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LII. *Application of the Variations of Temperature in Air that changes its Volume to account for the Velocity of Sound.* By J. IVORY, Esq. M.A. F.R.S.\*

THE formula for the velocity of sound investigated by Newton, having finally overcome all objections, it still remained to account for the remarkable discrepancy between the theory and observation. The difference, amounting to a sixth of the whole quantity, could hardly be thrown entirely upon incidental errors of the experiments. The author of the *Principia* led the way in the conjectures that were advanced for reconciling the calculated velocity of sound with the true velocity; but as all these attempts have shared the usual fate of hypotheses, and have lost all interest by the discovery of the real cause, it would be superfluous to mention them here. But it will be proper to observe, that the difficulty was occasioned by no inaccuracy or neglect of Newton. It arose from an inexact estimate of the air's elasticity, which he was unavoidably led to make from the state of natural science in his time, and which the progress of knowledge has enabled the philosophers of the present day to correct. When the exact elasticity is substituted for the inaccurate quantity, the discrepancy between theory and experiment disappears, without any change being required in the demonstration. At the time of the publication of the *Principia*, and long after that time, what could possibly have led any one to surmise, that nearly half as much heat enters into air when it dilates, and comes out of it when it contracts, as must be applied from some extraneous source, in order to produce the same change of volume?

The fact, that air absorbs heat when it expands, and evolves heat when it contracts, having been established by many experiments; and very notable effects being observed in some cases of great and sudden condensation; Laplace, between 20 and 30 years ago, first suggested that this property of air was the cause of the perplexing difference between the velocity of sound as determined by theory and observation. In the aërial undulations by which sound is conveyed to the ear, every small portion of air is first condensed and then dilated; and we may compare the elasticities on the two suppositions, that the temperature of the agitated air remains the same as in the quiescent state of equilibrium, and that it varies with the changes of volume. The external compressive force being always the same, it is manifest that, whenever the bulk of

\* Communicated by the Author.

the small parcel of air is less than in the quiescent state, the elasticity will be greater on the second supposition than on the first, on account of the extrication of heat; but, whenever the bulk is greater than in the quiescent state, the elasticity will be less on the second supposition than on the first, on account of the cold produced by the absorption of heat. Now the accelerating forces of the aerial particles are the differences between their actual elasticity and the elasticity of the quiescent medium; and as these forces are always greater on the second supposition than on the first, the velocity of the undulation must be swifter in that case than in this. The formula of Newton, being deduced from the law of Boyle and Mariotte, is consonant to the first supposition; and there is undoubtedly in the second supposition a tendency to diminish the difference between theory and experiment, by increasing the estimate of the velocity of sound. One circumstance however, it may be alleged, must in some degree modify the effect of the variations of heat in the agitated air; namely, the rapidity with which the small increments and decrements of free heat would be equalized to the temperature of the surrounding medium. But the whole time of an aerial vibration is extremely short; and we may safely consider every change of volume that takes place during its progress, and every variation of free heat, as enduring only for an indivisible instant of time. Every parcel of air as it is successively agitated retains the whole of its absolute heat; and the rapid evolution and absorption of free heat have no other effect than to increase the elasticity.

The principle suggested by Laplace, having a real existence in fact, and being adequate at least in a certain degree to reconcile the theory with experiment, it became important to ascertain the exact increase of velocity deducible from it. But here a difficulty occurred. It was known that heat was extricated from air when it is condensed, but there was no measure of the effect. It even seemed very difficult, if not impossible, to arrive at any tolerably precise estimation by direct experiment. MM. Biot and Poisson therefore reversed the question, and inquired in what degree the elasticity computed by the law of Boyle and Mariotte must be increased; or, which is the same thing, in what proportion the free heat must vary with respect to the volume; in order to bring out the true velocity of sound. By this means we might at least be able to judge whether the assigned cause would alone account for the observed deficiency. And, admitting that the effect fell within the limits of probability, there can be no doubt that the just rules of philosophizing would be nowise  
infringed

