

there was no change in the anomaly after long continued tension. Hence in this case the effect is probably due to the bifilar property of the phosphor-bronze strips.

In conclusion, I should like to express my obligation to Prof. Wilberforce for suggesting the investigation and for advice during the course of the work.

LXXXV. *The High-Frequency Spectra of the Elements.*

By IVAR MALMER, *Cand. phil.**

[Plate XIV.]

IN his excellent researches on the high-frequency spectra of the elements, Moseley † has analysed the characteristic types of X-radiation of a great many elements. For the elements aluminium–zinc in the periodic system he has *pro tempore* examined and measured the wave-lengths only of the characteristic K-radiation which has been found to consist of two definite wave-lengths for each element. For the elements yttrium–silver he has found and measured only one wave-length, corresponding to the weaker one of the two above-mentioned, and for these (with the exception of yttrium) and a great many of the following elements he has determined several wave-lengths of the characteristic radiation that has been named L-radiation. A relation of the utmost interest among the frequencies of the radiation of the different elements is, that they admit of being arranged in series such that the square root of the frequency in each series increases by a constant from element to element in the periodic system. In all cases this relation seems to hold good for the comparatively long waves, *i. e.* the so-called L-radiation and the K-series of the comparatively light atoms.

It must be of great interest to find out if the two K-series can be followed on to the heaviest atoms and if the constancy above-mentioned really continues throughout the system, even for these short waves. In his research on the line-spectrum emitted by platinum, M. de Broglie ‡ did not find any line corresponding to the K-series; this may depend either upon these series ending somewhere in the system or upon the exciting cathode rays not being fast enough.

This paper contains the description of an attempt to follow the two K-series further on in the periodic system than has

* Communicated by the Author.

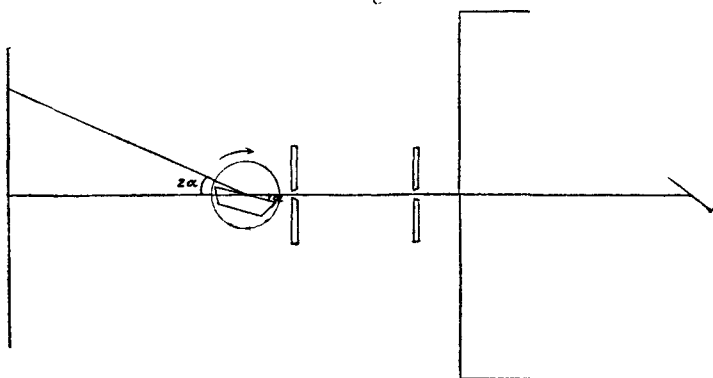
† *Phil. Mag.* xxvi. p. 1024; xxvii. p. 703.

‡ *C. R.* t. 158, p. 177.

hitherto been done by Moseley, and for this purpose the elements yttrium-silver, where Moseley has measured only one wave-length, and among the following elements, cadmium-antimony and barium-cerium, have been examined.

The experimental arrangement appears from fig. 1, a

Fig. 1.



revolving rocksalt crystal having been used in agreement with M. de Broglie's method * in his researches on the high-frequency spectra. The X-rays were limited to a comparatively fine beam by two lead-slits, 0.5 mm. wide, placed at a distance of 11 cm. from one another. Between the anticathode and the first slit the distance was about 30 cm.; from the second slit to the revolving axis of the crystal 4.5 cm., and from the second slit to the photographic plate about 26 cm.

The rocksalt crystal was mounted on the vertical axis of a clockwork, its reflecting surface containing the geometrical axis of the revolving movement, another cleavage-plane perpendicular to the axis. The angular velocity was 15° per hour. Only the space where the two lines were expected to be found was examined. The same space was passed over several times, from three times in the case of silver to from fifteen to twenty-five times for the earth-metals examined. At all exposures an intensifying screen of zincblende in close contact with the plate is used, in order to reduce the time of exposure required. Unfortunately the lines, by this method, show some want of sharpness and admit of less precision when measured. Owing to the comparatively feeble reflexion in the second order in the case of rocksalt, all spectra have been photographed only in the first order.

* *C. R.* 17 Nov. 1913.

