

ART. XV.—*On the wave lengths of the Spectral Lines of the Elements*; by WOLCOTT GIBBS, M.D., Rumford Professor in Harvard University. Read before the National Academy of Sciences, Aug. 16, 1867.

IN a memoir published in the Philosophical Transactions for 1864, Mr. Huggins has given for a particular scale the relative positions of a number of spectral lines. The scale selected was purely arbitrary. The number of elements examined was twenty-eight, and as the measurements were made with much accuracy it seemed to be desirable to extract from them all the information which they were capable of giving. For this purpose I have endeavored to determine the wave length of each line with as much precision as the present state of science permits, and in this manner to form tables which might enable me to determine whether the spectral lines are distributed according to definite laws, and if so, whether they can be considered as particular cases of the general principle of interferences.

The materials at my disposal were as follows: First, measurements by Angström of the wave lengths of 37 lines identified with particular elements and expressed in ten-millionths of a Paris inch. These were reduced to millionths of a millimeter by multiplying them by the constant 27.07. Second, measurements by Ditscheiner of 107 wave lengths, those of Fraunhofer's lines A, B, H and H' not being available. The entire number of spectral lines measured upon his scale by Mr. Huggins is about 1000, distributed among the elements as follows:

Oxygen	32	Thallium	16	Mercury	24
Hydrogen	1	Silver	18	Cobalt	75
Nitrogen	74	Tellurium	44	Arsenic	31
Sodium	9	Tin	18	Lead	22
Potassium	15	Iron	102	Zinc	28
Lithium	3	Cadmium	15	Chromium	65
Calcium	50	Antimony	67	Osmium	18
Barium	31	Gold	23	Palladium	43
Strontium	71	Bismuth	44	Platinum	20
Manganese	46				

With respect to Ditscheiner's measurements I may here state that I have reduced them as in my memoir on the construction of a normal map of the solar spectrum.* That is, I have taken the wave length of the more refrangible line of D, as 589.43, as determined by Angström, instead of 588.8, which is the

* This Journal, II, vol. xliii, p. 1, Jan., 1867.

value found by Fraunhofer. The reduced values as thus obtained correspond very closely with those of Angström, as I have shown in the paper referred to, excepting in the part of the spectrum between C and D.

With these materials it first became necessary to identify a sufficient number of lines upon Mr. Huggins's scale with lines the wave lengths of which had been measured. This proved to be a task of extraordinary difficulty and a severe tax upon my time and patience. Mr. Huggins's scale cannot in any part be superposed upon that of Kirchhoff. The identification of a few strongly marked lines, like those which Fraunhofer selected, is of course easy, but these do not suffice as data for interpolation. By very careful and laborious comparisons of the two scales; by observing the coincidences or close approximations of lines produced by different elements; by occasional graphical constructions, and by employing the numerical results obtained in my two papers on wave lengths already published, I at last succeeded in obtaining data covering all of Mr. Huggins's scale between 589.5, or C, and 4671, which lies about half way between G and H. Of the correctness of the identification of the lines I shall be able I think to furnish satisfactory proof.

In selecting the wave lengths to be used in interpolation, I have given the preference to the measurements of Angström. But in the part of the spectrum between C and D, these were not sufficiently numerous to be of service. For this portion I have employed exclusively the values given by Ditscheiner, reduced in the manner already pointed out. In other parts of the scale it has also been necessary occasionally to use Ditscheiner's measurements, but in all these portions the difference between the results of Angström and of Ditscheiner rarely amounts to a unit in the first decimal place.

The number of lines upon Mr. Huggins's scale identified with lines the wave lengths of which have been measured, amounts to forty-five. For the purpose of interpolation these were divided into nine groups. In table I, I have brought together for convenience both the data employed and the results obtained.

In this table the column H gives the scale-number of the line; λ the wave length as observed, λ' the wave length as calculated by the formula obtained, Δ the difference between the observed and calculated wave-lengths, and ϵ the probable error for each group of data. The wave-lengths marked † are those of Ditscheiner; the others are given by Angström.

TABLE I.