infringed by adopting the explanation deduced, by this inverted procedure, from the phænomenon itself. In 1816 Laplace published the following theorem, without the demonstration:—*The velocity of sound is equal to the velocity according to Newton's formula, multiplied by the square root of the proportion of the specific heat of air under a constant pressure, to the specific heat under a constant volume.* The investigation was first given in the *Conn. des Tems* 1825, and afterwards in the xiith book of the *Mécanique Céleste*. This theorem left nothing more to be done than to find a certain ratio in numbers; and this was accomplished by the ingenious experiment of MM. Clement and Desormes, from which we have deduced the proportion of the latent, to the free, heat, when air varies under a constant pressure. MM. Gay-Lussac and Welter improved a little the original procedure of the inventors, and repeated the experiment in a great variety of circumstances; by which means they not only determined the number sought more exactly, but they likewise showed that it was constant, or nearly so, in considerable diversity of temperature and pressure. The result of this long investigation, protracted for so many years, was a complete solution of the difficulty, and a satisfactory reconciliation of the theoretical, with the experimental, estimate of the velocity of sound.

The numerical value of the proportion indicated in Laplace's theorem is immediately deducible from what has been shown respecting the manner in which heat combines with elastic fluids. When air varies under a constant pressure, the absolute heat requisite to produce the rise of temperature  $\tau$ , is  $\tau + i$ ,  $i$  being the latent heat. But  $\tau$  is the heat that causes an equal rise of temperature when the volume is constant. It is manifest therefore that the proportion of the two specific heats in the theorem, is  $\tau + i$  to  $\tau$ , or  $1 + \frac{i}{\tau}$  to 1, that is,

$1 + \frac{\alpha}{\beta}$  to 1: and  $\sqrt{1 + \frac{\alpha}{\beta}}$  is the factor by which the Newtonian velocity of sound must be multiplied, in order to obtain the true velocity.

But the whole difficulty respecting the velocity of sound is overcome, when it has been found how much heat is extricated from air condensed in a given degree. This is the leading principle on which the investigation must turn, by whatever process the result is brought out. In Newton's formula the pressure and density are supposed to follow the law of Boyle and Mariotte; and the computation will be best rectified by searching out the true relation of the same quantities, and substituting it in the place of that inaccurately employed. It

remains, then, to investigate the relation between the elasticity and density of a mass of air that varies its temperature as it dilates and contracts, without losing or receiving any heat from the surrounding medium.

Put  $p'$ ,  $g'$ ,  $\theta$ , for the pressure, density, and temperature of a given mass of air; and suppose that these quantities are simultaneously changed into  $p$ ,  $g$ ,  $\theta + \tau$ ; then, we shall have,

$$\frac{p}{p'} = \frac{g}{g'} \cdot \frac{1 + \alpha \theta + \alpha \tau}{1 + \alpha \theta}.$$

Again,  $p'$  remaining the same, put  $D$  for the density at the beginning of the thermometrical scale; and let  $i'$  be the latent heat requisite to change  $D$  into  $g'$ : then

$$\frac{g'}{D} = \frac{1}{1 + \alpha \theta},$$

$$\frac{g'}{D} = \frac{1}{1 + \beta i'}.$$

Further let  $i' + i$  be the latent heat accompanying the change of  $D$  into  $g$ ; and,

$$\frac{g}{D} = \frac{1}{1 + \beta i' + \beta i} = \frac{1}{1 + \alpha \theta + \beta i}.$$

Hence, 
$$\frac{g}{g'} = \frac{1 + \alpha \theta}{1 + \alpha \theta + \beta i}.$$

From the values that have been found, we now get,

$$\left. \begin{aligned} \frac{p}{p'} &= \frac{1 + \alpha \theta + \alpha \tau}{1 + \alpha \theta + \beta i} \\ \frac{g}{g'} &= \frac{1 + \alpha \theta}{1 + \alpha \theta + \beta i} \end{aligned} \right\} \quad (C)$$