In spite of the zincblende sheet and the long exposure, the β -lines especially were in many cases so feeble as scarcely to be seen on a copy; for that reason the copies of Plate XIV. have been retouched in order to be reproducible and do not, I suppose, generally admit of being measured. Naturally the distances of the lines have been measured directly on the plate. Lumière plates, "Violet Label," were used and were developed with metol-hydrokinone. While exposed, the plates were covered with black paper.

The calculation of the incident angle was made in the following way. At first the incident angle for the strongest of the silver lines was measured directly with a spectrometer, whose prism had been replaced by the rocksalt crystal. The crystal was turned till the α -line appeared at its sharpest on a barium-platinocyanide screen in place of the plate. Greater accuracy was obtained with a micrometer screw. The crystal plate was then turned till the line appeared on the other side, and the angle turned through, equal to $180 - 2\alpha$, gave the incident angle α . The distances between the lines produced by the direct and the reflected X-ray beam for the element in question and silver may be a and b respectively, and the corresponding incident angles A and B; then A is calculated by the relation:

$$\frac{\tan 2A}{\tan 2B} = \frac{a}{b}.$$

Owing to altered arrangements during the research the plates must be compared with two different silver plates, on Plate XIV. marked Ag_I and Ag_{II} . With Ag_I ought to be compared Mo, Ru, Cd, and Sn, with Ag_{II} all others.

In the cases of Mo and Ru the incident angles, thanks to the strong reflexion, could also be directly measured for the α -lines. The values thus obtained were $7^\circ 16'$ and $6^\circ 34'$ respectively, while the values calculated from comparison with the silver plate were $7^\circ 16'$ and $6^\circ 34'.5$, which shows a good agreement. Generally, however, the errors of measuring must be valued at $3'$ to $4'$ owing to want of sharpness of the lines and owing to the incident beam possibly not falling quite on the centre of the rocksalt crystal. The "collimator" had been adjusted by an optical method, having been turned till a pencil of light-rays through the two slits was projected on the centre of the crystal.

The wave-lengths have been calculated from the well-known relation

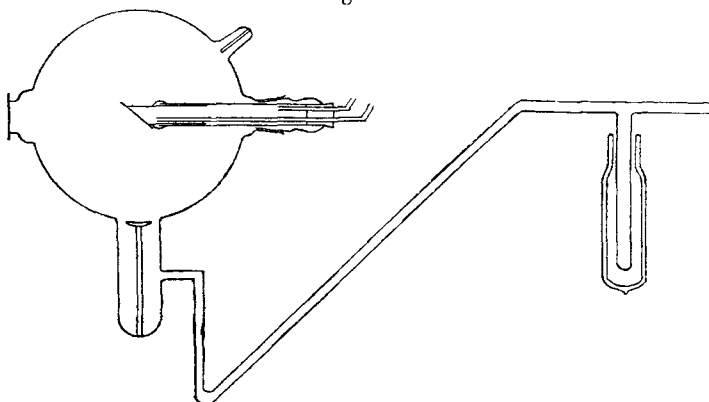
$$n\lambda = 2d \cdot \sin \alpha \quad (n \text{ here} = 1),$$

where

$$d = 2,814 \cdot 10^{-8} \text{ cm.}$$

The greatest difficulties have been to get the bulb sufficiently hard, several of the elements examined having a low melting-point, and, therefore, a comparatively high-vapour tension, even at not very high temperatures. Consequently an effective arrangement of cooling had to be used for all the elements examined except Mo, Ru, Pd, and Ag, the anticathode being cooled with a stream of cold water (fig. 2).

Fig. 2.



Thanks to this arrangement the bulb could be kept rather hard even with an anticathode of indium with a melting-point of less than 200° . Yet it was necessary to use an intermittent primary current closed for about a second and then interrupted for the same space of time, &c. In this way the bulb could, as a rule, be worked with for hours without softening. The pump (a Gaede mercury pump) then was working incessantly, cooling with charcoal in carbonic acid and ether also always being used.

