CORRELATION

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One object of these annual summaries on correlation, when they began in 1912, was to encourage the more frequent use of this important statistical tool. The deluge of studies using correlation in recent years makes it clear to the writer that this need has been met. The mere indexing of analagous studies does not justify the time required. The writer expects hereafter to cite papers that indicate extensions in new fields, and those in which contrasting points of view are involved. The main emphasis will be on the interpretation of coefficients and improvement in the statistical methods involved.

Interpretation of Coefficients.—Several papers inquire into the fundamental psychological assumptions on which measurements rest. The doubtful psychological equivalence of the units of measurement in tests and the clearer meaning of ranks in behavior, raises the question whether ranks might not be psychologically more sound and practically more fruitful even if they are biologically less broadly significant. Units of rank orders, Boring (9) concludes, are validly demonstrable for abilities. They afford the possibility of statistical treatment in terms of medians, quartiles, contingencies and correlation ratios, while we are not justified in assuming that the usual mental test scales are made up of equivalent units which can be treated with the statistics of averages, standard deviations, coefficients of correlation and linear regressions. See also McEwen and Michael (22) and the Scott Laboratory (32). Ruml (28) thinks that the failures of mental tests may in part be traced to "a willingness to accept statistical hypotheses as applied to intelligence simply to have statistical technique available for use." In particular he criticizes the assumptions: (1) That general intelligence can be expressed as a one-dimensional function. It is analogous to "size." (2) That regression of general intelligence upon test performances is linear. Evidences that it is linear which are based upon judged intelligence is "ordinarily worthless." Gross departures from linearity are found in age and in trade test relations.

(3) That an individual maintains a static level of intelligence from time to time. Correction for attenuation is made on this assumption, viz., that the difference between performances with the same test are due to errors in measuring rather than to actual variation of the ability from time to time. Mitchell (23) gives evidence that memory span is not static for the same individual. Myers (25) cautions against concluding that a close correlation between group and individual tests for school groups including several grades and wide ranges of mental ages would hold for a homogeneous group in any single school grade. Thorndike (38) indicates that a "halo" of general merit affects estimates of special traits so as to make their intercorrelations too large.

Thompson's paper (33) may upset the common explanation of the size of coefficients. It shows the possible effects of what he defines as "interference factors." These are groups of elements which operate in favor of one test and against another, a condition which is psychologically likely. If interference factors exist, zero correlation does not show that either general or group factors are not common to the two variates measured; high common factors may exist with low correlations; transfer of training is possible with no improvement of the second variate after the training and a reduction of the coefficient; after training the coefficient may remain the same in spite of the common factors having been much improved; the contemporary training of two abilities with common factors does not necessarily alter their correlation. Experiments might be arranged to discriminate some of these different effects. His method is an extension of Weldon's and of Thomson's dice patterns in which the factors producing correlation are additive. It gives splendid possibilities of visualizing possible sources of effects on the correlation coefficients and may be utilized by a non-mathematical experimenter who wishes to test whether his generalizations are justified. Other interpretations of correlations than those that assume the overlapping of common factors which are additive are to be taken up later. For example, the correlation may result from exclusion due to drawing from a limited pool, as in whist hands. Combination of elements by multiplying, instead of adding, would not affect the results provided that the form of function according to which the elements combine is the same in each variate, and the standard deviation of each element is the same.

Group Factors versus a General Factor.—A number of important papers treat of the relation of mental abilities to general, group, and specific factors. It is quite impossible in this summary to state the numerous qualifying clauses in these papers, but the drift of the discussion leaves the problem as follows: (1) A perfect hierarchy would demonstrate, as Spearman claims, that there is a *single* general factor and no group factors except for quite similar activities and these of small effect. (2) An imperfect hierarchy would be explained by group factors with or without general factors. (3) Interference elements included in general factors would account for any set of correlation coefficients, Thompson (33). (4) It is not certain whether the empirical data form a perfect hierarchy or only approach it. In this situation there is a general tendency to accept important group factors, types or levels, with possible universal factors. In the discussion Spearman defends his theory of the Two Factors, General and Specific, and believes his contention is proved. Thomson regards General Factors as unproved and unnecessary, although possible, and believes that the overlapping of important group factors best explains the available data. Garnett prefers Spearman's theory.