H	λ	λ'	Δ	ϵ	H	λ	λ'	Δ	ϵ
589.5	† 656.65	656.56	+0.09		1600	527.32	527.34	-0.02	
612	† 652.07	652.25	-0.18		1645	523.69	523.71	-0.02	
623	† 650.03	650.14	-0.11		1708	518.79	518.79	0.00	
709	† 634.23	634.03	+0.20		1723	517.66	517.65	+0.01	
772	† 623.59	623.39	+0.20		2036	† 496.11	496.05	+0.06	
795	† 619.60	619.90	-0.30		2092	492.23	492.31	-0.08	
813	† 617.48	617.39	+0.09		2147	489.50	489.45	+0.05	
818	† 616.71	616.71	0.00	± 0.18	2200	486.52	486.52	0.00	± 0.04
813	† 617.48	617.42	+0.06		2147	489.50	489.51	-0.01	
818	† 616.71	616.76	-0.05		2172	† 488.18	488.11	+0.07	
843	† 612.75	612.81	-0.06		2200	486.52	486.59	-0.07	
856	† 610.80	610.73	+0.07		2236	† 484.63	484.64	-0.01	
939	† 598.12	598.13	-0.01		2315	† 480.56	480.55	+0.01	± 0.04
1000	† 590.07	590.06	+0.01		2236	† 484.63	484.91	-0.28	
1005	† 589.43	589.45	-0.02	± 0.04	2315	† 480.56	480.26	+0.30	
1000	590.04	590.04	0.00		2785	457.75	457.74	+0.01	
1005	589.43	589.45	-0.02		3272	440.81	440.83	+0.02	
1236	561.99	562.00	-0.01		3341	438.63	438.67	-0.04	
1247	560.70	560.65	+0.05		3597	431.03	431.04	-0.01	± 0.18
1251	560.26	560.22	+0.04		3272	440.81	440.84	-0.03	
1274	557.65	557.53	+0.12		3341	438.63	438.69	-0.06	
1413	† 543.37	543.30	+0.07	± 0.06	3532	432.78	432.81	-0.03	
1413	† 543.37	543.44	-0.07		3597	431.03	431.00	+0.03	
1422	542.83	542.91	-0.08		3728	427.45	427.46	-0.01	
1445	† 540.95	541.09	-0.14		3773	426.26	426.28	-0.02	± 0.04
1491	† 537.55	537.42	+0.13		3597	431.03	430.97	+0.06	
1532	533.16	533.18	-0.02		3728	427.45	427.45	0.00	
1545	532.00	532.01	-0.01		3773	426.26	426.29	-0.03	
1582	528.73	528.69	+0.04		3812	425.21	425.29	-0.08	
1600	527.32	527.25	+0.07	± 0.07	3909	422.94	422.88	+0.06	
					4267	414.71	414.66	+0.05	
					4633	407.45	407.39	+0.06	
					4671	406.59	406.61	-0.12	± 0.06

The method of interpolation employed was that first given by Cauchy* and afterward reduced to a practical form by M. Yvon Villarceau, in the *Connaissance des Temps* for 1852. As in my recent reduction† of Kirchhoff's scale, I employed only expressions of the form

$$\lambda = a + bh + ch^2 + dh^3 \text{ \&c.}$$

these being found to give an approximation within the limits of the probable errors of observation.

Table II, gives the values of the constants, a , b , c , and d , as deduced from the data given in Table I.

In this table H is the initial and H' the terminal point of Mr. Huggins's scale in each group of data employed for discussion. For easy use in calculation it is more convenient to transfer the initial and terminal points, so as to make each parabolic curve

* Moigno. *Leçons de Calcul Differentiel et de Calcul Integral*. Tome I^{er}, 513.

† This Journal, II, May, 1868, vol. xlv, p. 1.

begin and end with a multiple of ten upon the scale. This has been done in Table III.*

TABLE II.

H	H'	a	b	c	d
589·5	818	656·5589	-19·0571	-0·4300	+0·4984
800	1018	619·5488	-15·6660	-0·3960	+0·4264
1000	1413	589·9750	-12·5440	+0·2369	+0·0165
1413	1600	543·5600	-7·8810	-0·8653	+0·2469
1600	2200	527·3240	-8·1670	+0·2276	-----
2147	2315	489·5078	-5·6093	+0·1622	-----
2230	3597	485·2950	-6·1092	+0·2398	-0·0061
3272	3773	440·8098	-3·3028	+0·1019	-0·0043
3597	4671	430·9556	-2·7232	+0·0433	-----

TABLE III.

