pupils from the start. At the time of this controversy Delamain supported himself by teaching mathematics in London and he advertised his ability to give instruction in mathematics, including the use of instruments. Delamain brought the

charge against Oughtred of unjustly calling "many of the [British] Nobility and Gentry doers of trickes and jugglers." To this Oughtred replies:<sup>2</sup>

"As I did to Delamain and to some others, so I did to William Forster: I freely gave him my helpe and instruction in these faculties: only this was the difference, I had the very first moulding (as I may say) of this latter: But Delamain was already corrupted with doing upon Instruments, and quite lost from ever being made an Artist: I suffered not William Forster for some time so much as speake of any Instrument, except only the Globe it selfe; and to explicate, and worke the questions of the Sphaere, by the way of the Analemma: which also himselfe did describe for the present occasion. And this my restraint from such pleasing avocations, and holding him to the strictnesse of percept, brought forth this fruit, that in short time, even by his owne skill, he could not onely use any Instrument he should see, but also was able to delineate the like, and devise others."

As representing Delamain's views, we make the following selection from his *Grammelogia* (London, about 1633), the part near the end of the book and bearing the title, "In the behalfe of vulgar Teachers and others," where Delamain refers to Oughtred's charge that the scholars of "vulgar" teachers are "doers of tricks, as it were iuglers." Delamain says:

"... Which words are neither *cautelous*, nor *subterfugious*, but are as downe right in their *plainnesse*, as they are touching, and *pernitious*, by two much derogating from many, and glancing upon many noble *personages*, with too *grosse*, if not too *base* an attribute, in tearming them *doers of tricks, as it were to iuggle*: because they perhaps make use of a necessitie in the furnishing of themselves with such knowledge by *Practicall Instrumentall operation*,

<sup>2</sup> Oughtred, *Apologeticall Epistle*, p. 27.

when their more weighty *negotiations* will not permit them for *Theoreticall figurative demonstration*; those that are guilty of the aspersion, and are touched therewith may answer for themselves, and studie to be more *Theoreticall*, than *Practicall*: for the *Theory*, is as the *Mother* that produceth the *daughter*, the very sinewes and life of *Practise*, the excellencie and highest degree of true *Mathematicall Knowledge*: but for those that would make but a step as it were into that kind of *Learning*, whose onely desire is expedition, and facilitie, both which by the generall consent of all are best effected with Instrument rather than with tedious regular demonstrations, it was ill to checke them so grosly, not onely in what they have *Practised*, but abridging them also of their liberties with what they may *Practise*, which aspersion may not easily be slighted off by any *glosse* or *Apologie*, without an Ingenuous *confession*, or some mentall reservation: To which vilification, howsoever, in the behalfe of my selfe, and others, I answer; That *Instrumentall* operation is not only the Compendiating, and facilitating of *Art*, but even the glory of it, whole demonstration both of the making, and operation is soly in the *science*, and to an *Artist* or disputant proper to be knowne and so to all, who would truly know the cause of the *Mathematicall operations* in their originall; But, for none to know the use of a *Mathematicall Instrumen[t]*, except he knowes the cause of its operation, is somewhat too strict, which would keepe many from affecting the *Art*, which of themselves are ready enough every where, to conceive more harshly of the difficultie, and impossibilitie of attayning any skill therein, than it deserves, because they see nothing but obscure propositions, and perplex and intricate demonstrations before their eyes, whose unsavoury tartnes, to an unexperienced palate like bitter pills is sweetned over, and made pleasant with an *Instrumentall compendious facilitie*, and made to goe downe the more readily, and

yet to retaine the same vertue, and working; And me thinkes in this queasy age, all *helpes* may bee used to procure a *stomacke*, all *bates* and invitations to the declining studie of so noble a *Science*, rather than by rigid Method and generall *Lawes* to scarre men away. All are not of like disposition, neither all (as was sayd before) propose the same end, some resolve to *wade*, others to put a *finger* in onely, or wet a *hand*: now thus to tye them to an obscure and *Theoricall* forme of teaching, is to crop their hope, even in the very bud. . . . The beginning of a *man's knowledge* even in the use of an *Instrument*, is first founded on *doctrinal precepts*, and these precepts may be conceived all along in its use: and are so farre from being excluded, that they doe necessarily *concomitate* and are contained therein: the *practicke* being better understood by the *doctrinall part*, and this later explained by the *Instrumentall*, making precepts obvious unto sense, and the *Theory* going along with the *Instrument*, better informing and inlightning the understanding, etc. *vis vnita fortior*, so as if that in *Phylosophy* bee true, *Nihil est [in] intellectu quod non prius fuit in sensu.*"

The difference between Oughtred and Delamain as to the use of mathematical instruments raises an important question. Should the slide rule be placed in the hands of a boy before, or after, he has mastered the theory of logarithms? Should logarithmic tables be withheld from him until the theoretical foundation is laid in the mind of the pupil? Is it a good thing to let a boy use a surveying instrument unless he first learns trigonometry? Is it advisable to permit a boy to familiarize himself with the running of a dynamo before he has mastered the underlying principles of electricity? These and similar questions are even more vital to-day than they were in the seventeenth century. Does the use of instruments ordinarily discourage a boy from mastery of the theory? Or does such manipulation con-



stitute a natural and pleasing approach to the abstract? On this particular point, who showed the profounder psychological insight, Oughtred or Delamain?

In July, 1914, there was held in Edinburgh a celebration of the three hundredth anniversary of the invention of logarithms. On that occasion there was collected at Edinburgh university one of the largest exhibits ever seen of modern instruments of calculation. The opinion was expressed by an experienced teacher that "weapons as those exhibited there are for men and not for boys, and such danger as there may be in them is of the same character as any form of too early specialization."

It is somewhat of a paradox that Oughtred who in his student days and during his active years felt himself impelled to invent sun-dials, planispheres and various types of slide rules—instruments which represent the most original contributions which he handed down to posterity—should discourage the use of such instruments in teaching mathematics to beginners. That without the aid of instruments he himself should have succeeded so well in attracting and inspiring young men constitutes the strongest evidence of his transcendent teaching ability. It may be argued that his pedagogic dogma, otherwise so excellent, here goes contrary to the course he himself followed instinctively in his self-education along mathematical lines. We read that Sir Isaac Newton, as a child, constructed sun-dials, wind-mills, kites, paper lanterns and a wooden clock. Should these activities have been suppressed? Ordinary children are simply Isaac Newtons on a smaller intellectual scale. Should their activities along these lines be encouraged or checked?

On the other hand it may be argued that the paradox alluded to above admits of explanation like all paradoxes, and that there is no inconsistency between Oughtred's pedagogic views and his own course of development. If he

invented sun-dials, he must have had a comprehension of the cosmic motions involved; if he solved spherical triangles graphically by the aid of the planisphere, he must have understood the geometry of the sphere, so far as it relates to such triangles; if he invented slide rules, he had beforehand a thorough grasp of logarithms. The question at issue does not involve so much the invention of instruments, as the use by the pupil of instruments already constructed, before he fully understands the theory which is involved. Nor does Sir Isaac Newton's activity as a child establish Delamain's contention. Of course, a child should not be discouraged from manual activity along the line of producing interesting toys in imitation of structures and machines that he sees, but to introduce him to the realm of abstract thought by the aid of instruments is a different proposition, fraught with danger. A boy may learn to use a slide rule mechanically and, because of his ability to obtain practical results, feel justified in foregoing the mastery of underlying theory; or he may consider the ability of manipulating a surveying instrument quite sufficient, even though he be ignorant of geometry and trigonometry; or he may learn how to operate a dynamo and an electric switchboard, and be altogether satisfied, though having no grasp of electrical science. Thus instruments draw a youth aside from the path leading to real intellectual attainments and real efficiency; they allure him into lanes which are often blind alleys. Such were the views of Oughtred.

