# The Cosmochrony Research Programme: A Structural Roadmap and Paper Inventory

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**Abstract**

The Cosmochrony programme develops a pre-geometric framework in which physical structure — spacetime, quantum mechanics, gauge symmetry, and Standard Model observables — arises from a single primitive: the local structure of admissible non-injective transitions between observable states. The corpus comprises 116 papers across three theory branches. Branch I (Foundation track) develops four admissibility axioms and their consequences. A finite $S_3$ countermodel now proves that the algebraic properties extracted from those axioms in the published carrier-selection argument do not force a central commutator, a finite Heisenberg group, or a Weil representation. The Heisenberg/Weil carrier is therefore a supplied realisation, not an axiomatic theorem. Branch II (O-series) develops the spectral admissibility sub-programme, studies the conjugate-pair fibre structure on a supplied Weil model and measures its capacity exponent $\delta_{\mathrm{pair}}$. The native Heisenberg span-growth law proves that the proposed conversion $\delta_{\mathrm{pair}} \to \beta^*$ has no derived carrier on the measurement substrate; its numerical agreement with the charged-lepton hierarchy remains a phenomenological coincidence check rather than a structural derivation. The colour factor and the complete Standard Model gauge group are open: O31 version 2.0 supplies neither. Branch III (Q-series and companion papers) develops spacetime geometry and gauge dynamics from the Branch I axioms and supplied Branch II spectral data, and states a universal singlet correlator conditional on an explicit, undemonstrated invariance hypothesis (paper Q3; the Born rule remains open), a conditional effective co-metric of rank three whose non-degenerate block is Lorentzian, under the open inputs [H-L], [H-lift] and [H-hyp] (the full-rank extension and every coefficient value remain open, and the co-metric $\mathrm{diag}(-2, 2, 2, 2)$ still asserted by some Lorentz-capacity consumers is pending revision); and the conditions for an Einstein ($a_2$) sector and the derivation of the local Yang–Mills ($a_4$) sector from a single spectral-entropy functional on a supplied metric, conditional on supplied renormalized matching data (papers Q12–Q13). This paper provides a complete inventory, a logical dependency map, and a structured account of what is proved, structural, heuristic, or open, intended both as an entry point for external readers and as an internal navigation reference. *Interpretive status.* The programme's unifying reading — that physical structure is organised by admissibility and by spectral (Seeley–DeWitt) order rather than by an enlarged symmetry group — is offered as an interpretive outlook, not a theorem; each individual result carries the epistemic label (proved, structural, heuristic, or open) recorded in the inventory below.

**Keywords.** Non-injective projection; admissibility; Born–Infeld constraint; spectral admissibility; Heisenberg group; Weil representation; emergent spacetime; lepton mass hierarchy; quantum mechanics; gauge structure; programme overview.

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# 1 Introduction and Purpose

The Cosmochrony programme was initiated with a white paper [18] positing that all physical structure emerges from a non-injective projection $\Pi : \Omega \to \mathcal{O}$ from a pre-geometric substrate $\chi$ onto observable space. The initial framework postulated a Born–Infeld (BI) saturation constraint, a relaxation mechanism, and an iterated projection cascade $U_{t+1} = \Pi \circ \sigma(U_t)$ as the engine of dynamics.

Over the course of 2026, the programme has evolved in two parallel directions. On one side, the spectral admissibility sub-programme (O-series) has progressively built the computational machinery for extracting observables from the spectral structure of the fibre. O24 [37] develops the conjugate-pair construction conditionally on a stated fibre structure for $\Pi$; the identification of that fibre structure from first principles remains open. The Span-Growth Note [32] proves, however, that the O4–O6 growth-equation closure, square-root frontier, pair observable, and exponent coordinates do not supply a native Heisenberg derivation of the downstream conversion $\delta_{\text{pair}} \to \beta^*$. On the other side, the Foundation paper [6] proposes a reformulation from four axioms (A1–A4), dispensing with an explicit substrate $\chi$ and relaxation mechanism. Its carrier-selection step is not derived: HeisenbergStructure [2] (version 2.1) contains a finite countermodel that satisfies the algebraic properties available before centrality while its commutator is non-central. The Q-series then builds quantum mechanics, SU(2) symmetry, and Lorentzian spacetime on top of the Foundation axioms and O-series spectral data.

The corpus comprises 116 papers with complex mutual dependencies. The interactive dependency graph covers all 116 entries of the inventory and distinguishes ordinary dependencies from sub-programme synthesis links. A reader entering the programme faces two difficulties: it is not obvious where to start, and the distinction between a theorem, a structural argument, and an open conjecture is not always transparent from individual paper titles. This roadmap addresses both difficulties.

**Conventions on result status.** Throughout this paper each result is classified as one of the following: **proved** (formally proved as a theorem within the stated assumptions); **structural** (derived from structural arguments that are internally consistent but not yet fully formalised); **numerical** (the principal contribution is a dataset or an exact finite computation with its reproduction code, and no structural or physical claim is asserted on its basis; a theorem proved about the data is a secondary label); **conditional** (a theorem whose conclusion is read as a claim about the programme, and at least one of whose hypotheses is a programme-level identification or postulate that this inventory lists as open, such as [H-L], [H-F], a carrier selection or a supplied selection rule; the label names that hypothesis; a theorem about a supplied object as such, whose conclusion does not depend on the object being the programme's, is **proved** with the supplied input stated, and the missing identification is recorded as a secondary **open**); **heuristic** (physically motivated and numerically supported but not yet rigorously derived: a physical claim is asserted beyond what the numerical or structural support establishes); **open** (explicitly conjectured or known to be unsettled). A seventh entry, **synthesis**, is reserved for the sub-programme presentation and synthesis notes: these papers consolidate and organise results established in their constituent papers and carry no new result of their own, so their epistemic content is that of the papers they map. Each paper carries one primary label, stated first in the inventory table, in the dependency figures and graph, and first in its synthesis paragraph where that paragraph states a status, followed where useful by the labels of its secondary components; the primary label is the status of the paper's principal contribution, the result named by its title; where the row's prose names an open or conditional item of the paper's own principal result, the status cell records it as a secondary label: a proved no-go or countermodel is **proved** as a negative result, and a withdrawal notice is **open**, any counterexample it proves being recorded as a secondary **proved**. These classifications record the current audited status. They may therefore correct an original paper's self-classification when a published countermodel or obstruction establishes that

a claimed implication does not follow.

## 2 Conceptual Architecture

The programme is organised in three structural levels.

**Level 1 — Axiomatic primitive.** The primitive is the local structure of admissible non-injective transitions. No background spacetime, no quantum postulate, and no substrate field are introduced at this level. The four axioms are: local projective admissibility (A1), structural non-injectivity (A2), admissibility as non-premature selection (A3), and projection locking (A4). From A1–A2, non-injective transitions lose information in their immediate image; neither permanent non-recovery nor the arrow of time follows, temporal order being conditional on the acyclicity hypothesis [H-acyc]. Under the published formalisation, A3 supplies irreducibility and a non-trivial commutator. It does not force centrality, class-two nilpotence, or the Weil representation. From A4, the discreteness of quantum transitions follows. Papers at this level: ENI [35], Foundation [6], HeisenbergStructure [2], noscale [33].

**Level 2 — Spectral fibre structure.** The O-series supplies the Heisenberg group, its irreducible Schrödinger representation $\pi_c$ on $\mathcal{H}_c = L^2(\mathbb{Z}/q\mathbb{Z})$, and the distinct associated Weil action of the symplectic automorphism group as its mathematical carrier data. Foundation and the earlier HeisenbergStructure versions claimed to derive this carrier, but HeisenbergStructure version 2.1 [2] refutes that selection proof. The O-series computes the spectral properties of the supplied fibre, including the capacity exponent $\delta_{\text{pair}}$. Its proposed conversion to the cascade exponent $\beta^*$ is not native to the Heisenberg substrate: the measured numerical agreement remains a conditional, cross-substrate phenomenological comparison rather than a derived mass-hierarchy mechanism. Papers at this level: SpectralAdmissibility [52], SpectralCapacity [53], SpectralGramRigidity [38], SpectralRelaxation [9], and O1–O33 ([44]–[76]).

**Level 3 — Physical observables.** Effective spacetime geometry, gauge structure, Standard Model charges and masses, gravitational waves, and cosmological observables are derived as consequences of Levels 1–2. The metric and co-metric results at this level are conditional on the continuum-limit hypothesis [H-L] of Q5a–Q5b (unestablished; Q5a version 3.2, Q5b version 2.2.1), on the lifting hypothesis [H-lift] and, for the signature, on the hyperbolicity hypothesis [H-hyp]. Papers at this level: the Q-series (Q1 [28], Q2 [7], Q3 [80], Q5a–Q7), the gravity track (Spectral2 [48], Gravity [17], Thermodynamics, Lorentz [34]), and the companion papers ([13, 11, 3, 16, 58, 4, 67]).

## 3 Branch I: Axiomatic Foundations

### 3.1 The White Paper

The white paper [18] ("Cosmochrony: A Non-Injective Projection Theory of Emergent Physics," v3.14) is the programme's primary overview document. It introduces the substrate field $\chi$, the non-injective projection $\Pi$, the BI saturation constraint, the relaxation mechanism, and the qualitative derivation of spacetime and quantum structure. The spectral entropy functional $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ appears here as the structural origin of the Einstein equation.

Several structural refinements informed by the O-series and Foundation results are incorporated. The white paper derives the first transition and a non-trivial kernel from the fibre commutator $[X, \sigma(X)] = Z \neq 0$; Foundation version 2.3 does not support that reading, and the status of each refinement against Foundation is:

- *Oriented state machine:* the admissible cascade is reformulated as a directed graph on admissible projections, and physical histories are directed paths; the Foundation paper does not guarantee the first transition, the existence of an admissible successor being a separate condition (Foundation Remark 4.11 [6]).
- *Initial conditions:* every non-injective admissible projection has a non-trivial fibre, hence a non-trivial kernel when it is represented linearly, by A2 alone and without any commutator (Foundation Corollary 4.9); this does not by itself make the cascade begin.
- *Inaccessibility of absolute zero:* the white paper reads the Heisenberg structure of a supplied carrier as forbidding projective stasis $U_{n+1} = U_n$; the Foundation axioms do not select that carrier (Foundation Theorem 5.7).
- *Structural fluctuations:* along non-injective steps the kernel is non-trivial, so fluctuations are structural there rather than contingent; A2 does not exclude injective steps.
- *Schrödinger as Mosco limit:* the continuous Schrödinger equation is derived as the Mosco limit of the discrete oriented admissibility cascade.
- *Extended numerical scope:* the PTO monotonicity is verified for $q \in \{29, 61, 101, 151, 211\}$, with 274 conjugate pairs and 360 post-burn-in steps.

*Role:* Programme overview; structural motivation; first formulation of the BI constraint and the relaxation cascade. *Evolution note:* The explicit substrate $\chi$ and relaxation mechanism of the white paper are not postulated in the Foundation reformulation: they emerge from axioms A1–A4. The white paper remains the primary narrative reference; Branch I papers are the formal foundation.

### 3.2 Non-Injectivity as a Structural Necessity of Genuine Emergence

The ENI paper [35] proves the following no-go theorem. Any genuinely emergent (non-isomorphic effective) physical description is necessarily non-injective and intrinsically compressive. The argument is purely structural: given a surjective $\Pi : \Omega \to \mathcal{O}$ such that $\mathcal{O}$ carries no independent microstate-resolving structure beyond $\Pi$, non-isomorphism of levels forces non-injectivity. A projection entropy $S_\Pi \geq 0$ is defined, and the equivalence genuine emergence $\Leftrightarrow$ $\Pi$ non-injective $\Leftrightarrow$ $S_\Pi > 0$ is established (**proved**, Theorem 2.1 and Corollary 3.1).

*Role:* Foundational justification for structural non-injectivity (Axiom A2). Corollary 6 establishes recursive non-injectivity: any tower of genuinely distinct emergent levels carries one independent non-injective projection per stratum; Section 5 covers towers of emergence and structural colour confinement as projective necessity. A Remark (algebraic route) shows that in frameworks whose fibre carries a Heisenberg algebra $[X, Y] = Z \neq 0$, $S_\Pi > 0$ also follows directly from fibre non-commutativity: a trivial-kernel projection would require simultaneous diagonalisation of $X$ and $Y$, which is structurally forbidden. The two derivations are complementary: Theorem 2.1 answers *what* is forced (framework-independently); the algebraic route answers *why* in Heisenberg-fibred frameworks. Cited by Foundation, HeisenbergStructure [2], Q1 [28], Bell, O32 [79].

### 3.3 Projection as Conditional Expectation and the Capacity Ceiling

The ProjCE note [90] (v1.0.0) derives the amplitude-level action of the non-injective projection: under four graded hypotheses — fibre-invariance (forced by the ENI emergence criterion), fixity on resolved content, linearity (the load-bearing structural transfer, at the tier of the ENI pivot lemma), and non-amplification (or a positivity/equivariance module variant) — the assignment of observed values is the conditional expectation onto the sigma-algebra of the

distinguishable-history record, by two independent classical characterisations (contractive projections; Moy–Rota–Douglas averaging operators), with the operator invariant under the degraded minimum-distortion selection (hypotheses **structural**; theorems **proved** given the hypotheses). Two consequences follow as mathematics: strict ensemble-power suppression equal to the erased intra-fibre power (law of total variance; the structural answer to the cosmic-variance objection), and the parameter-free capacity ceiling $(N-1)/N$ for a complex Gaussian mode supported on $N$ distinguishable histories (one-shot rate–distortion converse) — identically the Bessel envelope of LowLCapacity [85], upgraded from estimator heuristic to information-theoretic bound.

*Role:* Closes the entropy-to-power step of LowLCapacity as a conditional theorem chain; supplies the counting-to-power converter consumed by the low-$\ell$ note, the cosmological branch [3], and any $w(n)$ extraction from full-history compression [92].

### 3.4 Foundation: Admissible Non-Injective Transitions as the Primitive

The Foundation paper [6] (v2.3) states the axiomatic basis of the programme. From the four axioms A1–A4 it claims: (i) immediate information loss along non-injective transitions (A1+A2, Proposition 4.1), with temporal order conditional on [H-acyc] (Theorem 4.4); (ii) the proto-state as a physically real unresolved configuration retaining phase coherence (A3); (iii) a non-trivial commutator between conjugate generators, forced by A3 and the minimality of A1; (iv) the identification of the admissible fibre symmetry group as $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ for a prime $q$, together with BI parity (A1–A3); (v) the identification $F_n \simeq \mathcal{H}_c$ with the supplied Heisenberg–Schrödinger carrier, stated as a modelling dictionary (Remark 3.3), not as a theorem, and distinct from the Weil module $V_\rho$ used by the spectral construction; (vi) the discrete character of quantum transitions from A4; (vii) projective incompleteness (Corollary 4.9): under A2 alone, a non-injective realisation map $\Pi_n$ has a non-trivial fibre, so its observable description is incomplete. The paper also describes the threefold role of $\Pi_n$ (Remark 3.1): admissibility filter, generator of temporal order conditionally on [H-acyc], and revelation operator; these roles do not by themselves select the carrier group. Its results table lists the effective co-metric with no coefficient value established: $g^{\mu\nu} = \mathrm{diag}(-2,2,2,2)$ is not derived, since in Q10 the Casimir value 2 determines no coefficient and the uniformity hypothesis [U] is not established, and Q11 leaves $A_\tau/A_H$ unconstrained. A further section places the four axioms relative to established formalisms (Hamilton–Jacobi, symplectic geometry, WKB, functional renormalisation group). In its entanglement framework, fibres are defined as *infra-projectable cells* (unsplit configurations below the resolution of $\Pi$), the derivation of entanglement entropy is integrated into the foundation text by importing the admissible pair entropy of *Q3* [80] and *ENT-E2* [70], while the proposed Bell non-applicability implication is not available: a non-injective projection can possess deterministic local response functions, so A2 alone does not obstruct Bell factorisation.

Items (iv)–(v) are not established. The finite $S_3$ countermodel in HeisenbergStructure version 2.1 [2] has a faithful irreducible unitary carrier, a minimal generating pair exchanged by an involution, and a non-trivial commutator, but the commutator is not central. It also shows that the prime-order step reverses Schur's lemma.

*Status:* **structural** for the axiomatic architecture and the consequences that do not consume carrier selection; the Heisenberg/Weil identification is a postulated realisation; **open** for the continuum limit (Q5a version 3.2 establishes no spatial limit operator; the spatial limit is the unestablished hypothesis [H-L] of Q5b version 2.2.1). The SU(2) sector of the spin-$\frac{1}{2}$ fibre enters through the spinor carrier supplied conditionally by O23; version 2.3 lists the SU(2) gauge structure as conditional on that carrier and the Standard Model gauge group as open, O31 version 2.0 being a withdrawal notice. *Role:* Axiomatic spine. Cited by all Q-series papers, HeisenbergStructure [2] , noscale, and recent companion papers.

### 3.5 A Finite Obstruction to Heisenberg Carrier Selection from Admissibility Constraints

HeisenbergStructure [2] (version 2.1) records a proof attempt for the selection of the Heisenberg carrier from the Foundation axioms and refutes it by an obstruction theorem; Foundation states the obstruction as its Theorem 5.7. The central structural lemma — *admissible non-factorisability* — is a direct formalisation of Axiom A3. Its next step, "minimality implies central commutator", is false: a two-generator group need not be nilpotent of class two. The later prime-order argument is also invalid because an irreducible complex carrier contains one character of a central subgroup, not one non-zero eigenspace for every available character.

*Status:* **structural** for the non-factorisability setup; **open** for Heisenberg selection. *Role:* Documents the attempted carrier-selection proof and its conditional final Stone–von Neumann step.

HeisenbergStructure gives the six-element countermodel $G = S_3$, $X = (12)$, $Y = (23)$, with $\sigma$ given by conjugation by (13). The standard action on the two-dimensional zero-sum subspace of $\mathbb{C}^3$ is faithful, unitary, and irreducible; $\sigma$ exchanges the minimal generating pair; and $[X, Y]$ is non-trivial but non-central because $Z(S_3) = \{1\}$. The note independently corrects the Schur-lemma step and separates the failed selection edge from downstream mathematics internal to a supplied Heisenberg/Weil carrier (**proved**).

*Role:* Published evidentiary basis for the Found/Heis status correction.

### 3.6 No External Dimensional Scale at the Level of Admissibility

The noscale paper [33] establishes a structural rigidity result (Level 1): any deformation of the admissibility constraints that introduces a dimensional parameter independent of $c_{\mathrm{BI}}$ breaks at least one of BFS-consistent spectral scaling, structural non-injectivity (A2/ENI), or bounded admissibility closure (A3–A4). Dimensional analysis within the framework is therefore fully constrained by $c_{\mathrm{BI}}$ alone (**proved**).

*Role:* Closes the naturalness question for dimensional parameters.

### 3.7 Support and Preparation Criteria for Independent Effective Subsystems

EffComp [82] names the transversal contract

$$\mathrm{K4}: \quad \mathsf{Admissibility} \longrightarrow \mathsf{Composable\ effective\ subsystems},$$

whose source carries no supplied subsystem factorisation, and characterises its two prerequisite layers in the finite case. On a substrate $\Omega$ with reference admissible set, two effective projections, and cylindrical conditions combined by intersection, the independence blocks — outcome-set pairs that are margin-stable and jointly rectangular — are exactly the nonempty bicliques $S_A \times S_B \subseteq R$ of the projected support relation (**proved**). On a block, the full preparation simplex lifts every joint law, so the absence of a supplied factorisation alone obstructs nothing (**proved**); for the strictly positive cylindrical-conditioning families, closure under products of independently locally conditioned states holds iff the fibre-weight matrix has rank one (**proved**), and the counting countermodel $N_3$ shows that support rectangularity does not imply preparation independence (**proved**). A steering discipline separates locally prepared from steered conditional states; violating it manufactures spurious witnesses. The contract itself is proposed, not derived from A1–A4 or from an operational theory; the paper consumes no Heisenberg/Weil carrier and no continuum, and the general preparation layer and the states-and-effects layer of K4 remain open (**open**). Q3's bipartite singlet and the entanglement note's conjugate-pair closure are typed as consumers of a supplied composition structure.

*Role:* Opens the effective-composition track of Branch I; K4 joins carrier selection among the open structural deliverables.

### 3.8 Finite Criteria for Simplicial States and Local Tomography

STEFT [81] resolves K4's finite, mono-system, and bipartite-composition states-and-effects content, importing EffComp's Theorem 4.4 (rank-one product-closure) unchanged, as an already-proved hypothesis (**proved**). The raw cylindrical-conditioning family is never itself convex: it is finite while its own convex hull is a continuum, unconditionally, for any local outcome set of size at least two (**proved**). An explicit classical-randomisation postulate closes it onto the full probability simplex, locally and jointly, independently of the fibre-weight matrix's rank (**proved**). Affine effects on a simplex correspond bijectively to a cube of vectors, a bookkeeping fact special to simplices and not itself a claim about physical availability (**proved**); a weak coarse-graining axiom already yields the full Boolean effect algebra and the unit effect for free, while the full continuum of effects requires a further, dual randomisation postulate (**proved**). The two randomisation postulates are logically independent, and a common classical control resource reproduces both only under an explicit independence clause between its two interfaces, confirmed by an exact numeric witness (**proved**). Given physical existence of a composed bipartite effect, the pointwise product of two local effects is forced, not postulated (**proved**); local composition reaches only rectangle effects, strictly fewer than all joint sharp effects, and a CHSH-shaped separating witness shows this survives full local randomisation (**proved**). Local tomography holds exactly when each side's effect family linearly spans its ambient space, equivalently when it separates its local states and its linear span contains the unit effect (**proved**). All five governing axioms (classical randomisation of preparations and of effects, a common control resource, coarse-graining, and physical composition) are explicit imports, not derived from admissibility or from the simplicial geometry alone; no Born rule or quantum state geometry is derived, and the general preparation layer of K4, together with the tensor product as a supplied structure, remain exactly as open as EffComp left them (**open**).

*Role:* Continues the effective-composition track of Branch I opened by EffComp; resolves K4's finite states-and-effects layer while leaving its general preparation layer, the tensor product, and any phase/modulus law open for a future front outside the classical/simplicial regime.

### 3.9 Refinement Consistency Obstructs Nonlinear Power Lifts of Finite History Measures

HMPL [93] takes up the phase/modulus law that STEFT leaves open, without deriving it: for any finite branching system, the power-lift family $\mu^\alpha$ of its induced valuation is Kolmogorov-consistent, at a branching state reached with positive $\mu$-valuation, if and only if $\alpha = 1$ (**proved**), by an elementary, self-contained comparison of $x^\alpha$ against $x$ on $[0, 1]$ independent of the corpus or of any Cosmochrony-specific structure. A row-renormalised ("escort") lift restores Kolmogorov consistency trivially for every $\alpha$, but generally departs from the pointwise lift once path-dependent normalisation factors accumulate along the history (**proved**); this situates the result relative to the pre-existing escort-distribution/Rényi literature of nonextensive statistical mechanics, not in isolation. Specialised, conditional on an explicitly named, non-derived maximal-entropy/Parry postulate (H2), to the non-backtracking Heisenberg $b$-shadow automaton supplied by TrajectoryBranching [92]: the square-root failure is re-derived independently in exact $\mathbb{Q}(\sqrt{2})$ arithmetic (**proved**); the automaton's endpoint and $b$-shadow quotients are exactly incomparable, with explicit witnesses (**proved**); relabelling $X \leftrightarrow X^{-1}$ transports the Heisenberg endpoint by an exact group automorphism, forcing every uniform-probe phase sum weighted by a real shadow-only valuation, over histories sharing a final $b_n$, to be exactly real (**proved**); and a bounded exact search over all 13120 non-backtracking words of length up to 8 finds no destructive cancellation among the 64 qualifying classes, 62 of which carry genuine nontrivial relative phase (**proved**, calibrated negative on this range). No Born rule, complex-carrier selection, or coherent-sum construction is established; H2's dependency is confined to the concrete Parry kernel and the search's specific counts, not to the quotient-incomparability or covariance results, which hold for any shadow-only valuation on the same automaton.

*Role:* Bridges Branch I's effective-composition track (STEFT's open phase/modulus question) and Branch III's trajectory-branching automaton; identifies the exact obstruction any future amplitude-modulus construction on that automaton must confront or explicitly bypass, without constructing one itself.

### 3.10 Branch I Synthesis: NIF-Note

The papers ENI [35], Foundation [6], HeisenbergStructure [2], and noscale [33] form the *non-injective foundations sub-programme*, whose synthesis paper is the presentation note **NIF-Note** [62] (Presentation Note 5, working-paper version 1.5). NIF-Note separates ENI's framework-independent implication from the conditional representation route: under ENI's surjectivity, informational-completeness and non-isomorphism hypotheses, genuine emergence implies non-injectivity with $S_\Pi > 0$. Foundation proves immediate information loss along non-injective transitions and projective incompleteness under A2; temporal order requires the unestablished acyclicity hypothesis [H-acyc]. Irreducibility requires a representation-theoretic dictionary; non-commutation then requires dimension greater than one and a pair generating the acting group. HeisenbergStructure's finite $S_3$ countermodel blocks the inference from non-commutation to centrality and Heisenberg carrier selection. Given a supplied Heisenberg group and non-trivial central character, Stone–von Neumann identifies its irreducible Schr"odinger representation; the associated Weil action is separate supplied data. The note records the static substrate and the axiomatic admissibility formulation. Carrier selection, the Born rule in every sector, the non-trivial spatial limit [H-L] of Q5b, and Level 2 scale determination remain open. Q5a proves a zero-form Mosco limit on its canonical Fourier filtration and a normalisation no-go under [H-depth] and [H-w′].

*Role:* Synthesis of Branch I. Provides the explicit input/output table of the axiomatic foundations and is the canonical entry point for downstream sub-programmes (Notes 1–4 of the spectral admissibility, emergent geometry, gauge structure, and spectral gravity tracks).

## 4 Branch II: Spectral Admissibility Sub-Programme (O-Series)

The O-series investigates whether the charged-lepton mass hierarchy can be related to the spectral structure of the admissible fibre. The central quantity is the power-law exponent $\delta_{\text{pair}}$ of the pair-level capacity observable $\sigma_{\text{pair}}(n) = \sigma_c(n) \cdot \sigma_{q-c}(n)$ along the BFS cascade. The proposed capacity-to-rate chain is

$$c_{\text{BI}} \longrightarrow \delta_{\text{pair}} \dashrightarrow \beta^* \approx 0.126 \longrightarrow m_e : m_\mu : m_\tau. \tag{1}$$

The finite capacity segment is studied within the supplied Weil model, but Weil conjugation is not a derived Born–Infeld fibre identification (O18 [20]); O24 does not remove that open bridge. The dashed segment is not a native theorem: SGN [32] proves that both corpus-defined Heisenberg realisations of the O4 closure fail, the native frontier is $N^{3/4}$ rather than $p^{1/2}$, the block capacity rather than the pair product carries the exact span increment, and the exponent coordinates agree only in the non-native reduced filling model. O25 [55] establishes the numerical agreement, not the dashed implication.

The presentation note **SpAdm-Note** [64] is the synthesis paper of this sub-programme. Version 2.0 maps the five precursor papers, O1 and O3–O33, and the Span-Growth and Critical Coverage companion notes along the chain bounded flux ⇒ admissibility envelope ⇒ Weil-sector capacity ⇒ $\dim_{\text{pair}}$, on a supplied finite Heisenberg carrier and its associated Weil action, while separating proved mathematics, numerical measurements, conditional models, open bridges, and withdrawn claims. It records that the reciprocal prescription $\beta^* \approx 1/(\dim_{\text{pair}} + \frac{1}{2})$ has no carrier on the Heisenberg measurement substrate, so the agreement $\beta^* \approx 0.126$ with the charged-lepton window is a coincidence check and not a derivation; that the SU(2) carrier is supplied conditionally; the O31 withdrawal; O32's numerical-only status; O33's exact finite architecture; and

three open deliverables: hypothesis [H-color] (the two levels established in O32 stand), the identification of the spin-$\frac{1}{2}$ sector, and a full-breadth campaign beyond $q = 211$. Their individual contributions follow.

### 4.1 Precursor Papers: SU(2) Admissibility Sub-Programme

Three papers precede the O-series proper and explore spectral admissibility on Cayley graphs of finite SU(2) subgroups.

**SpectralAdmissibility** [52] (v2.0.3) carries out the spectral admissibility programme on two parallel chains. For the SU(2) chain $Q_8 \subset 2I \subset \mathrm{PSL}(2, \mathbb{F}_q)$, it derives the admissibility hierarchy $A_n^{\max} = c_{\mathrm{BI}}/\sqrt{\lambda_n}$ in the linearised regime and shows that the non-linear DBI correction preserves it. For the SU(3) chain $\Delta(27) \subset \Sigma(168)$, it establishes via character theory the hierarchy $\lambda_{\mathbf{3}} = \lambda_{\bar{\mathbf{3}}} = 28 < \lambda_{\mathbf{6}} = \lambda_{\mathbf{8}} = 42 < \lambda_{\mathbf{7}} = 48$ on $\mathrm{Cay}(\Sigma(168), C_4)$, proving that the fundamental and anti-fundamental representations are co-admissible and strictly dominate all competitors. The algebraic mechanism is uniform: $\chi_{\mathbf{3}}(g) = 1$ for all $g \in C_4$ plays the same structural role as $\chi_{\rho_4} = 0$ on $Q_8$ and the golden-ratio identity on $2I$ (**proved**).

**SpectralCapacity** [53] introduces the capacity functional $\mathcal{C}(G, S)$ and proves the binary-polyhedral maximality conjecture: the adjoint identity $\hat{A}_{\mathrm{adj}} = 2M - I$ (where $M$ is the second-moment tensor of the axis distribution) shows that icosahedral isotropy forces $\mu_{\mathrm{adj}}(2I) = -\frac{1}{3}$, while axis-plane confinement forces $\mu_{\mathrm{adj}}(\mathrm{Dic}_n) \geq 0$ for all $n \geq 3$; combined with a Plancherel uniform bound, this yields a strictly increasing capacity gap $\Delta(\alpha) > 0$ for all $\alpha > 0$ (**proved**).

**SpectralGramRigidity** [38] shows that the geometry of neutral generating sets on SU(2) is fully encoded in pairwise character values $G_{st} = -\frac{1}{2}\chi_\rho(st)$ (**proved**).

**Three Particle Generations** [66] derives the three-generation structure from the ADE stratigraphic levels of binary Cayley graphs (**structural**).

### 4.2 SpectralRelaxation: Expander Graph Dynamics

The SpectralRelaxation paper [9] establishes $\Lambda_{\mathrm{proj}} \asymp \lambda_2$ for LPS expander families (**conditional** on the capacity–isoperimetry closure and spectral-isoperimetric saturation hypotheses), providing the entry point for the O-series dynamics.

### 4.3 O1–O8: LPS Phase — Cascade Exponent $\beta$

**O1** [44]: LPS saturation is a fixed-valence artefact; growing valence restores the pre-saturation window (**proved**). (Note: O2 was not written; the O3 derivation path was chosen instead.)

**O3** [42]: First quantitative SM connection. The phenomenological window $\beta^* \in (0.09, 0.13)$ is derived from the charged-lepton mass ratios $m_e : m_\mu : m_\tau$ (**structural**).

**O4** [12]: Bounded relational flux and Ramanujan expansion do not yet yield a cascade-exponent bound: the former derivation of $\beta \leq 1$ does not follow from the stated argument, and any structural upper bound on $\beta$ by this route is open (**open**).

**O5** [26]: Obstruction theorem. The $\beta$ gap cannot be closed by any purely representation-theoretic or fixed-representation approach (**proved**).

**O6** [31]: Matrix-level dynamic redundancy, $R_n \to 0$ for the Steinberg fingerprint; the confinement of the cascade exponent is a corollary conditional on a power-law Ansatz that the paper's own fingerprint does not satisfy, a genuine power-law regime remaining open (**proved** / **conditional** on the power-law Ansatz / **open**).

**O7** [40]: Reformulates the target as a capacity exponent $\delta \in [7.4, 10.6]$ on three-step path fingerprints (**structural**).

**O8** [1]: Establishes the geometric obstruction closing the LPS phase. Exponential shell growth $|S_n| \sim p^n$ on LPS graphs compresses the pre-saturation window to $O(\log q)$ BFS steps, on an empirically observed shell-growth premise (**structural** / **numerical**).

### 4.4 O9–O15: Heisenberg Transition

The geometric obstruction identified in O8 motivates replacing LPS graphs by Cayley graphs of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$.

**O9** [41]: Structural transition. Polynomial ball growth $|B_n| \sim n^4$ provides $\Theta(q)$ usable BFS steps (**proved**). This is the key structural transition of the O-series.

**O10** [39]: First large-$q$ computation ($q = 101$). Measures $\hat{D} = 3.05$, continuing the convergence toward $D_{\text{hom}} = 4$; a TensorSketch representation obstruction is identified at large $q$ (**numerical**).

**O11** [72]: Bidimensional Fourier-character proxy overcomes the TensorSketch obstruction (**numerical**).

**O12** [22]: Exact Weil-block fingerprints. The incremental block capacity $\Sigma_n^{(c)}$ depends on the block only through a projective pair $(\alpha, \beta)$, exactly independent of the central coordinate $\gamma$. Finite-$q$ fitted slopes ($q \in \{29, 53, 61\}$) are crossover statistics of the block saturation-depth distribution; the intrinsic unfolded exponent is derived exactly, $\delta_{\text{exact}} = 3$ as $q \to \infty$, $n = o(\sqrt{q})$ (**proved**).

**O13** [10]: Extended prime range $q \in \{101, 151, 211\}$. The finite-$q$ exponent sequence ($4.42, 4.80, 4.51, 4.27, 3.59$ at $q \in \{29, 61, 101, 151, 211\}$) does not drift toward the phenomenological target at larger primes, consistent with O12's exact value $\delta_{\text{exact}} = 3$; measurement quality improves systematically with $q$ (**numerical**).

**O14** [74]: Diagnoses the observable-class mismatch in the proposed $\delta \mapsto \beta^*$ relation. Fixed-$q$ normalisation changes intercepts, not intra-$q$ slopes; the inter-$q$ estimator layer remains open (**structural**).

**O15** [36]: Heisenberg transfer no-go. The native Heisenberg identity is $\Delta r_n = |S_n|\Sigma_n$; the O6/O7 growth-law scalar observable is not equivalent to the exact Weil-block mean $\bar{\Sigma}_n$, so $\beta^* = 1/(\delta + \frac{1}{2})$ does not transfer to the exact-block setting. A checkable aggregation bound, $\alpha_{\text{dyn}} \leq \hat{\delta}_{\text{exact}}$ conditional on a non-negative OLS slope of $\log(1 + V_n)$, holds at two of five tested primes ($q = 151, 211$), open at the other three (**proved** for the non-equivalence theorem; conditional for the aggregation bound).

### 4.5 O16–O24: Pair Observable and the Fibre-Level Structure Programme

**O16** [24]: Fibre admissibility hypotheses (**conditional** on the parity and realisation hypotheses). Conjugate blocks $\{c, q-c\}$ constitute the fibres of $\Pi$ only under two independent, open premises: that $\Pi$ carries a parity involution, and that this involution is realised in the Weil representation as $c \mapsto q - c$. If a per-block observable's conjugate-pair ratio is $n$-independent, the pair-level exponent doubles, $\delta_{\text{pair}} = 2\delta_c$; on the real O12/O13 pipeline this holds trivially, since the exact conjugate-block identity forces the ratio to 1 (**structural** for the fibre hypotheses; algebraic identity for the doubling).

**O17** [21]: Exact block-independence, scalar toy model. The raw Gram-Schmidt redundancy count is exactly independent of the character and both initial-condition parameters, for any two blocks whatsoever, not only conjugate ones — too broad a fact to license a same-fibre criterion for conjugate pairs specifically. Scoped to this paper's own toy model; not shown to extend to the real O12/O13 pipeline (**proved** for the block-independence result within the model).

**O18** [20]: Born–Infeld parity covariance and the open fibre problem. The action is even, and parity acts on the response family with the character $(-1)^k$ of the derivative order: even-order responses are preserved under the transported-probe comparison, odd-order responses reverse sign (Prop. 2.6). An even countermodel refutes fixed-probe indiscernibility of $\chi$ and $-\chi$ (Prop. 2.4), so evenness does not imply $\Pi(\chi) = \Pi(-\chi)$; the fibre identification is a typed open classification problem (Problem 2.8), with (H-rank) one clause of it, and the Weil identification $\chi \mapsto -\chi$ as $c \mapsto q - c$ is an open bridge (**proved** for action parity and response covariance; fibre and Weil identifications: **open**).