The mathematical foundation on which Spearman erected his valiant hypothesis on the General Common Factor has been seriously shaken by Thomson (35, 36, 37). He shows that the criterion for determining the degree of perfection in a hierarchy of coefficients is mathematically incorrect and exaggerates the approach to perfection. In an artificial example, the known true degree of perfection, .59, was calculated by the Spearman criterion to be 1.00. Two sources of error in Spearman's proof of his criterion are demonstrated: (1) The arbitrary plan by which he rejects coefficients from a table happens to leave those which bring the average to about 1.00; (2) the equation for degree of perfection of the hierarchy assumes certain quantities uncorrelated when they are really strongly correlated and cannot be neglected. The use of Spearman's criterion mars Webb's and Garnett's deductions as to "general ability" and "will" factors. Thomson (35) also shows how overlapping group factors may be harmonized with a small transfer of training which otherwise tends to support the hypothesis of the absence of group factors. If improvement were due to the selection, mainly the economy, of elemental factors used in any activity, this might occur without the improvement of the elements themselves and so transfer only slightly to other group combinations of the elements. Similarly one might conceive a football team to gain by playing together, by its team work, by its elimination of useless factors, without the players becoming better individual players and without the improvement transferring to other teams in which the men play. In restating his hypothesis, under which "any performance is considered as being carried out by a sample of group factors," Thomson names it the Sampling Theory of Ability. A hierarchical order of coefficients is the natural order to expect on the theory alone of chance sampling of abilities. On the Mendelian theory any individual is a sample of unit qualities, so each of his activities may involve a further sampling of these qualities. A general factor, if found in Smith, may not be the same as a general factor found in Jones. He gives an admirable review in non-mathematical language of the entire discussion of the General Factor.

Regarding his theory of Two Factors, which was forecast in 1904, Spearman (31) says: "Hardly any writer (outside of those working in more or less intimate connection with myself) has so far uttered a sign of being convinced." He thinks that this obduracy may be due to a question in the minds of investigators who admit that his theory requires the data to conform to his equation and yet believe that other theories would also meet the same criterion. Now that Garnett (14, 15, 16) and Spearman (31) show that a perfect hierarchy requires a single general factor, Spearman hopes for a more general acceptance of his Theory. He also elaborates an earlier note regarding the method of showing by dice patterns that hierarchies could be produced from group factors. The dice hierarchy introduces the General Factor under another form. As shown also by Garnett and admitted by Thomson (16) the perfect hierarchy may mean either a single General Factor or group factors which are made up of an infinite number of interchangeable elements of the General Factor, which would still be interpretable as an underlying fund of brain energy. In discussing the group-factor hypothesis for explaining perfect hierarchies Garnett (15) is only concerned with variables that are distributed according to the normal law and measured in such units as will give each the same standard deviation. The correlations between three variables will always satisfy the conditions for a hierarchy expressed by terms of four independent variables of which one is a general factor while each of the others is a specific factor. There can always be found an infinite number of general factors of correlated variables. These general factors are to be distinguished from the unique general factor whose correlations satisfy the conditions for a perfect hierarchy. Garnett (13) finds that the data from Webb's monograph on Character and Intelligence indicate that several intellectual qualities (humor, originality and quickness) may be regarded as compounded of General Ability, and of an independent group factor "Cleverness." The latter, following Mercier, seems to be innate while General Ability may be acquired. A number of character traits (tendency not to abandon tasks in the face of obstacles or from mere changeability, kindness on principle, trustworthiness, conscientiousness, working with distant objects in view) show not only General Ability but "Purpose" which is a factor compounded of General Ability and an independent Group factor. He gives an equation for testing these relations and shows how the relations of General Ability and a group factor may be represented in three dimensions.

Other Contrasting Interpretations.—Pearson (26) finds that the resemblance of sibling orphans, published by Kate Gordon, .508, is very close to that found by the Galton Laboratory, .515, for siblings in general. It is new evidence that the sameness in environment of the non-orphan sibling pairs could hardly be the cause of their resemblance. Bagg (6) utilizes correlation, probably for the first time, in connection with the study of habit formation and family resemblance in animal behavior. No family resemblance appears in the behavior of mice, as judged by the non-reduction of the mean variations within the litters as compared with random groups of the same size. There is a correlation between early and late performances of the same mice. Rosenow (29) and Murchuson (24) discuss the relation of delinquency to intellectual deficiency. The former concludes, that, if the coefficient of correlation between intelligence and delinquency is +0.66 as Goring found in The English Convict, "the correct conclusion to be drawn is that it is exceedingly probable that factors other than intelligence are of *greater* importance as determinants of crime than intelligence." To the reviewer it does not seem that his demonstrations that all other causes combined correlate over .66 with delinquency, that there is a *possibility* of a score of larger causes and that these when combined with deficiency produce only a small addition to the total correlation with delinquency, do not raise the *probability* that there is one larger cause. Murchuson tested 3,328 criminals who could read and write and found a median intelligence score with the army tests of 62, which compares with that for the army and concludes "the difference between the average individual and the average criminal is not a difference which can be expressed in terms of intelligence." This conclusion is not clear from the data published. His exclusion of illiterates may have affected the comparison. Moreover the criminals should be compared with the intelligence average of their states not with the army generally.