Who was right, Oughtred or Delamain? It may be claimed that there is a middle ground which more nearly represents the ideal procedure in teaching. Shall the slide rule be placed into the student's hands at the time when he is engaged in the mastery of principles? Shall there be an alternate study of the theory of logarithms and of the slide rule—on the idea of one hand washing the other—until a mastery of both the theory and the use of the

instrument has been attained? Does this method not produce the best and most lasting results? Is not this Delamain's actual contention? We leave it to the reader to settle these matters from his own observation, knowledge and experience.

#### NEWTON'S COMMENTS ON OUGHTRED.

Oughtred is an author who has been found to be of increasing interest to modern historians of mathematics. But no modern writer has, to our knowledge, pointed out his importance in the history of the *teaching* of mathematics. Yet his importance as a teacher did receive recognition in the seventeenth century by no less distinguished a scientist than Sir Isaac Newton. On May 25, 1694, Sir Isaac Newton wrote a long letter in reply to a request for his recommendation on a proposed new course of study in mathematics at Christ's Hospital.<sup>3</sup> Toward the close of his letter, Newton says:

"And now I have told you my opinion in these things, I will give you Mr. Oughtred's, a Man whose judgment (if any man's) may be safely relied upon. For he in his book of the circles of proposition, in the end of what he writes about Navigation (page 184) has this exhortation to Seamen. And if, sayth he, the Masters of Ships and Pilots will take the pains in the Journals of their Voyages diligently and faithfully to set down in severall columns, not onely the Rumb they goe on and the measure of the Ships way in degrees, and the observation of Latitude and variation of their compass; but alsoe their conjectures and reason of their correction they make of the aberrations they shall find, and the qualities and condition of their ship, and the diversities and seasons of the winds, and the secret motions or agitations of the Seas, when they begin, and how long they continue, how farr they extend and with

<sup>3</sup>J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes*, London, 1850, pp. 279-292.

what inequality; and what else they shall observe at Sea worthy consideration, and will be pleased freely to communicate the same with Artists such as are indeed skilfull in the Mathematicks and lovers and enquirers of the truth: I doubt not but that there shall be in convenient time, brought to light many necessary precepts which may tend to the perfecting of Navigation, and the help and safety of such whose Vocations doe inforce them to commit their lives and estates in the vast Ocean to the providence of God. Thus farr that very good and judicious man Mr. Oughtred, I will add, that if instead of sending the Observations of Seamen to able Mathematicians at Land, the Land would send able Mathematicians to Sea, it would signify much more to the improvement of Navigation and safety of Mens lives and estates on that element."

May Oughtred prove as instructive to the modern reader as he did to Newton.

#### OUGHTRED AND HARRIOT.

Oughtred's *Clavis mathematicae* was the most influential mathematical publication in Great Britain which appeared in the interval between John Napier's *Mirifici logarithmorum canonis descriptio*, Edinburgh, 1614, and the time, forty years later, when John Wallis began to publish his important researches at Oxford. The year 1631 is of interest as the date of publication, not only of Oughtred's *Clavis*, but also of Thomas Harriot's *Artis analyticae praxis*. We have no evidence that these two mathematicians ever met. Through their writings they did not influence each other. Harriot died ten years before the appearance of his *magnum opus*, or ten years before Oughtred began to publish. Strangely, Oughtred who survived Harriot thirty-nine years, never mentions him. There is no doubt that, of the two, Harriot was the more original mind, more capable of penetrating into new fields

of research. But he had the misfortune of having a strong competitor in René Descartes, in the development of Algebra, so that no single algebraic achievement stands out strongly and conspicuously as Harriot's own contribution to algebraic science. As a text to serve as an introduction to algebra, Harriot's *Artis analyticae praxis* was inferior to Oughtred's *Clavis*. The former was a much larger book, not as conveniently portable, compiled after the author's death by others and not prepared with the care in the development of the details, nor with the coherence and unity, and the profound pedagogic insight, which distinguish the work of Oughtred. Nor was Harriot's position in life such as to be surrounded by so wide a circle of pupils as was Oughtred. To be sure, Harriot had such followers as Torperley, William Lower and Protheroe in Wales, but this group is small as compared with Oughtred's.

#### OUGHTRED'S PUPILS.

There was a large number of distinguished men who, in their youth, either visited Oughtred's home and studied under his roof or else read his *Clavis* and sought his assistance by correspondence. We permit Aubrey to enumerate some of these pupils in his own gossip style:<sup>4</sup>

"Seth Ward, M.A., a fellow of Sydney Colledge in Cambridge (now bishop of Sarum), came to him and lived with him halfe a yeare (and he would not take a farthing for his diet), and learned all his mathematiques of him. Sir Jonas More was with him a good while, and learn't; he was but an ordinary logist before. Sir Charles Scarborough was his scholar; so Dr. John Wallis was his scholar; so was Christopher Wren his scholar, so was Mr.... Smethwyck, Regiae Societatis Socius. One Mr. Austin (a most ingeniose man) was his scholar, and studied so

<sup>4</sup> Aubrey, *op. cit.*, Vol. II, 1898, p. 108.

much that he became mad, fell a laughing, and so dyed, to the great grieve of the old gentleman. Mr. . . . Stokes, another scholar, fell mad, and dream't that the good old gentleman came to him, and gave him good advice, and so he recovered, and is still well. Mr. Thomas Henshawe, Regiae Societatis Socius, was his scholar (then a young gentleman). But he did not so much like any as those that tugged and tooke paines to worke out questions. He taught all free.

"He could not endure to see a scholar write an ill hand; he taught them all presently to mend their hands."

Had Oughtred been the means of guiding the mathematical studies of only John Wallis and Christopher Wren—one the greatest English mathematician between Napier and Newton, the other one of the greatest architects of England,—he would have earned profound gratitude. But the above list embraces nine men, most of them distinguished in their day. And yet Aubrey's list is very incomplete. It is easy to more than double it by adding the names of William Forster who translated from Latin into English Oughtred's *Circles of Proportion*, Arthur Haughton who brought out the 1660 Oxford edition of the *Circles of Proportion*, Robert Wood, an educator and politician who assisted Oughtred in the translation of the *Clavis* from Latin into English for the edition of 1647, W. Gascoigne, a man of promise who fell, 1644, at Marston Moor, John Twysden who was active as a publisher, William Sudell, N. Ewart, Richard Shuttleworth, William Robinson, and Henry Frederick Howard who was the son of the Earl of Arundel, for whose instruction Oughtred originally prepared the manuscript treatise that was published in 1631 as the *Clavis mathematicae*.

Nor must we overlook the names of Lawrence Rook (who "did admirably well read in Gresham Coll. on the sixth chapt. of the said book," the *Clavis*), Christopher

Brooke (a maker of mathematical instruments who married a daughter of the famous mathematician), William Leech and William Brearly (who with Robert Wood "have been ready and helpfull incouragers of me [Oughtred] in this labour" of preparing the English *Clavis* of 1647) and Thomas Wharton who studied the *Clavis* and assisted in the editing of the *Clavis* of 1647.