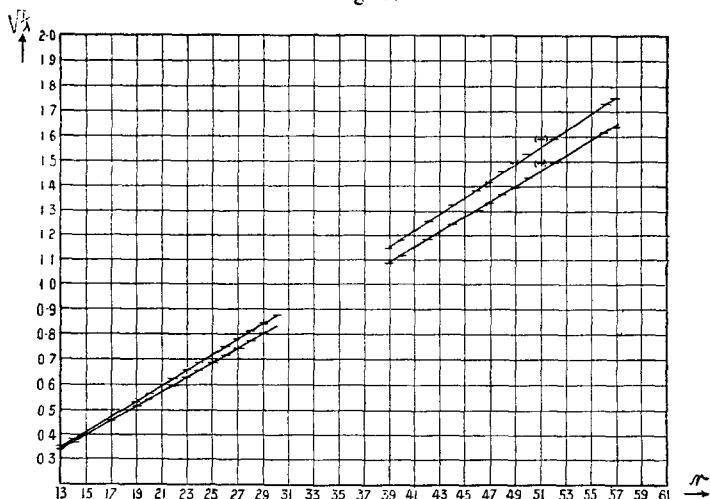
The fusible elements (Cd, In, and Sn) were melted on the anticathode, the metals more difficult of fusion were either tied with a platinum wire or soldered on to the anticathode. Some difficulties were caused by Y, Zr, Ba, La, and Ce, which were used in form of oxides. A good method of fastening them proved to be the making of a thick cream of the oxide and waterglass, which was then, as a thin layer, cemented on a rifled anticathode. If the layer was too thick, the heat easily caused splinters to be burst away and fall on the cathode, which instantly weakened the bulb.

The results are to be seen in the following Table.

N.	Element.	Line distance. nm.		α .	β .	$\lambda_{\alpha}, 10^6$.	$\lambda_{\beta}, 10^6$.	$\sqrt{\frac{1}{\lambda_{\alpha}}}, 10^4$.	$\sqrt{\frac{1}{\lambda_{\beta}}}, 10^4$.
		a .	b .						
39	Yttrium	63.5	56.4	8° 37'.5	7° 44'	0.844	0.757	1.088	1.149
40	Zirconium	60.4	53.5	8° 13'.5	7° 20'	0.805	0.718	1.115	1.180
42	Molybdenum.....	52.5	46.3	7° 16'	6° 26'.5	0.712	0.631	1.185	1.258
44	Ruthenium	47.3	41.6	6° 34'.5	5° 48'.3	0.644	0.569	1.245	1.326
46	Palladium	43.6	38.4	6° 1'	5° 19'	0.590	0.522	1.302	1.385
47 {	Silver I	41.2	36.2
	Silver II.....	41.6	36.8	5° 45'	5° 5'	0.564	0.499	1.332	1.416
48	Cadmium	39.0	34.2	5° 27'	4° 47'.5	0.535	0.471	1.368	1.458
49	Indium	37.6	32.9	5° 12'.5	4° 34'	0.511	0.448	1.398	1.494
50	Tin	35.3	31.0	4° 56'.5	4° 21'	0.485	0.427	1.436	1.531
51	Antimony	32.9	29.0	4° 34'.3	4° 2'	0.449	0.396	1.493	1.580
56	Barium	28.0	24.4	3° 54'	3° 24'	0.383	0.334	1.616	1.731
57	Lanthanum	27.3	23.8	3° 48'	3° 19'	0.373	0.326	1.637	1.753
58	Cerium

$\sqrt{\frac{1}{\lambda}}$ is graphically represented as a function of the atomic number N in fig. 3 (as well as the values found by

Fig. 3.



Moseley for the elements Al-Zn). The values are evidently ordered along two right lines—the deviations may be within the limits of the observation errors, which must here be relatively great because of the small incident angles. In the case of Sb, however, the deviation is too great to be explained by observation errors, all the more as the lines on this plate were rather sharp. As is seen from the figure, the agreement is rather good, if the atomic number of Sb is taken = 52 instead of 51. In the L-series measured by Moseley Sb must have the number 51—accordingly, it seems that such an anomaly is possible that an element has not the same number in all series.