H	H'	a	b	c	d
590	800	656·4637	-19·0614	-0·4225	+0·4984
800	1000	619·5488	-15·6660	-0·3960	+0·4264
1000	1410	589·9750	-12·5440	+0·2369	+0·0165
1410	1600	543·7956	-7·8284	-0·8875	+0·2469
1600	2200	527·3240	-8·1670	+0·2276	-----
2200	2310	486·5804	-5·4374	+0·1622	-----
2310	3270	480·5580	-5·7372	+0·2252	-0·0061
3270	3770	440·8759	-3·3069	+0·1022	+0·0043
3770	4670	426·3739	-2·5734	+0·0433	-----

From these tables it will be seen that in three out of the nine groups parabolas of the second order are sufficient.

If the results obtained in the discussion of Mr. Huggins's scale be compared with those given in my paper on the measurement of wave-lengths by the method of comparison in which Kirchhoff's scale is subjected to a similar treatment, the greater regularity of the former will be at once remarked. Thus in Kirchhoff's scale in two instances, parabolas of the 5th order are required. In two cases a straight line represents the observations best, the probable error being however unusually large. In the case of Mr. Huggins's scale, on the other hand, the probable error for each group appears to be much within the limits of the errors of observation. In the 6th and 7th groups of data discussed in the present paper, the intervals between the scale numbers are larger than I could have wished. Yet the differences between calculation and observation are not large, the curves here approaching a straight line. The greater regularity or relatively lower order of the curves in the case of Mr. Huggins's scale arises doubtless from the fact that the prisms employed were, during the whole series of observations, in a constant position, which was not the case with Kirchhoff's apparatus.

* In using the constants in Table III for computing wave-lengths the value of h is to be found by subtracting the initial number in column 1 from the scale number and dividing the remainder by 100.

By means of Table III the wave lengths of all the lines upon Mr. Huggins's scale between C and scale-number 4671 have been computed. These are given in Table IV. I have however not followed the order adopted by Mr. Huggins, but have put near each other as far as possible those elements which form members of natural groups.