The devotion of these pupils bears eloquent testimony not only of Oughtred's ability as a mathematician but also of his power of drawing young men to him—of his personal magnetism. Nor should we omit from the list Richard Delamain, a teacher of mathematics in London, who unfortunately had a bitter controversy with Oughtred on the priority and independence of the invention of the circular slide rule and a form of sun-dial. Delamain became later a tutor in mathematics to King Charles I, and perished in the civil war, before 1645.

#### OUGHTRED, THE "TODHUNTER OF THE SEVENTEENTH CENTURY."

To afford a clearer view of Oughtred as a teacher and mathematical expositor we quote some passages from various writers and from his correspondence. Anthony Wood<sup>5</sup> gives an interesting account of how Seth Ward and Charles Scarborough went from Cambridge University to the obscure home of the country mathematician, to be initiated into the mysteries of algebra:

"Mr. Cha. Scarborough, then an ingenious young student and fellow of Caius Coll. in the same university, was his [Seth Ward's] great acquaintance, and both being equally students in that faculty and desirous to perfect themselves, they took a journey to Mr. Will. Oughtred living then at Albury in Surrey, to be informed in many things in his *Clavis mathematica* which seemed at that

<sup>5</sup> Wood's *Athenae Oxonienses* (Ed. P. Bliss), Vol. IV, 1820, p. 247.

time very obscure to them. Mr. Oughtred treated them with great humanity, being very much pleased to see such ingenious young men apply themselves to these studies, and in short time he sent them away well satisfied in their desires. When they returned to Cambridge, they afterwards read the *Clav. Math.* to their pupils, which was the first time that that book was read in the said university. Mr. Laur. Rook, a disciple of Oughtred, I think, and Mr. Ward's friend, did admirably well in Gresham Coll. on the sixth chap. of the said book, which obtained him great repute from some and greater from Mr. Ward, who ever after had an especial favour for him."

Anthony Wood makes a similar statement about Thomas Henshaw:<sup>6</sup>

"While he remained in that coll. [University College, Oxford] which was five years. . . . he made an excursion for about 9 months to the famous mathematician Will. Oughtred parson of Aldbury in Surrey, by whom he was initiated in the study of mathematics, and afterwards retiring to his coll. for a time, he at length went to London, was entered a student in the Middle Temple."

Extracts from letters of W. Gascoigne to Oughtred, of the years 1640 and 1641, throw some light upon mathematical teaching of the time:<sup>7</sup>

"Amongst the mathematical rarities these times have afforded, there are none of that small number I (a late intruder into these studies) have yet viewed, which so fully demonstrates their authors' great abilities as your *Clavis*, not richer in augmentations, than valuable for contraction; . . ."

"Your belief that there is in all inventions aliquid divinum, an infusion beyond human cogitations, I am confident will appear notably strengthened, if you please to afford this truth belief, that I entered upon these studies acciden-

<sup>6</sup> Wood, *op. cit.*, Vol. II, p. 445.

<sup>7</sup> Rigaud, *op. cit.*, Vol. I, pp. 33, 35.



tally after I betook myself to the country, having never had so much aid as to be taught addition, nor the discourse of an artist (having left both Oxford and London before I knew what any proposition in geometry meant) to inform me what were the best authors."

The following extracts from two letters by W. Robinson, written before the appearance of the 1647 English edition of the *Clavis*, express the feeling of many readers of the *Clavis* on its extreme conciseness and brevity of explanation:<sup>8</sup>

"I shall long exceedingly till I see your *Clavis* turned into a pick-lock; and I beseech you enlarge it, and explain it what you can, for we shall not need to fear either tautology or superfluity; you are naturally concise, and your clear judgment makes you both methodical and pithy; and your analytical way is indeed the only way." . . .

"I will once again earnestly entreat you, that you be rather diffuse in the setting forth of your English mathematical *Clavis*, than concise, considering that the wisest of men noted of old, and said *stultorum infinitus est numerus*, these arts cannot be made too easy, they are so abstruse of themselves, and men either so lazy or dull, that their fastidious wits take a loathing at the very entrance of these studies, unless it be sweetened on with plainness and facility. Brevity may well argue a learned author that without any excess or redundancy, either of matter or words, can give the very substance and essence of the thing treated of; but it seldom makes a learned scholar; and if one be capable, twenty are not; and if the master sum up in brief the pith of his own long labours and travails, it is not easy to imagine that scholars can with less labour than it cost their masters dive into the depths thereof."

Here is the judgment of another of Oughtred's friends:<sup>9</sup>

"... with the character I received from your and my

\* Rigaud, *op. cit.*, Vol. I, pp. 16, 26.

\* Rigaud, *op. cit.*, Vol. I, p. 66.

noble friend Sir Charles Cavendish, then at Paris, of your second edition of the same piece, made me at my return into England speedily to get, and diligently peruse the same. Neither truly did I find my expectation deceived; having with admiration often considered how it was possible (even in the hardest things of geometry) to deliver so much matter in so few words, yet with such demonstrative clearness and perspicuity: and hath often put me in mind of learned Mersennus his judgment (since dead) of it, that there was more matter comprehended in that little book than in Diophantus, and all the ancients. . . ."

Oughtred's own feeling was against diffuseness in textbook writing. In his revisions of his *Clavis* the original character of that book was not altered. In his reply to W. Robinson, Oughtred said:<sup>10</sup>

"... But my art for all such mathematical inventions I have set down in my *Clavis Mathematica*, which therefore in my title I say is *tum logisticae cum analyticae adeoque totius mathematicae quasi clavis*, which if any one of a mathematical genius will carefully study, (and indeed it must be carefully studied,) he will not admire others, but himself do wonders. But I (such is my tenuity) have enough *fungi vice cotis, acutum reddere quae ferrum valet, exsors ipsa secundi*, or like the touchstone, which being but a stone, base and little worth, can shew the excellence and riches of gold."

John Wallis held Oughtred's *Clavis* in high regard. When in correspondence with John Collins concerning plans for a new edition, Wallis wrote in 1666-67, six years after the death of Oughtred:<sup>11</sup>

"... But for the goodness of the book in itself, it is that (I confess) which I look upon as a very good book, and which doth in as little room deliver as much of the fundamental and useful part of geometry (as well as of

<sup>10</sup> *Ibid.*, p. 9.

<sup>11</sup> *Ibid.*, p. 475.

arithmetic and algebra) as any book I know; and why it should not be now acceptable I do not see. It is true, that as in other things so in mathematics, fashions will daily alter, and that which Mr. Oughtred designed by great letters may be now by others be designed by small; but a mathematician will, with the same ease and advantage, understand  $A$ , and  $a^3$  or  $aaa$ . . . . And the like I judge of Mr. Oughtred's *Clavis*, which I look upon (as those pieces of Vieta who first went in that way) as lasting books and classic authors in this kind; to which, notwithstanding, every day may make new additions. . . .

"But I confess, as to my own judgment, I am not for making the book bigger, because it is contrary to the design of it, being intended for a manual or contract; whereas comments, by enlarging it, do rather destroy it. . . But it was by him intended, in a small epitome, to give the substance of what is by others delivered in larger volumes. . . ."