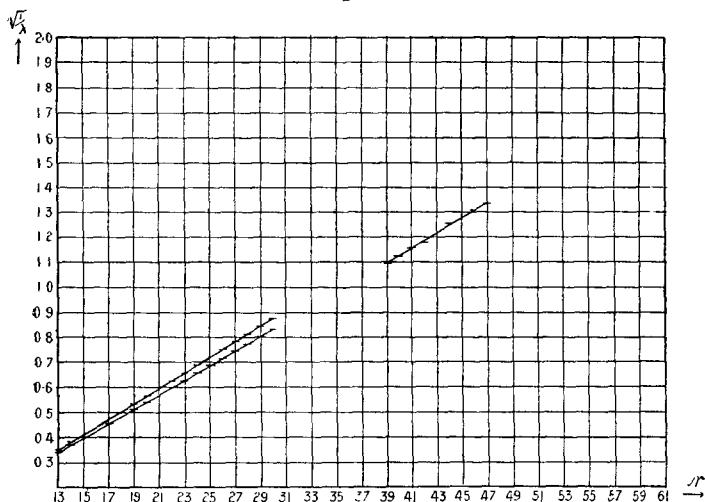
In the case of lanthanum the lines were rather broad (perhaps the oxide contained barium) and were both followed by a rather strong continuous spectrum. For cerium no lines could further be observed, although the bulb was extremely hard, and the plate was exposed twenty-five times, but only a continuous spectrum, which, however, was a little enfeebled immediately in front of the spot where the α -line ought to have been situated.

From fig. 3 it is evident that the formula given by Moseley for the α -line of the type

$$\nu = \frac{3}{4}\nu_0(N-1)^2$$

(ν being the frequency of the radiation, ν_0 Rydberg's general constant, and N the atomic number), which is in such good agreement with the values for Al-Zn, does not hold good throughout the series of the elements. In fig. 4 the values

Fig. 4.



found by Moseley for the elements Y-Ag are represented instead of mine, and, as seen, the curve shows the same tendency as that found by me. Possibly the formula

$$n\lambda = 2d \cdot \sin \alpha$$

is not quite accurate for the small angles dealt with, d being subject to a correction for these angles. This supposition might be examined by measuring the angles in the higher orders, but for this the method used here was not convenient.

With a silver target an exposure was made without the zincblende screen and with the slits only 0.2 mm. in width, in order to find out if the silver lines are double, as has been shown for rhodium by Bragg*, with an ionization method, and for nickel by W. F. Rawlinson† by a photographic method. Indeed, two different α -lines appeared on the plate, but on further examination even the band produced by the direct rays, showed on a less dark background two darker lines, closely situated, as well as three feebler ones. A supposition that these lines might be lines of interference

* Phil. Mag. xxvii.

† Phil. Mag. Aug. 1914, p. 274.

of the secondary radiation from the two edges of the slit had to be given up, exposures with slits of various width showing the lines at the same distance. The only possibility left was that the cathode rays were not focussed in only one point of the target. Consequently, the bulb was placed in such a position that the X-rays used were emitted at a minimal angle with the surface of the target, and now both the direct and the reflected lines were simple. Possibly former cases of double lines may be referred to the same cause.

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Sept. 1914.

LXXXVI. *Note on the Analysis of Energy Distribution for Natural Radiation.* By S. LEES, Reader in Applied Thermodynamics in the Manchester School of Technology and in the University of Manchester*.

§ 1. **T**HE following remarks are suggested by a paper of Lord Rayleigh's (Phil. Mag. xxvii. p. 466, 1889). Indeed, the theorem deduced in § 4 may be regarded as analogous to a theorem given by Rayleigh in his paper.

§ 2. At any point (x, y, z) in a medium traversed by natural radiation corresponding to some definite temperature, we may denote the component electric forces at time t by (X, Y, Z) . We shall consider any time-interval defined by $0 < t < T$, and suppose that

$$\begin{aligned} X &= 0 && \text{from } t = -\infty \text{ to } t = 0, \\ X &= X(t) && \text{,, } t = 0 \quad \quad \quad \text{,, } t = T, \\ X &= 0 && \text{,, } t = T \quad \quad \quad \text{,, } t = \infty. \end{aligned}$$

Then by Fourier's theorem,

$$X(t) = \frac{1}{\pi} \int_{p=0}^{p=\infty} \int_{u=0}^{u=T} X(u) \cos p(u-t) dp du, \quad . \quad (i.)$$

where we assume that X is finite, and only has a finite number of discontinuities in the interval $0 < t < T$. We may thus express X in the form

$$X = \int_0^\infty f(p) \cos (pt + \beta) dp, \quad . \quad . \quad (ii.)$$

where, of course, β may be a function of p , and the analysis is purely formal.

* Communicated by the Author.