**O19** [15]: Proposed canonical normalisation. Generic Gram–Schmidt phase invariance and the rank identity are valid, but the factor $r(c,q) \in \{1,2,3,4\}$ is not sourced for the real O12/O13 pipeline, the proposed normalisation is defined through an unspecified reference span, Tests P2/P3 are open, and the pipeline-independence proof is incomplete (**open** for the canonical observable; generic linear algebra: **proved**).

**O20** [43]: Conditional threshold-crossing template. Given a constant threshold and an exact decreasing power law, crossing within a fixed cascade window is algebraically equivalent to an exponent interval. The map from rank increments to a fibre amplitude, the inherited Born–Infeld envelope, and the threshold itself are open; the numerical interval $[7.4, 10.6]$ is not derived (**conditional** on the monotone-amplitude and capacity-bound hypotheses).

**O21** [51]: Local threshold-crossing diagnostic and open transfer. The two-point formula for $n_3^{\mathrm{obs}}$ is a valid extrapolated crossing rank under an exact power law and a fixed normalisation. The present registry's audit finds it not invariant under $\sigma \mapsto \lambda\sigma$, since the threshold base carries the factor and only the exponent is ratio-invariant: it scales as $\lambda^{1/\delta}$, so it does not eliminate amplitude or pipeline dependence. The threshold $9/q^2$, shell-alignment condition, and cross-substrate carrier remain external inputs; SGN excludes a native Heisenberg transfer for the corpus-defined objects (**conditional** on the shell-alignment conjecture and the external cross-substrate identification; the local algebra is **proved**; intrinsic rank and transfer **open**).

**O22** [50]: BI projection locking forces the O21-defined $n_3^{\mathrm{obs}}$ onto a BFS shell within O22's stated model (Theorem 3.2). This does not make the input rank invariant under amplitude rescaling and therefore does not close O21's intrinsic-definition problem (**structural** within the stated model).

**O23** [65]: Conditional adjoint-dimension theorem. For a supplied irreducible two-dimensional unitary carrier $V_\rho \cong \mathbb{C}^2$, the traceless neutral sector is $\mathfrak{su}(2) \cong \mathrm{Im}\,\mathbb{H}$, of real dimension exactly three; in $Q_8$ the neutral generating set $\{\pm i, \pm j\}$ spans only two axes, so generator-axis counting cannot produce the three. The carrier selection and the identification of $\Sigma_c$ with the module dimension are open bridges; the threshold $\Sigma_c(n_3) = 3$ is a supplied selection rule, definitional at the level of O21 (**conditional** on the supplied carrier and selection rule; the adjoint dimension is **proved**; threshold and bridges **open**). Version 2.1 determines what the deposited O12 shell-span filtration supplies on the carrier bridge. Its fingerprint vectors are pure Fourier modes, so every proper level is a Fourier-coordinate subspace whose Weyl support is one line $L$, and the exact stabiliser of that level in the Weil image is $U \rtimes M_n \subseteq B(L)$ with $M_n = \{s \in \mathbb{F}_q^\times : sF_n = F_n\}$ (**proved**, every odd prime, every generic block, every proper nonzero level). This excludes the *exceptional* binary polyhedral groups $2T$, $2O$, $2I$, including at primes where they exist in the ambient group (**proved**). It does not exclude a spinor carrier: at explicit generic levels at $q = 53$ and $q = 101$ the stabiliser is $\mathrm{Dic}_q$, whose restricted Weil module is multiplicity-free — one character plus four inequivalent two-dimensional irreducibles, exactly two of them faithful and SU(2)-valued (**structural**: exact computation on deposited witnesses with reproduction code, not a universal claim). The carrier bridge therefore splits into two open steps: selecting which prime, block and depth the projection singles out within the generic filtration, which no result addresses; and, inside a level already known to be dicyclic, choosing between the two admissible carriers, for which the candidate "unique odd member of $\{c_\Sigma, q - c_\Sigma\}$" succeeds on all 17 audited instances over 13 distinct subspaces with one step observed and not proved (**open**).

**O24** [37]: Capstone of the fibre-level construction. The Verticality Lemma ("$\ker \Pi$ is free, $\mathrm{Im}\,\Pi$ is constrained") closes the $c_{\mathrm{BI}} \to \delta_{\mathrm{pair}}$ segment with respect to the fibre structure of $\Pi$ (**conditional** on the Weil realisation and the stated fibre hypotheses). It does not derive the growth-equation conversion to $\beta^*$: SGN refutes the native Heisenberg realisation of its O4–O6 closure, frontier, pair-carrier, and exponent-coordinate inputs.

### 4.6 O25–O30, PTO, and Fibre Erasure: Numerical Validation, Carrier Diagnostics, a Conditional Covariance Model, and Information-Theoretic Axes

**O25** [55]: Systematic pair-level numerical campaign, extended to large primes (**numerical**). Computes $\delta_{\text{pair}}$ exhaustively across all conjugate pairs through $q = 211$; the capped-BFS extension — reproducing the observable exactly — uses 24 representative pairs at $q = 307$ and 12 at each of $q = 401, 601$. The inter-pair concentration establishes a stable fixed-$q$ pair statistic; it does not prove independence from $q$. Central result of the extension: the *raw* exponent $\delta_{\text{global}}(q)$ descends monotonically into the admissible window $[7.4, 10.6]$ on its own, reaching 7.61 at $q = 601$, with no correction; the O14 finite-size correction, needed at small $q$, becomes unnecessary and eventually *overcorrects* at large $q$ (sending $\delta_{\text{corr}}$ below 7.4). The asymptotic value $\delta_\infty$ is not pinned ($1/q$ and $1/\sqrt{q}$ laws remain equally viable at $q \le 601$). The reported $\log(n+1)$ convention inflates finite-window slopes relative to the paper-level $\log n$ definition. As a separate numerical comparison the legacy reciprocal image stays in the narrow band 0.108–0.123, near the mass-fitted window. SGN proves that this agreement is not a native capacity-to-rate derivation (**structural** as a phenomenological coincidence check).

**O26** [46]: Dictionary between conjugate Weil blocks and rank-1 matrices in $M_2(\mathbb{C})$, connecting the Heisenberg fibre to $2I \subset \mathrm{SU}(2)$ (**structural**). The note takes the pair-sector fibre structure as a structurally motivated hypothesis (O18 states the fibre identification as open): in the pre-saturation regime $\sigma_{\text{pair}}(n)$ has the growth exponent of the Hilbert–Schmidt norm of an associated matrix trajectory (Level I, **proved**); the quotient identification (Level II) is a conjecture whose open point is the $n$-independence of the proportionality constant, and the canonical representation-theoretic identification in an isotypic sector of $2I$ (Level III) is conditional on the admissible embedding $\Phi_{q,\rho}$. Version 2.1 proves that under Hypothesis 4.4(a), in a fixed conjugation basis, the embedded products are $uu^{\mathsf{T}}$ and span a subspace of $\mathrm{Sym}^2(V_\rho)$: their span has dimension at most $d_\rho(d_\rho+1)/2$, hence at most three for $d_\rho = 2$. The rank-four target is excluded under this contract, not for other embeddings or constructions. Tests 1–2 are implementation checks; the O28–O29 effective ranks are measured in $\mathrm{End}(H_{\text{eff}})$, not $\mathrm{End}(V_\rho)$, and occupy no row of the embedded-covariance decision table. This representation-side result does not supply the missing rate equation.

**O27** [5]: Quotient factorisation alone does not imply equivariance (**proved** / **open**). Saturation, the supplied carrier, and parity covariance are explicit inputs. Under the stated equivariance hypotheses, the Schur-level factorisation and multiplicity counts are proved; a canonical equivariant vector lift remains open. The result supplies no derived interpretation of $\beta^*$.

**O28** [8]: Two measurements from the Q5a-O5 checkpoints (**numerical**). Part A: out of sample (to $q = 601$) the calibrated linear extrapolation $n_1 \sim \hat{\alpha}\, q$ ($q \le 211$) fails, overpredicting $n_1$ by up to +88%; the finite measurements reject only that extrapolation, and the exact law is the interval theorem of the CC-Note [29], of which the measured depths are the transitional-regime data; the declared finite-size adjustment $\delta_{\text{corr}}(q)$ lies in $[7.4, 10.6]$ over $q \in \{29, 61, 101, 151, 211\}$, and O14 establishes that this term does not correct the intra-$q$ fitted exponent, so its stability is a diagnostic and not a corrected capacity exponent. Part B: formal effective-dimension computation of O26 Criterion 5.4 in $\mathrm{End}(H_{\text{eff}})$, $H_{\text{eff}} = \mathbb{C}^3$ (HEFF_DIM = 3, implementing the supplied selection rule $\Sigma_c(n_3) = 3$; the retained dimension is fixed in advance, so these ranks characterise the truncated carrier, not a measured admissible rank). O28 reported 1% threshold rank $r_{\text{eff}}^{1\%} = 3$ and resolved spectrum $[\lambda_1 : \lambda_2 : \lambda_3] = [1 : \frac{1}{2} : \frac{1}{2}]$ across its stated perimeter. O29's extended recomputation does not reproduce pairwise invariance: rank three occurs for 32/50 pairs at $q = 101$ and 103/105 at $q = 211$. It re-audits the finite-data rank and degeneracy without assigning them an exact axial origin. The gap from $d_\rho^2 = 4$ cannot be inverted to identify $V_\rho$: O29 distinguishes the measured span in $H_{\text{eff}}$, an auxiliary PCA truncation, and the separately supplied spin-$\frac{1}{2}$ module.

**CC-Note** [29]: Exact resolution of the saturation-depth asymptotics of O25–O28. Every

implemented Weil–BFS fingerprint is a non-zero scalar multiple of a single Fourier character, so the cumulative rank is the sumset cardinality $r_c(n) = |T'_c + C\, I_{n+1}|$, with universal shellwise ceiling $\Delta r_c(n) \le 2|T'_c| \le 18$ and a complete classification of the block resonances that lower it (**proved**). Exact sphere counts of the infinite Heisenberg group give the strict crossing $s_{22} = 16{,}934 < 18{,}000 < s_{23} = 19{,}412$: deterministically $n_1(q) \le 22$ and $5/q^2 \le x_1(q) \le 99{,}689/q^2$ for every prime $q \ge 311$ (**proved**), and under uniform generic block sampling $n_1(q) \to 22$ and $q^2 x_1(q) \to |B_{22}| = 99{,}689$ in probability (**proved**). The measured O28 depths are the transitional resonance regime of this law (effective exponent $\approx -0.76$ en route to the exact $-2$); the coverage-collapse ansatz is degenerate near the threshold front, and the implemented auto-calibrated window closes deterministically for $q > 937{,}024$. In the Q5a dual-window limit the potential coefficient vanishes, $\lambda = \lim x_1/C_{\mathrm{Heis}} = 0$: no oscillator route toward a spatial limit opens from this observable.

**Span-Growth Note** [32]: Native span growth and the failure of the Heisenberg capacity-to-rate transfer. The exact increment identity $\Delta r = \sigma_c\, |S_n|$ and the polynomial spheres ($D_{\mathrm{hom}} = 4$, Bass–Guivarc'h) yield a three-branch classification for abstract real-valued weights: growth, logarithmic growth, and saturation according as $\delta_c < D$, $\delta_c = D$, or $\delta_c > D$ (**proved**). Since the measured rank increments are integers, the pointwise hypothesis becomes unrealizable when $\delta_c > D - 1$; the production measurements therefore support only an exact finite-window formulation with two-sided bounds, not an asymptotic rank law. SGN proves a three-legged no-go for the proposed native transfer: both corpus-defined valence–exploration closures fail; the Heisenberg frontier is $N^{3/4}$ rather than $p^{1/2}$ and no Born–Infeld feedback carrier is defined in the corpus; the native block increment is neither the pair product nor expressed in the reduced-model exponent coordinates. Thus the $1/(\delta_{\mathrm{pair}} + \frac{1}{2})$ comparison is a cross-substrate phenomenological prescription, not a native derivation. The shell-growth factor $D - 1$ is recovered additively, while the inter-$q$ estimator correction remains open; fitting against $\log(n+1)$ instead of $\log n$ inflates the fitted exponent by 8–13% on the stated production windows. The window-effective exponents make contact near $q = 211$ and separate strictly for $q \ge 307$ ($\delta_c^{\mathrm{eff}} - D_{\mathrm{eff}} = -0.21$ to $-0.49$ at 6.7–14.5 inter-pair bootstrap standard errors, sign stable across all window-shift and knee-trimming variants and both logarithmic conventions): the contact is a transitional crossing in the accessible data, no current indication forces $\delta_c = D$, and the asymptotic value of $\delta_c$ remains open. The note's repository embeds the block-level data actually used (with frozen windows and per-checkpoint provenance), a single reproduction script, and the crossing figure: every quoted number is reproducible from the published deposit alone.

**O29** [71]: Veronese collapse in the measured trajectory span (**numerical**). The exact Weil-block identity $\rho_{q-c} = \bar{\rho}_c$ (an O17 theorem recalled in O18) motivates but does not prove the projected-coordinate relation $\pi_{q-c}(v) = \overline{\pi_c(v)}$, which the stored data satisfy only approximately; under it every outer product $M_j = \pi_c(v_j) \otimes \pi_{q-c}(v_j)^*$ is complex symmetric. Rank three is modal in $\mathrm{End}(H_{\mathrm{eff}})$, with seed-sensitive single-sample exceptions of four to nine at $q = 101$ and five at $q = 211$. None of these measurements is an embedded-covariance rank in $\mathrm{End}(V_\rho)$. On the O25/Q5a checkpoints the admissible trajectory spans all three complex dimensions of $H_{\mathrm{eff}}$. At $q \in \{29, 61\}$ the symmetric-square evaluation matrix has numerical rank 3 at the declared tolerance; fitted constraint commutators have numerical rank 3 in a chosen $\mathfrak{so}(3)$ basis and the computed commutant is one-dimensional. These finite-precision diagnostics are compatible with the adjoint module supplied conditionally by O23, but do not derive or identify it. Numerically: $r_{\mathrm{eff}} = 3$ universally for $q = 61$ (30/30 pairs) and $q = 151$ (75/75 pairs) with zero inter-pair variance; at $q = 211$, the single-sample run gives $r_{\mathrm{eff}} = 3$ for 103/105 pairs; at $q = 101$, modal $r_{\mathrm{eff}} = 3$ is confirmed by a dedicated multi-sample analysis ($M = 5$ resamples per anomalous pair, 76.7% of (pair, sample) cells satisfy $r_{\mathrm{eff}} = 3$, with modal $r_{\mathrm{eff}} = 3$ for all 18 pairs), without establishing that the remaining dispersion is an artefact. The two-dimensional PCA space $U_c^{(2)} \subset H_{\mathrm{eff}}$ is auxiliary and has no canonical representation status; rank 4 in $\mathrm{End}(U_c^{(2)})$ after broken locking identifies neither $V_\rho$ nor $d_\rho = 2$. The latter remains an O26 minimality selection, not an O29 data result.

**O30** [75]: Conditional $3 \times 3$ covariance model on $\mathrm{Sym}^2(V_\rho)$ (**conditional** on hypotheses (H0)–(H3)). Under the supplied hypotheses (H0)–(H3), including Veronese data, an inter-block

identification, weight exchange, and a fixed projected symmetric square, a lift satisfies $\beta_j = \varepsilon_j \bar{\alpha}_j$ with $\varepsilon_j \in \{\pm 1\}$. In the equatorial regime, the central component $\sqrt{2}\,\alpha_j\beta_j$ of $\mathrm{vec}(M_j)$ is phase-independent (the phases cancel exactly), while the diagonal components $\alpha_j^2$ and $\beta_j^2$ oscillate as $e^{\pm 2i\phi_j}$. The covariance diagonalises only under the two weighted conditions $\langle \varepsilon_j r_j^4 e^{2i\phi_j}\rangle = 0$ and $\langle r_j^4 e^{4i\phi_j}\rangle = 0$, yielding $\mathcal{C}_c = \langle r_j^4\rangle \cdot \mathrm{diag}(1/4, 1/2, 1/4)$ and spectral ratio $[1 : \frac{1}{2} : \frac{1}{2}]$ exactly. The factor of 2 is the squared symmetric-tensor normalisation coefficient in $e_0 = (v_+\otimes v_- + v_-\otimes v_+)/\sqrt{2}$, isolated by the equatorial constraint as the sole source of the unequal diagonal entries. The Carnot grading labels the direction that carries the factor; it is not its algebraic cause. O30's $3\times 3$ object is distinct from O28's measured $9\times 9$ covariance and does not derive its spectrum; the O28 ratio remains empirical.

**O31** [115]: Withdrawal notice for unsupported colour-triplet co-admissibility and gauge-group claims (**open** / **proved** for the three counterexamples). Version 2.0 withdraws eight result families: uniqueness of SU(3); irreducibility of the proposed fibre-preserving group; equality of the three individual capacity profiles; the arithmetic criterion $q \equiv 1 \pmod 3$ for triplet existence; the metaplectic step that changes central character while using a centre-fixing automorphism; the $\Delta(27)$ spectral proposition; the mistyped rank-eight test in $\mathrm{End}(\mathbb{C}^3)$; and every derivation of a gauge factor or of the complete Standard Model gauge group. The notice gives three explicit counterexamples: the proper irreducible subgroup $\mathrm{SO}(3) \subset \mathrm{SU}(3)$ refutes uniqueness; the diagonal torus $T^2 \subset \mathrm{SU}(3)$ satisfies the stated constraints while acting reducibly; and $\{1, 2, q-3\}$ is an admissible triplet for every prime $q \geq 7$, refuting the arithmetic criterion. The remaining scalar, arithmetic and discrete-group material is under re-audit. O31 supplies no positive conclusion about SU(3), a gauge factor, or the Standard Model gauge group. **O32** [79]: tests [H-color] numerically on $q \in \{61, 151, 211, 307\}$ (test, $q \equiv 1 \pmod 3$) and $q \in \{29, 101\}$ (controls). Noise floor $\texttt{ctrl_ref} = 1.657 \times 10^{-3}$. Normalised ratios: $R = 2.1$ ($q = 61$, 1/4 triplets valid), $R = 0.4$ ($q = 151$), $R = 1.0$ ($q = 211$), $R = 0.7$ ($q = 307$). All four pass $R \leq 3$; $\delta_{\mathrm{tri}} \approx 3\delta_c$ at the percent level (0.7% at $q = 151$, 1.3 to 2.2% elsewhere); $C_{\mathrm{color}}$ numerical rank 1 at $q \in \{61, 151, 211\}$. The rank and pointwise levels of [H-color] are open, O31 version 2.0 supplying neither; the averaged level ($O(q^{-1})$, O32 Proposition 1) and the effective level stand as O32's own results. The O32 numerical values remain reported measurements; they do not establish individual-profile co-admissibility or identify a gauge group. Status: **numerical**for the reported measurements; no structural interpretation of them is established, and their reading is under re-audit.

**O33** [76]: arithmetic Mackey–Hecke structure of the O12 three-character filtration (exact core, **proved**). The cumulative Fourier mode set is proved to be the cyclic sumset $K_n = c[-(n+1), n+1] + \{0, \pm(c_2 + c_3)\} + \{0, \pm c_3\}$. This yields an exact cyclic-spacing criterion for boundary porosity and reduces the full Weil stabiliser to the multiplicative symmetry group $A_K = \{a : aK = K\}$. Weyl quantisation realises the induced Mackey carrier internally in $\mathrm{End}(V_\rho) \cong \mathbf{1} \oplus \mathcal{D}_+ \oplus \mathcal{D}_-$. Generic principal-series pairs carry canonical $M_2(\mathbb{C})$ commutants; Harper dynamics gives the normalised split $1 \pm q^{-1/2}$. The eliminated residue supplies a second axis exactly when the short boundary character sum $C_\theta(K)$ is non-zero: a Galois separation theorem proves $\Delta_{\theta,c}(K) \neq 0 \iff C_\theta(K) \neq 0$ for every generic character, the hole pairs are geometrically bounded by four, and the zeros of $C_\theta(K)$ are completely classified into antipodal-pair, cube-triple, and double-pair mechanisms. The deposited parity-even operator algebra has zero projection onto $\mathcal{D}_-$, proving an odd-sector protection theorem. The stabiliser order divides $\gcd(q-1, |K|-1)$, and two uniform constructions give exactly $C_4$ from a square root of $-1$ and $C_6$ from a primitive cubic root. In the degenerate regime the depth-zero square extends to an infinite uniform Gaussian-box family $Q_r(i)$ with $C_4 \subseteq A_K$, realised at depths $r-1$ for $r \leq 4$, so the family statements are mechanisms rather than an exhaustion; an exhaustive audit through $q = 151$ classifies all 75 exceptional sets into Gaussian boxes, free four-point complement orbits, and Eisenstein $C_6$ sets, with no order eight at $q = 193, 241, 257$; completeness of the refined mechanisms remains conjectural. The exact stabiliser value is proved inside two of the three mechanisms. A fourth-moment invariant, resting on an exact Gaussian collision dichotomy and an explicit $\mathbb{Z}[i]$ division certificate, gives

$A_{Q_r(i)} = \langle i \rangle \cong C_4$ for every proper box, direct or wrapped; a free four-point complement orbit likewise has stabiliser exactly $C_4$, by the free action of $A_K$ on a four-element set. Completeness of the refined mechanisms, and a uniform proof of the Eisenstein top autocorrelation level, remain open; the unit groups of the Gaussian and Eisenstein integers are recorded as an organising principle for that residual question, not as a completeness theorem. The asymptotic porosity law is proved: the porous depths of a uniform admissible block converge to an explicit limiting point process, with count law $(3/20, 7/24, 9/40, 5/24, 1/8)$, mean $28/15$, piecewise affine intensity, and conditional mean normalised depth $491/10080$, machine-certified by an archived exact rational-polyhedral script. An exact audit of 31,166 porous cases through $q = 311$ confirms the zero classification with no exception; no Standard-Model or Dirac-mass identification is claimed.

**PTO** [45]: The cumulative projected capacity $I(n) = \sigma_{\text{pair}}(0) - \sigma_{\text{pair}}(n)$ is strictly non-decreasing and hence compatible with an arrow of time. Confirmed numerically (**numerical**) for $q \in \{29, 61, 101, 151, 211\}$. The admissible cascade is shown to carry the structure of an oriented state machine: nodes are admissible projections, edges are admissible transitions, and branching weights play the role of quantum probabilities. Foundation does not guarantee the first transition $U_0 \to U_1$: its projective incompleteness (Corollary 4.9) applies where a step is non-injective, the existence of an admissible successor is a separate condition (Remark 4.11), and temporal order is conditional on [H-acyc] (Theorem 4.4). Open: whether the characteristic shell $n^*(q)$ coincides with the locking depth $n_c(q)$ at which BFS coverage transitions from locally to universally dominated — which would connect the spectral ordering to the effective geometry results of Q10 [108].

**Fibre Erasure** [91]: Hub paper between ENI [35] and the BFS spectral cascade (**conditional** on [H-suff] and [H-sg]). Defines a residual fibre-information functional $I(c; \sigma(\ell))$ on the BFS coarse-graining axis $\ell$, distinct from and orthogonal to the temporal axis $n$ of PTO. Conditional on the structural sufficiency hypothesis [H-suff], the data-processing inequality implies that $I(c; \sigma(\ell))$ is non-increasing in $\ell$: fibre information is erased, not created, under admissible coarse-graining (Thm 4.1). [H-suff] is proved unconditionally at the BFS rank level (Prop. 3.6, label-blindness from Schur's lemma); for the full Born–Infeld capacity it is numerically supported for $q \in \{61, 151, 211\}$. Strictly not a $c$-theorem: no counting of effective degrees of freedom is claimed (listed as open problem [O-2]). Together with PTO, constitutes the information-theoretic architecture of the admissibility cascade: resolution axis $\ell$ (Fibre Erasure) and temporal axis $n$ (PTO) are structurally orthogonal.

# 5 Branch III: Q-Series and Physical Applications

## 5.1 Q-Series: Quantum Mechanics and Spacetime Emergence

The Q-series develops quantum-mechanical and geometric structure from the Foundation axioms, taking as supplied input the Heisenberg/Schrödinger carrier, its distinct associated Weil action, and selected O-series results; additional targets denoted $V_\rho$ are typed separately in the papers that use them.

**Q1** [28]: Proves an exact, unconditional Fourier-support rigidity theorem for the associated finite Weil action on the supplied Heisenberg/Schrödinger carrier (**proved**), and a precisely scoped no-go corollary for support-, rank-, and Gram-based diagnostics. Does not derive phase coherence, a singlet correlator, a Tsirelson bound, or the Born rule (**open**).

**Q2** [7]: Proves an exact finite-group fact (**proved**): on the binary icosahedral group $2I$, the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sectors share the Laplacian eigenvalue $\lambda_{1/2} = \lambda_{3/2} = 18$. Does not derive phase coherence, a Casimir/isotropy normalisation, the Born rule, a Tsirelson-type bound, or an SU(2) fixed-point/sector-selection argument (**open**).

**Q3** [80]: Proves a theorem conditional on an explicit, undemonstrated hypothesis (**conditional** on [H-inv] and the supplied composition): given a supplied bipartite carrier $V_{\chi_{2j+1}} \otimes V_{\chi_{2j+1}}$ and the hypothesis that the joint state is invariant under the diagonal $2I$-action —

*not* derived from admissibility or from Born–Infeld indiscernibility — Schur's lemma applied to $\mathrm{Hom}_{2I}(V^*_{\chi_{2j+1}}, V_{\chi_{2j+1}})$ uniquely identifies the state as the singlet $|\Omega_j\rangle$, with an explicit Casimir correlator $E(\hat{a},\hat{b}) = -j(j+1)/3 \cdot (\hat{a}\cdot\hat{b})$, for all five admissible sectors $j \in \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$ of $2I$. Does not derive the invariance hypothesis itself, phase coherence, the Born rule, or a CHSH/Tsirelson-type bound (**open**).

The papers Q1, Q2, Q3, and the Bell paper [13] form the *quantum structure sub-programme*, whose synthesis paper is the presentation note **QS-Note** [63] (Presentation Note 7). QS-Note states plainly the current, narrow scope of each constituent: Q1 proves an unconditional Fourier-support rigidity theorem unrelated to phase coherence or the Born rule; Q2 proves an exact finite-group eigenvalue degeneracy on $2I$; Q3 proves a theorem conditional on an explicit, undemonstrated diagonal-invariance hypothesis. No chain from Born–Infeld indiscernibility to phase coherence, a singlet correlator, a Tsirelson bound, the Born rule, or an SU(2) fixed point is currently established by these three papers. Bell [13] is published in *Quantum Reports*, but its claimed implication from non-injectivity to failure of Bell factorisation is disproved by a finite Bell-local countermodel. Its PR-box construction also has empty support. The general no-go theorem and the correct obstruction are proved in [87]. The Bell bridge is open: the programme does not derive an obstruction to a setting-independent global coupling. The note records the proved/structural/open status of every constituent result and states the open deliverables: the Born rule, phase coherence beyond Q1's narrow rigidity statement, and a derivation (rather than a supplied hypothesis) of Q3's diagonal-invariance premise.

The papers Q5a, Q5a-O2, Q5b, Q6b, Q7–Q11, U1, W1, and H2 form the *emergent geometry sub-programme*, whose synthesis paper is the presentation note **EG-Note** [57]. It maps these twelve papers into the chain $\Pi_q \to L_{\text{eff}} \to g^{\mu\nu} \propto \eta^{\mu\nu}$ on a supplied Heisenberg carrier, which the admissibility axioms do not select. Version 2.2 of the note states every link with its hypotheses: $L_{\text{eff}}$ and the rank-three co-metric rest on [H-L] and the open lifting hypothesis [H-lift]; the signature $(-,+,+)$ of the non-degenerate block needs [H-hyp$_{\text{red}}$]; the endpoint $g^{\mu\nu} \propto \eta^{\mu\nu}$ holds only for a supplied rank-four extension satisfying the hypotheses of Q11 Corollary 6.1, including a supplied negative ordering term, after rescaling $\tau$, with its overall scale free. No metric coefficient value is established: Q8 version 2.0 proves that invariance fixes the Casimir form only up to a free positive factor, Q11 version 2.0 leaves $A_\tau$ independent of the spatial scale, and Q5b version 2.2.1 and Q10 version 2.0.1 establish no coefficient value. The chain carries two distinct gaps. At its first link, Q5a version 3.2 establishes no spatial limit operator, which is the explicit, unestablished hypothesis [H-L] (Q5b version 2.2.1). At the Q7 step, Q7 version 2.0 records the identification of the measured space with that spin-1 module as supplied by no source, so the carrier of those four coefficients is itself open. The geometric convergence results are independent of both. The algebraic rigidities are independent of [H-L] but not of the identification: they are statements about the spin-1 module, and what is open is that the measured space is it. Their individual contributions follow.

**Q5a** [14]: Version 3.2 (*Canonical Fourier Filtration and the Obstruction to a Spatial Continuum Limit*) works on a supplied finite Heisenberg group and non-trivial central character; it does not select that carrier or its associated Weil action. It identifies the canonical filtration exactly — the stage is precisely the toric Fourier window $\mathrm{span}\{e^{2\pi i b x/q} : |b| \le n\}$ — proves that the published admissibility form converges to the zero form on it (rate $q^{-2}$), and proves a normalisation no-go: no common scalar normalisation yields a non-trivial toric differential operator; the only non-trivial rescaled limit is a conditional Dirichlet form in the rescaled *frequency* variable. The normalisation no-go is proved under [H-depth] and the measured positive finite weight limits; the dual-window limit is conditional on a subsequential critical-coverage limit. No Mosco derivation of $L_\Pi = -A\partial_x^2$ is established; the spatial limit operator is the hypothesis [H-L] and Q5 is open (**proved** for the negative results; **open** for the spatial continuum).

**Q5b** [110]: Version 2.2 (*Toward Four-Dimensional Lorentzian Geometry from BFS Shell Stratification*) proves that the BFS shells of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ converge, on balls of radius $n \ll \sqrt{q}$, to the sphere foliation of a sub-Finsler Carnot–Carathéodory metric on $\mathrm{Heis}_3(\mathbb{R})$, of homogeneous

dimension four (**structural**). Under the unestablished spatial-limit hypothesis [H-L] and the open lifting hypothesis [H-lift], the effective operator gives the co-metric $\mathrm{diag}(-A_\tau, A_H, A_H, 0)$ in the left-invariant frame: rank three, with a central slot that no lower-order term fills. The temporal term is stipulated; under the hyperbolicity hypothesis [H-hyp], applied to the sector independent of the central variable, in homogeneous and quasi-isotropic regimes, its non-degenerate block has signature $(-,+,+)$ with $\tau$ time-like, and an extension $\mathrm{diag}(-A_\tau, A_H, A_H, A_z)$ with $A_z > 0$ would be Lorentzian. Q9 [106] gives sufficient conditions [K] and [R] for a kinetic Mosco limit and does not discharge [H-lift]. Q5b-O1, Q5b-O2 (the full-rank extension) and Q5b-O3 (the coefficient values) are open, and no value of $A_\tau$, $A_H$ or $A_z$ is established (**structural** for the geometry; **conditional** on [H-L], [H-lift] and [H-hyp] for the co-metric and its signature).

**Q6a** [69]: Version 2.3. Gauge charges are the invariants of the projection-fibre equivalence classes, and the gauge group is the group of fibre-preserving transformations (**open** for the colour factor and the complete gauge group). U(1) is the residual phase symmetry when the projection is defined through span-based observables. On the spinor carrier supplied by O23, whose selection is open, and under the neutral-sector hypothesis $\mathrm{Im}\,\Pi_{\mathrm{adm}} = \mathrm{Ntrl}$ of O27, the group acting faithfully on the carrier is SU(2); this is a consistency statement about a supplied carrier, not a selection, and no uniqueness or minimality of SU(2) is established. SU(3) is a structural hypothesis whose identification is open. Q6a does not use O31 and determines no complete gauge group; its connection and field strength are conditional on two continuum hypotheses that it does not prove, the second being the open spatial continuum of Q5a, and the Yang–Mills action is a structural motivation.

**Q6b** [68]: Version 2.1 assembles, on a supplied Heisenberg carrier, the effective spacetime geometry that the admissibility framework yields conditionally. Under the unestablished spatial-limit hypothesis [H-L] and the open lifting hypothesis [H-lift] (Q9 gives sufficient conditions and does not discharge it), Q5b gives a rank-three co-metric $\mathrm{diag}(-A_\tau, A_H, A_H, 0)$ with a stipulated temporal term; under $[\text{H-hyp}_{\mathrm{red}}]$ its non-degenerate block has signature $(-,+,+)$ with stipulated signs, and $(-,+,+,+)$ holds only for a supplied rank-four extension (Q5b-O2); no coefficient value is established. Its Proposition 3.2 is **conditional**: for a supplied rank-four extension that is spherically symmetric and asymptotically flat in the exterior of a localised obstruction, flux conservation gives, when the exterior is asymptotically homogeneous, the leading Newtonian profile $\Phi_0 - C/r$, and the Schwarzschild form follows only with exterior homogeneity and, as external dynamical input, the closure $f_t f_r = 1$ and the exact temporal identification; no uniqueness is claimed. No extension diagonal in the left-invariant frame with $A_z > 0$ qualifies, and a qualifying extension would not preserve the large-scale Carnot structure; whether the admissibility structure supplies one is open. The horizon degeneracy is chart-relative and read only interpretively. The relation to the Einstein equations is conditional: the Einstein coefficient of the Gravity paper is a matching datum on Riemannian backgrounds, and the field equations of the spectral-stationarity paper assume an infrared response and a matter functional; applying either to the effective geometry also needs a Lorentzian formulation, which is not available. The chain $\Pi_q \to \mathcal{L}_\Pi \to g^{\mu\nu}_{(3)} \to g^{\mu\nu} \to G_{\mu\nu}$ is conditional at each arrow.

**Q7** [112]: Version 2.0 states what an equivariant bridge between the supplied admissible dimension $\dim H_{\mathrm{eff}} = 3$ (the O23 selection rule, implemented as HEFF_DIM = 3 in the O25–O29 pipeline) and the three spatial directions of Q5b would require, and proves the requirements that follow from representation theory on a supplied carrier. Proves $[\mathfrak{su}(2)] \ncong \mathrm{heis}_3$ unconditionally (the bridge cannot be a Lie algebra isomorphism), and, as unconditional representation theory, that $\mathrm{Sym}^2(V_\rho)$ is up to isomorphism the unique irreducible three-dimensional complex $\mathfrak{su}(2)$-module, isomorphic to the complexified adjoint. *Under an explicit identification hypothesis [ID] that no source supplies*, the measured space inherits that structure and an equivariant map to a three-dimensional adjoint module is either zero or unique up to one non-zero complex scalar (reality and positivity being a separate requirement and then a convention). That the real spatial sector *is* such a module is the separate hypothesis [ACT] below. Non-zero intertwiners exist once the two isomorphic modules are supplied. Their geometric compatibility is separate: real

rotation invariance [INV] yields horizontal isotropy, and the coefficient transport additionally requires an adapted unitary intertwiner [ADAPT], not implied by Schur or [ACT]. Q5b's spatial principal symbol has rank two everywhere and cannot equal a positive rank-three Casimir form under invertible transport. The revised conjecture instead asks for a new positive spatial target with a specified operator and compatible action; lower-order terms cannot change the existing principal rank. Proves analytically $[F_c, L_{\mathrm{Weil}}] = 0$; version 2.0 withdraws the application of its corollary to these data on two counts: the three stored rows are not $F_c$-eigenvectors, so the corollary's hypothesis is not met, and its conclusion would in any case concern a compression of the discrete operator rather than the cross terms of $\sigma_2(L_{\mathrm{eff}})$, whose identification is missing. Its numerical section is re-typed: the three stored basis rows of the O25 checkpoints are pure Fourier modes, so every *diagonal* entry of the compression *in the stored basis* is the Rayleigh quotient $4 - 2\cos(2\pi m/q)$ of $L_{\mathrm{Weil}} = 4I - W_a - W_a^\dagger - W_b - W_b^\dagger$, while an off-diagonal entry of modulus one appears exactly when two stored indices differ by $\pm c$ mod $q$, which is the paper's replacement account of the two pairs previously reported as anomalies. The entry $\tilde{L}_{00} = 4 - 2 - 0 = 2$, historically named $A_z$, is thus the flat-mode value. The pipeline reports it in that slot at eleven of the twelve pairs, the flat mode being ordered last at $(101, 3)$. Q7 version 2.0 records that this value follows from the $4I$ normalisation and is not normalisation-invariant, so its agreement with the Casimir carries no evidential weight; $A_z$ itself is left an unevaluated parameter. The difference between the two distinct entries, $4\sin^2(\pi k/q)$, is an identity, so the vanishing of the gap is equivalent to $\mathrm{dist}(k/q, \mathbb{Z}) \to 0$, a property of the pipeline's mode-selection rule that the corpus does not derive; the published fit $\approx 0.364\, q^{-0.52}$ tracks that rule rather than a spectral rate. What the pipeline reports is the compression after conjugation by the covariance eigenbasis, which as a matrix coincides with the stored-basis one at nine of the twelve pairs, agreeing in modulus but not in phase at a tenth and differing outright at the other two; the invariant content is its spectrum. Version 2.0 accordingly withdraws, as results and as supports, the Casimir reading, the isotropy fit, and the downstream values imported from Q8, Q10, U1 and Q11. The exact Q5b principal-symbol identification is ruled out by rank preservation. The broader bridge to a newly supplied positive target remains a missing construction. Separately, Q5a supplies a conditional obstruction under its depth and weight hypotheses and for common scalar normalisations on toric targets. (**proved** for the representation theory and scoped rank obstruction; **open** for the measured-space identification and for a geometric construction supplying a new target with [ACT], [INV] and [ADAPT].)

