



504. On the Circles of Curvature at B and C of the Conic $a^2 = \beta \gamma$

Author(s): N. M. Gibbins

Source: *The Mathematical Gazette*, Vol. 9, No. 127 (Jan., 1917), p. 12

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3602978>

Accessed: 06-01-2016 07:59 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

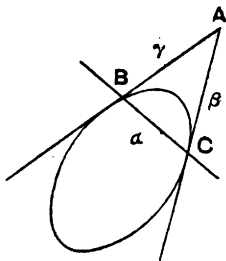
504. [L¹. 6. c.] On the circles of curvature at B and C of the conic $a^2 = \beta\gamma$.

1. Their equations are respectively

$$b(a^2 + \gamma^2 + 2a\gamma \cos B) = \gamma \cdot \Sigma a$$

and

$$c(a^2 + \beta^2 + 2a\beta \cos C) = \beta \cdot \Sigma a.$$



These may be written

$$\Sigma a\beta\gamma = \frac{ac}{b^2} \left(\frac{ba}{a} + \frac{b-c}{c} \beta \right) \Sigma aa$$

and

$$\Sigma a\beta\gamma = \frac{ab}{c^2} \left(\frac{ca}{a} - \frac{b-c}{b} \beta \right) \Sigma aa,$$

so that the radical axis of the circles is $bc(b+c)a = a(b^2\beta + c^2\gamma)$, on reduction.

2. The chords of intersection of the circles with the curve are

$$(a - 2b \cos B)a - (b - c)\gamma = 0$$

and

$$(a - 2c \cos C)a + (b - c)\beta = 0, \text{ respectively.}$$

These meet at the point given by

$$\frac{\alpha_0}{b-c} = -\frac{\beta_0}{(a-2c \cos C)} = -\frac{\gamma_0}{a-2b \cos B} \equiv \lambda,$$

while

$$\alpha_0^2 - \beta_0\gamma_0 \equiv 2\lambda^2(a^2 - bc)(1 - \cos A).$$

3. If $a^2 = bc$, the circles will intersect on the curve itself; and the radical axis will become $a(b+c)a = b^2\beta + c^2\gamma$, thus passing through the point $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ or the c.g. of the triangle of reference.

Moreover, the radical axis cuts the curve where $a^2(b+c)^2\beta\gamma = (b^2\beta + c^2\gamma)^2$ or $(b^3\beta - c^3\gamma)(b\beta - c\gamma) = 0$.

Thus the common point of the circles and the curve is $\left(\frac{1}{a^3}, \frac{1}{b^3}, \frac{1}{c^3}\right)$.

The other point of intersection of the circles is $(2a \cos A, b, c)$, which is collinear with the vertex A and the symmedian point (a, b, c) .

4. Note on the point $\left(\frac{1}{a^3}, \frac{1}{b^3}, \frac{1}{c^3}\right)$ in any triangle.

Let the above point be called P , and the Brocard points Ω and Ω' .

Also let $\lambda \equiv 2\Delta/\Sigma b^2c^2$.

Then the actual perpendiculars on the sides of the triangle of reference are:

for Ω ,	$c^2a\lambda$,	$a^2b\lambda$,	$b^2c\lambda$,
for Ω' ,	$ab^2\lambda$,	$bc^2\lambda$,	$ca^2\lambda$,
for P ,	$\frac{b^2c^2}{a}\lambda$,	$\frac{c^2a^2}{b}\lambda$,	$\frac{a^2b^2}{c}\lambda$.

By addition of the columns it is evident that the c.g. of the triangle of reference is also the c.g. of the triangle $P\Omega\Omega'$. Hence the point P is easily constructed.

N. M. GIBBINS.