That there continued to be a group of students and teachers who desired a fuller exposition than is given by Oughtred is evident from the appearance, over fifty years after the first publication of the *Clavis*, of a booklet by Gilbert Clark, entitled *Oughtredus Explicatus*, London, 1682. A review of this appeared in the *Acta Eruditorum*, Leipsic, 1684, p. 168, wherein Oughtred is named "clarissimus Angliae mathematicus." John Collins wrote Wallis<sup>12</sup> in 1666-67 that Clark, "who lives with Sir Justinian Isham, within seven miles of Northampton," "intimates he wrote a comment on the *Clavis*, which lay long in the hands of a printer, by whom he was abused, meaning Leybourn."

We shall have occasion below to refer to Oughtred's inability to secure a copy of a noted Italian mathematical

<sup>12</sup> *Ibid.*, p. 471.

work published a few years before. In those days the condition of the book trade in England must have been somewhat extraordinary. Dr. J. W. L. Glaisher throws some light upon this subject.<sup>13</sup> He found in the Calendar of State Papers, Domestic Series, 1637, a petition to Archbishop Laud in which it is set forth that, when Hooganhuisen, a Dutchman, "heretofore complained of in the High Commission for importing books printed beyond the seas," had been bound "not to bring in any more," one Vlacq (the computer and publisher of logarithmic tables) "kept up the same agency and sold books in his stead." . . . "Vlacq is now preparing to go beyond the seas to avoid answering his late bringing over nine bales of books contrary to the decree of the Star Chamber." Judgment was passed that, "Considering the ill-consequence and scandal that would arise by strangers importing and venting in this kingdom books printed beyond the seas," certain importations be prohibited, and seized if brought over.

This want of easy intercommunication of results of scientific research in Oughtred's time is revealed in the following letter, written by Oughtred to Robert Keylway, in 1645:<sup>14</sup>

"I speak this the rather, and am induced to a better confidence of your performance, by reason of a geometric-analytical art of practice found out by one Cavalieri, an Italian, of which about three years since I received information by a letter from Paris, wherein was praelibated only a small taste thereof, yet so that I divine great enlargement of the bounds of the mathematical empire will ensue. I was then very desirous to see the author's own book while my spirits were more free and lightsome, but I could not get it in France. Since, being more steep into

<sup>13</sup> J. W. L. Glaisher, "On Early Logarithmic Tables, and their Calculators," *Philosophical Magazine*, 4th. Ser., Vol. XLV, 1873, pp. 378, 379.

<sup>14</sup> Rigaud, *op. cit.*, Vol. I, p. 65.

years, daunted and broken with the sufferings of these disastrous times, I must content myself to keep home, and not put out to any foreign discoveries."

It was in 1655, when Oughtred was about eighty years old, that John Wallis, the great forerunner of Newton in Great Britain, began to publish his great researches on the arithmetic of infinites. Oughtred rejoiced over the achievements of his former pupil. In 1655, Oughtred wrote John Wallis as follows:<sup>15</sup>

"I have with unspeakable delight, so far as my necessary business, the infirmness of my health, and the greatness of my age (approaching now to an end) would permit, perused your most learned papers, of several choice arguments, which you sent me: wherein I do first with thankfulness acknowledge to God, the Father of lights, the great light he hath given you; and next I congratulate you, even with admiration, the clearness and perspicacity of your understanding and genius, who have not only gone, but also opened a way into these profoundest mysteries of art, unknown and not thought of by the ancients. With which your mysterious inventions I am the more affected, because full twenty years ago, the learned patron of learning, Sir Charles Cavendish, shewed me a paper written, wherein were some few excellent new theorems, wrought by the way, as I suppose, of Cavalieri, which I wrought over again more agreeably to my way. The paper, wherein I wrought it, I shewed to many, whereof some took copies, but my own I cannot find. I mention it for this because I saw therein a light breaking out for the discovery of wonders to be revealed to mankind, in this last age of the world: which light I did salute as afar off, and now at a nearer distance embrace in your prosperous beginnings. Sir, that you are pleased to mention my name in your never dying papers, that is your noble favour to

<sup>15</sup> *Ibid.*, p. 87.

me, who can add nothing to your glory, but only my applause. . . ."

The last sentence has reference to Wallis's appreciative and eulogistic reference to Oughtred in the preface. It is of interest to secure the opinion of later English writers who knew Oughtred only through his books. John Locke wrote in his journal under the date, June 24, 1681, "the best algebra yet extant is Outred's."<sup>16</sup> John Collins who is known in the history of mathematics chiefly through his very extensive correspondence with nearly all mathematicians of his day, was inclined to be more critical. He wrote Wallis<sup>17</sup> about 1667:

"It was not my intent to disparage the author, though I know many that did lightly esteem him when living, some whereof are at rest, as Mr. Foster and Mr. Gibson. . . . You grant the author is brief, and therefore obscure, and I say it is but a collection, which, if himself knew, he had done well to have quoted his authors, whereto the reader might have repaired. You do not like those words of Vieta in his theorems, *ex adjunctione plano solidi, plus quadrato quadrati*, etc., and think Mr. Oughtred the first that abridged those expressions by symbols; but I dissent, and tell you 'twas done before by Cataldus, Geysius, and Camillus Gloriosus.<sup>18</sup> who in his first decade of exercises, (not the first tract), printed at Naples in 1627, which was four years before the first edition of the *Clavis*, proposeth this equation just as I here give it you, viz.  $1ccc + 16qcc + 41qqc - 2304cc - 1836qc - 133000qq - 54505c + 3728q + 8064 N \text{ aequatur } 4608$ , finds *N* or a root of it to be 24, and composeth the whole out of it for proof, just in Mr. Oughtred's symbols and method. Catal-

<sup>16</sup> King's *Life of John Locke*, Vol. I, London, 1830, p. 227.

<sup>17</sup> Rigaud, *op. cit.*, Vol. I, pp. 477-480.

<sup>18</sup> *Exercitationum Mathematicarum Decas prima*, Naples, 1627, and probably Cataldus's *Transformatio Geometrica*, Bologna, 1612.

dus on Vieta came out fifteen years before, and I cannot quote that, as not having it by me."

"... And as for Mr. Oughtred's method of symbols, this I say of it; it may be proper for you as a commentator to follow it, but divers I know, men of inferior rank that have good skill in algebra, that neither use nor approve it. . . . Is not  $A^5$  sooner wrote than  $A_{qc}$ ? Let  $A$  be 2, the cube of 2 is 8, which squared is 64: one of the questions between Maghet Grisio and Gloriosus is whether  $64 = A_{cc}$  or  $A_{qc}$ . The Cartesian method tells you it is  $A^6$ , and decides the doubt. . . ."

There is some ground for the criticisms passed by Collins. To be sure, the first edition of the *Clavis* is dated 1631—six years before Descartes suggested the exponential notation which came to be adopted as the symbolism in our modern algebra. But the second edition of the *Clavis*, 1647, appeared ten years after Descartes's innovation. Had Oughtred seen fit to adopt the new exponential notation in 1647, the step would have been epoch-making in the teaching of algebra in England. We have seen no indication that Oughtred was familiar with Descartes's *Géométrie* of 1637.