**Q8–Q11, U1, W1, H2 (Geometric emergence cluster).** Q8 version 2.0 [105] proves that the Heisenberg commutator supplies no central coefficient: the sub-Laplacian has no first-order part and a rank-two principal symbol, which lower-order terms do not change, and a left-invariant completion with a positive constant central coefficient must contain a term $Z^2$ of homogeneous degree four; the ratio of central to horizontal coefficients scales as $r^2$ under the Carnot dilations, so isotropy selects a length scale. For a supplied positive rank-three target with full $\mathfrak{su}(2)$-invariance, $A_H = A_Z = 2\lambda$ with $\lambda > 0$ free: the Casimir eigenvalue fixes a normalisation, not a coefficient (**proved** for these statements; **open** for the target). Q9 [106] proves that nonnegative modulation energy leaves a kinetic Mosco limit unchanged when the kinetic forms converge under [K] and the same recovery sequences satisfy [R]. Its bound $\mathcal{V}_q(f_q) \le 8\pi^2 r_q a_q(X)\|k f_q\|^2/q^2$ retains the energy normalisation. The free kinetic form neither selects a nonzero Schrödinger character nor equals the full oscillator form. A relative moment bound gives an inequality for each qualifying vector, not a closed coercive form on a derived linear domain. Q5b-O1, [H-lift] and bridge non-obstruction remain open (**conditional** on [K] and [R]; version 2.0). Q10 version 2.0.1 [108] proves a sharp stability lemma, two averages of uniformly controlled positive data with a common depth weighting have a ratio within $2\varepsilon/(1-\varepsilon)$ of one, and a conditional isotropy theorem: under [U], a sector assignment [W], a linear response law [R] and a supplied rank-three target [T], $A_H/A_Z$ obeys the same bound, and an absolute value follows only from an independently supplied scale [S]. Exact conjugation parity holds for conjugately matched fingerprint data; the O25 campaign does not select such pairs. No value of $A_H$ is derived (**proved** for the lemmas; **conditional** on

[U], [W], [R], [T], [S] for the coefficient ratio). U1 version 2.0 [113] establishes no proof of [U] and no $O(q^{-1/2})$ rate. Its unconditional result is the exact equidistance of the finite Heisenberg multiplication generators at distinct central characters. A separate adjacent-character example excludes a uniform modulus for their summed operator. Neither result proves or refutes [U], which remains open (**proved** for the obstruction; **open** for [U]). W1 version 2.0.1 [114] establishes no proof of [H-w]. Its Theorem 4 gives the summation inequality only for a separately defined partial-sum proxy under a supplied relative profile estimate, which U1 does not provide. Its Lemma 6 identifies a depth-22 target when the cutoff follows the proved five-block calibration, while Corollary 7 requires a separate proxy-to-Q5a-response identification. Neither the current Q5a directional limits [H-$w'$] nor the stronger common [H-w] is proved by W1; fibre constancy alone does not give cross-fibre uniformity (**conditional** for the proxy bound; **open** for the weight bridge). H2's separate result does not close the weight hypotheses; [H-$\mathcal{E}1$] and [C] are not closed by the finite-rank route, their disposition being the zero-form one of Q5a version 3.2. Q5a-O2 [83] establishes the pure-Fourier-mode structure of the pre-saturation pipeline, the truncation status of its three-coordinate reduction (no rank is measurable from it), and an $O(q^{-2})$ upper bound on the admissibility form that rules out the $q^{-1}$ coercivity scale; [H-$\mathcal{E}1$] and [C] are therefore not closed by a finite-rank route, and their disposition is the one fixed by Q5a version 3.2: the canonical filtration is a growing Fourier window of dimension $\min(2n+1, q)$ on which the published form has zero-form Mosco limit. H2 [94] proves [H2] (strong convergence of the rescaled Weil generators $\hat{X}_q \to x$ and $\hat{P}_q \to -i\partial_x$), via a quantitative Poisson aliasing lemma and a discrete Sobolev identity (**proved**). Q5a version 3.2 proves that the published form has a zero-form Mosco limit, so this route gives no spatial limit operator; the chain's residual conditionality includes the spatial limit hypothesis [H-L] together with the open hypotheses [H-$\mathcal{E}1$] and [C] of a non-trivial Mosco-type route. **Q11** [107] (version 2.0): Spatial SU(2) invariance leaves the temporal coefficient free (**proved**). On the cotangent fibre $\mathbb{R}_\tau \oplus W$, with SU(2) trivial on the ordering line and spin one on the spatial sector (the supplied [ACT$_\tau$], built on the [ACT] of Q7 and Q8), every invariant form is $\alpha\, k_\tau^2 + \beta\, Q_{\mathrm{sp}}$ with $\alpha$ and $\beta$ independent: invariance excludes mixed temporal–spatial terms but relates the two scalars by no equation, so $A_H = 2\lambda$ gives no value of $A_\tau$. For every linear SU(2) action on four real dimensions the invariant forms are the multiples of one definite form or a family of dimension at least two, so no SU(2)-invariance condition fixes a Lorentzian co-metric up to an overall factor. The identification of $\tau$ with BFS depth is a modelling input, $\tau = n\,\Delta\tau$ with a free step, and temporal rescalings commute with the spatial action. For a supplied full-rank extension under [H-L], [H-lift], [T] and [INV$^+$], the co-metric is $\mathrm{diag}(-A_\tau, 2\lambda, 2\lambda, 2\lambda)$ with two independent scales (**conditional**); the temporal half of Q5b-O3 amounts to one datum coupling the temporal and spatial normalisations, which no source supplies (**open**). Q11 closes a derivation route, not the possibility $A_\tau = 2$. The derivation chain Q5a $\to$ Q5b $\to$ Q7 $\to$ Q8 $\to$ Q9 $\to$ Q10 $\to$ U1 $\to$ W1 $\to$ H2 is not a closure, and Q11 shows that the temporal link to Q5b-O3 cannot be closed by spatial SU(2) invariance. Q7 version 2.0 supplies no symmetric-square identification of the measured space; in Q8 version 2.0 and Q10 version 2.0.1 no scale is fixed by matching the other paper, Q10 taking the scale as an independent input and Q8 leaving the common value free; Q5b version 2.2.1 records [H-lift], Q5b-O2 and Q5b-O3 as open. The chain is a sequence of conditional steps with a missing identification at the Q7 step, a missing extended operator for the central direction, and no established coefficient value. Consumer entries that still assert the former values are pending revision.

**Q12** [116]: Version 1.7 derives Yang–Mills dynamics from the vertical variation of the projective spectral entropy at an arbitrary supplied fixed base metric (**proved**, conditional on a supplied compact gauge group and admissible connection; O31 supplies no SU(3) identification). This fixed-base theorem does not depend on Q11; only its interpretation on the Cosmochrony emergent base is conditional, on [H-L], [H-lift] and a supplied rank-four extension, and on a Riemannian-to-Lorentzian continuation that is not performed. The Gravity paper [17] states, for a supplied Riemannian metric, the conditions under which the Einstein tensor is the infrared-dominant horizontal response of $S_\Pi[g] = \frac{1}{2}\log\det' A_g$: a non-zero Einstein coefficient, which

is a matching datum whose cutoff contribution comes from $a_2$, and $RL_4^2 \ll 1$, $L_4$ being the renormalized higher-derivative scale (**conditional**). Q12 carries out the complementary vertical variation: the operator $A_g$ is extended to $A_{g,\mathcal{A}} = -(\nabla^{\mathcal{A}})^2 + E$ on the associated vector bundle of the admissible principal bundle $P_{G_\Pi}(M, G_\Pi)$, and the Seeley–DeWitt coefficient $a_4$ is computed:

$$a_4(A_{g,\mathcal{A}})\big|_{\text{gauge}} = \int \frac{1}{12}\,\mathrm{tr}_\rho(F_{\mu\nu}F^{\mu\nu})\sqrt{g}\,\mathrm{d}^4x\,,$$

the gauge component $\Omega^{\text{gauge}}_{\mu\nu} = F_{\mu\nu}$ being isolated from the total bundle curvature and the overall $(4\pi)^{-2}$ carried once by the heat-kernel prefactor. The extended spectral entropy $S_\Pi[g, \mathcal{A}]$ therefore contains a local logarithmic Yang–Mills term, and the vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed base metric yields the Yang–Mills equations $D_\mu F^{a\mu\nu} = 0$ in the current-free sector (**proved** for the $a_4$ derivation given $G_\Pi$). The coefficient of the induced logarithmic running of $g_{\text{YM}}^{-2}$ is $I_\rho/(3\cdot 16\pi^2)$, depending on the representation through the Dynkin index $I_\rho$ and hence not universal; the physical $g_{\text{YM}}$ and $G_N$ are renormalized matching data, of which only the logarithmic running is computed. What is structural, within the proper-time scheme, is the difference in UV divergence degree between the two sectors — quadratic at $a_2$ for gravity, logarithmic at $a_4$ for gauge; the contrast is not scheme-independent, the zeta-regularized determinant carrying no power divergences. No predictive $G_N$ formula and no tuning-free hierarchy is claimed (aligned with the Gravity paper, where $G_N$ is a matching datum). At fixed base metric the vertical variation isolates the Yang–Mills sector, but full horizontal–vertical decoupling does not hold: the metric variation of $F^2$ is the Yang–Mills stress tensor $T^{\text{YM}}_{\mu\nu}$, an $a_4$-order back-reaction treated in Q13. Beyond the technical derivation, Q12 offers an interpretive outlook (not a theorem): gravity and gauge dynamics are separated by geometric direction (horizontal vs. vertical variation on the admissible bundle) *and* by spectral order in the Seeley–DeWitt hierarchy, so the difficulty of reconciling gravity with the gauge-theoretic Standard Model may reflect a difference of Seeley–DeWitt order rather than the absence of a common symmetry group $G_{\text{unif}}$. The gauge–gravity synthesis is carried out in Q13.

**Q13** [84]: Version 1.6 gives the gauge–gravity spectral synthesis on supplied geometric and gauge data (**conditional** on the matched coefficients, and on Q12, the Gravity paper, matching coefficients and a supplied compact gauge group; O31 supplies no SU(3) identification). Its interpretation on the Cosmochrony emergent base is separately conditional on [H-L], [H-lift] and a supplied rank-four extension. Q13 joins the metric and connection variations of a single renormalized local action and obtains a *conditional* Einstein–Yang–Mills system. The derivation is Euclidean, and the Lorentzian continuation is open in the geometric sector as well as in the gauge sector. The metric variation of the matched gauge kinetic term supplies the Yang–Mills energy–momentum tensor through $(1/\sqrt{g})\,\delta\int\sqrt{g}\,\mathrm{tr}_\rho(F^2)/\delta g^{\mu\nu} = 2\tau_{\mu\nu}$ (**proved**), while the fixed-metric connection variation supplies $D_\mu F^{a\mu\nu} = 0$; jointly, $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = 8\pi G_N T^{\text{YM}}_{\mu\nu}$. The normalisation is matching, not prediction: since $\mathrm{tr}_\rho(F^2) = I_\rho F^a F^a$, canonical matching fixes $c_{\text{EH}} = 1/(16\pi G_N)$ and $c_F I_\rho = 1/(4g^2_{\text{YM}})$, so that $8\pi G_N = 1/(2c_{\text{EH}})$ — and *not* $c_{\text{YM}}/c_{\text{EH}}$, which would mix the gauge-action prefactor with the definition of the stress tensor. $G_N$, $g_{\text{YM}}$ and the cosmological coefficient $\Lambda_{\text{ren}}$ (the $a_0$ term does contribute to the metric equation) are independent renormalized matching data, so the ratio $G_N g^2_{\text{YM}}$ is not an output of the construction. The $a_6$ sector is given as a representative spanning list of the invariants that can occur, modulo integrations by parts and Bianchi identities: in four dimensions $a_6$ produces no ultraviolet divergence, its renormalized dimension-six coefficients are themselves matching data, the pure-gauge invariant at that order is $\mathrm{tr}_\rho(F^3)$, and no unique cross-term is determined (**structural**). No non-linear completion of the joint functional is asserted. The derivation is Euclidean; identification with the Lorentzian Einstein–Yang–Mills equations requires a continuation that is not performed. Within the proper-time scheme, the difference in ultraviolet divergence degree between the two sectors is structural ($a_2 \to$ gravity at quadratic UV, $a_4 \to$ gauge at logarithmic UV); this contrast is scheme-dependent and does not hold for the zeta-regularized determinant. The central conceptual contribution is an organising reading, not a theorem: the spectral stratification principle, in which gravity and

gauge dynamics are read not as unified by enlarging the symmetry group, but as *stratified by Seeley–DeWitt order* within the single operator $\mathcal{A}_{g,\mathcal{A}}$. The fermionic extension is addressed in Q14, conditionally on two explicit hypotheses.

The Gravity paper [17], Thermodynamics [19], Lorentz/CausalPropagation [34], and Born-Infeld [11] form the *spectral gravity sub-programme*, whose synthesis paper is the presentation note **SG-Note** [104] (Presentation Note 4). SG-Note (version 2.3) states, for a supplied Riemannian metric, the conditions under which the projective spectral entropy $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ supports an Einstein sector: the Einstein tensor locally dominates the renormalized horizontal response, by power counting, when the Einstein coefficient, a matching datum whose $a_2$ cutoff contribution is computed, is non-zero and $RL_4^2 \ll 1$. It recasts the coupling to matter as a conditional variational stationarity at leading local infrared order, analyses the posited four-derivative Lorentzian truncation (whose massless branch is exactly luminal), and presents the determinantal Born–Infeld completion as an admissible member of an explicitly parametrised family resting on the capacity axiom [A-cap], on the mixed Ricci carrier selected among linear two-derivative carriers by the saturation condition. The reading of the supplied metric as the emergent metric of Note 2 is conditional on [H-L], [H-lift], a supplied rank-four extension and a continuation from Riemannian signature that is not performed, and no metric coefficient value is established. This conditional Einstein sector is the gravitational input consumed by the gauge–gravity stratification sub-programme below (Note 8).

Q12 and Q13 form the *gauge–gravity stratification sub-programme*, whose synthesis paper is the presentation note **GGS-Note** [59] (Presentation Note 8). GGS-Note organises the two constituents into the spectral stratification chain $a_2 \to G_{\mu\nu}$ (horizontal $\delta_g$), $a_4 \to D_\mu F^{a\mu\nu} = 0$ (vertical $\delta_A$), $a_6 \to$ dimension-six invariants (spanning list only), all from the single functional $S_\Pi[g, A] = \frac{1}{2}\log\det' \mathcal{A}_{g,\mathcal{A}}$, an organising reading rather than a theorem, whose $a_2$ entry holds under the matching conditions of Note 4. Q12 is shared with the gauge-structure sub-programme (Note 3), whose version 2.2 records that the compact gauge group is supplied rather than derived; GGS-Note draws on Q12's $a_4$ vertical variation, on the fixed-metric isolation of the Yang–Mills sector (Lemma 1; full horizontal–vertical decoupling does *not* hold, the metric variation of $F^2$ being the $a_4$ back-reaction $T^{\mathrm{YM}}_{\mu\nu}$), and on the difference in UV divergence degree between the sectors, within the proper-time scheme ($a_2$ quadratic vs. $a_4$ logarithmic). Q13 supplies the conditional coupled Einstein–Yang–Mills system and the $a_6$ invariant inventory. It asserts no numerical hierarchy and no non-linear completion: $g_{\mathrm{YM}}$, $G_N$ and $\Lambda_{\mathrm{ren}}$ are independent renormalized matching data. The note isolates the upstream inputs consumed (Note 4 for $S_\Pi[g]$ and the conditions of its Einstein sector; Note 3 for the principal bundle of a supplied $G_\Pi$; a supplied Riemannian base metric, whose reading as the emergent base of Note 2 is conditional) and records the open deliverables: the quantitative $a_6$ coefficients, whether admissibility constrains any combination of the matching data, a possible non-linear completion of the joint functional, the Lorentzian continuation of the geometric and gauge sectors, and the coupled equations with fermionic matter currents from Q14 (Note 6). GGS-Note 2.1 is aligned with Q12 and Q13: on supplied geometric and gauge data it records the fixed-metric isolation of the gauge sector rather than a full horizontal–vertical decoupling, presents $a_6$ only as a representative spanning list of dimension-six invariants modulo integrations by parts and Bianchi identities, and asserts neither a non-linear completion nor a numerical coupling hierarchy; its proper-time scheme dependence and Euclidean-signature limitation are explicit. Only the emergent-base reading is conditional, on [H-L], [H-lift], a supplied rank-four extension and a continuation from Riemannian signature that is not performed. O31 version 2.0 withdraws the claimed pointwise result and every derivation of $G_\Pi = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The $a_4$ calculation of Q12 remains a conditional Yang–Mills result for a supplied compact gauge group; it does not identify the Standard Model group.

**Q14** [109]: Version 3.1 states what the fermionic sector rests on (**conditional**, on a supplied finite-Heisenberg carrier and central character, supplied model data, two explicit hypotheses [H-Spin] and [H-Weak], Q13, the Gravity and BI papers, and supplied gauge data; O31 leaves the colour sector conditional). The finite carrier acts on the Weil module through $\mathrm{SL}(2, \mathbb{Z}/q\mathbb{Z})$, which

has no Lie algebra, so a real metaplectic model and a doublet carrying the fundamental action of $\mathfrak{mp}(2,\mathbb{R})_{\mathbb{C}} \simeq \mathfrak{sl}_2(\mathbb{C})$ are supplied model data. Given them, Theorem A(a) is proved algebra: the symmetric square of the doublet is the adjoint, its exterior square the trivial line, and a Hermitian form selects $\mathfrak{su}(2)$. The physical readings rest on two hypotheses supplied by no source. [H-Spin] solders the $\mathrm{SL}(2,\mathbb{C})$ acting on the doublet to the spin group of a four-dimensional Lorentzian co-metric; Q5b, Q8 and Q11 supply such a co-metric only conditionally, and nothing relates the metaplectic Lie-algebra action to its frames. [H-Weak] supplies a distinct $U(2)$ weak factor $E_{\mathrm{weak}}$: by Schur's lemma a weak action commuting with Lorentz transformations cannot act on the soldered doublet, whose symmetric square is a Lorentz sector and whose exterior square carries no hypercharge. Under [H-Spin], the projected Dirac operator $\mathcal{D}_{\Pi,g,A}$ contains a canonical zero-order endomorphism $E_\Pi$, the spinorial lift of the supplied Born–Infeld datum (O30 (H1)–(H2)) is unique up to a phase and reverses chirality, and on an orientation-compatible branch taken as an input $E_\Pi$ is left-admissible; if $E_\Pi \preceq 0$, an explicit hypothesis of Q14 that is not inherited from PRS (whose Schur form concerns the compression remainder), a non-zero left-admissible $E_\Pi$ breaks that parity and the off-diagonal chiral blocks vanish. The chiral selection of the weak interaction ($V-A$) is the representation content of [H-Weak] and is not derived from left-admissibility. Under both hypotheses, the gauge-curvature $\gamma_5$-weighted $a_4$ terms impose anomaly constraints that select the hypercharge twists, given a supplied three-dimensional colour module, the minimal integral normalisation of $L_Y = \wedge^2(E_{\mathrm{weak}})$, the Standard Model content without $\nu_R$ and $Y_e \neq 0$, up to sign, rescaling and the exchange of $u_R$ and $d_R$ (the classification of Geng–Marshak and Minahan–Ramond–Warner is used, not proved); the $E_\Pi$-dependent traces are not analysed (**conditional**). The supplied saturation rule $\sigma_c(n_3) = 3$ (a selection rule, not a derived constant) admits a spinorial multiplicity reading, yielding a gauge-singlet three-generation factor $\mathbb{C}^3_{\mathrm{gen}} \subset \ker(\mathrm{ad}_{\mathrm{SU}(2)} \oplus Y)$ (**conditional**); no source supplies a geometric reading of that rule as three spatial directions. The colour-coupled quark sector is conditional on a supplied colour representation. O31 version 2.0 withdraws the claimed pointwise co-admissibility and does not derive that representation [115]. Q14 §6 is a conditional model of the inter-generation splitting on a distinct supplied generation doublet $V_{\mathrm{gen}}$ with $\mathcal{A}_3 \otimes \mathbb{C} \cong \mathrm{Sym}^2(V_{\mathrm{gen}})$ (after O30's supplied model data; neither O29 nor O30 derives it). A static $J_\Pi$-real, weight-preserving model operator has a degenerate outer pair; the per-step generator $\mathcal{G}_g = \log g$ of the ordered metaplectic step carries the exact $J_3$ component $\alpha = ts\,\theta/\sinh\theta$, $\cosh\theta = 1 + ts/2$, and no mixing component, since no element of $\mathfrak{sl}_2(\mathbb{C})$ connects the outer weights; a complex metaplectic phase is therefore ruled out as a mixing source within this model. The identification of $\mathcal{G}_g$ with an emergent ordering derivative is not supplied: Q11 defines the ordering derivative only on functions of $\tau$ (Q11 Modelling identification 3.1). The edge Q11→Q14 is conditional: Q11 Corollary 6.1 supplies a Lorentzian signature under its stated hypotheses, and Q14 uses only this signature; Q11 supplies neither coefficient values, nor a Lorentzian spin soldering, nor an order generator, so [H-Spin] remains an additional hypothesis and no identification of $\log g$ with $\partial_\tau$ follows. The reading of the §6 operators as restrictions of $E_\Pi^2$ is not supplied; the value $\varepsilon = 1/10$ is a matching value, and the amplitude is deferred to the cascade normalisation.

Q14 is the principal constituent of the *fermionic matter sub-programme*, whose synthesis paper is the presentation note **FM-Note** [101] (Presentation Note 6, working-paper version 3.1). FM-Note presents the layered conditional status of fermions, chirality, hypercharge and generations; each layer names its hypotheses. On a supplied Heisenberg carrier and supplied model data the algebra is proved and uses no metric: $\Pi \Rightarrow F_n \simeq V_\rho \overset{\text{model}}{\rightsquigarrow} \mathrm{Mp}(2,\mathbb{R}) \Rightarrow \mathrm{mp}(2,\mathbb{R})_{\mathbb{C}} \simeq \mathfrak{sl}_2(\mathbb{C})$, the symmetric square of the doublet is the adjoint and its exterior square the determinant line. The step $\mathfrak{sl}_2(\mathbb{C}) \rightsquigarrow \mathrm{Spin}(3,1) \simeq \mathrm{SL}(2,\mathbb{C}) \Rightarrow \mathcal{S}_\Pi$ is not an implication: it is [H-Spin]. [H-Spin] and [H-Weak] are separate hypotheses, supplied by no source and excluded by none. Under [H-Spin], $\mathrm{Sym}^2(S_L)$ is the Lorentz sector and $\wedge^2(S_L)$ a trivial line. Under [H-Weak], the distinct rank-two weak factor $E_{\mathrm{weak}}$ with structure group U(2) carries $\mathfrak{su}(2)_L \subset \mathrm{Sym}^2(E_{\mathrm{weak}})$ and the hypercharge line $L_Y := \wedge^2(E_{\mathrm{weak}})$, and the $V-A$ structure is its content. Lorentz chirality (the left-admissibility of $E_\Pi$, an input orientation branch) and weak chiral selection are different

statements, the second not derived from the first. The anomaly constraints on the hypercharge weights need both hypotheses; their rigidity carries a supplied three-dimensional colour module, and selecting the Standard Model pattern carries in addition the minimal integral normalisation of $L_Y$. The supplied saturation rule $\sigma_c(n_3) = 3$ gives conditionally a gauge-singlet three-generation factor $\mathbb{C}^3_{\mathrm{gen}}$ on a supplied rank-three carrier; the generation sector is a set of model operators on a distinct supplied doublet, their reading as restrictions of $E^2_\Pi$ is the named hypothesis [H-Res], not supplied, and the level-to-generation map is not established. Reading the residual reflection $R_b = W(-I)$ as commuting with $\gamma_5$ is a hypothesis on the chiral carrier, [H-WS] of Q11OF (part (i): $R_b$ acts as a scalar on $S_L \oplus S_R$; the value $-1$, part (ii), is used only by AOG-Note), which no source supplies; a group-level map from the finite Weil group to the $\mathrm{SL}(2,\mathbb{C})$ of [H-Spin] sending $W(-I)$ to $-I$ is not available at the primes $q = 61, 101, 151$ of the corpus. The independent fermionic weak doublet is not produced at this stratum: the finite-stratum obstructions of NSR [88] are not conditional on [H-Spin], and FM-Note reads $E_{\mathrm{weak}}$ as the role such a multiplicity would play, an explicit architectural extension. The upstream layers consumed are the Weil fibre (Notes 1, 5), the conditional Lorentzian geometry (Note 2), of which the geometric branch supplies only a rank-three degenerate part and which enters only through [H-Spin], the gauge bundle (Note 3), and Q13 [84] for the gauge–gravity synthesis. AOG-Note (version 1.8) takes the Lorentzian chiral lift as input and names it as the hypothesis [H-Spin] of Q14, which it does not derive. CHO and Q11OF, described below, take their dynamical-lift results from section 6 of Q14 and cite the $\mathfrak{sl}_2$ commutator $[E, F] = H$; neither constructs the Lorentzian chiral lift, which AOG names as the hypothesis [H-Spin] of Q14, and their use of $[\gamma_5, R_b] = 0$ is carried by the explicit hypothesis [H-WS] of Q11OF. PRS and the A4-Note (version 3.0 each) state their hypotheses explicitly ([H-Spin] and [H-Gen], [H-Mult], [H-Tr] in PRS; [H-Lor], [H-F], [H-Nd] and the PRS hypotheses in the A4-Note). The open deliverables include the explicit construction of $E_\Pi$ as a spectral operator and the full Lorentzian-signature matter content $S^{\mathrm{matter}}_\Pi = (\mathcal{S}_\Pi \oplus (\mathcal{S}_\Pi \otimes V_{\mathrm{color}})) \otimes \mathbb{C}^3_{\mathrm{gen}}$ with explicit Yukawa structure; the mass spectrum, Yukawa couplings, and CKM/PMNS mixing matrices remain open. The qualitative mechanism of the generation splitting is modelled in Q14 §6, conditionally on the supplied $\mathrm{Sym}^2(V_{\mathrm{gen}})$ identification; its value is dictionary-bound. PRS [103] separates what it proves from what it leaves open. Under a contraction hypothesis the compression remainder of the projected Dirac square is the Schur complement $-N^\dagger N \preceq 0$ (zero-order only under a symbol condition); it is the projective endomorphism $E_\Pi$ of Q14 only when $\Pi_S D^2 \Pi_S^* = \mathrm{Lich}$, an identity that no source states. Under the generation-reading hypothesis [H-Gen] a $J_\Pi$-commuting locking operator gives $u = 0$, so a non-zero split needs a locking operator that breaks $J_\Pi$-equivariance, and no mechanism for it is derived. The Schur-null and Schur-transverse loci of the locking defect form a dichotomy proved under explicit hypotheses, and which locus holds is open: the exact $\mathfrak{sl}_2$ opening $\alpha(x, y) = xy\, r/\sinh r$, produced by $[E, F] = H$, is transported to the locking operator only under the hypothesis [H-Tr], supplied by no source. Neither the sign of the Born–Infeld genus nor $u \neq 0$ follows [98]. The A4 companion extends the radicand to quartic order; the quartic coefficient depends on the parametrisation and selects no lock between a boundary contact and an interior stationary point, the first contact is chart-independent when it exists (for a collinear purely electric ray it is reached at $t_* = \beta/|\mathbf{E}_{F_0}|$), and the Lorentzian genus of a supplied two-form is not determined. The eliminated-block companion [100] constrains the jet of $1 - P(s)$ and determines one block of the $J_\Pi$-odd tangent; its finite witness has $J^2 = +1$ and lies outside [H-Gen](i), and the contact factorisation of the A4-Note is formal: no frame satisfying its hypotheses is exhibited, and it fixes no magnitude. The value $|u|$ is dictionary-bound through $\mathcal{N}_A$, so the first-principles front of the sub-programme is upstream and structural (the ADE case selection, the cascade exponent $\beta$, and the transfer constant $N_{\mathrm{casc}}$), and beyond it lies the Yukawa sector, treated conditionally in PYL and PYO.

**PRS** [103]: The projective residue as a Schur complement (**conditional**, algebra **proved** under its explicit hypotheses, physical selection **open**). Companion note of the fermionic matter sub-programme refining the Q14 §6 open deliverable on the inter-generation splitting. For the

reference operator $P = \Pi_S^* \Pi_S$, under a contraction hypothesis, the compression remainder of the projected Dirac square is $-N^\dagger N \preceq 0$ with $N = (1-P)^{1/2} D \, \Pi_S^*$, the Schur complement of the spinorial directions eliminated by the non-injective projection; it is zero-order under a symbol condition that forces it to vanish on irreducible Clifford fibres, and it is the projective endomorphism $E_\Pi$ of Q14 only when $\Pi_S D^2 \Pi_S^* = \mathrm{Lich}$ (an identity that forces $\Pi_S \Pi_S^* = \mathrm{id}$ at the symbol level and that no source states). For a general locking operator $0 \preceq P \preceq 1$ the Schur expression and its sign are unconditional, and the consequences for a non-zero split are conditional. Under the generation-reading hypothesis [H-Gen] (a reading map $\Phi$ applied to zero-order endomorphisms, with $J_\Pi$ compatible with $D$ and $\Pi_S$) the split $u$ vanishes for every $J_\Pi$-commuting locking operator, in particular for the reference operator; a non-zero $u$ requires $J_\Pi P J_\Pi^{-1} \neq P$, which on an interpretive reading of A1–A4 can come only from the projection-locking axiom A4. The multiplicative reading [H-Mult] is a separate hypothesis which, with a non-singular even sector, gives $u \equiv 0$ or admits no $\Phi$ in the chiral rank-two left-admissible frame. The antiunitary chiral equivariance defect $\Delta_\chi(P)$ is related to $u$ only as a structural reading, and the finite sector is conditional on an open identification of the parity $c \leftrightarrow q - c$ with $J_\Pi$. The Schur-null and Schur-transverse loci of the transported tilt of the locking operator form a dichotomy proved under explicit hypotheses. The oriented metaplectic step has the exact $\mathfrak{sl}_2$ opening $\alpha(x,y) = xy\, r/\sinh r$, $\cosh r = 1 + xy/2$, produced by $[E,F] = H$ and not by the Heisenberg bracket; that the transported tilt equals it up to a non-zero factor is the hypothesis [H-Tr], supplied by no source, and which locus holds is open. Neither the sign of the Born–Infeld genus nor the non-vanishing of $u$ follows, and no mechanism for a non-zero split is derived. Open: [H-Tr], the explicit Lorentzian block $1 - P(s)$, the amplitude $|u|$, and the projected Yukawa sector.

**AAR** [77]: Angular amplitude reduction: the oriented shear product of the $\mathfrak{sl}_2$ step and the central-phase observable of the Weil cascade (**structural**, working note; **conditional** on the input that $A_c$ carries the $J_3$ signal). Fixes the precise observable whose measurement tests the absolute normalisation of the Q14 §6 angular generation split and reports the Weil–BFS computations of it. A short Baker–Campbell–Hausdorff reduction in $\mathfrak{sl}_2$ identifies the per-step $J_\Pi$-odd $J_3$ component of the metaplectic cascade generator with the oriented product $ts$ of the two shear parameters, produced by the $\mathfrak{sl}_2$ commutator $[E,F] = H$ (Q14, Proposition 6.5), not a free shear parameter; in the SL(2)-on-$V$ model it uses only the $\mathfrak{sl}_2$ relations and no Heisenberg bracket, and no map between the two brackets is supplied. The observable proposed for the Weil–BFS cascade is the accumulated oriented central-phase increment $A_c$ of the Heisenberg lift, normalised by the projected capacity $\widehat{I}(n)$; that $A_c$ carries the $J_3$ signal is an input of the note, a formal analogy and not an identification. Under this input, at $n_3^{\mathrm{obs}}$ the normalised increment equals $\varepsilon_{\mathrm{Weil}}(n_3^{\mathrm{obs}})$, to be compared with the dictionary value $\varepsilon = 1/10$ without fitting. In the O12 fingerprint the two sectors are the two terms of the exponent: frequency $B_c$ carries the radial capacity $\sigma_c$, central phase $A_c$ carries the angular split under that input. A direct evaluation on symmetric BFS shells returns $\Theta_{\mathrm{raw}} = 0$ identically; the orientation prescription is an irreducible input prior to $\mathcal{N}_A$. A subsequent capacity audit on the exact Heisenberg–BFS frontier fixes the onset central-phase coefficient: the directed outgoing-frontier mean is an exact rational sequence ($\frac{1}{3}, \frac{5}{9}, \frac{2}{3}, \ldots$ per shell, identical across $q \in \{61, 101, 151, 211, 307\}$ over the first six shells, integer arithmetic, no fit) whose first-shell (onset) value is $\frac{1}{3}$, hence the maximal-locking bound at the onset shell $\theta_{\max}^{(1)} = 2\pi/(3q)$ (exact for the standard $\{\pm X, \pm Y\}$ campaign, generator-dependent; only $\varepsilon$ and $R = \frac{3}{2}$ are generator-independent). The $\widehat{I}$-normalised bound at $n_3^{\mathrm{obs}} = 2$ is $2.114/q$, $2.122/q$, $2.118/q$ for $q = 61, 101, 151$, 0.9–1.3 % above $2\pi/(3q)$: $\frac{1}{3}$ is an onset coefficient, not the value of the normalised bound. This fixes the onset coefficient, not the dictionary value: $\theta_{\max}^{(1)} \to 0$ with $q$, so the comparison with $\varepsilon = 1/10$ is a required-normalisation diagnostic, the geometric $\mathrm{Sym}^2$ normalisation $\mathcal{N}_A^{\mathrm{geom}} = 3q/(20\pi)$ being the algebra at the onset shell, gated by the same ADE datum [96]; and, within the input that $A_c$ carries the $J_3$ signal, no value of $\mathcal{N}_A$ turns $\Theta_{\mathrm{raw}} = 0$ into a split. The per-shell rationals are recomputed from the AAR repository alone.

**Q11OF** [102]: The oriented frontier observable (**structural**, diagnostic; chiral lift **conditional** on [H-WS]). Frontier-transfer diagnostic note on the recursive existence of the $J_3$-odd cascade signal underlying the Q14 §6 splitting mechanism. The symmetric-capacity obstruction makes the raw shell observable vanish identically ($\Theta_{\mathrm{raw}} = 0$): the involution $g \mapsto g^{-1}$ pairs opposite central phases on symmetric BFS shells, erasing the oriented $J_\Pi$-odd component before the radial recursion can act. The honest replacement is a frontier transfer observable on the directed outgoing edges $g \to gs$, whose central increment is the Heisenberg cocycle $a(g)\, s_b$, the discrete oriented increment of the cascade: a counterpart of the orientation datum, and not the $J_3$ coefficient of the metaplectic step, which in the SL(2) model of Q14 comes from the $\mathfrak{sl}_2$ commutator $[E, F] = H$. The orientation is the arrow of the Cayley graph from the projection origin to increasing BFS depth; no identification of this arrow with an ordering derivative $\partial_\tau$ is made (Q14 states that such an identification, for the generator $\log g$ of the metaplectic step, is a separate hypothesis that no source supplies). The first test is the existence of an oriented signal $\langle \Delta A_c \rangle_{\partial^+ S_m} \neq 0$; the discriminating anti-bias control is the symmetrised-frontier cancellation. No normalisation, no dictionary value, no $\varepsilon$ enters: the note tests only whether the recursion produces a non-zero, bias-free oriented signal. Its resolution section proves that the residual frequency reflection is the central Weil lift $R_b = W(-I) = \mathrm{FT}^2$ up to the metaplectic phase; its commutation $[\gamma_5, R_b] = 0$ is reduced to the named hypothesis [H-WS], supplied by no source ($R_b$ acts on the chiral carrier as the scalar $-1$: commutation is part (i), the value $-1$ part (ii), used only by AOG-Note), a group-level Weil-to-spin map not being available at the primes of the corpus.