TABLE IV.*

NITROGEN.		H	λ	H	λ	H	λ
H	λ						
629.5	2 649.08	2205	5 486.18	2181	5 487.56	1169	2 569.46 s
807	2 618.45h.d	2221	3 485.80	2213	3 485.74	1174	1 568.90 s
959	1 595.33	2305	2 480.84	2502	7 470.33 n	1746	1 515.90 n
967	6 594.26	2336	1 479.08	2512	7 469.84 n	1753	5 515.37 n
975	4 593.20	2350	7 478.29	2563	5 467.38 n	1991	498.87 h
978	1 592.80	2642	2 463.78 d	2597	5 465.82 n		
1100	5 577.53	2669	3 462.60	2626	2 464.50		
1118	1 575.37	2689	1 461.74	2642	2 463.78 d		
1135	2 573.37	2707	1 460.97	2748	1 459.27		
1150	2 571.64	2722	1-5 460.35	2766	1 458.54		
1171	1 569.24	2738	1-5 459.68	3086	446.75 h		
1177	8 568.55	2856	454.96 h	3238	1 445.03 n		
1180	1 568.21	2904	453.15 h	3241	1 441.75 n		
1187	7 567.43	2978	450.46 h	3395	1 436.85 n		
1294	5 555.56	3009	1 449.37	3456	2 435.00 n		
1302	5 554.71	3011	1 449.31	3560	1 431.95 n.d		
1310	1-5 553.85	3056	447.76 h	3710	427.87 h		
1314	5 553.37	3144	444.83 d	4059	1 419.29		
1319	2 552.91	3174	443.86 h	4087	1 418.65		
1349	1-2 549.78	3219	442.43 h	4232	415.43		
1366	1 548.04	3292	440.11 h	4395	2 412.01		
1383	5 546.33	3456	2 434.99 n	4615	1 407.82 n		
1394	7 545.43	3863	423.99 h	4639	1 407.31 n		
1502	2 535.81	3991	420.90 h				
1516	8 534.57	4145	417.34 h				
1537	2 532.69	4263	414.75 n				
1713	518.39 h	4330	413.34 h				
1718	518.01 h	4473	410.45 n				
1721	517.78 h	4505	1 409.82 n				
1860	5 507.64						
1900	3 504.88						
1929	7 502.93						
1941	5 502.12						
1951	5 501.46						
1956	5 501.14						
1960	5 500.84						
1967	10 500.42						
1978	5 499.71						
1990	3 498.94						
2079	5 493.43						
2140	5 489.86						
2168	5 488.28						
2192	3 486.96						
HYDROGEN.		H	λ				
				POTASSIUM.			
				H	λ	LITHIUM.	
						H	λ
				727	5 630.85 s		
				763	5 624.81 s		
				840	1-5 613.25 s		
				1049	1 583.78 n		
				1069	5 581.79 n		
				1073	1-5 580.80 n		
				1328	1 551.96 n		
				2260	2 483.18 n		
				3328	2 438.96 n		
				3591	1-5 431.16 n		
				3762	2 426.00 n		
				4082	3 418.77 n		
				SILVER.			
						H	λ
						690	2 637.48 s
						762	2 624.96 s
						899	5 604.06 s
						943	5 597.58 s
						1031.22	586.03 s
						1207	5 565.19 n
						1223	2 563.42 n
						1227	7 563.00 n
						1240	3 561.46 s
						1257	7 559.57 s
						1276.33	557.46 s
OXYGEN.		H	λ				
				SODIUM.			
				H	λ		
				818	1-5 616.72 s		
				821	1 616.56 s		
				1000	8 590.04 s		
				1005	8 589.43 s		

* In this table the small numerals after the scale numbers give the estimated intensities of the spectral lines upon an ascending scale. The letters s, n, h and d have the following significations; s, a sharply defined line, narrow when the slit is narrow; n, a line with indistinct edges even when the slit is narrow; h, a hazy line not resolvable into finer lines; d, double but too close for measurement.

TABLE IV (continued).