The year preceding Oughtred's death Mr. John Twysden expressed himself as follows in the Preface to his *Miscellanies*:<sup>19</sup>

"It remains that I should adde something touching the beginning, and use of these Sciences. . . . I shall only, to their honours, name some of our own Nation yet living, who have happily laboured upon both stages. That succeeding ages may understand that in this of ours, there yet remained some who were neither ignorant of these Arts, as if they had held them vain, nor condemn them as superfluous. Amongst them all let Mr. William Ought-

<sup>19</sup> *Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster, Sometime publike Professor of Astronomie in Gresham Colledge in London*, by John Twysden, London, 1659.

red, of Aeton, be named in the first place, a Person of venerable grey haire, and exemplary piety, who indeed exceeds all praise we can bestow upon him. Who by an easie method, and admirable Key, hath unlocked the hidden things of geometry. Who by an accurate Trigonometry and furniture of Instruments, hath inriched, as well geometry, as Astronomy. Let D. John Wallis, and D. Seth Ward, succeed in the next place, both famous Persons, and Doctors in Divinity, the one of geometry, the other of astronomy, Savilian Professors in the University of Oxford."

The astronomer Edmund Halley, in his preface to the 1694 English edition of the *Clavis*, speaks of this book as one of "so established a reputation, that it were needless to say anything thereof," though "the concise Brevity of the author is such, as in many places to need Explication, to render it Intelligible to the less knowing in Mathematical matters."

In closing this part of our monograph, we quote the testimony of Robert Boyle, the experimental physicist, as given May 8, 1647, in a letter to Mr. Hartlib:<sup>20</sup>

"The Englishing of, and additions to Oughtred's *Clavis mathematica* does much content me, I having formerly spent much study on the original of that algebra, which I have long since esteemed a much more instructive way of logic, than that of Aristotle."

#### WAS DESCARTES INDEBTED TO OUGHTRED?

This question first arose in the seventeenth century, when John Wallis of Oxford, in his *Algebra* (the English edition of 1685, and more particularly the Latin edition of 1693) raised the issue of Descartes's indebtedness to the English scientists, Thomas Harriot and William Oughtred.

<sup>20</sup> *The Works of the Honourable Robert Boyle in five volumes to which is prefixed the Life of the Author*, Vol. I, London, 1744, p. 24.



In discussing matters of priority between Harriot and Descartes, relating to the theory of equations, Wallis is generally held to have shown marked partiality to Harriot. Less attention has been given by historians of mathematics to Descartes's indebtedness to Oughtred. Yet this question is of importance in tracing Oughtred's influence upon his time.

On January 8, 1688-89, Samuel Morland addressed a letter of inquiry to John Wallis, containing a passage which we translate from the Latin:<sup>21</sup>

"Some time ago I read in the elegant and truly precious book that you have written on *Algebra*, about Descartes, this philosopher so extolled above all for having arrived at a very perfect system by his own powers, without the aid of others, this Descartes, I say, who has received in geometry very great light from our Oughtred and our Harriot, and has followed their track though he carefully suppressed their names. I stated this in a conversation with a professor in Utrecht (where I reside at present). He requested me to indicate to him the page-numbers in the two authors which justified this accusation. I admitted that I could not do so. The *Géométrie* of Descartes is not sufficiently familiar to me, although with Oughtred I am fairly familiar. I pray you therefore that you will assume this burden. Give me at least those references to passages of the two authors from the comparison of which the plagiarism by Descartes is the most striking."

Following Morland's letter in the *De algebra tractatus*, is printed Wallis's reply, dated March 12, 1688 ("Stilo Angliae"), which is, in part, as follows:

"I nowhere give him the name of a plagiarist; I would not appear so impolite. However this I say, the major part of his algebra (if not all) is found before him in other authors (notably in our Harriot) whom he does not

<sup>21</sup> The letter is printed in John Wallis's *De Algebra Tractatus*, 1693, p. 206.

designate by name. That algebra may be applied to geometry, and that it is in fact so applied, is nothing new. Passing the ancients in silence, we state that this has been done by Vieta, Ghetaldi, Oughtred and others, before Descartes. They have resolved by algebra and specious arithmetic [literal arithmetic] many geometrical problems. . . . But the question is not as to application of algebra to geometry (a thing quite old), but of the Cartesian algebra considered by itself."

Wallis then indicates in the 1659 edition of Descartes's *Géométrie* where the subjects treated on the first six pages are found in the writings of earlier algebraists, particularly of Harriot and Oughtred. For example, what is found on the first page of Descartes, relating to addition, subtraction, multiplication, division and root extraction, is declared by Wallis to be drawn from Vieta, Ghetaldi and Oughtred.

It is true that Descartes makes no mention of modern writers, except once of Cardan. But it was not the purpose of Descartes to write a history of algebra. To be sure, references to such of his immediate predecessors as he had read would not have been out of place. Nevertheless Wallis fails to show that Descartes made illegitimate use of anything he may have seen in Harriot or Oughtred.

The first inquiry to be made is, did Descartes possess copies of the books of Harriot and Oughtred? It is only in recent time that this question has been answered as to Harriot. As to Oughtred it is still unanswered. It is now known that Descartes had seen Harriot's *Artis analyticae praxis* (1631). Descartes wrote a letter to Constantin Huygens in which he states that he is sending Harriot's book.<sup>22</sup>

An able discussion of the question, what effect, if any,

<sup>22</sup> See *La Correspondance de Descartes*, published by Charles Adam and Paul Tannery, Vol. II, Paris, 1898, pp. 456 and 457.

Oughtred's *Clavis mathematicae* of 1631 had upon Descartes's<sup>23</sup> *Géométrie* of 1637, is given by H. Bosmans in a recent article. According to Bosmans no evidence has been found that Descartes possessed a copy of Oughtred's book, or that he had examined it. Bosmans believes nevertheless that Descartes was influenced by the *Clavis*, either directly or indirectly. Says he:<sup>24</sup>

"If Descartes did not read it carefully, which is not proved, he was none the less well informed with regard to it. No one denies his intimate knowledge of the intellectual movement of his time. The *Clavis mathematica* enjoyed a rapid success. It is impossible that, at least indirectly, he did not know the more original ideas which it contained. Far from belittling Descartes, as I much desire to repeat, this rather makes him the greater."

We ourselves would hardly go as far as does Bosmans. Unless Descartes actually examined a copy of Oughtred it is not likely that he was influenced by Oughtred in appreciable degree. Book reviews were quite unknown in those days. No evidence has yet been adduced to show that Descartes obtained a knowledge of Oughtred by correspondence. A most striking feature about Oughtred's *Clavis* is its notation. No trace of the Englishman's symbolism has been pointed out in Descartes's *Géométrie* of 1637. Only six years intervened between the publication of the *Clavis* and the *Géométrie*. It took longer than this period for the *Clavis* to show evidence of its influence upon mathematical books published in *England*; it is not probable that *abroad* the contact was more immediate than at home. Our study of seventeenth century algebra has led us to the conviction that Oughtred deserves a higher place in the development of this science than is usually accorded

<sup>23</sup> H. Bosmans, S. J., "La première édition de la *Clavis Mathematica* d'Oughtred. Son influence sur la *Géométrie* de Descartes," in *Annales de la société scientifique de Bruxelles*, 35th year, 1910-1911, Part II, pp. 24-78.

<sup>24</sup> H. Bosmans, *loc. cit.*, p. 78.

to him; but that it took several decennia for his influence fully to develop.