**CHO** [78]: Chiral orientation of the directed Heisenberg frontier (**conditional** on the inherited central-phase lift and on [H-WS]). Companion note resolving the chiral weight $\sigma_L$ left deferred by Q11OF. Under [H-WS], a parity argument closes the route $\sigma_L = f(T(e))$: edge inversion $U(e^{-1}) = U(e)^{-1}$ and the residual reflection both send the $J_\Pi$-odd cascade generator $T(e) \mapsto -T(e)$, so every function of $T$ has equal arrow- and $\varphi$-parity, whereas the admissible weight must be arrow-even and $\varphi$-odd (**proved** under [H-WS]). The resolution, under [H-WS], is that $\sigma_L$ is the Weyl representation type $\mathbf{2}$ versus $\bar{\mathbf{2}}$: group inversion preserves the representation and complex conjugation exchanges it (**structural**). In the inherited Heisenberg central-phase lift the edge phase is the linear cyclotomic character $\zeta_q^{\Delta A_c}$, with no Gauss, Maslov, or $\sqrt{5}$ component, and the natural branch gives $\sigma_L = \operatorname{sign} \Delta A_c$ on the cocycle support, hence, by this definition, $\rho_\chi = 1$ and $\mathcal{N}_A = \theta_{\max} \neq 0$, $q$-stable to 0.4 % ($\theta_{\max}\, q = 2.114, 2.122, 2.118$ at $n_3^{\mathrm{obs}} = 2$ for $q \in \{61, 101, 151\}$; the per-shell rationals are $q$-independent over the first six shells, $q \in \{61, 101, 151, 211, 307\}$) (**proved** within that convention and under [H-WS]). Thus [H-orient] is established in the inherited central-phase lift and under [H-WS], not unconditionally; the only remaining obstruction besides [H-WS], a possible additional spin-Galois phase orthogonal to $\zeta_q$ (a $\sqrt{5}$ or ADE-spin factor), is examined in AOG-Note [96], which, taking the Lorentzian chiral lift as an input ([H-Spin] of Q14), shows that under [H-Spin], [H-WS], its modelling premise [M], its explicit boost premise [B] (stated in the AOG-Note paragraph below) and the inherited central-phase convention the lift induces no action on $\mathbb{Q}(\sqrt{5})$. It fixes the frontier amplitude datum $\mathcal{N}_A \neq 0$ conditionally but does not derive $u \neq 0$, which would further need a transport of the oriented signal to the $J_\Pi$-odd generation sector, the only transport stated being the hypothesis [H-Tr] of [103], supplied by no source.

**A4-Note** [98]: The Born–Infeld saturation margin of the chiral modulus (**conditional**, algebra **proved** under its explicit hypotheses, genus and lock **open**). Companion note to PRS examining the Lorentzian saturation functional $\mathcal{B}_{\mathrm{sat}}(s)$ controlling the generation split $u$; it fixes no sign of the split. An antisymmetry lemma shows that the symmetric square of the $J_\Pi$-odd modulus is $J_\Pi$-even and cannot carry the orientation sign, so the carrier is taken, by hypothesis, to be an odd antisymmetric chiral two-form $F_{\chi,\mu\nu}(s)$; $\mathcal{B}_{\mathrm{sat}}(s)$ is defined as the Born–Infeld determinantal saturation margin of such a supplied two-form, and its second variation $\mu_\chi^2 = \frac{1}{2} P_{\chi,\mu\nu} P_\chi^{\mu\nu}$ is the Lorentzian genus of the chiral polarisation $P_\chi$. Hypotheses that no source supplies are stated where used: a Lorentzian co-metric [H-Lor], the construction of the two-form, a local-minimum reading of A4, the existence and non-degeneracy of a locked pair [H-Nd], the covariance of the

locking family under $J_\Pi$, and, from PRS, [H-Gen] and [H-Tr]; the Born–Infeld form is a conditional input, neither derived nor unique. The scope is that of PRS: no mechanism for a non-zero split is derived. The genus is not determined: the central commutator supplies no electric assignment, since the central direction is not temporal in the co-metrics of the corpus (Q5b's rank-three co-metric has a zero central entry, Q8's conditional completion is spacelike, Q11's temporal coefficient is free) and no map from the generation operator to a two-form is available. The quartic coefficient of the radicand is chart-dependent and selects no lock between a boundary contact and an interior stationary point; the first contact $\Delta_\chi = 0$ is chart-independent when it exists, and for a collinear purely electric ray under [H-F] it is reached at $t_* = \beta/|\mathbf{E}_{F_0}|$. The Front D factorisation of the contact split, $u_\partial = \sqrt{2}\,\eta\, s_*\, T_{J_3}(\partial_s \Delta_\chi(P)|_0)/\langle J_3, J_3\rangle$ to first order in $s_*$, is formal: it needs [H-Gen], [H-Mult], [H-Tr], [H-F], [H-Nd], the $J_\Pi$-covariance and diagonal-form premises of its Lemma on the odd split, and a localisation hypothesis that fails in the available witnesses of [100], which lie outside [H-Gen](i); no frame satisfying these hypotheses together is exhibited, and it fixes no magnitude. The magnitude $|u|$ is not predicted. In the interactive graph the edges PRS→A4-Note ([H-Gen], [H-Tr]) and A4-Note→EBJ (formal factorisation) are typed conditional.

**EBJ** [100]: The constrained jet of the Lorentzian eliminated block and the chiral defect-rate identity (**proved** / **open**). For every smooth family and every smooth unitary transport the projector jet $1-P(s) = Q_0+sQ_1+s^2Q_2+O(s^3)$ has $Q_1 = [A, Q_0]$ a purely off-diagonal Grassmann tangent and $Q_2$ diagonal blocks pinned to $Q_1^2$ with opposite signs; the parity grading holds under $J_\Pi$-equivariance, and with a covariant moving spinorial embedding and a $J_\Pi$-even Dirac operator the first-order $J_\Pi$-odd part of $E_\Pi^2$ is $\{E_0, E_1\}$. Antiunitary parity makes every odd-order generation-mixing coefficient vanish in every covariant family, with or without complex phases, even orders not being excluded; an explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with $u'(0) = 11\cdot 2^{1/4}/9 \neq 0$; this finite witness has $J^2 = +1$, lies outside [H-Gen](i) of PRS, meets the identity-reading clauses of the separate hypothesis [H-Mult] and conditions (ii)–(iii) of [H-Gen] (reading $J_\Pi$ as $J$ on its finite space), and illustrates a moving-embedding family without being an instance of [H-Gen]. In the chiral frame the left-handed block of the odd tangent is half the transported defect rate, one block of the tangent rather than a unique generator; the selection of the family, the localisation of the odd tangent, even-order mixing and the normalisation of $\partial_s \Delta_\chi(P)|_0$ remain open, $|u|$ dictionary-bound; neither the Schur transversality of the locking (a dichotomy under explicit hypotheses in PRS, the branch open) nor the Lorentzian genus is used.

**PYL** [111]: Toward projected Yukawa couplings: a conditional weak determinant line and generation-level ordering (**conditional**, algebra **proved** on supplied data). Companion note opening the mass-sector frontier; it derives no mass, Yukawa coupling or mixing, and records two statements, each with the hypothesis it needs. First, algebra proved on supplied model data: on a rank-two bundle with structure group in $\mathrm{SL}(2,\mathbb{C})$, $\wedge^2$ is the unique one-dimensional functor among the symmetric and exterior powers, and it is trivial; hence $\wedge^2(S_L)$ carries no abelian weight under [H-Spin]. Under [H-Weak], which is not supplied and not constructed, the line $L_Y := \wedge^2(E_{\mathrm{weak}})$ with structure group U(2) carries the abelian weight: it is a conditional weak determinant line, not a line of the spinor bundle. Second, on the generation triplet the model operator $\mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ has exit deficits $\{0, \frac{1}{2}-u, \frac{1}{2}+u\}$ that order three levels for $0 < u < \frac{1}{2}$, with a scale-independent ratio; reading it as $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$ is the named hypothesis [H-Res], not supplied (PRS reads the normal form through a reading map $\Phi$, itself a hypothesis there, of which [H-Res] is the restriction instance), and the level-to-generation map is not established. The Yukawa operator, which needs a weak linking carrier supplied by no source, its norm, the sign of $u$, and the mixing remain open; no mass value is claimed.

**PYO** [89]: Projected Yukawa operator and chiral polar factor (**conditional**, physical polar class **open**). Companion note taking the second step of the mass-sector frontier, a sharp partial no-go, on the generation block $Y_\Pi^{\mathrm{gen}}$ of the projected Yukawa morphism. Four premises, supplied by no source: [H-Res] (the model operator is $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$); [H-Sq] ($H_\Pi = Y_\Pi^\dagger Y_\Pi = \lambda_Y^2 E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$); [H-

Fac] (F1 a weak linking carrier with a non-zero space of invariant couplings, F2 that space of dimension at most one; independent); [H-Grad] (the $J_3$ grading of each generation carrier). Under [H-Res] and [H-Sq], $\mathrm{Spec}(H_\Pi)|_{\mathbb{C}^3_{\mathrm{gen}}} = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so $E_\Pi^2$ fixes the squared Yukawa levels, not the morphism. The polar decomposition $Y_\Pi = U_\Pi H_\Pi^{1/2}$ is linear algebra: the unitary chiral factor $U_\Pi$ and the norm $\lambda_Y$ are invisible to $E_\Pi^2$. Under [H-Grad], $[U_\Pi] \in U(1)^3_R\backslash U(3)/U(1)^3_L$ is a rephasing class, with invariants the moduli $|(U_\Pi)_{ij}|$ and a Jarlskog phase, non-trivial at first order iff its polar generator $\Omega$ has a transverse part. The projection $A_\Pi^{\mathrm{odd}}$ of the $\mathfrak{sl}_2$ lift onto its $J_\Pi$-odd anti-Hermitian part vanishes, by representation theory, for the projection as defined, and is not the polar generator. $\Omega$ solves $\Omega P + P\Omega = U^\dagger\dot{Y} - \dot{Y}^\dagger U$ (real parameter, fixed orthonormal frames, $P > 0$) and, at the identity, is the anti-Hermitian part of the lift, which is $J_\Pi$-even and generally non-zero. The polar factor of $\mathrm{Sym}^2 g$ for the real cascade step $g = e^{tE}e^{sF}$, which is not a Yukawa morphism, has the external entry $(t-s)^2/((2+ts)^2+(t-s)^2)$, equal to $1/17$ at $(t,s) = (2,1)$. The corpus supplies no family of Yukawa morphisms, so the physical polar class is open: the choice of the $J_\Pi$-odd part as polar generator is a modelling choice that no source justifies. On the orientation-compatible branch of Q14, an input, a non-zero diagonal split needs, in the framework of PRS (under [H-Gen]), a locking operator that breaks $J_\Pi$-equivariance, and no mechanism for it is derived (a conditional lead, not an established mechanism); the branch is not the $V-A$ structure. Under [H-Res] and [H-Sq], at the dictionary gate $u = 1/10$ the squared levels $\lambda_Y^2\{1, \frac{3}{5}, \frac{2}{5}\}$ are of order one, not the charged-fermion hierarchy, and the generation stabilisation depths are not derived.

**KUD** [99]: Charged-lepton Koide relation as a unit-dispersion constraint (**proved**). Source note fixing the phenomenological target of the mass sector, downstream of PYL/PYO, as a single scale-free invariant. The Koide quotient is exactly $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$ in the square-root mass vector $r_\ell$, so $Q_\ell = 2/3 \iff \mathrm{CV}(r_\ell) = 1$ (standard deviation equal to mean). This separates the dispersion constraint (Koide) from the concrete hierarchy (the phase placing one generation near a node of the square-root profile), and shows the cascade ladder $(1, \frac{1}{2}+u, \frac{1}{2}-u)$, taken by hypothesis as a pinned normal form of the projected generation levels, under-disperses, $\mathrm{CV} \le 1/\sqrt{2}$ ($Q \le 1/2$), so it cannot itself reach the Koide value. A maximum-entropy reading ($\mathrm{CV} = 1$ is the unit relative dispersion of the exponential) is recorded as an open test: it requires the projection to fix the first moment $\langle\sqrt{m}\rangle$ alone and a forced discrete-to-continuous bridge. Writing $r_\ell = r_S + r_D$, $s = \|r_S\|$, and $d = \|r_D\|$, KUD proves that no real-linear self-adjoint amplitude map in a positive metric can square to $(s^2 - d^2)P$ on a non-zero projected block; this closes only the linear self-adjoint determinant realisation of the singlet–doublet route, while the route itself and its nonlinear, Krein, and enlarged-tower extensions remain open. It is a target invariant, not a mass formula.

**TPC** [97]: Ternary phase carrier — colour, generations, and Koide as one carrier (**open**, speculative bridge). A single complex three-phase carrier $z_g = S + A\,e^{i(\delta+2\pi g/3)}$ read two ways: its pure-phase part sums to zero ($1+\omega+\omega^2 = 0$), a colour-neutral class; its real part is the square-root mass profile of KUD. The singlet amplitude $S$ is the master dial — $S = 0$ colour (confined), $S \neq 0$ masses, $A = \sqrt{2}\,S$ equipartition (Koide, $\mathrm{CV} = 1$), $\delta$ near a node the hierarchy — with a candidate capacity-balance filter $\|r_{\mathrm{dbl}}\| \le \|r_{\mathrm{sing}}\|$ saturated at $\mathrm{CV} = 1$, whose reading as the saturated admissible edge selected by projection locking is a further hypothesis that the cited sources (A4-Note, ENI) do not supply. Explicitly not a derivation: it reorganises the open questions and states the construction charge sheet (exact colour singlet, three generations after neutralisation, electroweak quantum numbers, anomaly cancellation, quark/lepton selection); the colour-to-generation breaking it needs along the neutralisation route is the one already closed by the exact colour degeneracy.

**NPI** [86]: Neutrino mixing and projective particle identity (**structural**). Structural companion note on the ontology of fermionic identity: the misalignment between the weak flavour basis and the mass propagation basis is the generic situation (neutral meson systems exhibit the same structure; the neutrino is the elementary case), while alignment requires an identity-stabilising

projection channel. The large PMNS mixing is then the unfixed default, consistent with the anarchy hypothesis, and the near-diagonality of CKM becomes the explanandum, conditional on the charged channels pinning the up-type and down-type mass bases to a common projective frame. The Dirac–Majorana alternative is reread as projective conjugation resolution. No mixing angles or masses are derived; NPI's reference to a trivial chiral polar class $[U_\Pi] = [I]$ cites PYO, in which the physical polar class is open, so that reference is an open consumer correction.

**NSR-Note** [88]: Operator doublets, erased history fibres, and the no-sequential-re-entry theorem (**proved** for the finite obstructions; corpus audit for the O33/EBJ levers). No-go note closing the two deposited candidates for the independent fermionic weak multiplicity $M_L \simeq \mathbb{C}^2$. The commutant of the finite Weil module is spanned by the two parity projectors, so the canonical Hecke $M_2(\mathbb{C})$ doublets of $\mathrm{End}(V_c)$ (O33) admit no fermionic lift to their own carrier; the erased history fibres of the Trajectory Branching canonical channel are structurally abelian ($C_4$ frame stabiliser, canonical $(\mathbb{Z}_2)^r$ torsor per $b$-shadow, near-regular induced multiplicities); the flip-antisymmetric fibre line transports exactly into the parity-odd operator sector, but the two odd Hermitian axes $i(A_X \pm A_Y)$ are exchanged by a semilinear automorphism of the cocycle-enriched Heisenberg–Weil datum $(RK)$, with only a coordinate-pinned conditional selector; and the central theorem ($\pi v = 0 \Rightarrow Q\pi v = 0$ for the canonical record quotient, extended to the conditional expectations of arbitrary non-degenerate measures) excludes every sequential re-entry. Corollary: realisation through the history-fibre route requires a parallel substrate channel with an oriented central/Weyl datum — an extension architecture, not a derivation; an independent new multiplicity carrier remains a distinct, unconstrained extension. Three archived scripts certify all computational claims.

**AOG-Note** [96]: Spin-stratum type-rigidity — an obstruction to the Galois action on the binary icosahedral stratum, with conditional consequences for the chiral lift (**conditional** as a whole, the character-table audit of the inner conjugations, $-I$ and $J_\Pi$ being **proved**; **conditional** for the other clauses: [H-WS] for $R_b$, the A4 hypotheses for the locking, modelling assertions for the tensor functors and the Schur projection, and the modelling premise [M] that the generator list is complete (it implies carrier-preservation of every admissible operation), and the base-fibre Born–Infeld parity, a candidate hypothesis of the foundations taken as a premise; the Lorentzian chiral lift is an input, the spin-solder hypothesis [H-Spin] of Q14, which AOG-Note does not construct, and its consequence rests on the explicit boost premise [B]; no link between the ADE gate and the Galois action is established). Transversal note isolating the logical status of the generation-split value $\varepsilon = 1/10$. It is shown to be a theorem modulo the ADE case-selection gate: the algebra $\frac{1/2+\varepsilon}{1/2-\varepsilon} = \frac{3}{2} \Rightarrow \varepsilon = 1/10$ is trivial and case-independent (**proved**), so the first-principles status of $\varepsilon$ is exactly that of the gate, which the note does not settle. In the compositum $\mathbb{Q}(\zeta_q, \sqrt{5})$ the Born–Infeld parity $c \mapsto q - c$ acts as complex conjugation ($\zeta_q \mapsto \zeta_q^{-1}$, $\sqrt{5} \mapsto \sqrt{5}$) and fixes $\sqrt{5}$, lying in a different direct factor of the Galois group from the Galois action $\sigma = (\zeta_q \mapsto \zeta_q,\ \sqrt{5} \mapsto -\sqrt{5})$ (**proved**); its status as the base-fibre involution is that of a candidate hypothesis of the foundations, taken by AOG-Note as a premise (not proved there, and not proved by ENI or O18). The proved result is an obstruction concerning $\sigma$: a character-table audit of $2I$ shows that inner conjugations, $-I$ and $J_\Pi$ act trivially on $\mathrm{Irr}(2I)$, whereas $\sigma$ induces $(2\,2')(3\,3')$ (**proved**). The central reflection $R_b$ acts as $-I$ under [H-WS] of Q11OF (the scalar $-1$ on the chiral carrier), the A4 locking contributes a parity-odd sign under the A4 hypotheses, and the tensor-functor and Schur-projection clauses are modelling assertions (not audited); relative to these clauses and to [M], no listed operation realises $\sigma$, and a finite recursive tower of admissible operations (carrier-preserving by [M]) preserves the arithmetic type; since $R_b$ belongs to the list that [M] makes complete, the tower statement inherits [H-WS]. No link between the gate and $\sigma$ is established: the outer automorphism fixes the order-four class and maps each of the order-five and order-ten loci to itself, so the gate stays open and no obstruction or no-go for the gate follows. The Lorentzian chiral lift being taken as input ([H-Spin] of Q14), AOG-Note shows, under [H-Spin], [H-WS], [M], the explicit boost premise [B] (the lift supplies only the central character and the complexification unit, by complex conjugation; asserted, not derived) and the inherited

central-phase convention of CHO, that the lift induces no action on $\mathbb{Q}(\sqrt{5})$, so that the residual spin-Galois caveat of CHO does not arise within the present spin stratum under [B] [78] ($\rho_\chi = 1$ holds by definition of $\sigma_L$; $\mathcal{N}_A \neq 0$ and the chiral-lift consequence are **conditional** on all of these inputs, in particular on [B]). A status sharpening records the concrete generator-set form of the gate. Once the order-five $\sqrt{5}$-locus is selected, $R = 3/2$ is not a fit but a forced Cayley-spectral invariant of $2I$, the outer deficits being $\{|\lambda_1 - 1|, |\lambda_3 - 1|\} = \{\frac{1}{6}, \frac{1}{4}\}$ with central level $\lambda_2 = 1$; the neutral order-four half-turn class — the programme's default admissibility convention — instead gives $\{\frac{1}{5}, \frac{1}{3}\}$, hence $R = \frac{5}{3}$ and $\varepsilon = \frac{1}{8}$, not $\frac{1}{10}$. The residual lock is therefore the order-five-versus-order-four selection (**proved** for the forced ratio, **open** for the selection); the Galois action $\sigma$ is not shown to be the missing ingredient.

**SRN** [95]: Spin as a transformation class of rotating wave modes (**proved** / **open**). Companion audit of the rotating-wave reading of spin: spin is fixed by how components transform under rotations of physical space, not by rotation alone; the central element acts as $(-1)^n$ on $V_\rho^{\otimes n}$; a Veronese-null versus compact-real-form lemma excludes the O28 outer-product real-form selector; the derived data fix the complex carrier and its univalence but neither the spatial realisation nor the rotation action; and, independently, $\overline{\psi_R}\psi_L$ carries exact Higgs-typed electroweak quantum numbers in every tree-level Yukawa sector, permitting but not deriving a composite Higgs condensate, with an ambient enlargement beyond a single copy of $V_\rho$, a canonical inter-sector identification and the condensation dynamics left open.

## 5.2 Gravity and Spacetime Track

**Spectral2** [48]: Derives operational metric distances from graph Laplacians on a relational substrate without any background manifold (**structural**).

**Gravity** [17] (*Conditions for an Infrared Einstein Sector from Spectral Geometry*): Determines, for a supplied Riemannian metric, the conditions under which the metric variation of $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ supports a local infrared Einstein sector (**conditional** on the supplied renormalised coefficients). Its reading as the emergent metric is conditional on [H-L], [H-lift], a supplied rank-four extension and a continuation from Riemannian signature that is not performed, and no metric coefficient value is established. It separates two regularisation statements that must not be conflated: a physical proper-time cutoff $\Lambda = \ell_{\mathrm{sp}}^{-1}$ carries the power-sensitive terms $a_0\Lambda^4$ and $a_2\Lambda^2$, while zeta regularisation carries the finite determinant and the logarithmic scale dependence governed in four dimensions by $a_4 = \zeta_A(0)$. One minimal scalar contributes $\Delta c_{\mathrm{EH}}^{\Lambda} = -\Lambda^2/[12(4\pi)^2]$, which is *negative*: the finite renormalised coefficient $c_{\mathrm{EH}}^{\mathrm{ren}} = (16\pi G_N)^{-1}$, its sign, and hence a positive Newton constant are *matching data* fixed by a renormalisation condition and the complete operator content — they are not predicted by the minimal determinant (**open**). Once the coefficients are supplied, the variation decomposes into Einstein, cosmological, higher-derivative and non-local sectors, the Einstein term dominating locally when the matched Einstein coefficient is non-zero and $RL_4^2 \ll 1$ with $L_4^2 = |\beta^{\mathrm{ren}}/c_{\mathrm{EH}}^{\mathrm{ren}}|$ (**structural**, conditional on the supplied coefficients: a power-counting statement that gives no general bound on the non-local terms). Under the separate coherence-extensivity hypothesis, the determinantal Born–Infeld density $\sqrt{-\det(g_{\mu\nu} + \ell_{\mathrm{sp}}^2 R_{\mu\nu})} - \sqrt{-g}$ is an admissible tensorial completion of a supplied Einstein infrared term, but not a uniquely selected one: the admissible completers form an explicitly parametrised family in a sector function, and the input a uniqueness theorem would require is stated (**structural**). Among linear two-derivative carriers $Y_\alpha = \frac{t}{d}I + X_0/(1 - \alpha d)$, the infrared limit and the heat-kernel coefficient leave the trace parameter $\alpha$ free, and second-order data determine at most $|1 - \alpha d|$; among the completers $\exp\sum_i h(\mu_i)$ built on these carriers, with $\mu_i$ their eigenvalues and $h$ a smooth sector function with $h(0) = 0$, $h'(0) = \frac{1}{2}$ and $h \to -\infty$ at $-1$, the saturation condition, applied as stated to the Ricci endomorphism, selects $\alpha = 0$ (**proved**, elementary; it does not use the coherence-extensivity hypothesis).

**Thermodynamics** [19] (*Einstein–Matter Coupling from Spectral Stationarity: A Conditional Variational Result and the Open Thermodynamic Bridge*): Establishes a conditional variational

core, and no thermodynamics (**conditional** on the supplied renormalised coefficients). Assuming a renormalised infrared metric response (supplied by Gravity, where the finite Einstein coefficient is a matching datum) and an effective matter functional $W_\Pi[g,\psi]$, stationarity of $\Gamma_\Pi = S_\Pi^{\rm ren} - W_\Pi$ yields $G_{\mu\nu} + \Lambda_{\rm ren} g_{\mu\nu} = 8\pi G_N T^{(\Pi)}_{\mu\nu}$ at leading derivative order (**proved** given the supplied coefficients); the factor $8\pi G_N$ is the ratio between the matter variation factor $\frac{1}{2}$ and the geometric coefficient $(16\pi G_N)^{-1}$, not an independent prediction. A dimensional audit of the diagonal heat-kernel quantity $u(x;t) = -\partial_t \log K(x,x;t)$ shows it carries dimension $L^{-2}$ — an eigenvalue scale — with $u = 2/t - R/6 + O(t\mathcal{R}^2, t\nabla^2 R)$ and normalised form $b(x;t) = t\,u(x;t)/2 = 1 - tR/12 + \cdots$. Neither is a temperature, an inverse temperature, or an energy density absent a calibration map; consequently *no local first law follows from the heat kernel alone.* A thermodynamic completion requires an independently defined heat one-form on configuration space, a physical energy notion, and a Carathéodory–Jauch–Roberts integrability theorem establishing $\delta Q = T\,\mathrm{d}S$ (**open**). Reading the stationarity as *equilibrium* is a programme-level interpretation, not a result.

**Lorentz** [34] (*No Quartic Graviton Dispersion from the Local Covariant Four-Derivative Truncation*): A negative result about a posited kernel (**proved** as a negative result; **open** for the Lorentzian construction). A Lorentzian counterpart of $S_\Pi$ is not available — an in-in construction on a closed time path is required and is **open** — so the local Lorentzian transverse-traceless kernel $\mathcal{O}_{\rm TT} = \Box(c_{\rm EH} + \beta\Box)$ is posited, not derived. For $c_{\rm EH} \neq 0$ its kernel is the *direct sum* $\ker\Box \oplus \ker(c_{\rm EH} + \beta\Box)$: the massless summand obeys $\omega^2 = |\vec{k}|^2$ exactly, with no correction at any order (**proved** given the posit). No quartic deformation of the graviton dispersion follows, hence $A_4 = 0$ on the massless branch, no mapping $A_4 \leftrightarrow \ell_\chi$, and no bound on $\ell_\chi$ resting on that identification. For $\beta \neq 0$ a helicity-2 pole appears with opposite residue — a ghost independently of the sign of $c_{\rm EH}/\beta$ — at signed mass squared $m_2^2 = -c_{\rm EH}^{\rm ren}/\beta^{\rm ren}$, a matching datum not tied to $\ell_\chi$. The resemblance to quadratic gravity is a comparison: the five-polarization massive spin-2 multiplet is not established, and the $R^2$ sector leaves a massive scalar unexcluded (**open**). The vector sector is not derived: the linearized vector equation keeps a $\Box\bar{h}_{0i}$ term that harmonic gauge does not remove, and only a stationarity assumption would reduce it to an elliptic constraint, so a derivation from the full linear system with spin projectors, and with it any frame-dragging reading, remains **open**; so does the first-principles construction of a projected momentum density $T^{(\Pi)}_{0i}$, awaiting the fermionic and coupled-matter sector.

### 5.3 Lorentz Capacity Sub-Programme

The Lorentz-capacity (LC) sub-programme isolates a single structural question: where does the Lorentz factor $\gamma$ originate, given that the Lorentzian metric is itself an effective object reconstructed from admissible non-injective projection? The answer is that $\gamma$ is not a kinematic postulate but the temporal residual of a bounded Born–Infeld capacity budget, lifted to special-relativistic observer transformations through the effective co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$ taken from Q11, whose coefficient values are not established: Q11 shows that spatial SU(2) invariance leaves $A_\tau$ free, and this entry is pending revision. A guiding distinction runs throughout: the Born–Infeld capacity sphere $(F^\tau)^2 + \beta^2 = 1$ supplies the load $\beta$ and the clock factor $1/\gamma$, while the Lorentz group acts as the isometry group of the effective metric, not of the capacity sphere. The chain assembled by the sub-programme is

$$\text{BI budget} \to F^\tau = 1/\gamma \to g^{\mu\nu} = 2\eta^{\mu\nu} \to \Lambda(\beta) \to R(x)^2 = 1 - \epsilon(x) \to \text{finite-}q\text{ domain},$$

with the co-metric step carrying the coefficient values now pending revision. The synthesis paper **LC-Synthesis** [61] (**synthesis**) gathers the six constituent papers below into this single account; their individual contributions follow.

**LorentzCapacity** [54]: Derives the Lorentz factor and relativistic mobility $\beta\gamma$ from the O7 saturation parameter $\eta_n^{\rm O7}$ under the Born–Infeld admissibility budget (**conditional** on that budget). The analytic factorization (central coordinate $c$ contributes only a global phase) shows the total O7 residual satisfies a linear filling law and does not by itself yield the Lorentz factor. The

conditional mechanism via the temporally projected residual $\Sigma_n^\tau := \mathcal{P}_\tau^{\mathrm{Q5b}}\,\Sigma_n^{\mathrm{tot}}$ and a quadratic normalization gives $\eta_n^\tau \to \beta\gamma$ and $\Phi(\eta_n^\tau) \to 1/\gamma$ (**proved** for the algebraic BI target and the factorization; $\mathcal{P}_\tau^{\mathrm{Q5b}}$ construction: closed by TempProj [56]).

**TempProj** [56]: Closes the open construction of $\mathcal{P}_\tau^{\mathrm{Q5b}}$ from LorentzCapacity (**conditional** on [U] and the Born–Infeld lift). Constructs the temporal residual map as the unique positive function compatible with the Lorentzian splitting of the effective co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$ taken from Q11, whose coefficient values are not established (Q11 leaves $A_\tau$ free; this entry is pending revision), the Born–Infeld unit capacity-flow sphere, and the exact Weil filling law $\Sigma_n^{\mathrm{tot}} = 1 - B_n$. The explicit formula $\mathcal{P}_\tau^{\mathrm{Q5b}}\Sigma_n^{\mathrm{tot}} = \sqrt{1 - B_n^2}$ follows from a fibre-by-fibre uniqueness argument; Lemma 1 and Theorem 1 of LorentzCapacity become unconditional (**proved** conditional on [U]; three-input necessity: **proved**).

**LCII** [25] (version 1.3): Extends TempProj to the non-homogeneous fibrewise regime (**conditional** on the postulated occupancy–lapse identification and on a supplied Lorentzian co-metric whose flat temporal reference $A_\tau^\infty$ is postulated, Q11 leaving $A_\tau$ free). Its weak-field corollary restates the general-relativistic lapse of a supplied weak-field metric in the capacity language; no paper of the programme supplies that metric. A localised spectral occupancy $\epsilon(x)$ reduces the local BI capacity radius to $R(x)^2 = 1 - \epsilon(x)$; the fibrewise map factorises as $\mathcal{P}_{\tau,x}^{\mathrm{Q5b}}\Sigma^{\mathrm{tot}}(x,n) = R(x)\sqrt{f(x,n)(2 - f(x,n))}$. Gravitational time dilation is identified as $d\tau/dn|_x = R(x)/\gamma$; LCIIo1 [23] refutes the free-fraction identification of the capacity radius with the metric radius, which agree only at $\epsilon(x) = 0$, while leaving scalar closure of the bridge open. The paper establishes a unified capacity arbitration picture: special-relativistic time dilation ($1/\gamma$) and gravitational time dilation ($R(x)$) are two independent manifestations of the same BI capacity trade-off, separated in the product $R(x)/\gamma$ (**structural**; open problem LC-O2-O1 remains open, see [23]).

**LCIIo1** [23] (version 2.0): Establishes a *no-go* for the scalar free-fraction reduction of the capacity–metric bridge of LCII. Version 1.0 claimed to close that bridge via $A_\tau(x) = A_\tau^\infty(1-\epsilon(x))$ together with $g^{\tau\tau}(x) = -A_\tau(x)/4$; that claim is withdrawn. With the identification $g^{\tau\tau} = -A_\tau$ (flat-space value $-2$ taken from Q11, not established: Q11 leaves $A_\tau$ free) and a diagonal temporal block, the free-fraction law gives $g_{\tau\tau}(x)/g_{\tau\tau}^\infty = 1/(1 - \epsilon(x))$, which agrees with $R(x)^2 = 1 - \epsilon(x)$ only at $\epsilon(x) = 0$. The paper also shows that the partial-trace form of the local coefficient is a fresh postulate rather than a consequence of [114], and that it is mistyped (a norm-valued occupancy is not a trace fraction). What survives is the co-metric statement $g^{\tau\tau}/g_\infty^{\tau\tau} = 1 - \epsilon$, which does not govern proper-time dilation. The result is narrow: a reciprocal scalar law would close the bridge algebraically. **LC-O2-O1 is reopened** (**proved**, as a negative result).

**LCo1** [30]: Closes LC-O1, the reconstruction of effective Lorentz transformations between inertial observers from projective temporal ordering data. Given a projective temporal capacity load $\beta$ from LorCap/TempProj, the unique proper orthochronous transformation between the corresponding inertial splittings of the effective co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision, is the standard Lorentz boost; length contraction $\ell = \ell_0/\gamma$ follows as a corollary (**proved**).

**LCo3** [27]: Closes LC-O3, the finite-$q$ correction problem near the saturation boundary. Setting $R := (q-1)/2$ and $n = R + m$, the parabolic Carnot formula deviates from the exact torus capacity by $\Delta_q(n) = 4m^2/q^2$; the quadratic identity $B_n^2 + (F_n^\tau)^2 = 1$ receives no correction, while the Carnot identification holds for $n \le R$ and asymptotically as $m/q \to 0$ (**proved**). With LC-O1 and LC-O3 closed, the Lorentz-capacity sub-programme is complete for *inertial* kinematics, up to the co-metric coefficient values it takes from Q5b–Q11, which are pending revision. Its non-homogeneous extension is not: LC-O2-O1 is reopened, and the identification of the local capacity radius with the metric lapse remains the declared postulate of Remark 3.2 of [25].

### 5.4 Standard Model Companion Papers

The papers Foundation, TopologicalInvariants, SMchargemass, O23, O27–O32, Q6a, Q7, and Q12 form (in their gauge-group identification aspect) the *gauge structure sub-programme*, whose synthesis paper is the working paper **The Gauge Structure Sub-Programme** (GS-Note, Presentation Note 3, version 2.2) [60]. It identifies no gauge group. O31 withdraws every derivation of SU(3) and of the composite $G_\Pi = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$; the colour factor and complete group are open, and O32 measurements do not supply the missing group-identification bridge. U(1) is structural on premises carried as input, and SU(2) is conditional on a supplied carrier. Q12 and Q13 take $G_\Pi$ as a supplied compact group, and the note takes no metric coefficient from Note 2. The note covers gauge group identification only; the derivation of Yang–Mills dynamics belongs to the gauge–gravity spectral stratification sub-programme.

**BornInfeld** [11]: capacity axiom [A-cap] bounding the per-node relaxation rate, with the BI action as the saturation candidate compatible with it; the constitutive, propagation, and gauge-sector bridges to continuum physics are stated as open problems (**structural**).

**TopologicalInvariants** [67]: Charge quantisation ($\pi_1(S^1) \cong \mathbb{Z}$) and absence of magnetic monopoles from the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$ of the projection fibre (**proved**; $\pi_2(\mathcal{C}_{\mathrm{eff}}) = 0$ beyond the canonical model: **open**, Remark 4.2).