H	λ	H	λ	H	λ	CALCIUM.
1286 ⁵³	556 ³⁶ s	2859	¹ 454 ⁸⁴ h	2529	³ 469 ⁰¹ n	H λ
1372	² 547 ⁴⁴ s	2897	¹ 453 ⁴⁰ h	2687	¹ 461 ⁸³ h	622 ⁵ 650 ³⁴ s
1380	⁷ 546 ⁶³ s	3006	¹⁻⁵ 449 ⁴⁸ h	2740	¹ 459 ⁶⁰ h	625 ¹⁻⁵ 649 ⁷⁷ s
1421	³ 542 ⁸¹ s	3097	²⁻⁵ 446 ³⁸ h	2763	² 458 ⁶⁶ n	637 ⁶ 647 ⁴⁷ s
1435	³ 541 ⁶⁴ s	3381	¹ 437 ³⁴ n	2977	² 450 ⁵¹ n	642 ² 646 ⁵¹ s
1446	⁵ 540 ⁷² h	3497	¹ 433 ⁷⁷ n	3115	¹ 445 ⁷⁹ n	649 ⁶ 645 ¹⁷ s
1675	⁵ 521 ³⁴ n			3359	¹ 437 ⁹⁷ h	655 ⁶ 644 ⁰³ s
		ANTIMONY.		3446	⁴ 435 ²⁹ n	699 ² 635 ⁸³ s
	GOLD.	H λ		3756	³ 426 ⁶⁸ n	709 ² 634 ⁰³ s
	H λ	614	¹ 651 ⁹² n	3819	¹ 425 ⁰⁸ n	723 ² 631 ⁵⁵ s
643	⁵ 646 ³² s	620	¹ 650 ⁷² n	4043	¹⁻⁵ 419 ⁶⁷ n	813 ¹ 617 ⁵¹ s
659	⁵ 643 ²⁸ s	640	¹ 646 ⁸⁹ n			818 ⁴ 616 ⁶⁷ s
727	⁵ 630 ⁸⁵ s	679	² 639 ⁵¹ h	BISMUTH.		843 ³ 612 ⁷⁸ s
734	⁵ 629 ⁶³ s	719	¹ 632 ²⁴ n	H λ		859 ² 611 ⁰⁶ s
747	³ 627 ⁴³ s	729	² 630 ⁴⁹ n	621	⁴ 650 ⁵³ s	863 ² 609 ⁶³ s
951	⁵ 596 ⁴⁵ s	739	² 628 ⁷⁷ n	837 ³⁵	613 ⁷² n	882 ⁶ 606 ⁶⁷ h
956	⁵ 595 ⁷⁵ s	765	³ 624 ⁴⁹ n	884	⁵ 606 ³⁶ n	921 ² ² 600 ⁶³ s
981	⁷ 592 ⁴¹ s	787	¹ 621 ⁰⁹ h	887	⁵ 605 ⁹⁵ s	934 ⁶ 598 ⁸⁷ h
1011	¹ 588 ⁶² n	796	⁵ 619 ⁷⁷ h	899	¹ 604 ⁰⁶ s	1031 ³⁻⁵ 586 ⁰⁵ s
1025	² 586 ⁸² s	819	² 616 ⁵⁶ n	939	¹ 598 ¹⁴ h	1247 ⁵ 560 ⁶³ s
1045	⁴ 584 ³⁸ s	837	⁷ 613 ⁷⁰ n	943	¹ 597 ⁵⁸ h	1249 ⁵ 560 ³⁹ s
1081 ⁵⁵	579 ⁷⁶ s	871	⁷ 607 ⁴⁵ n	1026	² 586 ⁶⁸ n	1252 ¹⁻⁵ 560 ¹³ s
1109	¹ 576 ⁴⁴ s	889	³ 605 ⁵⁹ n	1059	¹ 582 ⁵³ n	1256 ²⁻⁵ 559 ⁶⁹ s
1199	⁵ 566 ⁰⁷ s	921 ¹⁸	600 ⁷⁵ n	1143	⁶ 572 ³⁶ n	1258 ⁵⁵ ⁵ 559 ⁴¹ s
1266	⁵ 558 ⁵⁹ n	937 ⁵²	598 ⁷² n	1197	³ 566 ³¹ n	1260 ⁴ 559 ²⁵ s
1647	⁵ 523 ⁵⁵ s	981 ⁵¹	592 ³⁵ n	1293	¹ 555 ⁶⁷ s	1265 ⁷ 558 ⁵⁹ s
1869	⁵ 506 ⁸⁷ s	988 ⁵³	591 ⁴⁵ n	1305	⁵ 554 ³⁹ s	1335 ⁷ 551 ²³ s
2291	⁵ 481 ⁵¹ s	1000 ⁵³	589 ⁹⁶ n	1395	¹ 545 ¹³ h	1506 ¹ 535 ⁴⁵ s
2326	⁴ 479 ⁶⁴ s	1041	⁵ 584 ⁷⁸ s	1453	¹ 540 ¹² h	1599 ⁵⁴ ⁵ 527 ³⁶ s