#### THE SPREAD OF- OUGHTRED'S NOTATIONS.

An idea of Oughtred's influence upon mathematical thought and teaching can be obtained from the spread of his symbolism. This study indicates that the adoption was not immediate. The earliest use that we have been able to find of Oughtred's notation for proportion,  $A.B::C.D$ , occurs nineteen years after the *Clavis mathematica* of 1631. In 1650 John Kersey brought out in London an edition of Edmund Wingate's *Arithmetique made easie*, in which this notation is used. After this date publications employing it become frequent, some of them being the productions of pupils of Oughtred. We have seen it in Vincent Wing (1651),<sup>25</sup> Seth Ward (1653),<sup>26</sup> John Wallis (1655),<sup>27</sup> in "R. B.," a schoolmaster in Suffolk,<sup>28</sup> Samuel Foster (1659),<sup>29</sup> Jonas Moore (1660),<sup>30</sup> and Isaac Barrow (1657).<sup>31</sup> In the latter part of the seventeenth century Oughtred's notation,  $A.B::C.D$ , became the prevalent, though not universal, notation in Great Britain. A tremendous impetus to their adoption was given by Seth Ward, Isaac Barrow, and particularly by John Wallis who was rising to international eminence as a mathematician.

In France we have noticed Oughtred's notation for proportion in Franciscus Dulaurens (1667),<sup>32</sup> J. Prestet

<sup>25</sup> Vincent Wing, *Harmonicon coeleste*, London, 1651, p. 5.

<sup>26</sup> Seth Ward, *In Ismaelis Bullialdi astronomiae philolaicae fundamenta inquisitio brevis*, Oxford, 1653, p. 7.

<sup>27</sup> John Wallis, *Elenchus geometriae Hobbiana*, Oxford, 1655, p. 48.

<sup>28</sup> *An Idea of Arithmetick, at first designed for the use of the Free Schoole at Thurlow in Suffolk...* By R. B., Schoolmaster there, London, 1655, p. 6.

<sup>29</sup> *The Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster* ... by John Twysden, London, 1659, p. 1.

<sup>30</sup> *Moore's Arithmetick in two Books*, London, 1660, p. 89.

<sup>31</sup> Isaac Barrow, *Euclidis data*, Cambridge, 1657, p. 2.

<sup>32</sup> *Francisci Dulaurens Specima mathematica*, Paris, 1667, p. 1.

(1675),<sup>33</sup> R. P. Bernard Lamy (1684),<sup>34</sup> Ozanam (1691),<sup>35</sup> R. P. Petro Nicolas (1697).<sup>36</sup>

In the Netherlands we have noticed it in R. P. Bernard Lamy (1680),<sup>37</sup> and in an anonymous work of 1690.<sup>38</sup> In German and Italian works of the seventeenth century we have not seen Oughtred's notation for proportion.

In England a modified notation soon sprang up in which ratio was indicated by two dots instead of a single dot, thus  $A : B :: C : D$ . The reason for the change lies probably in the inclination to use the single dot to designate decimal fractions. W. W. Beman pointed out that this modified symbolism (:) for ratio is found as early as 1657 in the end of the trigonometric and logarithmic tables that were bound with Oughtred's *Trigonometria*.<sup>39</sup> It is not probable however that this notation was used by Oughtred himself. The *Trigonometria* proper has Oughtred's  $A.B :: C.D$  throughout. Moreover in the English edition of this trigonometry which appeared the same year, 1657, but subsequent to the Latin edition, the passages which contained the colon as the symbol for ratio, when not omitted, are recast, and the regular Oughtredian notation is introduced. In Oughtred's posthumous work, *Opuscula mathematica hactenus inedita*, 1677, the colon appears quite often but is most likely due to the editor of the book.

We have noticed that the notation  $A : B :: C : D$  antedates the year 1657. Vincent Wing, the astronomer, published in 1651 in London the *Harmonicon coeleste* in which is found not only Oughtred's notation  $A.B :: C.D$  but also

<sup>33</sup> *Elémens des mathématiques*, Paris, 1675, Preface signed "J. P."

<sup>34</sup> *Nouveaux élémens de géométrie*, Paris, 1692 (permission to print 1684).

<sup>35</sup> Ozanam, *Dictionnaire mathématique*, Paris, 1691, p. 12.

<sup>36</sup> Petro Nicolas, *De conchoidibus et cissoidibus exercitationes geometricae*, Toulouse, 1697, p. 17.

<sup>37</sup> R. P. Bernard Lamy, *Elémens des mathématiques*, Amsterdam, 1692 (permission to print 1680).

<sup>38</sup> *Nouveaux élémens de géométrie*, 2d. ed., The Hague, 1690, p. 304.

<sup>39</sup> W. W. Beman in *L'intermédiaire des mathématiciens*, Paris, Vol. IX, 1902, p. 229, question 2424.

the above modified form of it. The two are used interchangeably. His later works, the *Logistica astronomica* (1656), *Doctrina spherica* (1655) and *Doctrina theorica*, published in one volume in London, all use the symbols  $A : B :: C : D$  exclusively. The author of a book entitled, *An Idea of Arithmetick at first designed for the use of the Free Schoole at Thurlow in Suffolk... by R. B., School-master there*, London, 1655, writes  $A : a :: C : c$ , though part of the time he uses Oughtred's unmodified notation.

We can best indicate the trend in England by indicating the authors of the seventeenth century whom we have found using the notation  $A : B :: C : D$  and the authors of the eighteenth century whom we have found using  $A.B :: C.D$ . The former notation was the less common during the seventeenth but the more common during the eighteenth century. We have observed the symbols  $A : B :: C : D$ , (besides the authors already named) in John Collins (1659),<sup>40</sup> James Gregory (1663),<sup>41</sup> Christopher Wren (1668-69),<sup>42</sup> William Leybourn (1673),<sup>43</sup> William Sanders (1686),<sup>44</sup> John Hawkins (1684),<sup>45</sup> Joseph Raphson (1697),<sup>46</sup> E. Wells (1698)<sup>47</sup> and John Ward (1698).<sup>48</sup>

Of English eighteenth century authors the following still clung to the notation  $A.B :: C.D$ : John Harris's translation of F. Ignatius Gaston Pardies (1701),<sup>49</sup> George

<sup>40</sup> John Collins, *The Mariner's Plain Scale New Plain'd*, London, 1659, p. 25.

<sup>41</sup> James Gregory, *Optica promota*, London, 1663, pp. 19, 48.

<sup>42</sup> *Philosophical Transactions*, Vol. III, London, p. 868.

<sup>43</sup> William Leybourn, *The Line of Proportion*, London, 1673, p. 14.

<sup>44</sup> *Elementa geometriæ... a Gulielmo Sanders*, Glasgow, 1686, p. 3.

<sup>45</sup> *Cocker's Decimal Arithmetick*,...perused by John Hawkins, London, 1695 (preface dated 1684), p. 41.

<sup>46</sup> Joseph Raphson, *Analysis Aequationum universalis*, London, 1697, p. 26.

<sup>47</sup> E. Wells, *Elementa arithmeticae numerosae et speciosae*, Oxford, 1698, p. 107.

<sup>48</sup> John Ward, *A Compendium of Algebra*, 2d ed., London, 1698, p. 62.

<sup>49</sup> *Plain Elements of Geometry and Plain Trigonometry*, London, 1701, p. 63.

Shelley (1704),<sup>50</sup> Sam Cobb (1709),<sup>51</sup> John Craig (1718),<sup>52</sup> Jo. Wilson (1724).<sup>53</sup> During the seventeenth century the notation  $A : B :: C : D$  acquired almost complete ascendancy in England.