These formulæ express the elasticity and density of the air by means of the initial quantities  $p'$ ,  $g'$ ,  $\theta$ , and the variations of temperature and latent heat represented by  $\tau$  and  $i$ . It must be observed, however, that the mass of air is supposed to vary in an unlimited supply of heat; so that the small increments and decrements of free heat arising from the changes of volume produce no effect on the thermometer, being continually equalized to the temperature of the surrounding bodies. In this case the quantities  $\tau$  and  $i$  are independent on one another; the first being the temperature as shown by the thermometer, and the second the latent heat connected with the change of bulk. But if the supply of heat were limited, it would be requisite to take into account the free heat evolved or absorbed by the contraction and dilatation of the air. For this purpose we must write  $\tau - i$  for  $\tau$  in the first of the formulæ (C); supposing that  $\tau$  is all the heat derived from extraneous sources, and  $+ i$  the variation of the latent heat. In a parcel

a parcel of air agitated in an aërial undulation, there is no extraneous heat, and  $\tau = 0$ : the foregoing equation, therefore, will become,

$$\frac{p}{p'} = \frac{1 + \alpha \theta - \alpha i}{1 + \alpha \theta + \beta i},$$

$$\frac{\rho}{\rho'} = \frac{1 + \alpha \theta}{1 + \alpha \theta + \beta i}.$$

And, by exterminating  $i$ ,

$$\frac{p}{p'} = \frac{\rho}{\rho'} \left( 1 + \frac{\alpha}{\beta} \right) - \frac{\alpha}{\beta}. \quad (\text{D})$$

This equation expresses the relation between the elasticity and density in the circumstances supposed, and it is that which must be employed in the investigation of the velocity of sound

in place of the equation  $\frac{p}{p'} = \frac{\rho}{\rho'}$ , resulting from the law of Boyle and Mariotte, and forming the basis of Newton's formula.

In the *Philosophical Magazine* for June 1825, p. 12, the following equation is obtained in considering the motion of a line of air, viz.

$$\frac{\rho}{\rho'} = 1 - \frac{dz}{dx}:$$

Substitute, now, this value in *equat. (D)*; thus

$$\frac{p}{p'} = 1 - \left( 1 + \frac{\alpha}{\beta} \right) \frac{dz}{dx}:$$

and if we put  $k = 1 + \frac{\alpha}{\beta}$ , and go through the rest of the calculation as at the place cited, we shall obtain,

$$\frac{d dz}{d \tau^2} = k \times \frac{p'}{\rho'} \times \frac{d dz}{d x^2}.$$

The true velocity of sound is therefore  $\sqrt{k \times \frac{p'}{\rho'}}$ : but the Newtonian velocity, deduced from the law of Boyle and Mariotte, is  $\sqrt{\frac{p'}{\rho'}}$ : and these two formulæ contain the demonstration of Laplace's theorem.

The theory we have attempted to give of the combination of heat with elastic fluids is founded on acknowledged facts. It is general, extending as far as experience has enabled us to reduce the effects of heat to precise rules. It follows from it that the quantity  $k$ , on which the velocity of sound depends, has the same value for air and all the gases; and likewise that it remains constant in every diversity of pressure and density: all which consequences are known to be consonant to observation.