3026 ²⁻⁵	448 ⁷⁸ s	1057	⁵ 582 ⁷⁷ s	1495	¹ 536 ⁴³ h	1605 ³ 526 ⁹³ s
		1081	¹ 579 ⁸² n	1598	⁷ 527 ⁴⁸ n	1609 ⁵ 526 ⁶¹ s
		1145	⁵ 572 ²² n	1675	¹⁰ 522 ¹² n	1612 ¹⁻⁵ 526 ³⁶ s
	ARSENIC.	1158	¹ 570 ⁷² n	1685	¹ 520 ⁵⁶ n	1702 ¹ 519 ²⁴ s
	H λ	1189	⁵ 567 ²⁶ n	1759	⁷ 514 ⁹² n	1907 ⁷ 504 ⁴⁰ s
672	¹ 640 ⁸³ n	1207	¹ 565 ¹⁹ h	1787	⁶ 513 ⁸⁸ n	1935 ³ 502 ⁵² s
707	¹ 634 ³⁷ n	1214	⁴ 564 ⁴¹ n	1834	¹ 509 ⁴⁶ n	2172 ⁷ 493 ⁸⁵ s
759	¹ 625 ⁴⁵ n	1220	¹ 563 ⁷⁵ n	1851 ³⁵	508 ⁴¹ n	2777 ⁵ ⁵ 458 ¹⁰ s
812	⁶ 617 ⁶⁷ n	1279	⁴ 557 ¹⁸ n	1979	¹ 499 ⁵² h	2784 ³ 457 ⁸² n
833	¹ 614 ³⁵ n	1383	³ 546 ³³ n	2015	⁴ 497 ³⁶ h	2792 ² 457 ⁵⁰ s
850	³ 611 ⁶⁷ n	1457	⁵ 539 ⁷⁷ h	2105	⁵ 491 ⁸⁹ n	3124 ³ 445 ⁴⁹ s
870	² 608 ⁵³ n	1471	³ 538 ⁵⁵ n	2119	¹ 491 ⁰⁷ n	3181 ³ 443 ⁶⁴ s
908 ⁸³	602 ⁵⁹ n	1501	⁵ 535 ⁹⁰ n	2317	¹ 480 ¹⁵ n	3212 ² 442 ⁶⁶ s
1042	¹ 584 ⁶⁶ n	1636	³ 524 ⁴² h	2408	¹ 475 ¹⁴ n	3561 ³ 431 ⁹⁴ s
1090	⁵ 578 ⁷³ n	1661	¹ 522 ⁴⁴ s	2467	⁵ 472 ⁰⁹ n	3602 ⁵²⁻⁵⁴ ³⁹ 79 s
1203	⁷ 565 ⁶³ n	1715	³ 518 ²⁴ h	2502	¹ 470 ³³ n	3617 ⁴ 430 ³⁶ s
1231	⁵ 562 ⁵⁴ n	1765	³ 514 ⁴⁷ h	2837	⁵ 455 ⁶⁸ n	3628 ²⁻⁵ 430 ⁰⁸ s
1257	⁵ 559 ⁵⁸ n	1803	³ 511 ⁶⁴ h	3060	¹ 447 ⁶² n	3665 ³ 429 ⁰⁶ s
1291	⁶ 558 ⁸⁸ n	1849	¹ 508 ⁴¹ s	3315	⁴ 439 ³⁹ n	3692 ²⁻⁵ 428 ³⁶ s
1348	⁶ 549 ⁸⁸ n	1900	² 504 ⁸⁸ h	3481	³ 434 ²⁴ n	3909 ⁶ 422 ⁸⁶ s
1443	¹ 540 ⁹⁷ n	1919	² 503 ⁶⁰ h	3519	⁴ 433 ¹³ n	
1465	¹ 539 ⁰⁸ n	2051	³ 495 ¹³ n	3619	⁵ 430 ³¹ n	BARIUM.
1529	⁶ 533 ⁴¹ n	2171	³ 488 ¹¹ n	3778	⁵ 426 ¹¹ n	H λ
1577	¹ 529 ²⁰ n	2251	² 483 ⁶⁷ n	4378	⁴ 412 ⁸⁵ n	608 ¹ 653 ⁰² s
1648	⁴ 523 ⁴⁷ n	2339	² 478 ⁹⁰ h	4603	³ 407 ⁹⁷ n	621 ⁵⁷ ⁵ 650 ⁴⁴ s
1737	¹ 517 ⁶³ n	2377	² 476 ⁸¹ h			645 ¹ 645 ⁵⁴ s
1814	⁴ 510 ⁸⁹ n	2397	² 475 ⁷⁴ n			704 ¹ 634 ⁸³ s
1993	² 498 ⁷⁵ n	2440	² 473 ⁴⁶ n			847 ² 612 ¹⁵ s
2153	¹ 489 ¹² n	2488	³ 471 ⁰³ n			879 ² 607 ¹³ s
2450	¹ 472 ⁹⁵ n					