**SMchargemass** [16]: Mass and electric charge as BI-saturated responses (**structural**; baryon-to-photon ratio $\eta$: **open**).

**Bell** [13]: The claimed derivation of Bell-inequality violations from non-injectivity alone is refuted by a finite Bell-local countermodel; the required cross-context global-coupling obstruction is not derived (**open**).

**BellNoGo** [87]: Proves that non-injectivity alone has no Bell-discriminating power (redundant-extension and universal pushforward theorems), exhibits the explicit Bell-local countermodel, disproves output entropy as a non-injectivity measure, and identifies the correct obstruction as the absence of a setting-independent global coupling across measurement contexts (Fine's theorem) (**proved**).

The Bell paper opens the *entanglement sub-programme*, whose presentation paper is **ENT-Note** [49] (v1.2). The note locates entanglement *inside* the spectral admissibility cascade: for a closed conjugate Weil pair $\{c, q-c\}$, the reduced entropy on the residual fibre-level support of the Gram–Schmidt span is exactly $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$, the logarithm of the residual Gram–Schmidt rank, with no surviving spectral weighting. Structural monotonicity of $S_{\mathrm{ent}}$ along the cascade follows from the Gram–Schmidt construction alone. The note integrates the analytical results of ENT-E1 (Weil fingerprint orthogonality theorem) and ENT-E2 (separation of the admissible pair entropy $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ from the full residual-rank entropy), upgrading several previously numerical hypotheses to theorems and recording the residual epistemic stratification. The note thus reduces entanglement to a pure O-series observable and connects the pair-capacity exponent $\delta_{\mathrm{pair}}$ — whose numerical comparison with the mass hierarchy has no native capacity-to-rate derivation — to a downstream trace of the stabilization profile.

**ENT-E1** [73]: closes the open analytic strengthening of ENT-Note by explicit computation (**proved** / **open**). The $k$=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ for any block $(c_1, c_2, c_3)$ factors as a global phase times a single discrete Fourier mode at frequency $A = c_1 b_1 + c_2 b_2 + c_3 b_3 \bmod q$. Since distinct Fourier modes are orthogonal, every newly admitted fingerprint is already orthogonal to the current span, so the admission residual $w_j = 1$ is an *exact* algebraic identity — not a numerical accident. For matched single-character blocks the conjugate pair samples sign-reflected frequencies and the closure hypothesis of ENT-Note becomes a theorem; the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ is then derived analytically (**proved**; literal Schmidt pairing for independently sampled canonical O25 blocks: **open**).

**ENT-E2** [70]: separates two genuinely distinct entanglement entropies that ENT-Note and ENT-E1 establish on two different projections of the conjugate Weil pair. The *admissible pair entropy* lives on a hypothesised carrier: the explicit hypothesis (H-carrier), stated by E2 it-

self and derived nowhere, fixes the admissible sector of each factor as the atomic Fourier triple $\mathrm{ran}(\Pi_q^{(c)}) = \mathrm{span}\{e_0, e_{\xi_c}, e_{q-\xi_c}\}$, whose parity-pair component $\{e_{\xi_c}, e_{q-\xi_c}\}$ is identified with the spin-$\frac{1}{2}$ representation. The hypothesis is motivated by the pure-mode structure of the pre-saturation pipeline (*Q5a-O2* [83]); its rank-three input is the supplied selection rule of *O23* [65], within the open fibre-identification problem of *O18* [20]; no mean-zero theorem removes the zero mode (the passage to the pair sector is part of the sector definition); the singlet identification of *Q3* [80] then gives $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ for general canonical blocks, conditional on both (H-carrier) and Q3's explicit, undemonstrated diagonal-invariance hypothesis [H-inv] (**conditional** on (H-carrier) and [H-inv]). Q1 does not establish a singlet correlator or a Tsirelson bound. The *full residual-rank entropy* $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ lives on the full Gram–Schmidt basis (rank $q$ at saturation, not rank 3) and is a pipeline-level contraction observable, unconditional for matched blocks (ENT-E1) and otherwise conditional on the residual admissibility indiscernibility hypothesis [$\mathrm{H_{res}}$] and on extending *O17*'s equal-residual-dimension result beyond its own toy model to the real pipeline; the metaplectic dilation of *O31* [115] is excluded as a bridge for [$\mathrm{H_{res}}$] (**proved**; [$\mathrm{H_{res}}$] for general canonical blocks: **open**).

**SchwingerLamb** [58]: Fine-structure constant $\alpha$ as a projective invariant; Lamb shift and Schwinger correction rederived in the pre-geometric framework (**structural**; quantitative radiative corrections: **heuristic**).

### 5.5 Cosmological and Condensed-Matter Applications

**Cosmology** [3]: Flat galaxy rotation curves and the Hubble tension from a common effective mechanism; no dark matter or modified gravity (**heuristic**; quantitative fit: **heuristic**). Version 2.0 adds a structural expansion sector: under the rank–time dictionary stated as an explicit hypothesis ($dt = H^{-1}dn$, hence $n = \log a$, bridging the temporal-ordering sector and the operational closure of its Appendix A), the accelerated component is carried by projective history branching [92] rather than by a dark-energy fluid, and the naive state-volume reading is quantitatively excluded ($N_{\mathrm{dist}} \sim a^{0.881} \neq a^3$); the QUMOND relation is repositioned precisely (interpolation structure as phenomenological input; falsifiable content in the $a_\star \sim \kappa c H(z)$ closure, the environmental $H_0$ bias, and void signatures); the dimensional chain of the saturation scale is corrected ($a_\star \sim \kappa\, c_{\mathrm{BI}}^2/\ell$); $H_\infty$, $w(n)$ and the low-$\ell$ D4 re-audit are left open, with the phenomenological core independent of them (H-dict: explicit hypothesis; expansion reading: **structural**; open items: **open**). Version 2.2 upgrades the asymptotic de Sitter limit from a free late-time fit to a calibrated spectral steady state driven by the history-entropy production rate (structural carrier: **structural**, conditional on the [H-ent-id] identification; numerical value of $H_\infty$: explicitly not derived, with the two extraction routes closed conditionally), and gates the $w(n)$ candidate by the conditional no-go of [92]. Cosmology 2.2 also records the interpretive consequence of the low-rank Pell count: the earliest projectable epoch is naturally read as finite-resolution rather than infinitely hot, while the standard hot classical regime is left unchanged.

**TrajectoryBranching** [92]: Projective trajectory branching and the de Sitter limit (endpoint no-gos, $b$-only reduction and generic-probe Pell rate: **proved**; full-history compression rate $\log 2$: **proved**; exact class count: **proved**; cosmological reading: **structural**). The LPS $\to$ Heisenberg transfer relocates exponential growth from endpoint volumes (polynomial by Bass–Guivarc'h, degree 4) to projectively distinguishable histories (D4): any endpoint-based distinguishability collapses, while the canonical O12-compatible residual channel is $b$-only (the central charge is unconditionally erased from mono-path norms) and counts histories by the Pell recurrence $N(n) = 2N(n-1) + N(n-2)$, rate $h_b = \log(1+\sqrt{2})$, proved for a generic probe and witnessed exactly at $q = 211$. Supplies the structural bridge from the admissibility cascade to a de Sitter-like component independent of matter content. Since v1.1.0 the full-history Gabor channel is itself a theorem: the Gabor span exhausts the abelian $\ell^1$ disk, so nonzero symbols are confined to $\ell^1$-outward geodesics (geodesic death), and for a generic probe (Lawrence–Pfander–Walnut full spark plus a Chebotarëv-based separation lemma) the compression rate is exactly pinned at

$\log 2 < \log(1+\sqrt{2})$. Version 1.2.0 upgrades the class count itself to a theorem, $N_{\mathrm{dist}}^{\mathrm{V3}}(n) = 2^{n+2} - 4n - 1$ ($n \geq 2$, window $q > (4n-1)(2n+3)$): the last separation step ($a$-mirror pairs at equal $|b|$) is closed by the generic non-vanishing of a mixed two-anchor coefficient — a Hermitian jet at the orthogonal point of an anchor moment domain — combined with a Klein rigidity property of outward geodesics, so the $b$-mirror is the only generic degeneracy of the projective history record. Version 1.3.0 gates the equation-of-state candidate by an explicit conditional no-go: the finite-$n$ rate correction decays like $n\,2^{-n}$, so under the rank–time dictionary hypothesis any transient built on the full-history profile is confined to the early low-rank window, and under the minimal count-to-density mapping class the exact counts do not supply an observable evolving equation of state (conditional no-go: **proved** given the stated hypotheses; remaining evolving-$w$ routes: rank-dependent dictionary corrections or the spectral-stationarity route [19]); it also records the arithmetic identity of the canonical count (Pell equations, $\sqrt{2}$ convergents) and its effective memory depth one (transfer matrix).

**LowLCapacity** [85]: Low-multipole suppression from distinguishable-history capacity (**numerical** for the re-audited 8/3 fibre-to-base stiffness; capacity ceiling: conditional theorem chain in ProjCE [90]; saturation: **open**; no-fit grid: numerical benchmark; TE/EE transfer: **open**). Replaces the white-paper phenomenological low-$\ell$ envelope (two free parameters, illustrative comparison) by a small-$N$ mechanism: the largest angular modes probe the first admissibility ranks, which contain only 3, 7, 17 distinguishable histories (canonical Pell count [92]). Under a separate saturation hypothesis, a quadrupole-anchored, parameter-free $2 \times 2$ grid of envelopes and angular dictionaries defines a saturation benchmark, not a derived prediction. Version 1.5.0 applies the Planck PR3 map calibration and the real-$m = 0$ full-sky multiplet correction to the TT Bessel cells. The leading TT diagnostic changes from the historical $27.39 \to 17.29$ reported in version 1.4.0 to the active $27.26 \to 17.70$. TE is neutral and EE mildly disfavoured under the uncorrected minimal-transfer convention; the envelope is a staircase in $\ell$ (integer ranks), a discrete signature. The $S^3$ 8/3 stiffness witness is re-audited (geometric, not LPS-bound) and carries the separate modulation observable $\sqrt{8/3} \simeq 1.633$.

**Superconductivity** [4]: Projective phase locking as the structural origin of superconducting coherence across BCS, cuprate, and nickelate systems (**structural**).

**Quantum Fisher Information** [47]: The scale-free growth of the quantum Fisher information density in the strange metal $Ce_3Pd_{20}Si_6$ reduces to projective spectral data; it is a property of the full nilpotent $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ substrate (edge spectral dimension $d_s \approx 4$, $f_Q(T) \sim T^{-1}$) and only marginal for the single $w = 2$ sector, with a thermal closure $\Delta_\Pi(T) = \frac{d_s}{2} T$ derived from the heat-kernel response (regime **structural**; observed exponent **heuristic**; asymptotic value **open**). Condensed-matter companion to the superconductivity note, and a new spectral entanglement witness complementary to the entanglement sub-programme.

## 6 Dependency Structure

Figure 1 presents the logical dependency DAG. Node status and edge status are deliberately independent. The interactive graph types audited claim-bearing edges as *proved*, *conditional*, *postulated*, *interpretive*, or *withdrawn*, and renders synthesis links separately. Every substantive edge not yet audited defaults to *dependency*: it records use or citation and carries no derivational status. Thus a proved source node and a proved target node never make their connecting edge proved by default. Three entry paths emerge for external readers.

**Foundational path.**

$$\mathrm{ENI} \to \mathrm{Found} \to \mathrm{HeisStr} \to \mathrm{noscale}.$$

Requires least prior knowledge; separates the axiomatic core from the open carrier-selection edge.

**Spectral path.** BornInfeld → SpAdm → SpRel → O1 → ⋯ → O24 → O25 → O28 → O29. Develops the spectral machinery, the fibre-level chain, and the SGN no-go for its proposed capacity-to-rate continuation.

**Physical path.** Foundation → Q1 → Q2 → Q3, with O18 + O22 + O23 supplying a proposed auxiliary fibre chain whose O18 bridge is open; Q2 → Q5a → Q5b → Q6b and Q5a → Q6a → Q6b; O23 + O28 + Q5b ⇢ Q7 (requirements for an equivariant bridge; the identification is missing, verdict 3, with O29 auditing the measured rank rather than supplying it); parallel: Spectral2 → Gravity → Thermodynamics → Lorentz; Q5b + Q11 + PTO + BI → LorCap → TempProj → LCII → LCIIo1; LCo1, LCo3.

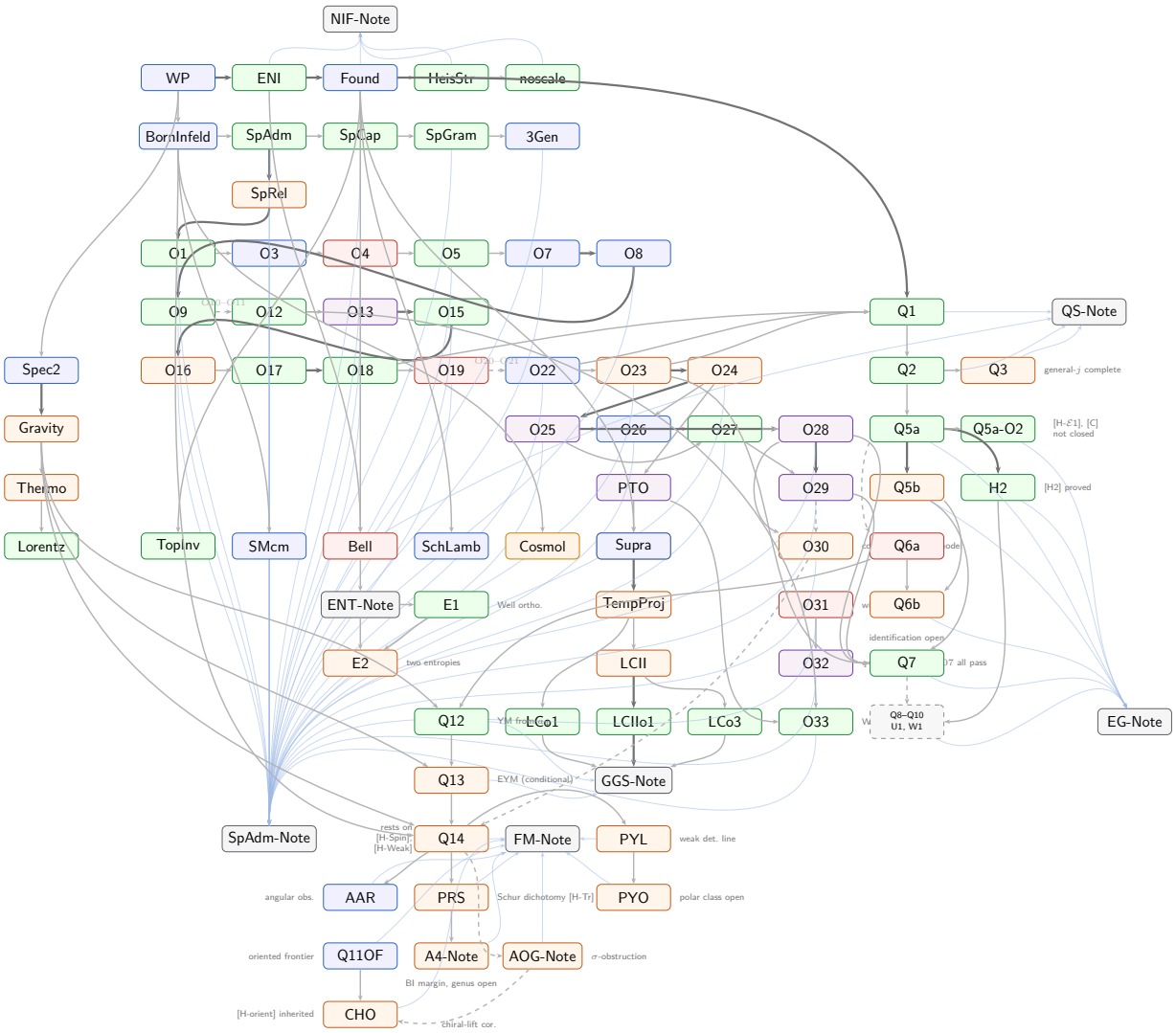



Figure 1: Logical dependency DAG of the Cosmochrony programme. Colour coding: ■ **proved**, ■ **structural**, ■ **numerical**, ■ **conditional**, ■ **heuristic**, ■ **open**, ■ **synthesis**; each single-paper node carries the primary label of its inventory row, while the dashed composite block gathers a sub-chain and carries no paper label. In both figures the grey side annotations and the dashed result boxes carry no status colour: where their text states a status it is that of the component named, not the primary label of the node. The interactive graph's conditional-reading and postulated edges (EG-Note to Q12, Q13, Q14, SG-Note and GGS-Note), its conditional edge Q11→Q14 (Lorentzian signature only; spin soldering remains an input) and the conditional typing of the edges into PYL and PYO ([H-Weak], [H-Res]; including the dependencies PRS→PYL, PRS→PYO and Q14→PYO, typed in the graph) are not drawn in this figure. Thick arrows indicate the main logical spine; thin arrows indicate supporting dependencies. Dashed arrows with labels abbreviate linear sub-chains; the dashed arrow Q5a→Q6a is a conditional edge. The faint converging edges gather the constituent papers of each thematic sub-programme into its synthesis node: the Branch I foundational papers (ENI, Foundation, HeisStr, noscale) into **NIF-Note**, the spectral admissibility papers into **SpAdm-Note**, the Lorentz-capacity papers into **LC-Synth**, the geometric emergence papers into **EG-Note**, Q1/Q2/Q3 and the Bell paper into the quantum-structure **QS-Note**, Q12 and Q13 into the gauge–gravity stratification **GGS-Note**, and Q14 with the companion notes PRS, AAR, Q11OF, CHO, A4-Note, AOG-Note, PYL and PYO into the fermionic-matter **FM-Note**. The dashed uncoloured arrow from Q7 leads to the geometric emergence block Q8–Q10, U1 and W1, detailed in Figure 2; it carries no status colour because what Q7 version 2.0 supplies forward is conditional on an identification its own inventory row records as open. The interactive version with clickable nodes is provided as a companion HTML file.

Table 1 lists all 116 papers of the programme, the present registry excepted. The **Key** column gives the citation key used in `cosmochrony-bibliography`. The final block gathers the sub-programme presentation and synthesis notes, marked **synthesis**: they organise results established elsewhere and are not themselves sources of new results.

Table 1: Complete inventory of the 116 programme papers, excluding the present registry.

| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| WP | White Paper | [18] | Programme overview; substrate $\chi$; iterated projection; BI constraint; relaxation cascade | **structural** |
| ENI | Emergence & non-injectivity | [35] | No-go: genuine emergence $\Leftrightarrow$ $\Pi$ non-injective; projection entropy $S_\Pi$ | **proved** |
| Found | Foundation | [6] | Axioms A1–A4; the published Heisenberg/Weil selection theorem is not established; version 2.3: continuum-limit passages conditioned on the unestablished spatial-limit hypothesis [H-L] (Q5b); no co-metric coefficient value established; SU(2) conditional on a supplied carrier; Standard Model gauge group open; relation to HJ, symplectic, WKB, FRG formalisms | **structural** / **conditional** on [H-L] / **open** |
| HeisStr | Heisenberg carrier obstruction | [2] | Contains the explicit $S_3$ counter-model to the carrier-selection implication and corrects the central-subgroup/Schur step (version 2.1); the conditional Stone–von Neumann endpoint remains valid when the Heisenberg carrier and central character are supplied | **proved** / **conditional** on a supplied carrier and central character / **open** |
| noscale | No external scale | [33] | No independent dimensional parameter beyond $c_{\mathrm{BI}}$ | **proved** |
| EffComp | Effective composition prerequisites (K4) | [82] | Independence blocks = bicliques of the projected support; full simplex lifts all products; cylindrical-conditioning closure iff $\operatorname{rank} W = 1$; support rectangularity does not imply preparation independence; contract proposed, not derived from A1–A4 | **proved** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| STEFT | Simplicial states & local tomography (K4 cont'd) | [81] | Primitive family never convex ($m \geq 2$, unconditional); classical randomisation $P1$ closes it rank-independently; affine effects = cube (representation, not availability); coarse-graining gives the Boolean skeleton, AX-RND the full cube; $P1$/AX-RND independent, unified only under an explicit control-independence clause; pointwise product forced given composed-effect existence; local tomography iff local families span their ambient spaces | **proved** |
| HMPL | Power-lift refinement obstruction (STEFT cont'd) | [93] | $\mu^\alpha$ Kolmogorov-consistent iff $\alpha = 1$ (general, self-contained); escort renormalisation restores it trivially but departs from the pointwise lift; exact $\mathbb{Q}(\sqrt{2})$ instantiation under its own postulate H2 on the Trajectory-Branching automaton, distinct from the Q5a [H2]; endpoint/shadow quotients incomparable; $X \leftrightarrow X^{-1}$ transports the endpoint by an exact automorphism, forcing real co-$b_n$ phase sums; bounded exact search finds no cancellation among 62 nontrivial-phase classes of 64 | **proved** / **conditional** on its postulate H2 for the instantiation |
| Born-Infeld | Bounded relational capacity | [11] | Capacity axiom [A-cap]; BI action as saturation candidate; bridges open | **structural** / **open** |
| SpAdm | Spectral admissibility ($Q_8$) | [52] | $A_n^{\max} = c_{\mathrm{BI}}/\sqrt{\lambda_n}$; first tractable instance | **proved** |
| SpCap | Spectral capacity | [53] | Capacity functional $\mathcal{C}(G, S)$; binary-polyhedral maximality proved via adjoint identity $\hat{A}_{\mathrm{adj}} = 2M - I$ and icosahedral isotropy | **proved** |
| SpGram | Gram rigidity | [38] | Neutral-generator geometry from $G_{st} = -\frac{1}{2}\chi_\rho(st)$ | **proved** |
| 3Gen | Three generations | [66] | Three-generation structure from ADE stratigraphic levels | **structural** |
| SpRel | Spectral relaxation | [9] | $\Lambda_{\mathrm{proj}} \asymp \lambda_2$ for LPS expander families | **conditional** on the capacity–isoperimetry closure and spectral-isoperimetric saturation hypotheses |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| O1 | Variable-valence graphs | [44] | LPS saturation is a fixed-valence artefact; growing valence restores window | **proved** |
| O3 | Mass hierarchy | [42] | First SM connection: $\beta^* \in (0.09, 0.13)$ from lepton mass ratios | **structural** |
| O4 | No cascade-exponent bound from bounded flux | [12] | Bounded relational flux and Ramanujan expansion do not yet yield a cascade-exponent bound: the former derivation of $\beta \leq 1$ does not follow from the stated argument, and any structural upper bound on $\beta$ by this route is open | **open** |
| O5 | Rep-theoretic obstruction | [26] | Fixed-repr. approaches cannot close the $\beta$ gap | **proved** |
| O6 | Matrix redundancy | [31] | Matrix-level dynamic redundancy: $R_n \to 0$ for the Steinberg fingerprint (proved); the confinement corollary $\beta_{\text{eff}} \in \beta^*$ is conditional on a power-law Ansatz that the paper's own fingerprint does not satisfy, so a genuine power-law regime remains open | **proved** / **conditional** on the power-law Ansatz / **open** |
| O7 | Continuum target | [40] | Target: $\delta \in [7.4, 10.6]$ on 3-step fingerprints | **structural** |
| O8 | LPS geometric obstruction | [1] | Exponential shell growth $\Rightarrow$ usable window $= O(\log q)$ BFS steps, on a shell-growth premise the paper records as an empirical observation for its family | **structural** / **numerical** |
| O9 | Heisenberg graphs | [41] | Structural transition: $\text{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ with $\lvert B_n\rvert \sim n^4$ replaces LPS | **proved** |
| O10 | Large-$q$ computation | [39] | Ball growth $\hat{D} = 3.05$ at $q = 101$, continuing the convergence toward $D_{\text{hom}} = 4$; TensorSketch obstruction identified | **numerical** |
| O11 | Fourier-character proxy | [72] | Bidimensional proxy overcomes TensorSketch obstruction | **numerical** |
| O12 | Exact Weil blocks | [22] | $\delta_{\text{exact}} = 3$ exactly (unfolded, $q \to \infty$); finite-$q$ fitted slopes are crossover statistics, independent of the central coordinate $\gamma$ | **proved** |
| O13 | Extended prime range | [10] | Exponent sequence at $q \in \{29, 61, 101, 151, 211\}$ does not drift toward the phenomenological target; consistent with $\delta_{\text{exact}} = 3$ | **numerical** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| O14 | Observable-class mismatch | [74] | Fixed-$q$ normalisation $q^{-\eta}$ changes intercepts, not intra-$q$ slopes, and defines no endpoint correction from a fitted slope alone; the central-phase correction vanishes identically; no inter-$q$ estimator or $\beta^*$ diagnostic follows; SGN places the shell factor additively | **structural** |
| O15 | Block-level no-go | [36] | $\beta^* = 1/(\delta + \frac{1}{2})$ does not transfer to the exact-block setting; aggregation bound $\alpha_{\mathrm{dyn}} \leq \hat{\delta}_{\mathrm{exact}}$ holds conditionally, verified at $q = 151, 211$ | **proved** / **conditional** on the stated aggregation hypothesis |
| O16 | Pair observable | [24] | Conjugate pairs $\{c, q-c\}$ are fibres of $\Pi$ under two independent open hypotheses; exponent doubling $\delta_{\mathrm{pair}} = 2\delta_c$ holds conditionally, trivially on the real pipeline | **conditional** on the parity and realisation hypotheses |
| O17 | Block-independence, toy model | [21] | Raw Gram-Schmidt redundancy is exactly independent of block parameters for any two blocks, not only conjugate ones; scoped to a scalar toy model, not shown for the real pipeline | **proved** |
| O18 | BI action parity | [20] | $S_{\mathrm{BI}}[-\chi] = S_{\mathrm{BI}}[\chi]$; neither $\Pi(\chi) = \Pi(-\chi)$ nor the character pairing $c \leftrightarrow q-c$ follows | **proved** / **open** |
| O19 | Proposed canonical normalisation | [15] | Generic Gram–Schmidt identities survive; the residual factor, completed tests, and pipeline-independent observable are not established for the real pipeline, the conjugate-block identity of O16 [24] forcing $r(c,q) \equiv 1$ exactly on the O12/O13 exact block observable | **open** / **proved** |
| O20 | Conditional persistence template | [43] | Threshold-crossing algebra under explicit hypotheses; no intrinsic threshold or selected exponent window | **conditional** on the monotone-amplitude and capacity-bound hypotheses |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| O21 | Local crossing diagnostic | [51] | Two-point crossing formula under a fixed normalisation; it scales under amplitude rescaling, while shell alignment and the capacity-to-rate carrier remain open | **conditional** on the shell-alignment conjecture and the cross-substrate identification / **open** |
| O22 | Projection locking | [50] | BI projection locking forces the O21-defined $n_3^{\mathrm{obs}}$ onto a BFS shell within the stated model, on a sketch-level continuous decay law and the observable-space shell decomposition; this does not make that rank amplitude-invariant | **structural** / **open** |
| O23 | Conditional adjoint dimension | [65] | For a supplied irreducible $\mathbb{C}^2$ carrier, the traceless neutral sector is $\mathfrak{su}(2) \cong \mathrm{Im}\,\mathbb{H}$, dimension three; $Q_8$'s two-axis neutral generating set rules out generator-axis counting; carrier selection and the $\Sigma_c$-identification are open, so $\Sigma_c(n_3) = 3$ is a supplied selection rule | **conditional** on the supplied carrier and the $\Sigma_c$ selection rule / **open** |
| O24 | Observable rank rigidity | [37] | Verticality Lemma develops $c_{\mathrm{BI}} \to \delta_{\mathrm{pair}}$ under the stated fibre hypotheses; no first-principles fibre identification and no native derivation of $\delta_{\mathrm{pair}} \to \beta^*$ | **conditional** on the supplied fibre and rank-three hypotheses / **open** |
| O25 | Numerical campaign (extended to $q = 601$) | [55] | Exhaustive pair sampling through $q = 211$; representative sampling of 24 pairs at $q = 307$ and 12 at $q = 401, 601$; raw $\delta_{\mathrm{global}} \to 7.61$ at $q = 601$ under the frozen $\log(n+1)$ estimator; fixed-$q$ pair concentration; O14 correction overcorrects at large $q$; $\delta_\infty$ not pinned; reciprocal image 0.108–0.123 only as a phenomenological comparison | **numerical** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| O26 | Quadratic completion | [46] | Dictionary: Weil blocks $\leftrightarrow$ rank-1 matrices in $M_2(\mathbb{C})$; connects $\mathrm{Heis}_3$ to $2I \subset \mathrm{SU}(2)$; pair-sector fibre structure taken as a motivated hypothesis; Level I growth-exponent identification of $\sigma_{\mathrm{pair}}$ with a Hilbert–Schmidt trajectory norm proved; Level II a conjecture ($n$-independence of the proportionality open); Level III conditional on the admissible embedding $\Phi_{q,\rho}$, version 2.1 proves the symmetric-square rank bound under Hypothesis 4.4(a) (at most three for $d_\rho = 2$, excluding the rank-four target under that contract); measured $H_{\mathrm{eff}}$ ranks do not test the embedded covariance; no rate equation follows | **structural** / **conditional** on the admissible embedding / **open** |
| O27 | Quaternionic rigidity | [5] | Quotient factorisation does not imply equivariance; under explicit saturation, carrier, parity and equivariance inputs, the Schur-level counts are proved; canonical equivariant vector lift and any $\beta^*$ interpretation remain open | **proved** / **open** |
| O28 | Window calibration & $r_{\mathrm{eff}}$ | [8] | Calibrated linear extrapolation of $n_1$ fails out of sample (to $q = 601$, up to $+88\%$); the declared adjustment $\delta_{\mathrm{corr}}$ lies in $[7.4, 10.6]$ over $q \le 211$ and, by O14, does not correct the intra-$q$ fitted exponent (a diagnostic, not a corrected exponent); O28 reports the 1% threshold rank $r_{\mathrm{eff}}^{1\%} = 3$ and the resolved spectrum $[1 : \frac{1}{2} : \frac{1}{2}]$ in its stated perimeter; O29's extended recomputation finds rank three modal rather than invariant (32/50 pairs at $q = 101$, 103/105 at $q = 211$) | **numerical** |
| CC-Note | Inverse-square coverage & constant depth | [29] | Fingerprints are Fourier characters, so $r_c(n) = \lvert T_c' + C\, I_{n+1}\rvert$ with shellwise ceiling 18; deterministically $n_1(q) \le 22$ and $x_1(q) \asymp q^{-2}$ for $q \ge 311$; under generic sampling $n_1(q) \to 22$ and $q^2 x_1(q) \to 99{,}689$; O28's depths are the transitional regime | **proved** |