The

The equation (D) does not coincide with what is elsewhere given for expressing the relation of the same quantities. It is different from the equation published by M. Poisson in the *Conn. des Tems* 1826, p. 264. In order to clear away all clouds of obscurity from a matter of considerable importance, I shall now examine particularly, what it is that occasions the difference. For this purpose I shall set out from M. Poisson's equation (6), p. 263, viz.

$$\omega = (k - 1) (1 + \alpha \theta) \frac{\gamma}{\alpha}.$$

Here  $\omega$  is the variation of latent heat corresponding to the small condensation  $\gamma$ ; and, in our notation,  $\omega = di$ ,  $\gamma = \frac{d\varrho}{\varrho}$ ,

$k - 1 = \frac{\alpha}{\beta}$ : the equation may now be put in this form, viz.

$$\frac{d\varrho}{\varrho} = \frac{\beta di}{1 + \alpha \theta},$$

which is nowise different from what M. Poisson obtains in p. 264, except that he writes  $d\theta = \omega$ , instead of  $di = \omega$ . Differentiate the second of the formulæ (C), changing the sign of  $i$  in order to agree with M. Poisson's supposition, that the density increases; then,

$$\frac{d\varrho}{\varrho} = \frac{\beta di}{1 + \alpha \theta - \beta i}.$$

Now this equation is identical with M. Poisson's only at one point, namely, when  $i = 0$ . The latter is therefore true only in a particular state of the variables, and is inexact in all other circumstances. When the density and latent heat of a mass of air vary together, M. Poisson's equation expresses the true relation of the differentials only initially; and it ceases to be exact when the variable quantities have changed their original magnitudes. The integral formulæ deduced from such a process cannot be accurate results, although they may be approximations. The truth of what has been observed must be so evident to any one that will consider with attention the manner in which the author obtains the equation in question, that it would be a waste of words to attempt any further explanation.

The investigation I have given in pp. 7 and 8 of the *Phil. Mag.* for June 1825, is liable to the same objection that has just been urged against M. Poisson. The relation of the differentials is obtained only in a particular state of the variables. The experiment of MM. Clement and Desormes, although it enables us to ascertain the value of  $\frac{\alpha}{\beta}$ , is, nevertheless, in-

sufficient

sufficient for finding, generally, the relation between the density and latent heat, when these quantities vary together.

It must not, however, be imagined that the damage arising from the inadvertency that has been noticed, is ruinously great. The formulæ obtained are true to quantities of the second order with respect to  $\alpha$  and  $\beta$ . They are sufficiently exact for investigating the velocity of sound; and they can hardly lead to any error of moment in any practical inquiry. But it is always best to square our speculations according to experience and the laws actually followed in nature; and, in a case like the present, when it may be supposed that we have returned into the right path after having deviated a little from it, it is instructive to look back and examine what led us astray.

In further illustration of what has been said, it may not be improper to add a few words concerning the equations in the xiith book of the *Mécanique Céleste*, pp. 123, 127. For this purpose I seek the values of  $\tau$  and  $i$  from the foregoing equations (C); then, by taking the sum, we get,

$$\tau + i = V = \left( \frac{p}{p'} \cdot \frac{g'}{g} - 1 \right) \frac{1 + \alpha\theta}{\alpha} + \left( \frac{g'}{g} - 1 \right) \frac{1 + \alpha\theta}{\beta}.$$

Put  $k = 1 + \frac{\alpha}{\beta}$  as before, and differentiate: then

$$\frac{dV}{dg} g + k \frac{dV}{dp} p = \frac{1 + \alpha\theta}{\beta} \cdot \frac{g'}{g} \cdot \frac{p - p'}{p'}.$$

We have initially,  $p = p'$ ,  $g = g'$ ; and if we suppose that the mass of air undergoes only a small variation from the initial state, we shall have,

$$\frac{dV}{dg} g + k \frac{dV}{dp} p = 0$$

$$k = \frac{-\frac{dV}{dg} g}{\frac{dV}{dp} p}.$$

These equations are true only at one point, and in a particular state of the variables, as has been mentioned. They can have nothing to do with integration, which supposes that the differential equations are exact for all values of the flowing quantities within the limits of their variation. They merely express that the two specific heats, under a constant pressure and under a constant volume, have to one another the same invariable proportion, whatever be the condition of the mass of air.

March 5, 1827.

J. IVORY.

LIII. *Theory*