The Statistical Methods of Correlation.—A more general method than multiple correlation or than least squares has been provided by McEwen and Michael (22) for determining the functional relation of one variable to each of a number of correlated variables. The method has been found to be practically more useful in making predictions when the forms of the functions are unknown, as is frequently the case in dealing with biological material, especially social material. It avoids the assumption of practical linearity made in multiple correlation or that of any pre-determined mathematical function as in the method of least squares. The method consists of determining the relation of the independent variable to each of the dependent variables by a successive approximation to group averages. The coefficient is comparatively easily calculated by following their model. An illustrative case is worked out in the prediction of wheat yield per acre in South Dakota on the basis of the season's temperature and precipitation. The writers were collaborating on problems concerning the quantitative relation between variations in the number of certain marine organisms and fluctuations in the elements of their environment. The paper is introduced by Wm. E. Ritter as an important step forward in the methodology of natural science. Smith (30) develops the method for proper choice of distributions of observations for two variates connected by a linear relation. Isserlis' paper (18) is described by its title. Kornhauser, Meine and Ruml (21) explain the construction of two three-dimension models which materially assist in visualizing and understanding the meaning and relations of the coefficients of correlation and regression, the standard error of the variates, and the standard error of estimation.

Short Methods.—The Scott Company Laboratory (32) gives tables for facilitating the calculation of correlation by the rank difference method. They include the squares of differences up to a difference of 80, and the corresponding coefficients. Burtt (8) shows how to calculate partial correlations with a slide rule and finds it as rapid and accurate as using tables. Chapman (11) supplements Thurstone's method of calculating the product-moment coefficient without the use of deviations by also avoiding cross multiplying of the two variates. The necessary operations can then be performed by tables of squares and an adding machine. A splendid series of papers by Ayres shows short and easy ways of computing the product-moment coefficient (4, 1, 5), the coefficient of regression (2) and the correlation ratio (3). His simple method for computing the product-moment coefficient (4) saves from half to three fourths of the time of the common method. The method is based in the principle that in any series of numbers the sum of the squares of the deviations from the average is equal to the sum of the squares of the numbers in the series, minus the product of the total of the series and its average.

New Applications.—Thomson (34) shows that the problem of right and wrong cases in psycho-physics is a special case of the application of Pearson's Criterion of Goodness of Fit. Kelley (20) shows how partial correlation indicates principles for the selection of tests for classifying men. He also calculates that the army mental tests correlate .484 with vocational choice, and that all factors not measured by the tests would correlate .875. This method of estimating the relation to other factors than those tested is suggestive. It is also followed by Rosenow (29) who gives a table for facilitating the calculation, assuming various intercorrelations of the variables. Toops and Pintner (40) find a rank correlation of the intelligence level of trades with the grades at leaving school amounting to .79. Pressey and Ralston (27) trace the relation of the occupations of fathers to the intelligence of their children. Burtt (7) shows how he utilized regression equations for weighting test scores to place men at the work for which they were best fitted in an auto tire factory. Kelley (19) gives a method for correcting the measure of overlapping in school grades for a tested ability. All other measures have been incorrect, he claims, in not allowing for the reliability coefficients of the tests. Thurstone (39) derives a formula for weighting the right and wrong answers of a test in order to obtain the highest correlation with the criterion chosen. Courtis and Thorndike (12) set forth a method for developing correction formulæ for a test, in this case an addition test, under which the results are tested for various methods of giving and scoring the test. Through the intercorrelations it was found that scoring this test by the rate of performance at 75 per cent. accuracy corrected for motor ability was best. Rate scores are better than accuracy. The relations between interests and abilities are shown by (10) and (17), between mentality and school progress by (41). Yoakum and Yerkes (42) give the best summary of the application of the army tests to educational and industrial problems.

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