*continued...*

| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| SGN | Native span growth and transfer no-go | [32] | $\Delta r = \sigma_c \lvert S_n \rvert$; real-weight three-branch theorem and rigorous integer-rank finite-window law; corpus-native valence–exploration, frontier, pair-carrier and exponent-coordinate obstructions refute a Heisenberg derivation of $1/(\delta_{\mathrm{pair}} + \frac{1}{2})$ | **proved** |
| O29 | Numerical Veronese and carrier diagnostics | [71] | Finite-checkpoint numerical $6 \to 3$ collapse; fitted commutators and commutant are compatible with a supplied adjoint carrier but do not identify it; $W_{\mathrm{adm}}$, the auxiliary PCA truncation $U_c^{(2)}$, and $V_\rho$ remain distinct; neither $d_\rho = 2$ nor a Veronese root is measured | **numerical** |
| O30 | Conditional $3 \times 3$ model | [75] | Under (H0)–(H3), $\beta_j = \varepsilon_j \bar{\alpha}_j$; two weighted cancellations give $\mathcal{C}_c = \langle r^4 \rangle \, \mathrm{diag}(1/4, 1/2, 1/4)$; this object is distinct from O28's measured $9 \times 9$ covariance and does not derive its spectrum; factor 2 comes from symmetric-tensor normalisation | **conditional** on (H0)–(H3) |
| O31 | Withdrawal notice for colour-triplet and gauge-group claims | [115] | Withdraws the proposed individual-profile co-admissibility, arithmetic criterion, metaplectic step, $\Delta(27)$ proposition, rank-eight test, SU(3) uniqueness, irreducibility of the Born–Infeld-compatible fibre-preserving group, and every gauge-group derivation; gives explicit SO(3), $T^2$ and $\{1, 2, q-3\}$ counterexamples; remaining material under re-audit | **open** / **proved** |
| O32 | [H-color] numerical test | [79] | [H-color] passes for all $q \in \{61, 151, 211, 307\}$ ($R \leq 3$, ctrl_ref $= 1.657 \times 10^{-3}$; $R = 2.1$, 0.4, 1.0, 0.7 respectively); $C_{\mathrm{color}}$ rank 1 at $q \in \{61, 151, 211\}$; $\delta_{\mathrm{tri}} \approx 3\delta_c$ at the percent level (0.7% at $q = 151$, 1.3 to 2.2% elsewhere); the rank and pointwise levels of [H-color] are open (O31 supplies neither), the averaged ($O(q^{-1})$, O32 Proposition 1) and effective levels stand; no gauge-group reading is established | **numerical** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| O33 | Arithmetic Weil doublets and mixing | [76] | Closed sumset formula and spacing theorem proved; stabiliser reduced exactly to $A_K$, with an orbit-divisibility sieve, uniform exact $C_4, C_6$ families, and an additive rigidity theorem $A_K = \{\pm 1\}$ outside a degenerate boundary regime (completeness open only there); internal Mackey–Hecke $M_2(\mathbb{C})$ doublets and Harper split derived; boundary mixing reduced to the character sum $C_\theta(K)$ by Galois separation, with its zeros completely classified on at most four hole pairs; degenerate regime: infinite Gaussian-box $C_4$ family, all 75 exceptional sets through $q = 151$ in three mechanisms, no order eight at $q = 193, 241, 257$; porous depths converge to an explicit point process with mean 28/15; every proper Gaussian box, direct or wrapped, and every free four-point complement orbit proved to have exact stabiliser $C_4$ by a fourth-moment argument, leaving completeness and the Eisenstein top level open; odd sector protected by parity closure | **proved** / **open** |
| PTO | Temporal ordering | [45] | $I(n) = \sigma_{\text{pair}}(0) - \sigma_{\text{pair}}(n)$ is a non-decreasing temporal proxy; confirmed for $q \in \{29, 61, 101, 151, 211\}$ | **numerical** |
| Fibre-Erase | Fibre erasure | [91] | Monotone $I(c; \sigma(\ell))$ on BFS coarse-graining axis $\ell$ via DPI (Thm 4.1, conditional on [H-suff]); [H-suff] proved at the rank level (Prop. 3.6); capacity-level numerical evidence for $q \in \{61, 151, 211\}$; explicitly not a $c$-theorem | **conditional** on [H-suff] and [H-sg] |
| Q1 | Fourier-support rigidity | [28] | Exact, unconditional conjugate-sector subspace identity and its rank/support no-go corollary; no phase coherence, singlet correlator, Tsirelson bound, or Born rule | **proved** |
| Q2 | $2I$ eigenvalue degeneracy | [7] | Exact finite-group fact: spin-$\frac{1}{2}$/spin-$\frac{3}{2}$ share $\lambda = 18$ on $2I$; no phase coherence, Born rule, Tsirelson bound, or fixed point | **proved** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q3 | Conditional universal singlet | [80] | Given an explicit, undemonstrated diagonal-$2I$-invariance hypothesis [H-inv], Schur's lemma identifies the proto-state as the singlet $\|\Omega_j\rangle$ for all $j \in \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$, with an explicit Casimir correlator; [H-inv] itself, phase coherence, and the Born rule are not derived | **conditional** on the diagonal-invariance hypothesis [H-inv] and a supplied composition |
| Q5a | Canonical Fourier filtration | [14] | On a supplied finite-Heisenberg carrier and non-trivial central character (version 3.2): canonical filtration identified exactly (toric Fourier window span$\{e^{2\pi i b x/q} : \|b\| \le n\}$); published form converges to the zero form (rate $q^{-2}$); under [H-depth] and measured positive finite weight limits, normalisation no-go (no common scalar normalisation yields a non-trivial toric differential operator); conditional dual-window Dirichlet limit in the rescaled frequency variable. No Mosco derivation of $L_\Pi = -A\partial_x^2$; the spatial limit is left as the hypothesis that Q5b names [H-L]; Q5 open | **proved** (negative, under [H-depth] and [H-w']); **open** (continuum) |
| Q5a-O2 | Pipeline Fourier structure & truncation status | [83] | Pre-saturation fingerprints and selected GS vectors are pure Fourier modes; displacements are $o(q)$, frequencies $o(q)$ for fixed blocks but macroscopic per-$q$ under the implemented sampling (no $q$-independent limit); the three-coordinate reduction is a truncation from which no rank is measurable (HEFF_DIM = 3 fixed in advance); the form is $O(q^{-2})$ on the whole unit sphere (normalisation fact), ruling out the $q^{-1}$ coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route (zero-form limit on the canonical filtration) | **proved** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q5b | Toward 4D Lorentzian geometry | [110] | Version 2.2: BFS stratification of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z}) \to$ Carnot foliation with $D_{\mathrm{hom}} = 4$; under [H-L] and [H-lift] a rank-three co-metric $\mathrm{diag}(-A_\tau, A_H, A_H, 0)$, whose non-degenerate block has signature $(-,+,+)$ under [H-hyp] applied to the reduced sector. [H-lift] open (Q9 does not discharge it); Q5b-O2 open (no lower-order term fills the central slot); no coefficient value established | **conditional** on [H-L], [H-lift], [H-hyp] (metric chain); **structural** (geometry) |
| Q6a | Toward gauge structure (version 2.3) | [69] | Gauge charges as fibre invariants; U(1) residual phase symmetry (span-based projection); SU(2) read off a supplied carrier (O23, O27), not selected; no use of O31; SU(3), colour factor and complete gauge group open | **open** |
| Q6b | Toward effective spacetime (version 2.1) | [68] | Rank-three co-metric; block $(-,+,+)$ under [H-hyp$_{\mathrm{red}}$], signs stipulated; leading Newtonian exterior for a supplied spherically symmetric, asymptotically flat extension with asymptotically homogeneous exterior; Schwarzschild form only with external dynamical input, no uniqueness; horizon interpretive; Einstein relation conditional (matching data, assumed matter functional, Lorentzian formulation not available); chain conditional at each arrow; no coefficient value | **conditional** on [H-L], [H-lift], [H-hyp$_{\mathrm{red}}$], a stipulated temporal term and a supplied rank-four extension; exterior statements also on its spherical symmetry and asymptotic flatness, Schwarzschild on external dynamical input |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q7 | Requirements for an equivariant bridge | [112] | Proves $[\mathfrak{su}(2)] \not\cong \mathfrak{heis}_3$, the uniqueness of the spin-1 module and $[F_c, L_{\mathrm{Weil}}] = 0$, and gives the closed form of the compressions on the O25 data; version 2.0 withdraws the Casimir reading, the isotropy fit and the transfer to the continuum symbol. The identification $H_{\mathrm{eff}} \cong \mathrm{Sym}^2(V_\rho)$ is supplied by no source. The exact Q5b principal-symbol target is excluded by rank; a new compatible positive target remains open | **proved** (requirements on the supplied carrier); **open** (the identification, and the new target under [ACT], [INV] and [ADAPT]) |
| Q8 | No central coefficient; scale-free Casimir rigidity | [105] | Version 2.0: the sub-Laplacian has no first-order part and a rank-two principal symbol, so the commutator supplies no central coefficient; $Z^2$ has homogeneous degree four and $A_Z/A_H$ scales as $r^2$, so isotropy selects a length scale; for a supplied rank-three target with full $\mathfrak{su}(2)$-invariance, $A_H = A_Z = 2\lambda$ with $\lambda$ free; no value of $A_Z$ derived | **proved** / **open** (target) |
| Q9 | Kinetic limit under [K] and [R] | [106] | A nonnegative modulation term leaves a supplied kinetic Mosco limit unchanged when compatible recovery sequences suppress it. The bound is $\mathcal{V}_q(f_q) \le 8\pi^2 r_q a_q(X)\|kf_q\|^2/q^2$. The free form selects no nonzero central character; a relative moment bound gives only an elementwise inequality on a cone. Q5b-O1, [H-lift] and bridge non-obstruction remain open (version 2.0) | **conditional** on [K] and [R] |
| Q10 | Toward coefficient isotropy | [108] | Version 2.0.1: sharp ratio lemma ($2\varepsilon/(1-\varepsilon)$); under [U], [W], [R], [T], $\lvert A_H/A_Z - 1\rvert \le 2\varepsilon/(1-\varepsilon)$; absolute value only from an independent scale [S]; exact parity for conjugately matched data; the inputs cited for [U] do not imply it; no value of $A_H$ derived | **proved** (lemmas); **conditional** on [U], [W], [R], [T], [S] |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| U1 | Obstruction to the generator route to [U] | [113] | Version 2.0 proves the exact equidistance of finite Heisenberg multiplication generators at distinct characters. A separate example excludes the claimed modulus for their sum. It establishes no proof of [U], no $O(q^{-1/2})$ rate, no small-$\theta$ conclusion and no closure of Q10 or Q7. The O25 observable is a sampled-block rank increment, not an identified compressed operator norm. [U] remains open | **proved** / **open** |
| W1 | Finite-window proxy; [H-w] open | [114] | Version 2.0.1 establishes no common positive limit of Q5a's fingerprint-response weights. Theorem 4 proves only a summation inequality for a separate proxy under a supplied relative profile bound, never proved by U1. Lemma 6 gives a depth-22 target under the specified five-block calibration, not an infinite-series coefficient; Corollary 7 exposes the missing proxy-to-weight identification. Q5a's directional [H-$w'$] and the stronger common [H-w] remain open. H2 [94] does not discharge them | **conditional** / **open** |
| H2 | [H2] proved | [94] | Strong convergence $\hat{X}_q \to x$, $\hat{P}_q \to -i\partial_x$ on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: quantitative Poisson aliasing lemma (rate $O(q^{-1})$), discrete Sobolev identity, quasi-isometry of sinc embedding; proves hypothesis [H2] of a Mosco-type framework that Q5a does not establish, whose [H-$\mathcal{E}1$] and [C] are not closed by the finite-rank route | **proved** |
| Q11 | Temporal coefficient free under spatial SU(2) (version 2.0) | [107] | Invariant forms on $\mathbb{R}_\tau \oplus W$ are $\alpha\, k_\tau^2 + \beta\, Q_{\mathrm{sp}}$ with independent scalars and no mixed term; no linear SU(2) action on four dimensions fixes a Lorentzian form up to scale; $\tau = n\,\Delta\tau$ is a modelling input; the cometric $\mathrm{diag}(-A_\tau, 2\lambda, 2\lambda, 2\lambda)$ under [H-L], [H-lift], [T], [INV$^+$]; the coupling datum fixing $A_\tau/A_H$ is missing | **proved** / **conditional** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q12 | Yang–Mills from $a_4$ (version 1.7) | [116] | Seeley–DeWitt $a_4 \supset \frac{1}{12}\mathrm{tr}_\rho(F^2)$ (gauge component); vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed metric yields $D_\mu F^{a\mu\nu} = 0$; induced log running $\propto I_\rho$ (non-universal), $g_{\mathrm{YM}}$ and $G_N$ renormalized matching data; sectors differ in UV divergence degree (proper-time scheme), no tuning-free hierarchy claimed (**proved** for the $a_4$ derivation at a supplied fixed metric and supplied compact gauge data; only the emergent-base reading consumes [H-L], [H-lift] and a supplied rank-four extension, and a Riemannian-to-Lorentzian continuation that is not performed) | **proved** / **conditional** on [H-L], [H-lift] and a supplied rank-four extension for the emergent-base reading |
| Q13 | Gauge–gravity spectral synthesis (version 1.5) | [84] | On supplied geometric and gauge data, joint $\delta_{g,\mathcal{A}} S^{\mathrm{ren}} = 0$ yields a conditional Einstein–Yang–Mills system; metric variation of the gauge kinetic term $= 2\tau_{\mu\nu}$; $a_6$ given as a spanning list modulo IBP and Bianchi (no unique cross-term, no UV divergence in $d = 4$); no non-linear completion asserted; $8\pi G_N = 1/(2c_{\mathrm{EH}})$ by canonical matching with $c_F I_\rho = 1/(4g^2_{\mathrm{YM}})$; couplings are matching data, no hierarchy claimed; Euclidean; Lorentzian continuation open; only the emergent-base reading consumes [H-L], [H-lift] and a supplied rank-four extension | **conditional** on the matched coefficients and, for the emergent-base reading, on [H-L], [H-lift] and a supplied rank-four extension |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q14 | Fermionic matter & chirality (version 3.1) | [109] | On a supplied Heisenberg carrier and supplied model data (real metaplectic model, doublet), the $\mathfrak{sl}_2(\mathbb{C})$ tensor algebra is proved; the Lorentz reading rests on [H-Spin] (spin solder, not supplied) and the electroweak reading on a distinct $U(2)$ weak factor [H-Weak] (not supplied); under [H-Spin], $E_\Pi$ is left-admissible on an input orientation branch, and with the explicit hypothesis $E_\Pi \preceq 0$ (not inherited from PRS) its off-diagonal chiral blocks vanish; $V-A$ is the representation content of [H-Weak]; anomaly constraints select the hypercharge twists up to sign, rescaling and $u_R \leftrightarrow d_R$, given a supplied colour module and minimal $L_Y = \wedge^2(E_{\mathrm{weak}})$ normalisation; a supplied rank-three selection rule yields $\mathbb{C}^3_{\mathrm{gen}}$; colour, masses, Yukawa couplings and CKM/PMNS mixing remain open | **conditional** on [H-Spin], [H-Weak] and the supplied colour module / **proved** (algebra on supplied model data) / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| PRS | Schur form of the compression remainder + Schur dichotomy | [103] | Under a contraction hypothesis the compression remainder of the projected Dirac square is the Schur complement $-N^\dagger N \preceq 0$, $N = (1-P)^{1/2} D\,\Pi_S^*$, zero-order only under the symbol condition (which forces 0 on irreducible Clifford fibres), and equal to Q14's $E_\Pi$ only when $\Pi_S D^2 \Pi_S^* =$ Lich; for any locking operator $0 \preceq P \preceq 1$ the Schur expression and its sign are unconditional; under [H-Gen] ($J_\Pi$ compatible with $D$ and $\Pi_S$, reading map $\Phi$) $u[P] = 0$ for $J_\Pi$-commuting $P$, automatic for the reference operator, so a non-zero $u$ needs $J_\Pi P J_\Pi^{-1} \neq P$, on an interpretive reading of A1–A4 only A4 can supply it; a $J_\Pi$-commuting locking has $E[P] = 0$ on the left-admissible branch; [H-Mult] separate (in the chiral rank-two left-admissible frame with a non-singular even sector, $u \equiv 0$ or no $\Phi$); the chiral equivariance defect is related to $u$ only as a structural reading; finite sector conditional on the open identification of $c \leftrightarrow q - c$ with $J_\Pi$; Schur-null / Schur-transverse dichotomy proved under explicit hypotheses, which locus holds open: the exact $\mathfrak{sl}_2$ opening $\alpha(x, y) = xy\,r/\sinh r$ (from $[E, F] = H$) reaches the locking operator only under [H-Tr] (supplied by no source); no sign of the Born–Infeld genus, no $u \neq 0$ and no mechanism for a non-zero split derived; open: [H-Tr], explicit $1 - P(s)$ (structurally constrained by EBJ), amplitude $\lvert u \rvert$, Yukawa | **conditional** / **proved** (algebra under explicit hypotheses) / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| AAR | Angular amplitude reduction: oriented shear product and central-phase observable (version 1.3) | [77] | BCH reduction in $\mathfrak{sl}_2$ identifies the per-step $J_\Pi$-odd $J_3$ component of the metaplectic cascade generator with the oriented shear product $ts$, produced by $[E, F] = H$ (Q14 Proposition 6.5), no Heisenberg bracket entering (no map between the two brackets supplied); observable is the accumulated oriented central-phase increment $A_c$ along the Weil–BFS cascade normalised by $\widehat{I}(n)$, that $A_c$ carries the $J_3$ signal being an input (analogy of form, not identification); equals $\varepsilon_{\mathrm{Weil}}(n_3^{\mathrm{obs}})$ to be compared with dictionary $\varepsilon = 1/10$ without fitting; O12 fingerprint: $B_c$ carries radial $\sigma_c$, $A_c$ carries angular split; symmetric BFS shells give $\Theta_{\mathrm{raw}} = 0$ identically; capacity audit fixes the first-shell (onset) coefficient $\frac{1}{3}$, $\theta_{\max}^{(1)} = 2\pi/(3q)$ (generator-dependent; the $\widehat{I}$-normalised bound at $n_3^{\mathrm{obs}}$ is $2.114/q$–$2.122/q$, 0.9–1.3 % above), the comparison with $\varepsilon = 1/10$ being a required-normalisation diagnostic, still gated | **structural** / **conditional** on the input that $A_c$ carries the $J_3$ signal |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Q11OF | Oriented frontier observable (version 1.1) | [102] | Symmetric-capacity obstruction makes raw shell observable vanish identically ($g \mapsto g^{-1}$ pairs opposite central phases); frontier transfer observable on directed outgoing edges $g \to gs$ with central increment Heisenberg cocycle $a(g)\, s_b$ (a counterpart of the orientation datum, not the $J_3$ coefficient of the metaplectic step, which comes from $[E, F] = H$); orientation is the arrow of the Cayley graph from the projection origin, no identification with $\partial_\tau$ made; first test is existence of oriented signal $\langle \Delta A_c \rangle_{\partial^+ S_m} \neq 0$; anti-bias control via symmetrised-frontier cancellation; the residual frequency reflection is the central Weil lift $R_b = W(-I) = \mathrm{FT}^2$ up to the metaplectic phase (proved), and $[\gamma_5, R_b] = 0$ is reduced to the hypothesis [H-WS] part (i) ($R_b$ acts on the chiral carrier as a scalar; the value $-1$ is part (ii), used only by AOG-Note), supplied by no source | **structural** / **conditional** on [H-WS] for the chiral lift |
| CHO | Chiral orientation of the directed frontier (version 1.1) | [78] | Resolves the chiral weight $\sigma_L$ deferred by Q11OF: under [H-WS] the route $\sigma_L = f(T(e))$ is closed by a parity argument (edge inversion and residual reflection both send $T(e) \mapsto -T(e)$, whereas the admissible weight is arrow-even and $\varphi$-odd, proved under [H-WS]); $\sigma_L$ is instead the Weyl representation type $\mathbf{2}$ vs $\bar{\mathbf{2}}$, the unique label separated by conjugation; edge phase is the linear cyclotomic character $\zeta_q^{\Delta A_c}$, with no Gauss or Maslov factor; [H-orient] established in the inherited central-phase lift and under [H-WS], not unconditionally; $u \neq 0$ not derived (would need the transport hypothesis [H-Tr]) | **conditional** on the inherited central-phase lift and [H-WS] / **proved** under [H-WS] |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| A4-Note | BI saturation margin of the chiral modulus (version 3.0) | [98] | Defines Lorentzian $\mathcal{B}_{\mathrm{sat}}(s)$, the Born–Infeld determinantal saturation margin of a supplied odd antisymmetric chiral two-form $F_{\chi,\mu\nu}(s)$ (antisymmetry lemma: the symmetric square is $J_\Pi$-even and cannot carry the orientation sign); second variation $\mu_\chi^2 = \frac{1}{2} P_{\chi,\mu\nu} P_\chi^{\mu\nu}$, the Lorentzian genus of $P_\chi$, not determined (the central direction is not temporal in the co-metrics of the corpus; no map from the generation operator to a two-form); hypotheses [H-Lor], [H-F], [H-Min], [H-Nd], [H-Coin] and, from PRS, [H-Gen], [H-Tr]; Born–Infeld a conditional input, neither derived nor unique; chart dependence of the quartic radicand coefficient and chart independence of the first contact proved, no selection between a boundary contact and an interior stationary point; for a collinear purely electric ray under [H-F], $t_* = \beta/\lvert\mathbf{E}_{F_0}\rvert$; Front D factorisation of $u(s_*)$ formal (needs also the $J_\Pi$-covariance and diagonal-form premises; no frame satisfies its hypotheses; the EBJ witnesses lie outside [H-Gen](i)), fixing no magnitude; no mechanism for a non-zero split, $\lvert u\rvert$ not predicted | **conditional** / **proved** (algebra under explicit hypotheses) / **open** (genus, lock, factorisation formal) |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| EBJ | Constrained jet of $1 - P(s) +$ chiral defect-rate identity | [100] | Constrains PRS open deliverable 1, the explicit Lorentzian $1 - P(s)$, structurally: for every smooth family and every smooth unitary transport the self-adjoint projector jet $1 - P(s) = Q_0 + sQ_1 + s^2Q_2 + O(s^3)$ is constrained — $Q_1 = [A, Q_0]$ is a purely off-diagonal Grassmann tangent ($Q_0Q_1Q_0 = 0 = (1 - Q_0)Q_1(1 - Q_0)$), $Q_2$ diagonal blocks pinned to $Q_1^2$ with opposite signs; the parity grading holds under $J_\Pi$-equivariance, and with a covariant moving spinorial embedding and a $J_\Pi$-even Dirac operator the residue jet is graded $E_0$ even, $E_1$ odd, the first-order $J_\Pi$-odd part of $E_\Pi^2$ is $\{E_0, E_1\}$, and $u'(0) = \langle J_3, \{E_0, E_1\}\rangle / \langle J_3, J_3\rangle$; antiunitary parity makes every odd-order generation-mixing coefficient vanish in every covariant family, with or without complex phases (even orders not excluded); an explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with $u'(0) = 11 \cdot 2^{1/4}/9 \neq 0$ (a finite witness with $J^2 = +1$, outside [H-Gen](i) of PRS, meeting the identity-reading clauses of [H-Mult] and (ii)–(iii) of [H-Gen]; not an instance of [H-Gen]); for the parity written in the chiral frame as $\Sigma_\tau \circ \mathrm{conj}$ with $\Sigma_\tau = \left(\begin{smallmatrix} 0 & \tau \\ \tau^{-1} & 0 \end{smallmatrix}\right)$, $[\Pi_{J_\Pi\text{-odd}}(1 - P)]_{LL} = \frac{1}{2}\Delta_\chi(P)$ and $[\Pi_{J_\Pi\text{-odd}}\dot{Q}(0)]_{LL} = \frac{1}{2}\partial_s\Delta_\chi(P)|_0$ determine one block of the odd tangent, not a unique generator; selection of the family, localisation of the odd tangent, normalisation of $\partial_s\Delta_\chi(P)|_0$, and generation mixing at even orders open, $|u|$ dictionary-bound | **proved** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| PYL | Conditional weak determinant line + generation-level ordering (version 3.0.1) | [111] | Derives no mass, Yukawa coupling or mixing. Algebra proved on supplied data: $\wedge^2$ is the unique one-dimensional functor among the symmetric and exterior powers, trivial on $\mathrm{SL}(2,\mathbb{C})$, so $\wedge^2(S_L)$ carries no abelian weight under [H-Spin]; under [H-Weak] (not supplied) $L_Y := \wedge^2(E_{\mathrm{weak}})$ carries $\mathrm{U}(1)_Y$; the model operator $\mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ orders three levels for $0 < u < \frac{1}{2}$, read as $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$ only under [H-Res] (not supplied; PRS's reading map $\Phi$ is itself a hypothesis there); level-to-generation map not established; open: weak linking carrier, $Y_\Pi$, norm, sign$(u)$, mixing | **conditional** / **proved** (algebra on supplied data) / **open** |
| PYO | Projected Yukawa operator + chiral polar factor (version 2.1) | [89] | Partial no-go on the generation block under [H-Res], [H-Sq], [H-Fac], [H-Grad] (supplied by no source): $\mathrm{Spec}(H_\Pi) = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so $E_\Pi^2$ fixes the squared Yukawa levels, not the morphism; $Y_\Pi = U_\Pi H_\Pi^{1/2}$ leaves $U_\Pi$ and $\lambda_Y$ invisible to $E_\Pi^2$; $[U_\Pi]$ is a rephasing class (moduli, Jarlskog phase); $A_\Pi^{\mathrm{odd}}$ vanishes, by representation theory, for the projection as defined, and is not the polar generator $\Omega$, which is $J_\Pi$-even and generally non-zero at the identity ($\mathrm{Sym}^2 g$ of the real cascade step, not a Yukawa morphism, has external entry 1/17); no Yukawa-morphism family in the corpus: physical polar class open, the odd-part choice a modelling choice no source justifies; in the framework of PRS (under [H-Gen]) a non-zero diagonal split needs a $J_\Pi$-breaking locking operator and no mechanism for it is derived (a conditional lead, not an established mechanism); depth hierarchy not derived | **conditional** / **proved** (linear algebra) / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| KUD | Charged-lepton Koide relation as unit dispersion | [99] | Mass-sector target note: $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$, so Koide $\Longleftrightarrow$ $\mathrm{CV}(r_\ell) = 1$ ($\sigma_r = \bar{r}$); separates the dispersion constraint from the hierarchy phase (node placement); the cascade ladder $(1, \frac{1}{2} + u, \frac{1}{2} - u)$, taken by hypothesis as a pinned normal form, underdisperses ($\mathrm{CV} \le 1/\sqrt{2}$, $Q \le 1/2$), so it cannot by itself reach $Q = 2/3$; maximum-entropy reading ($\mathrm{CV} = 1$) recorded as an open test (first-moment constraint + forced discrete-to-continuous bridge); target invariant, not a mass formula; scoped no-go excludes the linear self-adjoint determinant realisation of $s^2 - d^2$ (equipartition route otherwise open) | **proved** / **open** |
| TPC | Ternary phase carrier: colour, generations, Koide as one carrier | [97] | Speculative bridge: one complex three-phase carrier $z_g = S + A\, e^{i(\delta + 2\pi g/3)}$ — pure-phase part sum-zero ($1 + \omega + \omega^2 = 0$, colour-neutral), real part the square-root mass profile of KUD; singlet amplitude $S$ the master dial ($S = 0$ colour, $S \neq 0$ masses, $A = \sqrt{2}S$ equipartition/Koide, $\delta$-node the hierarchy); candidate capacity filter $\|r_{\mathrm{dbl}}\| \le \|r_{\mathrm{sing}}\|$ saturated at $\mathrm{CV} = 1$, whose selection of the saturated admissible edge by projection locking is a further hypothesis the cited sources do not supply; not a derivation, states the construction charge sheet | **open** |
| NPI | Neutrino mixing and projective particle identity | [86] | Structural note: interaction–propagation basis misalignment is generic, alignment requires an identity-stabilising projection channel; PMNS large mixing the unfixed default (anarchy-consistent), CKM near-diagonality the explanandum (conditional: common projective frame); Dirac–Majorana as conjugation resolution; falsifiable expectation: fewer stabilising channels $\Rightarrow$ larger mixing; no angles or masses derived | **structural** / **conditional** on a common projective frame |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| NSR-Note | No-go: operator doublets and erased history fibres | [88] | Weil carrier commutant $= \mathbb{C}P_+ \oplus \mathbb{C}P_-$ (no fermionic lift of the Hecke doublets); $C_4$ frame stabiliser; erased fibres canonical $(\mathbb{Z}_2)^r$ torsors; semi-linear $RK$ exchange of the odd axes; no-sequential-re-entry theorem $\pi v = 0 \Rightarrow Q\pi v = 0$ (all measures); history-fibre route requires a parallel oriented substrate channel (extension, not derivation) | **proved** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| AOG-Note | Spin-stratum type-rigidity: obstruction to the Galois action on the binary icosahedral stratum, conditional consequences for the chiral lift | [96] | $\varepsilon = 1/10$ is a theorem modulo the ADE case-selection gate: $\frac{1/2+\varepsilon}{1/2-\varepsilon} = \frac{3}{2} \Rightarrow \varepsilon = 1/10$ trivial and case-independent (proved), the first-principles status of the gate being left open; BI parity acts as complex conjugation ($\zeta_q \mapsto \zeta_q^{-1}, \sqrt{5} \mapsto \sqrt{5}$), in a distinct Galois factor from $\sigma = (\zeta_q \mapsto \zeta_q, \sqrt{5} \mapsto -\sqrt{5})$ (proved); its status as the base-fibre involution is a candidate hypothesis of the foundations, taken as a premise; character-table audit of $2I$: inner conjugations, $-I$ and $J_\Pi$ act trivially on $\mathrm{Irr}(2I)$ whereas $\sigma$ induces $(2\,2')(3\,3')$ (proved); no listed operation realises $\sigma$ under [H-WS] for $R_b$, the A4 hypotheses for the locking, modelling assertions for the tensor functors and Schur projection, and the modelling premise [M] that the generator list is complete, which implies carrier-preservation (conditional; the tower statement inherits [H-WS]); no link between the gate and $\sigma$ is established, the gate stays open; chiral-lift consequence: the lift, an input ([H-Spin] of Q14, not constructed in AOG-Note), induces no $\sqrt{5}$ action under the boost premise [B], so the residual caveat of CHO does not arise in the present spin stratum under [B], with $\mathcal{N}_A \neq 0$ under [H-Spin], [H-WS], [M], [B] and the inherited central-phase convention ($\rho_\chi = 1$ by definition of $\sigma_L$; conditional); concrete form: $R = 3/2$ a forced Cayley invariant on the order-five locus ($\{\frac{1}{6}, \frac{1}{4}\}$) vs $R = \frac{5}{3}$, $\varepsilon = \frac{1}{8}$ on neutral order-four, residue = the order-five selection (proved/open) | **conditional** (primary), the audit of the inner conjugations, $-I$ and $J_\Pi$ being **proved** and the other clauses and the chiral-lift consequence **conditional** / **open** for the gate |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| SRN | Spin as a transformation class of rotating wave modes | [95] | Conceptual and structural audit of the rotating-wave reading of spin, companion to the fermionic-matter sub-programme: spin is fixed by how components transform under rotations of physical space, not by rotation alone (scalar counterexample); central grading on tensor powers of the admissible module, with the central element acting as $(-1)^n$ on $V_\rho^{\otimes n}$ (proved); Veronese-null vs compact-real-form lemma excludes the O28 outer-product real-form selector (proved); spinorial-soldering indeterminacy: the derived data fix the complex carrier and its univalence but neither the spatial realisation, the rotation action $\rho_{\mathrm{sp}}$, nor its normalisation; corrects Q7 version 1.3 ($e_0$ is the weight $m = 0$ of the $j = 1$ module, not a $j = 0$ state; version 2.0 carries the corrected reading); Version 1.4 adds an independent downstream reconnaissance: $\overline{\psi_R}\psi_L$ carries exact Higgs-typed electroweak quantum numbers ($H$ or $\widetilde{H}$) in every tree-level Yukawa sector under $\mathrm{Spin}(3,1) \times SU(3)_c \times SU(2)_L \times U(1)_Y$ (proved, exact representation audit), permitting but not deriving a composite Higgs condensate; independent of the soldering audit above; three data left explicitly open (an ambient enlargement beyond a single copy of $V_\rho$, a canonical inter-sector identification, and the condensation dynamics itself) | **proved** / **open** |
| Spec2 | Relational spacetime | [48] | Operational metric from graph Laplacians; spacetime from spectral correlations | **structural** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| Gravity | Conditions for an IR Einstein sector | [17] | Cutoff ($a_0\Lambda^4$, $a_2\Lambda^2$) and zeta ($a_4 = \zeta_A(0)$) sectors separated; $c_{\rm EH}^{\rm ren}$, its sign and $G_N > 0$ are matching data, not predicted; supplied Riemannian metric; Einstein term locally dominant for $c_{\rm EH}^{\rm ren} \neq 0$ and $RL_4^2 \ll 1$ (power counting); BI determinantal UV completion admissible, not unique, under coherence-extensivity | **conditional** on supplied renormalised coefficients; **structural** (dominance); **open** (gravitational coupling) |
| Thermo | Conditional Einstein–matter coupling | [19] | $G_{\mu\nu} + \Lambda_{\rm ren} g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(\Pi)}$ from stationarity of $\Gamma_\Pi$, given supplied coefficients; $u \sim L^{-2}$ is not a temperature; no local first law | **conditional** on the supplied renormalised coefficients |
| Lorentz | No quartic graviton dispersion | [34] | Posited local four-derivative TT kernel factorizes; massless branch exactly $\omega^2 = \lvert\vec{k}\rvert^2$, no $A_4$, no $\ell_\chi$ bound; pole mass and in-in construction open; vector sector and any frame-dragging reading open (a $\Box\bar{h}_{0i}$ term survives harmonic gauge) | **proved** (negative, about the posited truncation); **open** (Lorentzian construction, vector sector) |
| LorCap | Lorentz factor from capacity | [54] | $\eta_n^\tau \to \beta\gamma$; $\Phi(\eta_n^\tau) \to 1/\gamma$; BI factorization and central-phase factorization (**proved**); $\mathcal{P}_\tau^{\rm Q5b}$ construction: closed by TempProj [56] | **conditional** on the Born–Infeld mobility budget |
| TempProj | Temporal residual map | [56] | $\mathcal{P}_\tau^{\rm Q5b}\Sigma_n^{\rm tot} = \sqrt{1 - B_n^2}$; uniqueness (three-input necessity); Lemma 1 of LorCap proved under [U] and the Born–Infeld lift | **conditional** on [U] and the Born–Infeld lift |
| LCII | Fibrewise capacity lapse | [25] | Fibrewise extension to non-homogeneous regime; $d\tau/dn\vert_x = R(x)/\gamma$; unified capacity arbitration: $1/\gamma$ (kinematic) and $R(x)$ (gravitational) as two facets of the same BI trade-off | **conditional** on the postulated occupancy–lapse identification and a supplied Lorentzian metric |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| LCIIo1 | No-go for the scalar free-fraction closure of the capacity–metric bridge | [23] | The free-fraction law $A_\tau = A_\tau^\infty(1-\epsilon)$ with $g^{\tau\tau} = -A_\tau$ gives $g_{\tau\tau}/g_{\tau\tau}^\infty = 1/(1-\epsilon)$, matching $1-\epsilon$ only at $\epsilon = 0$; LC-O2-O1 reopened | **proved** |
| LCo1 | Lorentz transformations from projective temporal ordering | [30] | Effective Lorentz transformations between inertial observers are uniquely fixed by the projective temporal capacity load $\beta$; length contraction $\ell = \ell_0/\gamma$ as a corollary | **proved** |
| LCo3 | Finite-$q$ corrections to the Carnot capacity identification | [27] | Parabolic Carnot formula deviates from the exact torus capacity by $\Delta_q(n) = 4m^2/q^2$ ($n = R + m$, $R = (q-1)/2$); quadratic identity exact, Carnot identification holds for $n \le R$ | **proved** |
| TopInv | Topological invariants | [67] | Charge quantisation ($\pi_1 \cong \mathbb{Z}$) and no monopoles on the canonical model (proved); projective chirality (Hopf fibration) established at the framework level, qualitatively; the extension to the full effective configuration space is open | **proved** / **open** |
| SMcm | Charge, mass, inertia | [16] | Mass and charge as BI-saturated responses | **structural** |
| Bell | Bell factorisation bridge | [13] | Non-injectivity alone refuted as sufficient; global-coupling obstruction not derived | **open** |
| BellNoGo | Redundant-extension no-go theorem | [87] | Non-injectivity has no Bell-discriminating power; correct obstruction is cross-context global-coupling absence (Fine's theorem) | **proved** |
| ENT-E1 | Weil fingerprint orthogonality | [73] | The $k$=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ factors as a global phase times a single discrete Fourier mode, so the admission residual $w_j = 1$ is an exact algebraic identity and not a numerical accident; for matched single-character blocks the closure hypothesis of ENT-Note becomes a theorem and the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ is derived analytically under the companion's supplied purity of the joint state (literal Schmidt pairing for independently sampled canonical O25 blocks: open) | **proved** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| ENT-E2 | Two entanglement entropies of the conjugate pair | [70] | Separates the admissible pair entropy $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ on a hypothesised atomic carrier (the explicit hypothesis (H-carrier), stated by E2 itself and derived nowhere; the singlet identification further conditional on Q3's [H-inv], for general canonical blocks) from the full residual-rank entropy $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ on the full Gram–Schmidt basis (unconditional for matched blocks by ENT-E1, conditional on $[\mathrm{H_{res}}]$ and the O17 toy-model transfer gap for general blocks); excludes the metaplectic dilation as a bridge for $[\mathrm{H_{res}}]$ | **conditional** on (H-carrier) and [H-inv]; **proved** for matched blocks |
| SchLamb | Fine-structure constant | [58] | $\alpha$ as projective invariant; Lamb shift and Schwinger correction | **structural** |
| Cosmol | Rotation curves / Hubble | [3] | Flat rotation curves and Hubble tension from a common effective mechanism; v2.0 adds the history-branching expansion sector under the explicit rank–time dictionary hypothesis ($n = \log a$), with the state-volume reading quantitatively excluded and QUMOND repositioned as input | **heuristic** / **conditional** on the rank–time dictionary hypothesis |
| Supra | Superconductivity | [4] | Projective phase locking as origin of superconducting coherence | **structural** |
| QFI | Quantum Fisher information as spectral witness | [47] | Condensed-matter companion to Supra: the scale-free growth of the quantum Fisher information density in $Ce_3Pd_{20}Si_6$ reduces to projective spectral data of the full nilpotent $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ substrate (edge spectral dimension $d_s \approx 4$, $f_Q(T) \sim T^{-1}$), only marginal for the single $w = 2$ sector, with the thermal closure $\Delta_\Pi(T) = \frac{d_s}{2} T$ from the heat-kernel response (regime structural; observed exponent heuristic; asymptotic value open) | **structural** / **heuristic** / **open** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| TrajBr | Trajectory branching / de Sitter limit | [92] | Exponential growth relocated from endpoint volumes (polynomial, Bass–Guivarc'h) to projectively distinguishable histories; canonical O12-compatible channel is $b$-only with exact Pell count, $h_b = \log(1+\sqrt{2})$ (generic probe, exact $q = 211$ witness); full-history channel proved strictly compressive by geodesic death, pinned rate $\log 2$ and exact class count $2^{n+2} - 4n - 1$ (generic $a$-mirror separation via the mixed two-anchor coefficient); v1.3.0 gates the $w(n)$ candidate by a conditional no-go (early low-rank window only, under dictionary and minimal mappings) | **proved** / **conditional** on the dictionary and minimal mappings |
| LowLCap | Low-$\ell$ from history capacity | [85] | Conditional small-$N$ capacity ceiling evaluated under a separate saturation hypothesis: Planck-calibrated, real-$m = 0$-corrected TT benchmark changes $\chi^2$ 27.26 $\to$ 17.70 in the leading Bessel+$n_{\mathrm{H}}$ cell; TE/EE transfer and saturation remain open; staircase envelope; 8/3 witness re-audited with observable $\sqrt{8/3}$ | **numerical** / **conditional** on the saturation hypothesis / **open** |
| ProjCE | Projection kernel / capacity ceiling | [90] | Amplitude-level projection derived as conditional expectation onto the distinguishable-record algebra (two classical characterisations, graded hypotheses, operator invariant under degraded selection); strict ensemble-power suppression by total variance, and the parameter-free ceiling $(N-1)/N$ — identically the Bessel envelope of the low-$\ell$ note | **structural** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| SpAdm-Note | The spectral admissibility sub-programme (Note 1) | [64] | Version 2.0 maps the five precursor papers, O1 and O3–O33 and the SGN/CC companions along bounded flux ⇒ envelope ⇒ Weil-sector capacity ⇒ $\dim_{\text{pair}}$ on a supplied carrier; the reciprocal $\beta^*$ prescription has no carrier on the measurement substrate (coincidence check); SU(2) carrier supplied conditionally; O31 withdrawal; O32 measurements; O33 exact finite structure; open: [H-color] beyond O32's two levels, the spin-$\frac{1}{2}$ sector, campaign beyond $q = 211$ | **synthesis** |
| EGNote | The emergent geometry sub-programme (Note 2) | [57] | Maps the twelve constituents (Q5a, Q5a-O2, Q5b, Q6b, Q7–Q11, U1, W1, H2). Q9 gives a conditional kinetic-limit criterion under [K] and [R] but does not discharge [H-lift]. Version 2.1: a rank-three co-metric under [H-L] and [H-lift], block signature $(-,+,+)$ under [H-hyp$_{\text{red}}$], endpoint $g^{\mu\nu} \propto \eta^{\mu\nu}$ only for a supplied extension under the Q11 Corollary 6.1 hypotheses and after rescaling $\tau$, no coefficient value; the identification Q7 version 2.0 records as supplied by no source remains an input; the exterior and Einstein statements taken from Q6b are aligned with Q6b version 2.1 | **synthesis** |
| GSNote | The Gauge Structure Sub-Programme (Note 3, version 2.2) | [60] | Organises Foundation, TopInv, SMcm, O23, O27–O32, Q6a, Q7 and Q12. Identifies no gauge group: U(1) structural, SU(2) conditional on a supplied carrier, SU(3) and the composite withdrawn at the source (O31); Q12 and Q13 take a supplied compact group. Gauge group identification only; Yang–Mills dynamics belongs to Note 8. Consolidation of the constituent papers; no new result | **synthesis** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| SGNote | The spectral gravity sub-programme (Note 4, version 2.3) | [104] | Organises Gravity, Thermo, Lorentz and BornInfeld into the conditions under which a supplied Riemannian metric supports an Einstein sector, its emergent reading being conditional on [H-L], [H-lift], a supplied rank-four extension and a continuation: $a_2$ infrared response of the projective spectral entropy, its variational restatement, the four-derivative Lorentzian truncation, and the determinantal BI completion. Consolidation of the constituent papers; no new result | **synthesis** |
| NIFNote | The non-injective foundations sub-programme (Note 5, version 1.5) | [62] | Synthesises ENI, Foundation, HeisenbergStructure and noscale with separate proved, conditional and supplied ingredients: ENI's one-way implication under its own hypotheses; immediate information loss and projective incompleteness under A2; temporal order conditional on [H-acyc]; irreducibility conditional on a representation dictionary, with non-commutation requiring dimension greater than one and a generating pair. The finite $S_3$ obstruction is sourced in HeisenbergStructure 2.1. The Heisenberg group, nontrivial central character and associated Weil action are supplied; carrier selection, the Born rule and Q5b's spatial limit [H-L] remain open. Consolidation of the constituent papers; no new result | **synthesis** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| FMNote | The fermionic matter sub-programme (Note 6, version 3.1) | [101] | Layered conditional status of fermions, chirality, hypercharge and generations, each layer naming its hypotheses: the $\mathfrak{sl}_2(\mathbb{C})$ tensor algebra is proved on supplied model data; [H-Spin] (Lorentz reading) and [H-Weak] (distinct U(2) factor: $\mathfrak{su}(2)_L$, $L_Y = \wedge^2(E_{\text{weak}})$, $V-A$) are separate hypotheses supplied by no source; Lorentz chirality (an input branch) is distinct from weak $V-A$; hypercharge constraints need both hypotheses, rigidity a supplied colour module; the generation factor uses a supplied rank-three carrier, its model operators read as restrictions of $E^2_{\Pi}$ only under [H-Res]; $R_b = W(-I)$ commutes with $\gamma_5$ only under the hypothesis [H-WS] part (i) (a scalar on the chiral carrier; the value $-1$, part (ii), is used only by AOG-Note), supplied by no source; the Schur form concerns a compression remainder, which is the projective endomorphism only under a symbol identity that no source states; a non-zero split, its Schur transport [H-Tr] and its Born–Infeld genus are hypotheses, and no mechanism for such a split is derived; PYL and PYO summarised with their typing (physical polar class open). Masses, Yukawa couplings and CKM/PMNS mixing remain open. Consolidation of the constituent papers; no new result | **synthesis** |
| QSNote | The quantum structure sub-programme (Note 7) | [63] | States the current, narrow scope of Q1 (unconditional rigidity theorem), Q2 (exact eigenvalue degeneracy), and Q3 (conditional singlet theorem) and records that no chain from Born–Infeld indiscernibility to phase coherence, a Tsirelson bound, or the Born rule is currently established. Consolidation of the constituent papers; no new result | **synthesis** |