In France Oughtred's unmodified notation  $A.B :: C.D$ , having been adopted later, was also discarded later than in England. An approximate idea of the situation appears from the following data. The notation  $A.B :: C.D$  was used by M. Carré (1700),<sup>54</sup> M. Guisnée (1705),<sup>55</sup> M. de Fontenelle (1727),<sup>56</sup> M. Varignon (1725),<sup>57</sup> M. Robillard (1753),<sup>58</sup> M. Sebastien le Clerc (1764),<sup>59</sup> Clairaut (1731),<sup>60</sup> M. L'Hopital (1781).<sup>61</sup>

In Italy Oughtred's modified notation  $a : b :: c : d$  found entrance the latter part of the eighteenth century. In Germany the symbolism  $a : b = c : d$ , suggested by Leibniz, found wider acceptance.<sup>62</sup>

It is evident from the data presented that Oughtred proposed his notation for ratio and proportion at a time

<sup>50</sup> George Shelley, *Wingate's Arithmetick*, London, 1704, p. 343.

<sup>51</sup> *A Synopsis of Algebra, Being a posthumous work of John Alexander of Bern, Switzerland... Done from the Latin by Sam. Cobb*, London, 1709, p. 16.

<sup>52</sup> John Craig, *De Calculo fluentium*, London, 1718, p. 35. The notation  $A : B :: C : D$  is given also.

<sup>53</sup> *Trigonometry*, 2d ed., Edinburgh, 1724, p. 11.

<sup>54</sup> *Méthode pour la mesure des surfaces, la dimension des solides... par M. Carré de l'académie r. des sciences*, 1700, p. 59.

<sup>55</sup> *Application de l'algèbre à la géométrie...* Paris, 1705.

<sup>56</sup> *Elémens de la géométrie de l'infini*, by M. de Fontenelle, Paris, 1727, p. 110.

<sup>57</sup> *Eclaircissemens sur l'analyse des infiniment petits*, by M. Varign, Paris, 1725, p. 87.

<sup>58</sup> *Application de la géométrie ordinaire et des calculs différentiel et intégral*, by M. Robillard, Paris, 1753.

<sup>59</sup> *Traité de géométrie théorique et pratique*, new ed., Paris, 1764, p. 15.

<sup>60</sup> *Recherches sur les courbes à double courbure*, Paris, 1731, p. 13.

<sup>61</sup> *Analyse des infiniment petits*, by the Marquis de L'Hopital. New ed. by M. Le Fèvre, Paris, 1781, p. 41. In this volume passages in fine print, probably supplied by the editor, contain the notation  $a : b :: c : d$ ; the parts in large type give Oughtred's original notation.

<sup>62</sup> The tendency during the eighteenth century is shown in part by the following data: *Jacobi Bernoulli Opera, Tomus primus*, Geneva, 1744, gives  $B.A :: D.C$  on page 368, the paper having been first published in 1688; on page 419 is given  $GE : AG = LA : ML$ , the paper having been first published in 1689. *Bernhardi Nieuwentiit analysis infinitorum*, Amsterdam, 1695, has on page 276,

when the need of a specific notation began to be generally felt, that his symbol for ratio  $a.b$  was temporarily adopted in England and France but gave way in the eighteenth century to the symbol  $a:b$ , that Oughtred's symbol for proportion  $::$  found almost universal adoption in England and France and was widely used in Italy, the Netherlands, the United States and to some extent in Germany; it has survived to the present time but is now being gradually displaced by the sign of equality  $=$ .

Oughtred's notation to express aggregation of terms has received little attention from historians but is nevertheless interesting. His books, as well as those of John Wallis, are full of parentheses but they are not used as symbols of aggregation in algebra; they are simply marks of punctuation for parenthetical clauses. We have seen that Oughtred writes  $(a+b)^2$  and  $\sqrt{a+b}$  thus,  $Q:a+b:$ ,  $\sqrt{a+b}$ , or  $Q:a+b$ ,  $\sqrt{a+b}$ , using on rarer occasions a single dot in place of the colon. This notation did not originate with Oughtred but, in slightly modified form, occurs in writings from the Netherlands. In 1603 *C. Dibvadii in geometriam Euclidis demonstratio numeralis*, Leyden, contains many expressions of this sort,

$x:c-x::s:r$ . Paul Halcken's *Deliciae mathematicae*, Hamburg, 1719, gives  $a:b::c:d$ . Johannis Baptistae Caraccioli, *Geometria algebraica universa*, Rome, 1759, p. 79 has  $a.b::c.d$ . *Delle corde ouverte fibre elastiche schediasmi fisico-matematici del conte Giordano Riccati*, Bologna, 1767, p. 65 gives  $P:b::r:ds$ . "Produzioni mathematiche" del Conte Giulio Carlo di Fagnano, Vol. I, Pesario, 1750, p. 193, has  $a.b::c.d$ . *Géométrie du compas*, by L. Mascheroni, translated by A. M. Carette, Paris, 1798, p. 188, gives  $\sqrt{3}:2::\sqrt{2}:Lp$ . Danielis Melandri and Paulli Frisi, *De theoria lunae commentarii*, Parma, 1769, p. 13, has  $a:b::c:d$ . *Institutiones analyticae*, Vicentio Riccato and Hieronymo Saladino, Vol. I, Bologna, 1765, p. 47, gives  $x:a::m:n+m$ . R. S. Boscovich, *Opera pertinentia ad opticam et astronomiam*, Bassani, 1785, p. 409, uses  $a:b::c:d$ . Jacob Bernoulli, *Ars Conjectandi*, Basel, 1713, has  $n-r.n-1::c.d$ . Pavlini Chelvicii, *Institutiones analyticae, editio post tertiam Romanum prima in Germania*, Vienna, 1761, p. 2,  $a.b::c.d$ . Christiani Wolfii, *Elementa matheseos universae*, Vol. III, Geneva, 1735, p. 63, has  $AB:AE=1:q$ . Johann Bernoulli, *Opera omnia*, Vol. I, Lausanne and Geneva, 1742, p. 43, has  $a:b=c:d$ . *Analyse des mesures des rapports et des angles*, by D. C. Walmesley, Paris, 1749, uses extensively  $a.b::c.d$ , later  $a:b::c:d$ . *Institutiones geometriae sublimioris*, by G. W. Krafft, Tübingen, 1753, p. 194, has  $a:b=c:d$ . J. H. Lambert, *Photometria*, 1760, p. 104, has  $C:\pi=BC^2:MH^2$ . *Meccanica sublimis del Dott. Domenico Bartaloni*, Naples, 1765, has  $a:b::c:d$ . Occasionally ratio is not designated by  $a.b$ , nor by  $a:b$ , but by