H		λ		H		λ		H		λ		H		λ	
908	1-5	602-70	s	1638	2	524-26	s	1225	5	563-20	s	2147	1	489-47	s
925	1-5	600-18	s	1651	1-5	523-23	s	1236	1-5	561-93	s	2781	5	458-02	s
943	1-5	597-58	s	1656	1-5	522-83	s	1247	5	560-69	s	3272	1-5	440-81	s
993	5	590-89	s	1659	1-5	522-60	s	1261	2	560-24	s	3341	2	438-55	s
1005	1	589-43	n	1665	1	522-14	s	1261	2	559-14	s	3532	1-5	432-75	s
1034	6	585-67	s	1745	1	515-98	s	1274	7	557-71	s	3597	1-5	430-93	s
1057	1	582-77	s	1817	5	510-68	s	1276	2	557-49	s	3610	1-5	430-65	s
1096	1	578-00	s	2021	1	496-98	s	1338	7	550-91	s,d	3623	5	430-21	s
1119	1	575-26	s	2029	2	496-48	s	1383	5	546-33	s	3645	5	429-61	s
1308	4	554-06	s	2060		494-58	h	1391	7	545-53	s	3728	2	427-40	s
1327	1	552-06	s	2145	1	489-57	s	1400	7	544-64	s	3773	1-5	426-24	s
1351	5	549-58	s	2176	1-5	487-84	s	1413	5	543-37	s	3812	1-5	425-26	s
2075	9	493-67	n	2180	1-5	487-62	s	1419	7	542-97	s	4009	1	420-46	s
2133	4	490-23	n	2185	1	487-35	s	1421-57		542-78	s	4019	1	420-23	s
2459	5	472-49	s	2213	1	485-74	s	1434	5	541-73	s	4221	1	415-70	s
2535	5	468-71	s	2254	1-5	483-51	s	1438	5	541-39	s	4267	1	414-67	s
2856	9	454-96	s	2291	1-7	481-51	s	1445	7	540-80	s	4323	1	413-49	s
2931	3	452-23	n	2343	1-5	478-69	s	1446	7	540-71	s	4633	1	407-41	s
4127	2	417-74	n	2410	5	475-04	s	1456	7	539-86	s	4671	1	406-73	s
4332	5	413-30	n	2427	1	474-14	s	1459-52		539-55	s				
				2469	1	471-98	s	1467	7	539-00	s				
				2726	5	460-18	n	1481	1	537-67	s,d				
				3169	1	444-02	s	1485-35		537-28	s				
				3389	5	437-03	n	1486	2	537-24	s	704	2	634-93	s
				3409	5	436-41	n	1486-52		537-18	s	837		613-72	
				3489	1	434-01	n	1488	2	537-05	s	843		612-78	
				3553	1	432-16	h	1532	2	533-14	s	909	3-5	602-55	s
				3604	5	430-73	n	1537	7	532-69	s	913	2-5	601-96	s
				3952	5	421-83	n	1541	5	532-34	s	915-52		601-66	s
				4181	3	416-58	n	1545	5	531-98	s	1289	1	556-00	s
				4599	3	408-05	n	1560	2	530-67	s	1329	1-5	551-85	n
								1574	2	529-45	s	1376	1	547-04	s
								1582	7	528-77	s,d	1413	7	543-37	s