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| Label | Short title | Key | Main contribution | Status |
|---|---|---|---|---|
| GGSNote | The gauge–gravity stratification subprogramme (Note 8, version 2.1) | [59] | On supplied geometric and gauge data, organises Q12 and Q13 around the spectral stratification principle ($a_2 \to$ gravity, $a_4 \to$ gauge), an organising reading rather than a theorem, with fixed-metric isolation of the gauge sector, $a_6$ as a representative spanning list, and neither a non-linear completion nor a numerical coupling hierarchy claimed; only the emergent-base reading is conditional, on [H-L], [H-lift], a supplied rank-four extension and a continuation. Consolidation of the constituent papers; no new result | **synthesis** |
| ENTNote | The entanglement subprogramme presentation note | [49] | Locates entanglement inside the spectral admissibility cascade: reduced entropy of a closed conjugate Weil pair as $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$, with structural monotonicity along the cascade; integrates the ENT-E1 and ENT-E2 results (version 1.2), the pair-sector $\log 2$ being conditional on E2's atomic-carrier hypothesis (H-carrier) and [H-inv], and records the residual epistemic stratification. Consolidation of the constituent papers; no new result | **synthesis** |
| LC-Synthesis | The Lorentz-capacity subprogramme synthesis | [61] | Gathers LorCap, TempProj, LCII, LCIIo1, LCo1 and LCo3 into the single chain BI budget $\to F^\tau = 1/\gamma \to g^{\mu\nu} = 2\eta^{\mu\nu} \to \Lambda(\beta) \to R(x)^2 = 1 - \epsilon(x) \to$ finite-$q$ domain, distinguishing the BI capacity sphere from the Lorentz group as isometry group of the effective metric; the co-metric coefficient values are pending revision. Consolidation of the constituent papers; no new result | **synthesis** |

# 7 Open Problems

## 7.1 Branch II (O-Series)

**O29 at larger primes.** The effective rank $r_{\mathrm{eff}} = 3$ is observed for all tested pairs at $q \in \{61, 151\}$, modally with seed sensitivity at $q = 101$, and for 103/105 pairs in the single-sample run at $q = 211$. Extension of the carrier diagnostics — Veronese collapse, $\mathfrak{so}(3)$ span and Schur commutant — to $q \in \{307, 401\}$ is pending.

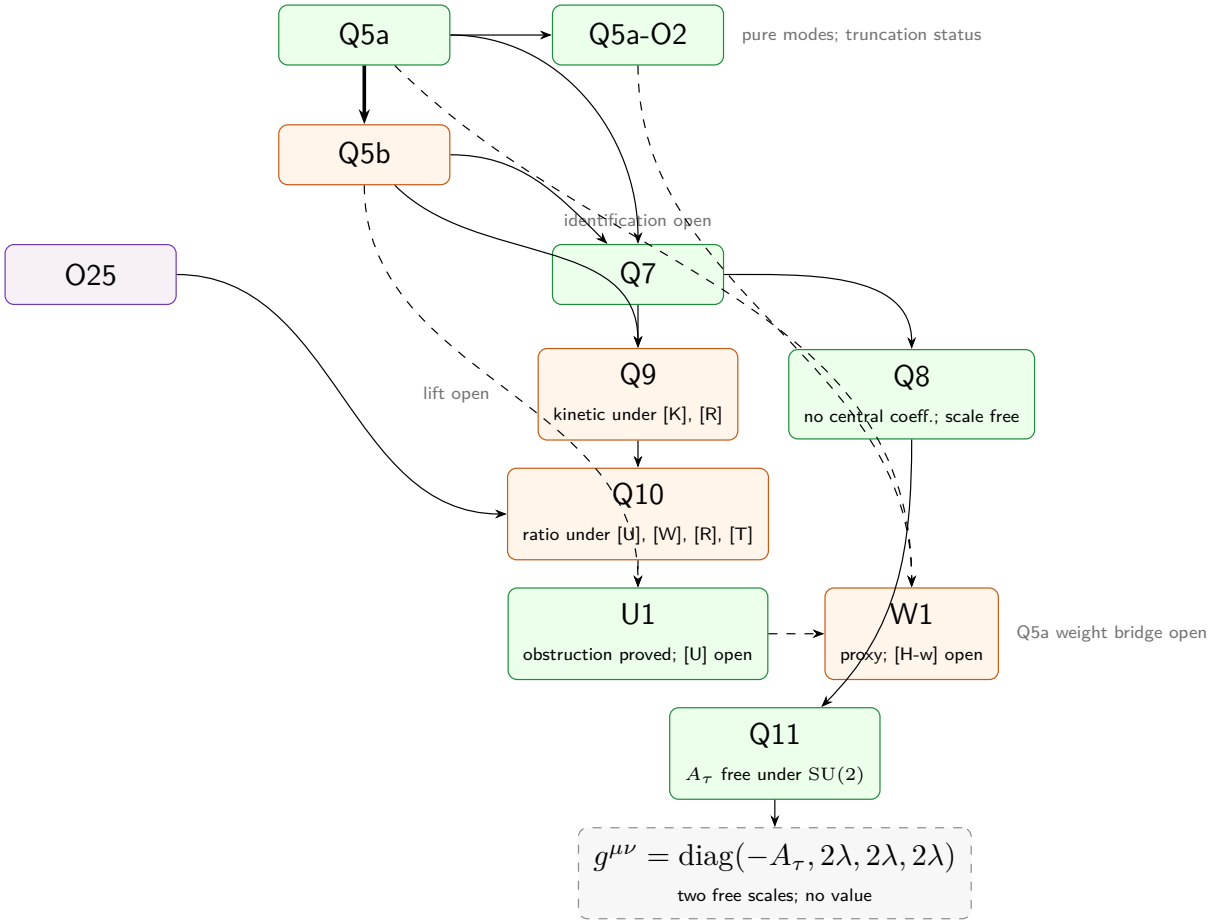



Figure 2: Geometric emergence sub-programme: dependency map from Q5a to the proposed effective Lorentzian co-metric. **Q5a-O2** [83] (**proved** for its stated results): pure Fourier modes of the pre-saturation pipeline; truncation status of the three-coordinate reduction; rules out the $q^{-1}$ coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route (zero-form limit on the canonical filtration, Q5a version 3.2). **Q9** (**conditional** on [K] and [R]): a nonnegative modulation term leaves a supplied kinetic Mosco limit unchanged when each finite-energy vector has one kinetic recovery sequence on which the modulation energy vanishes. The bound retains its energy scale; the free limit does not select a Schrödinger character or close [H-lift]. **Q8** (version 2.0; **proved**): the commutator supplies no central coefficient; a supplied fully invariant rank-three form has $A_H = A_Z$ with a free scale. **Q10** (version 2.0.1; **proved** for its lemmas, **conditional** on [U], [W], [R], [T] for the ratio): $A_H/A_Z \to 1$; an absolute value needs an independent scale [S]; no value derived. **U1** (**proved** for an unconditional obstruction; **open** for [U]): exact equidistance of distinct-character Heisenberg multiplication generators; a separate adjacent-character example obstructs the claimed summed-operator modulus. No proof of [U] and no $O(q^{-1/2})$ rate is established. **W1** (**conditional** for its proxy bound; **open** for [H-w]): a supplied relative profile estimate bounds a separately defined finite-window sum. No convergence of Q5a's response-average weights is proved; fibre constancy alone does not close their identification. **Q11** (version 2.0; **proved**): invariant forms are $\alpha\, k_\tau^2 + \beta\, Q_{\mathrm{sp}}$ with independent scalars; the datum coupling the temporal and spatial normalisations is **open**. No value of the co-metric $\mathrm{diag}(-A_\tau, 2\lambda, 2\lambda, 2\lambda)$ is established, and Q11 shows that spatial SU(2) invariance leaves $A_\tau$ and $\lambda$ independent: besides the unestablished spatial-limit hypothesis [H-L] of Q5a version 3.2/Q5b version 2.2.1, the open [H-lift], a full-rank extension that Q5b-O2 leaves open and the identification of the measured admissible space with the spin-1 module, which Q7 version 2.0 records as supplied by no source, the value 2 would need full $\mathfrak{su}(2)$-invariance of a supplied target and an independent scale (Q8, Q10), and the spatial value 2 would still leave $A_\tau$ free (Q11): Q7 is **proved** for the requirements it states on the supplied carrier and **open** for that identification. [H-F] is also unestablished.

**Non-conjugate block protocol.** A measurement protocol using non-conjugate block data remains an open construction. To probe $\mathrm{End}(V_\rho)$ it would require a typed bridge to that module; measured $H_{\mathrm{eff}}$ ranks alone cannot supply it. O26 2.1 excludes the rank-four target for $d_\rho = 2$ under its conjugate-product Hypothesis 4.4(a), without excluding other constructions.

**Notation consolidation: $c_{\mathrm{BI}} \to c_{\mathrm{BI}}$.** A coordinated rename of the BI saturation constant across all papers is planned and deferred pending consistency with already-deposited O-series papers.

### 7.2 Branch III (Q-Series and Applications)

**Bridge existence at finite $q$ (Q7–Q10).** Q10 version 2.0.1 gives the coefficient ratio only under [U], which U1 version 2.0 leaves open, and under the further inputs [W], [R], [T]; it derives no value of $A_H$. Q8 version 2.0 shows that a positive central coefficient needs a new operator of homogeneous degree four and a length scale. The exact Q5b rank-two principal symbol cannot realise the positive rank-three target. The revised Conjecture 4.7 asks instead for a new spatial construction supplying the target, action and compatible adapted transport. Abstract intertwiner existence is already guaranteed for the supplied isomorphic modules.

**$\pi_2(\mathcal{C}_{\mathrm{eff}}) = 0$ beyond the canonical model (TopologicalInvariants, Remark 4.2).** General proof of this topological property is open.

**Full Standard Model gauge group (Q6a, O31–O32).** O31 version 2.0 withdraws every derivation of $\mathrm{SU}(3)$ and of $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. O32 reports finite-sample numerical observations, but these do not identify a group or restore the withdrawn individual-profile claim. The colour factor, the complete gauge group, the interpretation of the O32 measurements, a transfer chain to $\alpha_s$, and the identification of quarks remain open.

**Baryon-to-photon ratio (SMchargemass).** Quantitative derivation of $\eta \sim 10^{-10}$ is open.

**$V - A$ chiral structure and fermionic matter (Q14).** Q14 [109] separates what the $V-A$ coupling rests on. Under the spin solder [H-Spin], the projected Dirac endomorphism $E_\Pi$ is left-admissible on an input orientation branch (Lorentz chirality); the chiral selection of the weak interaction is the representation content of the distinct weak factor [H-Weak] and is not derived from left-admissibility. Under both hypotheses, anomaly-cancellation constraints on the gauge-curvature $\gamma_5$-weighted $a_4$ terms select the hypercharge twists, given a supplied three-dimensional colour module, the minimal integral normalisation of $L_Y = \wedge^2(E_{\mathrm{weak}})$, the Standard Model content without $\nu_R$ and $Y_e \neq 0$, up to sign, rescaling and $u_R \leftrightarrow d_R$. The three-generation multiplicity follows, conditionally, from the supplied saturation rule $\sigma_c(n_3) = 3$. A qualitative mechanism for the inter-generation splitting is modelled (Q14 §6, on a distinct supplied generation doublet with $\mathcal{A}_3 \otimes \mathbb{C} \cong \mathrm{Sym}^2(V_{\mathrm{gen}})$: static obstruction, oriented $J_\Pi$-odd lift, exact $J_3$ component of the step generator). Residual open points: the amplitude of the splitting (the cascade normalisation $\varepsilon = \mathcal{N}_{\mathrm{casc}}|\alpha|$; the $c_{\mathrm{BI}} \to \delta_{\mathrm{pair}} \to \beta^*$ route has no native Heisenberg derivation), the explicit mass generation mechanism, generation mixing (which Q14 3.0 shows no $\mathfrak{sl}_2(\mathbb{C})$ step generator can produce on the outer weights), and the two hypotheses [H-Spin] and [H-Weak]. In the mass sector, PYL records on supplied model data that $\wedge^2$ is the unique one-dimensional functor among the symmetric and exterior powers and is trivial on $\mathrm{SL}(2,\mathbb{C})$: $\wedge^2(S_L)$ carries no abelian weight under [H-Spin], and under [H-Weak] the line $L_Y = \wedge^2(E_{\mathrm{weak}})$ carries $\mathrm{U}(1)_Y$ [111]. It also records that the model operator $\mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ has exit deficits that order three levels for $0 < u < \frac{1}{2}$, read as $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$ only under the named hypothesis [H-Res], which is not supplied; the level-to-generation map is not established and no mass value is claimed [111]. Under the premises [H-Res] and [H-Sq], supplied by no source, the Yukawa operator splits further: its hermitian

square $H_\Pi = Y_\Pi^\dagger Y_\Pi$ has $\mathrm{Spec}(H_\Pi)|_{\mathbb{C}^3_{\mathrm{gen}}} = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so $E_\Pi^2$ fixes the squared Yukawa levels, not the morphism [89]; what stays open is the chiral polar factor $U_\Pi$ in $Y_\Pi = U_\Pi H_\Pi^{1/2}$ (chiral orientation and complex mixing phase), the separate norm $\lambda_Y$, and hence the absolute masses and mixing. Under the grading premise [H-Grad], the polar factor is not canonical but a rephasing class $[U_\Pi] \in U(1)^3_R\backslash U(3)/U(1)^3_L$ (the moduli and a Jarlskog phase), non-trivial at first order iff its polar generator $\Omega$, the solution of $\Omega P + P\Omega = U^\dagger\dot{Y} - \dot{Y}^\dagger U$, has a transverse part [89]. The projection $A_\Pi^{\mathrm{odd}}$ of the $\mathfrak{sl}_2$ lift onto its $J_\Pi$-odd anti-Hermitian part vanishes, by representation theory, for the projection as defined, and does not determine the polar factor. At the identity $\Omega(0)$ is $J_\Pi$-even and generally non-zero, and the polar factor of $\mathrm{Sym}^2 g$ for the real cascade step $g = e^{tE}e^{sF}$, which is not a Yukawa morphism, has the external entry $1/17$ at $(t,s) = (2,1)$ [89]. The corpus supplies no family of Yukawa morphisms, so the physical polar class is open, and the choice of the $J_\Pi$-odd part as polar generator is a modelling choice that no source justifies [89]. The transverse datum has an internal block $e_0 \leftrightarrow e_\pm$ and an external block $e_+ \leftrightarrow e_-$; the external block has no matrix element in the image of $\mathfrak{sl}_2(\mathbb{C})$ in the step model of Q14 (Remark 6.4), and the internal block is neither excluded nor constructed as a physical statement [89]. The oriented shear-product normalisation $N_A$ is the real Cartan channel and may fix the diagonal split normalisation without determining the polar class [89]. In the A4-Note the first Born–Infeld contact of a collinear purely electric ray is $t_* = \beta/|\mathbf{E}_{F_0}|$ under [H-F], in a parametrisation-dependent form that selects no lock and makes no numerical prediction [98]. The angular onset coefficient is fixed, not the normalisation: a capacity audit on the exact Heisenberg–BFS frontier returns the exact onset $\frac{1}{3}$, $q$-independent over the verified primes $q \in \{61, 101, 151, 211, 307\}$, hence $\theta^{(1)}_{\max} = 2\pi/(3q)$ at the onset shell for the standard campaign, while the dictionary value $\varepsilon = 1/10$ is neither a capacity fit nor fixed by $\mathrm{Sym}^2(\mathbb{C}^2)$ alone [77, 96]. The mass-sector frontier thus separates into three layers: the present stratum does not derive a non-zero diagonal split (the angular coefficient datum $N_A \neq 0$ is conditional on [H-WS] in the inherited central-phase lift; $u \neq 0$ would need the transport hypothesis [H-Tr] of PRS, a $J_\Pi$-breaking locking operator and a determination of the Born–Infeld genus, none of which is supplied), it does not derive polar mixing (the physical polar class is open), and the dictionary value reduces — once the order-five $\sqrt{5}$-locus is selected, giving the forced invariant $R = 3/2$ against the neutral order-four alternative $R = \frac{5}{3}$, $\varepsilon = \frac{1}{8}$ — to the single ADE / ADE datum $R = 3/2$, equivalently the prime 5 or the $\sqrt{5}$-gate, which AOG-Note leaves open [96]. The third layer is a status statement: the projected-Yukawa split parameter $u$ and the angular datum $\varepsilon$ are the same functional of the level ratio, $\varepsilon(R_\Pi) = u$ with $R_\Pi = (\frac{1}{2}+u)/(\frac{1}{2}-u)$, so the equality $u = \varepsilon = 1/10$ is exactly the dictionary gate $R_\Pi = R_{\mathrm{ADE}} = 3/2$; the representation $\mathrm{Sym}^2(\mathbb{C}^2)$ leaves $u$ free, and the recursion of admissible operations, carrier-preserving relative to the modelling premise [M] of [96] (with [H-WS] for $R_b$), does not supply the Galois action $\sigma$, no link between $\sigma$ and the selection of the order-five value being established [89, 96]. The split is moreover not the mass hierarchy: under [H-Res] and [H-Sq], at the gate the squared Yukawa levels are $\lambda_Y^2\{1, \frac{3}{5}, \frac{2}{5}\}$, order-one ratios that cannot be the observed charged-fermion hierarchy by direct assignment, and the single-level cascade factor is itself order one [9]. The hierarchy is therefore carried, if at all, by generation-dependent cascade depths $n_g$ in the projective amplification law $\Lambda_{\mathrm{proj}}(n)$ in the exponential growth regime: the cascade exponent fixes the amplification rate, while the generation stabilisation depths remain the next structural datum, with $\lambda_Y$ excluded as a hierarchy carrier [89, 9]. The linear-growth route to these depths is closed: every depth extracted from the growth law $N(\lambda; n) \sim F_{\mathrm{KM}}(\lambda)\, n$ [9] — the level-crossing depth, the saturation rank, or any intermediate fraction — is a function of the same Kesten–McKay coefficients $c_g(p)$, hence inherits both the band-edge divergence and the symmetry pin $c_2 = \frac{1}{2}$, and cannot be at once stable in the cascade parameter and correctly ordered, for the inter-block and the stratigraphic inter-level partition alike [89]. The remaining positive route — a decaying capacity-controlled per-level profile $\sigma_{\lambda_g}(n)$ transported onto the cascade — is likewise tested and closed as a static route. Quantising the per-level resolution density rather than the count removes the band-edge denominator obstruction, the depth gap becoming independent of $c_g(p)$; but an exact audit shows

the Weil–Schur transport is scale-free, so the required stabilisation window is reached only by re-inserting the Laplacian valency $|S|$, the count normalisation, which returns to the already closed law $n_g = \Delta I_g / c_g(p)$ [89]. The static admissibility spectrum therefore orders three levels of the model operator under [H-Res], with an order-one level split (the level-to-generation map is not established), but not the charged-fermion hierarchy. The remaining dynamical route — reading the depths off the relaxation trajectory rather than the static spectrum — is also tested at first order and reproduces the same obstruction [89]. A parallel reading, with thresholds fixed by the generation carrier, makes the depth gap independent of the band-edge coefficient but leaves it on the order-one level-crossing scale; the sequential Born–Infeld and renewed-floor readings reach the hierarchy scale only through the cascade amplification itself or an unpinned ceiling-to-floor ratio, both of which reintroduce a free normalisation. The hierarchy is thus carried by no forced static or first-order dynamical quantity of the projective cascade: the cascade orders three levels of the model operator under [H-Res], with an order-one splitting and a band-edge-independent order-one depth gap, but does not fix the charged-fermion hierarchy at this stratum.

**Gauge–gravity synthesis (Q12 → Q13).** Gravity and Q12 together give, conditionally on supplied matching data, the Einstein ($a_2$) and Yang–Mills ($a_4$) sectors of the single functional $S_\Pi[g, \mathcal{A}]$ — the former from the horizontal variation, the latter from the vertical one: at fixed base metric the vertical variation isolates the Yang–Mills equation, but the two sectors are *not* independent under a joint variation, the metric variation of $F^2$ being the Yang–Mills back-reaction $T^{\mathrm{YM}}_{\mu\nu}$. Q13 [84] carries out the joint variation of a single matched local action and obtains the conditional coupled Einstein–Yang–Mills system, with $8\pi G_N = 1/(2c_{\mathrm{EH}})$ fixed by canonical matching. No numerical hierarchy ratio and no non-linear completion are asserted: $g_{\mathrm{YM}}$, $G_N$ and $\Lambda_{\mathrm{ren}}$ are independent renormalized matching data. Residual open points: whether a well-defined non-linear completion of the joint functional exists at all, the quantitative $a_6$ coefficients (after reduction of the spanning list to a minimal independent set), whether admissibility constrains any combination of the matching data, the Lorentzian continuation in the geometric and gauge sectors, and the extension to projected matter currents $J^{a\nu}$.

**Gauge-group identification upstream of Yang–Mills dynamics (Q12, O31).** Q12 derives the Yang–Mills equations for a supplied compact $G_\Pi$ from the $a_4$ heat-kernel coefficient. O31 version 2.0 withdraws the proposed SU(3) identification, so the colour factor and the complete gauge group remain inputs rather than derived results.

**Lorentz Capacity Sub-Programme.** The Lorentz-capacity sub-programme is closed for inertial kinematics, up to the co-metric coefficient values it takes from Q5b–Q11, which are pending revision; it is open in its non-homogeneous extension. TempProj closes the $\mathcal{P}_\tau^{\mathrm{Q5b}}$ construction of LorCap; LCIIo1 [23] (version 2.0) *reopens* LC-O2-O1, establishing that the scalar free-fraction law $A_\tau(x) = A_\tau^\infty(1 - \epsilon(x))$ does not derive $R(x)$ from $\epsilon(x)$: with $g^{\tau\tau} = -A_\tau$ it yields $g_{\tau\tau}/g^\infty_{\tau\tau} = 1/(1-\epsilon)$, agreeing with the capacity definition only in the homogeneous vacuum. LCo1 [30] closes LC-O1: the effective Lorentz transformations between inertial observers are uniquely fixed by the projective temporal capacity load $\beta$, yielding length contraction $\ell = \ell_0/\gamma$ as a corollary. LCo3 [27] closes LC-O3: the parabolic Carnot identification holds for $n \le R := (q-1)/2$ with deviation $\Delta_q(n) = 4m^2/q^2$ for $n = R + m$, while the quadratic identity $B_n^2 + (F_n^\tau)^2 = 1$ is exact.

**Thermodynamics: the integrability bridge, Lorentzian extension and matter coupling.** The v1.1 local spectral first law was withdrawn in v2.0: the heat-kernel quantity $u \sim L^{-2}$ is an eigenvalue scale, not a temperature, so no first law follows from the determinant alone. Establishing one requires an independently defined heat one-form $\xi_{\mathrm{sp}}$ on configuration space, a physical energy notion, and a Carathéodory–Jauch–Roberts integrability result showing $\ker \xi_{\mathrm{sp}}$ is

flat, hence $\delta Q = T\,\mathrm{d}S$. Analytic continuation and the explicit form of $W_\Pi$ for matter fields also remain open.

# 8 Conclusion

The Cosmochrony corpus is organised as a coherent, multi-level programme toward the derivation of physical structure from a single primitive. Branch I (Foundation, ENI, and HeisenbergStructure [2]) is not formally closed. The $S_3$ countermodel proves that the published inputs do not force centrality or a Heisenberg/Weil carrier. The carrier remains a supplied realisation for the downstream mathematical programme. The binary-polyhedral maximality conjecture (SpectralCapacity [53]) is proved: $2I$ uniquely maximises $\mathcal{C}_\alpha(G,S)$ for all $\alpha > 0$ among finite SU(2) subgroups with neutral generating sets, closing the precursor programme for the O-series. Branch II (O-series) has measured the pair exponent on the supplied conjugate-pair sector through $q = 601$ (O24, O25), the Born–Infeld fibre identification remaining open (O18). SGN proves that the subsequent conversion to $\beta^*$ is not derivable natively on the Heisenberg measurement substrate: its two corpus-defined closures, square-root frontier, pair-growth carrier, and exponent-coordinate identification fail. The observed numerical agreement with the mass-fitted window therefore remains a phenomenological coincidence check rather than a mass-hierarchy derivation. O29 reports finite-checkpoint numerical rank, Veronese, commutator, and commutant diagnostics that are compatible with a supplied adjoint carrier but do not identify it. Its auxiliary PCA plane is not the spin-$\frac{1}{2}$ module, and neither $d_\rho = 2$ nor a Veronese square root follows from the measurements. O30 studies a distinct conditional $3\times3$ model on $\mathrm{Sym}^2(V_\rho)$: (H0)–(H3) give the equatorial form, and two weighted cancellations give the ratio $[1 : \frac{1}{2} : \frac{1}{2}]$ within that model. It does not derive O28's measured $9\times9$ spectrum, which remains empirical. O31 version 2.0 is a withdrawal notice: the proposed colour-triplet co-admissibility, arithmetic and group-identification results do not survive audit. The colour factor and complete Standard Model gauge group are open, and O32 numerical measurements supply no replacement group derivation [115]. Branch III (Q-series and companions) develops conditional mathematical models toward quantum mechanics, spacetime, and Standard Model observables. It contains proved internal results but does not derive those physical structures from the foundational axioms as a closed chain. The SU(2) quantum sector is not established: Q3 proves, conditional on an explicit, undemonstrated diagonal-$2I$-invariance hypothesis, that the proto-state is the singlet for all five admissible sectors of $2I$ via Schur's lemma, with the universal correlator $E(\hat{a},\hat{b}) = -j(j+1)/3 \cdot (\hat{a}\cdot\hat{b})$ (**structural**); the invariance hypothesis itself, phase coherence, and the Born rule remain open (**open**). The geometric emergence sub-programme has advanced substantially: Q9 gives a kinetic Mosco limit under [K] and [R], with no derived [H-lift] or $A_H > 0$ coefficient; Q10 version 2.0.1 bounds the ratio $A_H/A_Z$ under [U] and three further inputs and derives no value; Q8 version 2.0 shows that the commutator supplies no central coefficient and that invariance leaves the scale free; Q5b version 2.2.1 records [H-lift], the full-rank extension and the coefficient values as open; U1 version 2.0 establishes no proof of [U] and no $O(q^{-1/2})$ rate; it proves an unconditional equidistance obstruction for finite Heisenberg generators and leaves [U] open. W1 version 2.0.1 establishes no proof of [H-w]: its conditional sum bounds a proxy, while the identification with Q5a's response-average weights and their positive limits remain open. Q11 [107] version 2.0 proves that invariance under a spatial SU(2) action leaves the temporal coefficient $A_\tau$ independent of the spatial scale, so no value of the co-metric $\mathrm{diag}(-A_\tau, 2\lambda, 2\lambda, 2\lambda)$ is established and fixing $A_\tau/A_H$ needs a datum coupling the temporal and spatial normalisations. Any metric interpretation additionally requires the independent, unestablished spatial-limit hypothesis [H-L] and the identification of the measured admissible space with the spin-1 module, which Q7 version 2.0 records as supplied by no source; [H-F] remains separately open. Q5b version 2.2.1, Q8 version 2.0, Q10 version 2.0.1 and Q11 version 2.0 carry the current statements; the consumer entries that still assert the former coefficient values are pending revision, and the marker is carried on the surfaces that assert the affected results. Q12 [116] completes the spectral derivation of gauge dynamics: the gauge component of the Seeley–DeWitt $a_4$ coefficient of the operator $A_{g,\mathcal{A}}$ on the admissible vector bundle contains the Yang–Mills density $\frac{1}{12}\mathrm{tr}_\rho(F^2)$, and the vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed metric yields $D_\mu F^{a\mu\nu} = 0$; the induced logarithmic running is non-universal (Dynkin index $I_\rho$), while $g_{\mathrm{YM}}$

and $G_N$ remain renormalized matching data. Q13 [84] carries out the gauge–gravity synthesis, obtaining the conditional coupled Einstein–Yang–Mills system from the joint variation of a single matched local action, with $8\pi G_N = 1/(2c_{\mathrm{EH}})$ fixed by canonical matching. What is structural, within the proper-time scheme, is the difference in UV divergence degree between the sectors ($a_2$ quadratic vs. $a_4$ logarithmic); the numerical hierarchy $G_N g_{\mathrm{YM}}^2$ is not a prediction, both couplings being independent matching data. Q14 [109] states what the fermionic sector rests on: on supplied model data the $\mathfrak{sl}_2(\mathbb{C})$ tensor algebra is proved, its Lorentz reading rests on the spin solder [H-Spin] and its electroweak reading on a distinct $U(2)$ weak factor [H-Weak], neither supplied (the independent fermionic weak doublet is a distinct object, excluded at the operator and history strata [88]); under [H-Spin], $E_\Pi$ is left-admissible on an input orientation branch, while the chiral selection of the weak interaction is the representation content of [H-Weak]; under both, anomaly constraints select the hypercharge twists, with a supplied colour module and the minimal normalisation of $L_Y = \wedge^2(E_{\mathrm{weak}})$, up to sign, rescaling and $u_R \leftrightarrow d_R$; and the supplied saturation rule $\sigma_c(n_3) = 3$ yields a gauge-singlet three-generation factor $\mathbb{C}^3_{\mathrm{gen}}$. A qualitative mechanism for the inter-generation splitting is modelled in Q14 §6 on a distinct supplied generation doublet (a static $J_\Pi$-protected degeneracy which the $J_3$ component of the step generator $\log g$ can lift), its value being dictionary-bound. Together, Q12–Q14 organise a conditional common spectral framework for supplied geometric, gauge, colour, and carrier data. They do not derive the complete Standard Model field content from non-injective projection: the colour factor, several carrier selections, [H-L], masses, Yukawa couplings, and mixing data remain open or supplied. *Interpretive outlook.* Beyond the individually labelled results above, the programme's unifying reading can be stated in one sentence — it is a *spectrally stratified theory of emergent interactions*, in which the type of a fundamental interaction is determined by the Seeley–DeWitt order at which the admissible operator responds. This reading is an interpretive synthesis, not a theorem: its force rests on the proved and structural results catalogued above, and specifically on the qualitative separation of spectral orders, not on any quantitative claim about the couplings, which are renormalized matching data throughout. Q5a-O2 [83] establishes the pure-Fourier-mode structure of the pre-saturation pipeline, the truncation status of its three-coordinate reduction (no rank measurable from it), and the $O(q^{-2})$ upper bound ruling out the $q^{-1}$ coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route, their disposition being the zero-form Mosco limit on the canonical Fourier-window filtration of Q5a version 3.2. The H2 paper proves [H2] (strong convergence of the rescaled Weil generators $\hat{X}_q \to x$ and $\hat{P}_q \to -i\partial_x$) via a quantitative Poisson aliasing lemma and a discrete Sobolev identity.

For an external reader, three entry sequences are suggested. For the quantum structure thread: ENI → Foundation → Q1, with the proposed O18 → O22 → O23 fibre chain treated as an open auxiliary route rather than a proved foundation. For the spectral mass-hierarchy question: ENI → Foundation → O16 → O24 → SGN, ending with the native-transfer no-go and leaving the numerical mass comparison phenomenological. For the spacetime thread: Spectral2 → Gravity → (Thermodynamics or Lorentz); for the geometric emergence thread: Q5a, Q5b, Q7–Q11, U1, W1 and H2, with Q9's kinetic result independent of Q7 and Q8 and with the geometric and coefficient steps still open.

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[46] J. Beau. Quadratic Completion of Admissible Spectral Pairs via Binary-Icosahedral Representation, 2026. O26 (version 2.1): dictionary between conjugate Weil blocks and rank-one matrix coefficients; Level I growth-exponent identification proved in the pre-saturation regime; Level II conjectural; Level III conditional on the admissible embedding. Under Hypothesis 4.4(a), embedded products lie in $\mathrm{Sym}^2(V_\rho)$, with rank at most three for $d_\rho = 2$, excluding the rank-four target under this contract. Tests 1–2 are implementation checks; measured ranks in $\mathrm{End}(H_{\mathrm{eff}})$ are not ranks in $\mathrm{End}(V_\rho)$ and occupy no row of the embedded-covariance decision table, doi:10.5281/zenodo.19488845.

[47] J. Beau. Quantum Fisher Information as a Spectral Entanglement Witness: the Projective Stability Gap and the Origin of Scale-Free Growth, 2026. Cosmochrony condensed-matter companion note, doi:10.5281/zenodo.21001565.

[48] J. Beau. Relational Reconstruction of Spacetime Geometry from Graph Laplacians, 2026. Cosmochrony companion paper A, doi:10.5281/zenodo.18356037.

[49] J. Beau. Residual-Rank Entanglement of Conjugate Weil Pairs in the Spectral Admissibility Cascade, 2026. Entanglement sub-programme presentation note: computes the entanglement entropy of a closed conjugate Weil pair as $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$, the logarithm of the residual fibre-level Gram–Schmidt rank, with structural monotonicity along the spectral admissibility cascade, doi:10.5281/zenodo.20498835.