$\sqrt{\cdot 136} + \sqrt{2048}$ , signifying  $\sqrt{(136 + \sqrt{2048})}$ . The dot is used to indicate that the root of the binomial (not of 136 alone) is called for. This notation is used extensively in *Ludolphi à Cevlen de circulo*, Leyden, 1619, and in *Willebrordi Snellii De circuli dimensione*, Leyden, 1621. In place of the single dot Oughtred used the colon (:), probably to avoid confusion with his notation for ratio. To avoid further possibility of uncertainty he usually placed the colon both before and after the algebraic expression under aggregation. This notation was adopted by John Wallis and Isaac Barrow. It is found in the writings of Descartes. Together with Vieta's horizontal bar, placed over two or more terms, it constituted the means used almost universally for denoting aggregation of terms in algebra. Before Oughtred the use of parentheses had been suggested by Clavius<sup>63</sup> and Girard.<sup>64</sup> The latter wrote for instance  $\sqrt{(2 + \sqrt{3})}$ . While parentheses never became popular in algebra before the time of Leibniz and the Bernoullis they were by no means lost sight of. We are able to point to the following authors who made use of them: I. Errard de Bar-le-Duc (1619),<sup>65</sup> Jacobo De Billy (1643),<sup>66</sup> one of whose books containing this notation was translated into English, and also the posthumous works of Samuel Foster.<sup>67</sup>

$a, b$ , as for instance in A. de Moivre's *Doctrine of Chance*, London, 1756, p. 34, where he writes  $a, b :: 1, q$ . A further variation in the designation of ratio is found in James Atkinson's *Epitome of the Art of Navigation*, London, 1718, p. 24, namely,  $3.2 :: 72.48$ . Curious notations are given in Rich, Balam's *Algebra*, London, 1653.

<sup>63</sup> *Chr. Clavii Operum mathematicorum tomus secundus*, Mayence, 1611, algebra, p. 39.

<sup>64</sup> *Invention nouvelle en l'algèbre*, by Albert Girard, Amsterdam, 1629, p. 17.

<sup>65</sup> *La géométrie et pratique générale d'icelle*, par I. Errard de Bar-le-Duc, Ingénieur ordinaire de sa Majesté. 3d ed., revised by D. H. P. E. M., Paris, 1619, p. 216.

<sup>66</sup> *Novae geometriae clavis algebra*, authore P. Jacobo de Billy, Paris, 1643, p. 157; also an *Abridgement of the Precepts of Algebra*. Written in French by James de Billy, London, 1659, p. 346.

<sup>67</sup> *Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster, sometime publike Professor of Astronomie in Gresham Colledge in London*, London, 1659, p. 7.

The symbol for the arithmetical difference between two numbers,  $\sim$ , is usually attributed to John Wallis but it occurs in Oughtred's *Clavis Mathematicae* of 1652, in the tract on *Elementi decimi Euclidis declaratio*, at an earlier date than in any of Wallis's books. As Wallis assisted in putting this edition through the press it is possible though not probable that the symbol was inserted by him. Were the symbol Wallis's, Oughtred would doubtless have referred to its origin in the preface. During the eighteenth century the symbol found its way into foreign texts even in far off Italy.<sup>68</sup> It is one of three symbols presumably invented by Oughtred and which are still used at the present time. The other two are  $\times$  and  $::$

The curious and ill-chosen symbols,  $\sqsupset$  for "greater than," and  $\sqsubset$  for "less than," were certain to succumb in their struggle for existence against Harriot's admirably chosen  $>$  and  $<$ . Yet such was the reputation of Oughtred that his symbols were used in England quite extensively during the seventeenth and beginning of the eighteenth centuries. Considerable confusion has existed among algebraists and also among historians as to what Oughtred's symbols really were. Particularly is this true of the sign for "less than" which is frequently written  $\sqsubset$ . Oughtred's symbols, or these symbols turned about in some way, have been used by Seth Ward,<sup>69</sup> John Wallis,<sup>70</sup> Isaac Barrow,<sup>71</sup> John Kersey,<sup>72</sup> E. Wells,<sup>73</sup> John Hawkins,<sup>74</sup> Tho.

<sup>68</sup> Pietro Cossali, *Origine, trasporto in Italia primi progressi in essa dell'algebra*, Vol. I, Parmense, 1797, p. 52.

<sup>69</sup> In. Is. Bullialdi *astronomiae philolaicae fundamenta inquisitio brevis*, Auctore Setho Wardo, Oxford, 1653, p. 1.

<sup>70</sup> John Wallis, *Algebra*, London, 1685, p. 321, and in some of his other works. He makes greater use of Harriot's symbols.

<sup>71</sup> *Euclidis data*, 1657, p. 1; also *Euclidis elementorum libris XV*, London, 1659, p. 1.

<sup>72</sup> John Kersey, *Algebra*, London, 1673, p. 321.

<sup>73</sup> E. Wells, *Elementa arithmeticae numerosae et speciosae*, Oxford, 1698, p. 142.

<sup>74</sup> Cocker's *Decimal Arithmetick*, perused by John Hawkins, London, 1695 (preface dated 1684), p. 278.

Baker,<sup>75</sup> Richard Sault,<sup>76</sup> Richard Rawlinson,<sup>77</sup> Franciscus Dulaurens,<sup>78</sup> James Milnes,<sup>79</sup> George Cheyne,<sup>80</sup> John Craig<sup>81</sup> and Jo. Wilson.<sup>82</sup>

General acceptance has been accorded to Oughtred's symbol  $\times$ . The first printed appearance of this symbol for multiplication in 1618 in the form of the letter  $x$  hardly explains its real origin. The author of the "Appendix" (be he Oughtred or some one else) may not have used the letter  $x$  at all but may have written the cross  $\times$ , called the St. Andrews cross, while the printer, in the absence of any type accurately representing that cross, may have substituted the letter  $x$  in its place. The hypothesis that the symbol  $\times$  of multiplication owes its origin to the old habit of using two directed bars to indicate that two numbers are to be combined, as for instance in the multiplication of 23 and 34, thus,

$$\begin{array}{r} 2 \quad \quad 3 \\ | \quad \quad | \\ 3 \quad \quad 4 \\ \hline 7 \quad 8 \quad 2 \end{array}$$

has been advanced by two writers, C. Le Paige<sup>83</sup> and Gravelaar.<sup>84</sup> Bosmans is more inclined to the belief that

<sup>75</sup> Th. Baker, *The geometrical Key*, London, 1684, p. 15.

<sup>76</sup> Richard Sault, *A New Treatise of Algebra*, London (no date).

<sup>77</sup> Richard Rawlinson in a pamphlet without date, issued sometime between 1655 and 1668, containing trigonometric formulas. There is a copy in the British Museum.

<sup>78</sup> F. Dulaurens, *Specma mathematica*, Pars, 1667, p. 1.

<sup>79</sup> J. Milnes, *Sectionum conicarum elementa*, Oxford, 1702, p. 42.

<sup>80</sup> Cheyne, *Philosophical Principles of Natural Religion*, London, 1705, p. 55.

<sup>81</sup> J. Craig, *De calculo fluentium*, London, 1718, p. 86.

<sup>82</sup> Jo. Wilson, *Trigonometry*, 2d ed., Edinburgh, 1724, p. v.

<sup>83</sup> C. Le Paige, "Sur l'origine de certains signes d'opération," in *Annales de la société scientifique de Bruxelles*, 16th year, 1891-1892, Part II, pp. 79-82.

<sup>84</sup> Gravelaar, "Over den oorsprong van ons maalteeken ( $\times$ )," *Wiskundig Tijdschrift*, 6th year. We have not had access to this article.

Oughtred adopted the symbol somewhat arbitrarily, much as he did the numerous symbols in his *Elementi decimi Euclidis declaratio*.<sup>85</sup> In the absence of any further facts the mind is quite free to indulge in the sweets of unrestricted speculation as to the origin of this symbol.

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<sup>85</sup> H. Bosmans, *loc. cit.*, p. 40.