[50] J. Beau. Shell-Alignment from Projection Locking: A Discrete Admissibility Theorem under Born-Infeld Saturation, 2026. Spectral admissibility sub-program "O22" paper, doi:10.5281/zenodo.19367375.

[51] J. Beau. Shell-Level Saturation and the Conditional Fibre-to-Rate Prescription, 2026. Spectral admissibility sub-program "O21" paper, doi:10.5281/zenodo.19358961.

[52] J. Beau. Spectral Admissibility under Bounded Flux: Non-Abelian Mode Selection on Cayley Graphs of SU(2) and SU(3) Subgroups, 2026. Step A of Cosmochrony Spectral Admissibility sub-Program, doi:10.5281/zenodo.18944985.

[53] J. Beau. Spectral Capacity Functional and the Binary-Polyhedral Maximality Conjecture, 2026. Step B of Cosmochrony Spectral Admissibility sub-Program, doi:10.5281/zenodo.18945273.

[54] J. Beau. Spectral Mobility and the Lorentz Factor from Projective Capacity Saturation, 2026. Cosmochrony technical note N, doi:10.5281/zenodo.20299014.

[55] J. Beau. Systematic Pair-Level Campaign for $\delta_{\mathrm{pair}}$: Convergence, Inter-Pair Concentration, and Normalization Structure, 2026. O25 (version 1.5): systematic pair-level campaign for $\delta_{\mathrm{pair}}$ over all conjugate pairs through $q = 211$, extended by capped BFS to $q \in \{307, 401, 601\}$; the raw exponent descends into $[7.4, 10.6]$ on its own (7.61 at

$q = 601$) while the O14 correction overcorrects at large $q$; $\delta_\infty$ not pinned; the reciprocal image 0.108–0.123 is a phenomenological coincidence, not a capacity-to-rate inference, doi:10.5281/zenodo.19462132.

[56] J. Beau. Temporal Residual Map from the Completed Principal Symbol: Construction, Uniqueness, and the Lorentz Identification, 2026. Lorentz Capacity sub-programme: temporal residual map construction, doi:10.5281/zenodo.20316526.

[57] J. Beau. The Emergent Geometry Sub-Programme, 2026. Presentation Note 2: maps the twelve constituent papers (Q5a, Q5a-O2, Q5b, Q6b, Q7–Q11, U1, W1, H2) on a supplied Heisenberg carrier; a rank-three co-metric under [H-L] and [H-lift], block signature $(-,+,+)$ under the hyperbolicity hypothesis, endpoint $g \propto \eta$ only for a supplied extension under the hypotheses of Q11 Corollary 6.1 and after rescaling $\tau$, and no coefficient value, doi:10.5281/zenodo.20417961.

[58] J. Beau. The Fine-Structure Constant as a Projective Invariant: Lamb Shift and Schwinger Effect in a Pre-Geometric Framework, 2026. Cosmochrony companion paper F, doi:10.5281/zenodo.18602702.

[59] J. Beau. The Gauge–Gravity Stratification Sub-Programme, 2026. Presentation Note 8: organises the two constituent papers (Q12, Q13) around the spectral stratification principle $a_2 \to$ gravity, $a_4 \to$ gauge, an organising reading. On a supplied Riemannian base metric, whose emergent reading is conditional on [H-L], [H-lift], a supplied rank-four extension and a continuation, Q12 gives the Yang–Mills equations $D_\mu F^{a\mu\nu} = 0$ from the vertical $a_4$ variation at fixed metric; Q13 gives the conditional Einstein–Yang–Mills system from the joint variation of a single matched local action, with $8\pi G_N = 1/(2c_{\mathrm{EH}})$ fixed by canonical matching and the couplings as independent renormalized data. It records fixed-metric isolation of the gauge sector rather than full horizontal–vertical decoupling, $a_6$ as a representative spanning list of dimension-six invariants modulo integrations by parts and Bianchi identities, and neither a non-linear completion nor a numerical coupling hierarchy, doi:10.5281/zenodo.20563349.

[60] J. Beau. The Gauge Structure Sub-Programme, 2026. Presentation Note 3: audits the gauge-structure chain. The phase-fibre and supplied SU(2) sectors are status-separated, while O31 version 2.0 withdraws the claimed colour-triplet derivation; SU(3) and the complete Standard Model gauge group remain open, doi:10.5281/zenodo.20435029.

[61] J. Beau. The Lorentz-Capacity Sub-Programme: Projective Origin of the Lorentz Factor in the Cosmochrony Programme, 2026. Lorentz Capacity sub-programme synthesis: projective origin of the Lorentz factor, unifying LorCap, TempProj, LCII, LC-O1, LC-O2-O1, and LC-O3, doi:10.5281/zenodo.20386737.

[62] J. Beau. The Non-Injective Foundations Sub-Programme, 2026. Presentation Note 5, version 1.5: synthesises ENI's one-way implication under its hypotheses, Foundation's immediate information loss and A2 projective incompleteness, temporal order conditional on [H-acyc], and irreducibility conditional on a representation dictionary. Non-commutation requires dimension greater than one and a pair generating the acting group. HeisenbergStructure 2.1 supplies the finite carrier-selection obstruction. The Heisenberg group, non-trivial central character and associated Weil action remain supplied inputs; the Born rule and Q5b spatial-limit hypothesis [H-L] remain open, doi:10.5281/zenodo.20548383.

[63] J. Beau. The Quantum Structure Sub-Programme, 2026. Presentation Note 7: status-separated map of Q1, Q2, Q3, the published Bell paper, and BellNoGo. Q1 proves Fourier-support rigidity and a no-go result; Q2 proves one finite Laplacian degeneracy; Q3's singlet and Casimir correlator require its explicit undemonstrated invariance hypothesis; and

BellNoGo proves that bare non-injectivity does not explain Bell violations. Phase coherence, the Born rule, a Tsirelson bound, sector selection, and an SU(2) fixed point remain open, doi:10.5281/zenodo.20562949.

[64] J. Beau. The Spectral Admissibility Sub-Programme, 2026. Presentation Note 1, version 2.0: maps the five precursor papers, O1 and O3–O33 and the Span-Growth and Critical Coverage companion notes along bounded flux ⇒ admissibility envelope ⇒ Weil-sector capacity ⇒ $\dim_{\text{pair}}$ on a supplied Heisenberg carrier and its associated Weil action; records that the reciprocal $\beta^*$ prescription has no carrier on the Heisenberg measurement substrate (coincidence check), the conditionally supplied SU(2) carrier, the O31 withdrawal, O32 measurements and O33 exact finite structure; open: [H-color] beyond O32's two levels, the spin-$\frac{1}{2}$ sector, and a campaign beyond $q = 211$, doi:10.5281/zenodo.20402946.

[65] J. Beau. Three Neutral Directions from a Spinor Carrier: Conditional Status of the Threefold Admissibility Threshold, 2026. Spectral admissibility sub-program "O23" paper, doi:10.5281/zenodo.19375136.

[66] J. Beau. Three Particle Generations and Mass Hierarchy from Spectral Stratigraphy, 2026. Spectral admissibility sub-program paper, doi:10.5281/zenodo.19040490.

[67] J. Beau. Topological Invariants of Admissible Configurations: Charge Quantisation, Absence of Magnetic Monopoles, and Projective Chirality in Cosmochrony, 2026. Cosmochrony companion paper K, doi:10.5281/zenodo.19282962.

[68] J. Beau. Toward Effective Spacetime Geometry from Admissible Non-Injective Projection: A Conditional Rank-Three Co-Metric and Newtonian Exterior, 2026. Emergent-geometry paper Q6b (version 2.1): under [H-L] and the open [H-lift], a rank-three co-metric; a leading Newtonian exterior for a supplied spherically symmetric, asymptotically flat rank-four extension with an asymptotically homogeneous exterior, and a Schwarzschild form only with external dynamical input, doi:10.5281/zenodo.19744005.

[69] J. Beau. Toward Gauge Structure from Non-Injective Projection: Fibre-Invariant Charges, a Supplied SU(2) Carrier, and a Conditional Connection, 2026. Gauge-structure paper Q6a (version 2.3): gauge charges as fibre invariants; U(1) as the residual phase symmetry for a span-based projection; SU(2) on the spinor carrier supplied by O23 under the neutral-sector hypothesis of O27, not selected; SU(3) a hypothesis; connection and curvature conditional on two continuum hypotheses, doi:10.5281/zenodo.19655294.

[70] J. Beau. Two Entanglement Entropies of the Conjugate Weil Pair: the Admissible Pair Sector and the Full Residual Rank, 2026. Entanglement sub-programme "E2" paper: separates the admissible pair entropy $S_{\text{ent}}^{\text{adm}} = \log 2$ from the full residual-rank entropy $S_{\text{ent}}(n) = \log r_{\text{pair}}(n)$. The first requires E2's own explicit hypothesis [H-carrier] and Q3's independent [H-inv]; the rank-three threshold is a supplied O23 selection rule and no mean-zero theorem removes the zero mode. The second is unconditional for matched blocks by E1 and conditional on [$\text{H}_{\text{res}}$] for general blocks; the metaplectic dilation is excluded as a bridge, doi:10.5281/zenodo.20499440.

[71] J. Beau. Veronese Collapse in the Measured Trajectory Span: What the O28 Covariance Does and Does Not Identify, 2026. O29 (version 2.0): audits what the O28 rank identifies; rank three modal, not invariant (32/50 at $q = 101$, 103/105 at $q = 211$); measured span, PCA truncation and supplied spin-$\frac{1}{2}$ module separated; the projected-coordinate relation is satisfied only approximately; finite-precision Veronese, commutator and commutant diagnostics compatible with the supplied adjoint reading but deriving no carrier, doi:10.5281/zenodo.19787221.

[72] J. Beau. Weil-Block Projective Capacity on Heisenberg Graphs: Resolving the Representation Obstruction and Extracting the Capacity Exponent, 2026. Spectral admissibility sub-program "O11" paper, doi:10.5281/zenodo.19169987.

[73] J. Beau. Weil Fingerprint Orthogonality and the Flat Schmidt Spectrum of Conjugate Pairs, 2026. Entanglement sub-programme "E1" paper: proves analytically that the $k$=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ is a discrete Fourier mode, upgrading the numerical hypothesis $w_j = 1$ of the entanglement-note to a theorem and deriving the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ for matched conjugate-pair blocks, doi:10.5281/zenodo.20499309.

[74] J. Beau. Why Exact Heisenberg Capacity Does Not Determine the Phenomenological Cascade Rate: Observable Heterogeneity, Normalisation, and the Transfer Boundary, 2026. O14 (version 1.1): a fixed-$q$ normalisation factor $q^{-\eta}$ changes intercepts, not slopes, and defines no endpoint correction from a fitted slope alone; the central-phase correction vanishes identically; no endpoint or inter-$q$ estimator and no $\beta^*$ diagnostic follow from the exact pipeline, doi:10.5281/zenodo.19226272.

[75] Jérôme Beau. A Conditional $3\times3$ Model on $\mathrm{Sym}^2(V_\rho)$: Equatorial Fibres from a Fixed Symmetric Square, and the Exact Decorrelation Conditions for its Spectrum, 2026. O30 (version 2.0): conditional $3 \times 3$ model on $\mathrm{Sym}^2(V_\rho)$, distinct from O28's measured $9 \times 9$ covariance; under the Veronese-cone, canonical inter-block identification and fixed projected-point assumptions the equatorial form follows, and the spectrum $[1 : \frac{1}{2} : \frac{1}{2}]$ holds exactly under two weighted cancellation conditions not implied by the unweighted second-harmonic condition, doi:10.5281/zenodo.20014707.

[76] Jérôme Beau. A Finite Spectral Architecture for Mass-Square Splitting and Mixing: Exact Doublets and Protected Sectors from Weil Admissibility, 2026. O33: exact cyclic-sumset closed form of the three-character Weil filtration, spacing criterion for porosity, full stabiliser through $A_K$, Mackey–Hecke doublets with $M_2(\mathbb{C})$ commutant and Harper split $1 \pm q^{-1/2}$, Galois separation of the boundary sum with its zero locus classified, odd-sector parity protection, exact $C_4$ and $C_6$ families with completeness and the Eisenstein top level open, doi:10.5281/zenodo.21360722.

[77] Jérôme Beau. Angular Amplitude Reduction: the Oriented Shear Product of the sl(2) Step and the Central-Phase Observable of the Weil Cascade, 2026. Fermionic matter subprogramme working note: fixes the numerical observable for the absolute normalisation of the angular generation split of Q14 §6 and reports the Weil–BFS computations of it. A Baker–Campbell–Hausdorff reduction in $\mathfrak{sl}_2$ identifies the per-step $J_\Pi$-odd $J_3$ component of the metaplectic cascade generator with the oriented product $ts$ of the two shear parameters, produced by the $\mathfrak{sl}_2$ commutator $[E, F] = H$ (Q14 Proposition 6.5), not a free shear parameter. The computation uses only the $\mathfrak{sl}_2$ relations of the SL(2)-on-$V$ model; no map from the Heisenberg bracket is supplied. The proposed observable is the accumulated oriented central-phase increment $A_c$ of the Heisenberg lift, normalised by the projected capacity $\widehat{I}(n)$; that $A_c$ carries the $J_3$ signal is an input of the note, not a consequence of the reduction. Under this input, at $n_3^{\mathrm{obs}}$ the normalised increment equals $\varepsilon_{\mathrm{Weil}}(n_3^{\mathrm{obs}})$, to be compared with the dictionary value $\varepsilon = 1/10$ without fitting. Direct evaluation on symmetric BFS shells returns $\Theta_{\mathrm{raw}} = 0$ identically; the orientation prescription is an irreducible input prior to $\mathcal{N}_A$. doi:10.5281/zenodo.20601228.

[78] Jérôme Beau. Chiral Orientation of the Directed Heisenberg Frontier, 2026. Chiral orientation of the directed frontier: under the hypothesis [H-WS] of Q11OF ($R_b$ acts as the scalar $-1$ on the chiral carrier, supplied by no source) the chiral weight $\sigma_L$ is not a function of the cascade generator but the Weyl representation type $\mathbf{2}$ versus $\bar{\mathbf{2}}$; the edge phase is the linear cyclotomic character with no Gauss, Maslov or $\sqrt{5}$ component, giving $\rho_\chi = 1$ and $\mathcal{N}_A = \theta_{\max}$ in the inherited central-phase lift; $u \neq 0$ not derived, doi:10.5281/zenodo.20735907.

[79] Jérôme Beau. Colour Triplet Co-admissibility on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: Numerical Test of Hypothesis [H-color], 2026. O32: numerical test of [H-color] on $q \in \{61, 151, 211, 307\}$ with controls $q \in \{29, 101\}$; inter-triplet variance ratio at or below the noise floor, $\delta_{\mathrm{tri}} \approx 3\delta_c$ at the percent level, triplet covariances of numerical rank one at three primes; the averaged level of [H-color] is proved to $O(q^{-1})$ (Proposition 1) and the effective level in the $q \to \infty$ limit, both standing, while the pointwise and rank levels rested on the superseded O31 and are open; no group identification, doi:10.5281/zenodo.20145607.

[80] Jérôme Beau. Diagonal 2I-Invariance Selects the Universal Spin-j Singlet and Its Casimir Correlator, 2026. "Q3" paper, doi:10.5281/zenodo.19979535.

[81] Jérôme Beau. Finite Criteria for Simplicial States and Local Tomography toward Emergent Operational Composition. Working paper, 2026. Companion to Effective Composition, resolving K4's finite, mono-system, and bipartite-composition states-and-effects content: the raw cylindrical-conditioning family is never itself convex; an explicit classical-randomisation postulate closes it onto the full simplex rank-independently; affine effects correspond to a cube of vectors (representation, not physical availability); the pointwise product of two local effects is forced given physical existence of a composed effect; local tomography holds exactly when local effect families linearly span their ambient spaces.

[82] Jérôme Beau. Finite Support and Preparation Criteria for Independent Effective Subsystems: A Contract for Non-Injective Admissibility. Working paper, 2026. Proposed finite prerequisite contract for effective composition (K4): independence blocks are exactly the bicliques of the projected support relation; on a block the full simplex lifts every product preparation; for strictly positive cylindrical conditionings, product closure holds iff the fibre-weight matrix has rank one; support rectangularity does not imply preparation independence.

[83] Jérôme Beau. Fourier Structure of the Pre-Saturation Admissible Pipeline: What the Three-Coordinate Projection Does and Does Not Measure, 2026. "Q5a-O2" paper, doi:10.5281/zenodo.19915951.

[84] Jérôme Beau. Gauge–Gravity Spectral Synthesis: A Conditional Einstein–Yang–Mills System from Projective Spectral Entropy, 2026. "Q13" paper, doi:10.5281/zenodo.20209859.

[85] Jérôme Beau. Low-Multipole Suppression from Distinguishable-History Capacity, 2026. Replaces the white-paper phenomenological low-$\ell$ envelope by a parameter-free small-$N$ distinguishable-history mechanism (canonical Pell count). The conditional capacity ceiling is evaluated under a separate saturation hypothesis, so the comparison is a benchmark rather than a derived prediction. Version 1.5.0 applies the Planck PR3 map calibration and the real-$m = 0$ multiplet correction to the TT Bessel cells: the leading diagnostic changes from the historical 27.39 to 17.29 in version 1.4.0 to the active 27.26 to 17.70. TE/EE transfer and saturation remain open; staircase envelope as discrete signature; $S^3$ 8/3 witness re-audited with its own observable $\sqrt{8/3}$. doi:10.5281/zenodo.21204352.

[86] Jérôme Beau. Neutrino Mixing and Projective Particle Identity: A Structural Note, 2026. Fermionic matter sub-programme companion note: basis misalignment as the generic situation, alignment as requiring an identity-stabilising projection channel; PMNS large mixing as the unfixed default and CKM near-diagonality as the explanandum, conditional on a common projective frame; Dirac–Majorana as projective conjugation resolution. Structural throughout, no mixing angles or masses derived, doi:10.5281/zenodo.21207750.

[87] Jérôme Beau. Non-Injective Observable Maps Do Not Explain Bell Violations: A Redundant-Extension No-Go Theorem, 2026. Cosmochrony no-go theorem paper, doi:10.5281/zenodo.21837950.

[88] Jérôme Beau. Operator Doublets, Erased History Fibres, and the No-Sequential-Re-entry Theorem, 2026. Fermionic-matter no-go note. Exact finite obstructions: abelian carrier commutant of the Weil module (no fermionic lift of the Hecke doublets); $C_4$ frame stabiliser and canonical $(\mathbb{Z}_2)^r$ erased-fibre torsor; semilinear $RK$ exchange of the two odd Hermitian axes; the no-sequential-re-entry theorem ($\pi v = 0 \Rightarrow Q\pi v = 0$) and the O33/EBJ lever audit; corollary: realisation through the history-fibre route requires a parallel substrate channel with an oriented central/Weyl datum, doi:10.5281/zenodo.21381089.

[89] Jérôme Beau. Projected Yukawa Operator and Chiral Polar Factor, 2026. Fermionic matter sub-programme companion note, stated on the generation block $Y_\Pi^{\mathrm{gen}}$ of the projected Yukawa morphism $Y_\Pi$: under the premises [H-Res] and [H-Sq], the squared observable $H_\Pi = (Y_\Pi^{\mathrm{gen}})^\dagger Y_\Pi^{\mathrm{gen}}$ has $\mathrm{Spec}(H_\Pi)|_{\mathrm{gen}} = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so $E_\Pi^2$ fixes the squared levels, not the block $Y_\Pi^{\mathrm{gen}} = U_\Pi H_\Pi^{1/2}$; transfer to the full coupling needs the premise [H-Fac], the existence of a weak linking carrier and multiplicity at most one of the invariant couplings, doi:10.5281/zenodo.20767498.

[90] Jérôme Beau. Projection as Conditional Expectation and the Capacity Ceiling, 2026. Derives the amplitude-level action of the non-injective projection as the conditional expectation onto the distinguishable-record sigma-algebra, under four graded hypotheses and by two independent classical characterisations (contractive projections; Moy–Rota–Douglas averaging operators); converts it into strict ensemble-power suppression (law of total variance) and the parameter-free capacity ceiling $(N-1)/N$ for a complex Gaussian mode on $N$ distinguishable histories — identically the Bessel envelope of the low-$\ell$ capacity note. doi:10.5281/zenodo.21227547.

[91] Jérôme Beau. Projective Information Loss and Fibre Erasure in Admissible Non-Injective Projection, 2026. doi:10.5281/zenodo.20466459.

[92] Jérôme Beau. Projective Trajectory Branching and the de Sitter Limit, 2026. Result note: the LPS $\to$ Heisenberg transfer relocates exponential growth from endpoint volumes (polynomial, Bass–Guivarc'h degree 4) to projectively distinguishable histories. Endpoint-based distinguishability (D1–D3) provably collapses; the canonical O12-compatible residual channel is $b$-only (the central charge is unconditionally erased from mono-path norms) and its distinguishable-history count is the Pell count $N(n) = 2N(n-1) + N(n-2)$, rate $h_b = \log(1+\sqrt{2})$, proved for a generic probe and witnessed exactly at $q = 211$. Structural bridge from the admissibility cascade to a de Sitter-like component independent of matter content; the full-history Gabor channel keeps a projective-compression profile as the open candidate for an evolving effective equation of state. doi:10.5281/zenodo.21197757.

[93] Jérôme Beau. Refinement Consistency Obstructs Nonlinear Power Lifts of Finite History Measures. Working paper, 2026. Takes up the phase/modulus law left open by STEFT: for any finite branching system, the power-lift family $\mu^\alpha$ of its induced valuation is Kolmogorov-consistent, at a branching state reached with positive $\mu$-valuation, iff $\alpha = 1$, by a self-contained comparison of $x^\alpha$ against $x$ on $[0,1]$; a row-renormalised escort lift restores consistency trivially but generally departs from the pointwise lift. Specialised, conditional on a named, non-derived maximal-entropy/Parry postulate H2, to the trajectory-branching automaton: exact instantiation in $\mathbb{Q}(\sqrt{2})$; endpoint/shadow quotients exactly incomparable; $X \leftrightarrow X^{-1}$ transports the Heisenberg endpoint by an exact automorphism, forcing real co-$b_n$ phase sums; bounded exact search over 13120 words finds no destructive cancellation among 62 nontrivial-phase classes of 64.

[94] Jérôme Beau. Semiclassical Consistency of the Weil Representation on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: Quantitative Sinc Embedding, Generator Convergence, and Aliasing Control, 2026. "H2" paper, doi:10.5281/zenodo.19962051.

[95] Jérôme Beau. Spin as a Transformation Class of Rotating Wave Modes, 2026. Spin as a transformation class of rotating wave modes: spin is fixed by how components transform under spatial rotations; central grading $(-1)^n$ on tensor powers of the admissible module; Veronese-null versus compact-real-form lemma; soldering indeterminacy; version 1.4 adds the exact Higgs-typed quantum numbers of the Dirac bilinear in every tree-level Yukawa sector, permitting but not deriving a composite condensate, doi:10.5281/zenodo.21380026.

[96] Jérôme Beau. Spin-Stratum Type-Rigidity: An Obstruction to the Galois Action on the Binary Icosahedral Stratum, with Conditional Consequences for the Chiral Lift, 2026. Arithmetic orthogonality of the Galois action $\sqrt{5} \mapsto -\sqrt{5}$ and the spin-stratum operations: $\varepsilon = 1/10$ is a theorem modulo the ADE case-selection gate, whose first-principles status this note does not settle. A character-table audit of $2I$ proves that inner conjugations, $-I$ and $J_\Pi$ act trivially on $\mathrm{Irr}(2I)$ whereas $\sigma$ induces $(2\,2')(3\,3')$; the Born–Infeld parity $c \mapsto q - c$, acting as complex conjugation, fixes $\sqrt{5}$ (proved), but its status as the base-fibre involution is that of a candidate hypothesis of the foundations, taken here as a premise. Modulo [H-WS] for $R_b$, the A4 hypotheses for the locking, modelling assertions for the tensor functors and Schur projection, and the modelling premise [M] that the generator list is complete (which implies carrier-preservation of every admissible operation), no listed operation realises $\sigma$. No link between the gate and $\sigma$ is established. The Lorentzian chiral lift is an input not constructed here ([H-Spin] of Q14). The chiral-lift conclusion (no action on $\mathbb{Q}(\sqrt{5})$) is conditional on [H-Spin], [H-WS], the modelling premise [M], an explicit boost premise [B] (the lift supplies only the central character and the complexification unit, by complex conjugation) and the inherited central-phase convention [C]. doi:10.5281/zenodo.20693084.

[97] Jérôme Beau. Ternary Phase Carrier: Colour Confinement, Generation Splitting, and Koide Dispersion, 2026. Fermionic matter sub-programme note (speculative bridge): one complex three-phase carrier $z_g = S + A\,e^{i(\delta+2\pi g/3)}$ read two ways – pure-phase part sums to zero ($1+\omega+\omega^2 = 0$, colour-neutral class), real part $S + A\cos(\delta + 2\pi g/3)$ is the square-root mass profile of the unit-dispersion note. The singlet amplitude $S$ is the master dial: $S = 0$ pure-phase (colour, confined); $S \neq 0$ real offset (masses); $A = \sqrt{2}S$ singlet-doublet equipartition (Koide, CV=1); $\delta$ near a node gives the hierarchy. Candidate admissibility filter: capacity balance $\|r_{dbl}\| \leq \|r_{sing}\|$ saturated at CV=1, whose selection of the saturated admissible edge by projection locking is a further hypothesis the cited sources do not supply. Not a derivation; states the construction charge sheet. doi:10.5281/zenodo.21115977.

[98] Jérôme Beau. The Born–Infeld Saturation Margin of the Chiral Modulus: Antisymmetric Structure and Lorentzian Genus of the Generation Split, 2026. Fermionic matter sub-programme companion note to PRS: the chiral modulus is $J_\Pi$-odd and the primary variable is an antisymmetric chiral two-form; the Born–Infeld determinantal saturation margin of a supplied such two-form has second variation $\mu_\chi^2 = \frac{1}{2}P_{\mu\nu}P^{\mu\nu}$, the Lorentzian genus of the chiral polarisation. The genus is not determined: the construction of the two-form, the coincidence of the Born–Infeld modulus with the Schur tilt, the transport of the metaplectic opening to the locking operator, and the non-degeneracy of the split are hypotheses; the amplitude analysis is stated for a supplied purely electric two-form. doi:10.5281/zenodo.20633931.

[99] Jérôme Beau. The Charged-Lepton Koide Relation as a Unit-Dispersion Constraint, 2026. Fermionic matter sub-programme source note: recasts the charged-lepton Koide relation as a unit coefficient-of-variation condition on the square-root mass vector, $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$, hence $Q_\ell = 2/3 \iff \mathrm{CV}(r_\ell) = 1$ ($\sigma_r = \bar{r}$); separates the dispersion constraint from the hierarchy phase (node placement); records the dispersion obstruction $\mathrm{CV} \leq 1/\sqrt{2}$, $Q \leq 1/2$ of the cascade ladder, taken by hypothesis as a pinned normal form; states the maximum-entropy reading as an open $S_\Pi$-variational test. Target invariant for the fermionic-mass sector, not a mass formula. doi:10.5281/zenodo.21109812.

[100] Jérôme Beau. The Constrained Jet of the Lorentzian Eliminated Block and the Chiral Defect-Rate Identity: A Moving-Schur Existence Theorem for the Generation Split, 2026. Fermionic matter sub-programme companion note, logically between the Schur reduction (PRS) and the physical identification of the chiral generator. For every smooth self-adjoint projector family $1 - P(s) = Q_0 + sQ_1 + s^2Q_2 + O(s^3)$ the projector constraints force $Q_1 = [A, Q_0]$ to be a purely off-diagonal Grassmann tangent and pin the diagonal blocks of $Q_2$ to $Q_1^2$ with opposite signs; the jet parities alternate under $J_\Pi$-equivariance. With a covariant moving spinorial embedding and a $J_\Pi$-even Dirac operator the first-order $J_\Pi$-odd part of $E_\Pi^2$ is $\{E_0, E_1\}$, and antiunitary parity makes every odd-order generation-mixing coefficient vanish, mixing at even orders not being excluded. An explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with a non-zero split rate $u'(0) = 11 \cdot 2^{1/4}/9$. The finite witness has $J^2 = +1$ and lies outside hypothesis [H-Gen](i) of the Schur note. In the chiral frame $[\Pi_{J_\Pi\text{-odd}} \dot{Q}(0)]_{LL} = \frac{1}{2}\partial_s \Delta_\chi(P)|_0$ determines one block of the odd tangent, not a unique generator; the selection of the family, the localisation, and the normalisation of $|u|$ remain open, doi:10.5281/zenodo.20763532.

[101] Jérôme Beau. The Fermionic Matter Sub-Programme, 2026. Presentation Note 6 of the fermionic-matter sub-programme, on a supplied Heisenberg carrier: the $\mathfrak{sl}_2(\mathbb{C})$ tensor algebra of a supplied doublet is proved on supplied model data; its Lorentz reading rests on the spin solder [H-Spin] and its electroweak reading on a distinct $U(2)$ weak factor [H-Weak], neither supplied; $\mathfrak{su}(2)_L \subset \mathrm{Sym}^2(E_{\mathrm{weak}})$ and $L_Y = \wedge^2(E_{\mathrm{weak}})$ hold under [H-Weak], and $V-A$ is the content of [H-Weak]; the hypercharge constraints need both hypotheses; the generation factor $C^3_{\mathrm{gen}}$ uses a supplied rank-three selection rule and its levels are model operators, read as restrictions of $E_\Pi^2$ only under [H-Res]; no mass spectrum, Yukawa coupling or mixing matrix is derived, doi:10.5281/zenodo.20562665.

[102] Jérôme Beau. The Oriented Frontier Observable: a Recursive Q11 Diagnostic for the Angular Generation Split, 2026. Fermionic matter sub-programme diagnostic note. The raw shell observable of the angular generation split vanishes identically on symmetric BFS shells ($\Theta_{\mathrm{raw}} = 0$); this note defines an oriented replacement, a frontier transfer observable on the directed outgoing edges $g \to gs$ whose central increment is the Heisenberg cocycle $a(g)\, s_b$. It is not the $J_3$ coefficient of the metaplectic step, which in the SL(2) model of Q14 is produced by the $\mathfrak{sl}_2$ commutator $[E, F] = H$ (Q14 Proposition 6.5). The first test is the existence of an oriented signal, with the symmetrised-frontier cancellation as anti-bias control. The chiral lift of the residual reflection $R_b = \mathrm{FT}^2$ is reduced to one named hypothesis [H-WS]: (i) $R_b$ acts on the chiral carrier $S_L \oplus S_R$ as a scalar, so that $[\gamma_5, R_b] = 0$; (ii) the scalar is $-1$, used only in AOG; supplied by no source. No normalisation, no dictionary value, no $\varepsilon$ enters. doi:10.5281/zenodo.20601245.

[103] Jérôme Beau. The Projective Residue as a Schur Complement: Reduction of the Generation Split to the Chiral Asymmetry of Projection Locking, 2026. Fermionic matter sub-programme note: under a contraction hypothesis the compression remainder of the projected Dirac square is, as a quadratic form, the Schur complement $-N^\dagger N$ of the eliminated spinorial directions; it is zero-order under a separate symbol condition, which forces $E_\Pi = 0$ on irreducible Clifford fibres; chiral block reduction, finite $J_\Pi$-equivariance under an equivariant construction, and the Schur-null and Schur-transverse loci of the locking defect, which locus holds being open and conditional on a transport hypothesis; no sign of the Born–Infeld genus and no non-vanishing of $u$ is derived. doi:10.5281/zenodo.20601040.

[104] Jérôme Beau. The Spectral Gravity Sub-Programme, 2026. Presentation Note 4: conditions under which the projective spectral entropy supports an Einstein sector for a supplied Riemannian metric, with the Einstein coefficient a matching datum; its emergent reading is conditional on [H-L], [H-lift], a supplied rank-four extension and a continuation, doi:10.5281/zenodo.20533351.

[105] Jérôme Beau. Toward a Full-Rank Effective Metric: No Central Coefficient from the Heisenberg Commutator, and Scale-Free Casimir Rigidity, 2026. "Q8" paper, version 2.0, doi:10.5281/zenodo.19879909.

[106] Jérôme Beau. Toward a Kinetic Limit on a Supplied Schrödinger Fibre: Conditions for Generator Suppression, 2026. "Q9" paper, version 2.0, doi:10.5281/zenodo.19880574.

[107] Jérôme Beau. Toward a Temporal Scale for Emergent Spacetime: Spatial SU(2) Invariance Leaves the Temporal Coefficient Free, 2026. "Q11" paper, version 2.0, doi:10.5281/zenodo.20098387.

[108] Jérôme Beau. Toward Coefficient Isotropy from Character Uniformity: A Stability Lemma and Its Missing Geometric Inputs, 2026. "Q10" paper, version 2.0.1, doi:10.5281/zenodo.19880900.

[109] Jérôme Beau. Toward Fermionic Matter from Projective Dirac Admissibility: Chirality and Electroweak Structure Rest on a Lorentzian Spin Solder and a Distinct Weak Factor, 2026. Fermionic-matter paper Q14: on a supplied Heisenberg carrier and supplied model data, the $\mathfrak{sl}_2(\mathbb{C})$ tensor algebra is proved; the Lorentz reading rests on a spin solder [H-Spin] and the electroweak reading on a distinct $U(2)$ weak factor [H-Weak], neither supplied; $V-A$ selection is the representation content of [H-Weak], hypercharge twists are selected by anomaly constraints under both hypotheses with a supplied colour module and minimal $L_Y$ normalisation, and the three-generation factor uses a supplied rank-three selection rule; the sign $E_\Pi \preceq 0$ of the projective endomorphism, used for its chiral block structure on the left-admissible branch, is an explicit hypothesis. doi:10.5281/zenodo.20218409.

[110] Jérôme Beau. Toward Four-Dimensional Lorentzian Geometry from BFS Shell Stratification, 2026. Emergent-geometry paper Q5b (version 2.2.1): on balls of radius $n \ll \sqrt{q}$ the BFS shells of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ converge to a sub-Finsler Carnot–Carathéodory sphere foliation with $D_{\mathrm{hom}} = 4$; under the unestablished [H-L] and the open [H-lift] the effective co-metric has rank three, the full-rank extension and the coefficient values being open, doi:10.5281/zenodo.19686700.

[111] Jérôme Beau. Toward Projected Yukawa Couplings: A Conditional Weak Determinant Line and Generation-Level Ordering, 2026. Fermionic matter sub-programme companion note: under the distinct weak factor [H-Weak] of Q14, not supplied, the determinant line $L_Y = \wedge^2(E_{\mathrm{weak}})$ of a rank-two weak bundle with structure group $U(2)$; and, for the model operator $\mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ on $C^3_{\mathrm{gen}}$, whose reading as $E^2_\Pi|_{\mathrm{gen}}$ is the named hypothesis [H-Res], the ordering of three exit-deficit levels for $0 < u < \frac{1}{2}$, without any mass value, doi:10.5281/zenodo.20767265.

[112] Jérôme Beau. Toward Three Spatial Directions from a Supplied Spinor Carrier: Requirements for an Equivariant Bridge, 2026. Q7 (version 2.0): states the requirements an equivariant bridge would have to meet and proves those that follow from representation theory on a supplied carrier; the identification of the measured space with the spin-one module is an explicit hypothesis supplied by no source. The stored basis rows are pure Fourier modes, so the flat-mode entry of the compression is the Rayleigh quotient $4 - 2 - 0 = 2$, $A_z$ is an unevaluated parameter, and the gap between the two distinct entries is the identity $4\sin^2(\pi k/q)$. Verdict on the bridge: missing identification, neither derived nor ruled out, doi:10.5281/zenodo.19802123.

[113] Jérôme Beau. Uniformity Requirements for Weil Fingerprint Energies: Obstructions to the Lipschitz Route to [U], 2026. "U1" paper, version 2.0, doi:10.5281/zenodo.19881146.

[114] Jérôme Beau. What Weight Stabilisation Would Require: A Finite-Window Audit of Admissibility Weights, 2026. W1 paper, version 2.0.1, doi:10.5281/zenodo.19886319.

[115] Jérôme Beau. Withdrawal Notice for O31: Unsupported Colour-Triplet Co-admissibility and Gauge-Group Claims, 2026. O31 withdrawal notice, doi:10.5281/zenodo.20030491.

[116] Jérôme Beau. Yang–Mills Dynamics from Projective Spectral Entropy: Vertical Heat-Kernel Variation on the Admissible Fibre, 2026. "Q12" paper, doi:10.5281/zenodo.20189137.