TABLE IV (continued).

H	λ	H	λ	H	λ	H	λ
3120 1	445·62 s	1626 3	525·23 s	1657 7	522·76 s	2562 6	467·43 s
3131·53	445·31 s	1642 5	523·94 s	1677 4	521·18 s	3239 4	441·81 s
3133 2	445·19 s	1650 5	523·31 s	1680 3	520·95 s		
3141 1	444·99 s	1670 1·5	521·73 s	1681·53	520·87 s		ZINC.
3180 1	443·67 s	1685 5	520·56 s	1749 2	515·68 n		H λ
3242 1	441·72 s	1699 5	519·46 s	1815 7	510·82 n	696 7	637·37 s
3691 1	428·44 s	1707 5	518·85 s	2097 1	492·35 s	782 1	621·84 s
3749 1	426·85 s	1743 5	516·13 s	2156 1	488·95 s	855 5	610·89 n
3782 1	426·11 s	1756 5	515·15 s	2175 7	487·89 s	895 1	604·69 s
3870 3	423·84 s	1781 1	513·30 n	2181 3	487·57 s	909 5	602·55 n
		1813 1	510·96 h	2198 5	486·63 s	1001 2	589·90 s
		1857 5	507·85 h	2257 5	483·35 s	1062 5	582·16 s
	COBALT.	1876 5	506·52 s	2266 1·5	482·86 s	1110 5	576·33 s
	H λ	1887 3	505·77 s	2336 7	479·07 s	1122 6	574·90 n
645 1	645·94 n.d	1925 5	503·19 s	2400 1	475·57 s	1269 5	558·26 n
701 5	635·47 s	2021 5	496·98 s	2406 5	475·25 s	1283 1	557·75 n
731 2	630·75 s	2186 3	487·29 s	2435 7	473·73 s	1519 5	534·30 n
745 5	627·77 s	2236 3	484·47 s	2452 7	472·85 s	1626 1	525·23 n
763 2	624·81 s	2286 2	481·78 s	2474 1	471·72 s	1645 5	523·70 n
844 1	612·62 h	2325 1	479·63 s	2619 1	464·81 s	1743 5	516·13 n
865 7	609·32 s	2409 5	475·09 s	2627 7	464·46 s	1790 5	512·64 n
891 7	605·29 s	2438 5	473·56 s	2632 1·5	464·21 s.d	1797 5	512·13 n
921 1	600·76 s	2471 5	471·98 s	2663 5	462·86 s	1845 5	508·69 n
923 2	600·47 s	2550 5	468·01 s	2701 7	461·31 s	1859 1	507·71 n
931 5	599·30 s	2785 2	457·78 n	2740 7	459·60 s	1893 5	505·35 n
937 5	598·45 s	2823 1	456·22 n	2768 7	458·45 s	2016 5	497·29 n
985 2	591·90 s	2862 3	454·73 s	2840 7	455·56 s	2091 8	492·71 n
1039 2	585·04 s	2910 3	452·92 s	2871 7	454·38 s	2110 8	491·60 n
1043 2	584·53 s	4388 3	412·15 n	2887 7	453·78 s	2191 5	487·02 s
1207 5	565·19 s	4394 3	412·07 n	2899 7	453·33 s	2294 7	481·35 s
1217 5	564·08 s	4437 1	411·16 n	2914 1	452·78 s	2469 6	471·98 s
1257 7	559·58 s	4523 1	409·48 n	2927 1	452·30 s	2559 4	467·58 s
1361 1	548·55 s			3007 1	449·45 s		
1401 1·5	544·54		CHROMIUM.	3444 2	435·35 s		LEAD.
1470 5	538·64 s		H λ	3465 1	434·72 s		H λ
1483 1	537·50 s	621 5	650·53 n	3473 1	434·48 s	898 1	604·22 s
1491 1	536·79 s	640 5	646·89 n	3489 1	434·01 s	924 2	600·33 n
1496 2	536·34 s	654 1	644·22 n	3663 3	429·13 s	1000 1	590·02 n
1500 52	535·75 s	817 1	616·86 n	3719 3	427·64 s	1015 2	588·14 n
1501·52	535·66 s	843 1·5	612·77 n	3797 3	425·64 s	1031·14	586·05 s
1508 2	535·27 s	856 7	610·73 n	3905 1	422·96 s	1055 5	583·03 n
1514 4	534·74 s	1081 1	579·82 h	3951 1	421·90 n	1094 1	578·25 n
1525 5	533·77 n.d	1087 1	579·10 h			1240 9	561·46
1534 5	532·96 s	1090 1	578·73 h		CADMIUM.	1279 5	557·18 n
1539 3	532·52 s	1212 1	564·63 s		H λ	1479 7	537·85 n
1543 2	532·17 s	1242 1	561·23 s	639 2	647·08 n	1593·35	527·84 n
1549 5	531·64 s	1439 1·5	542·14 s	656 8	643·84 s	1685 1	520·57 s
1573 2	529·54 s	1507·57	535·31 s	889·55	605·22 n	1698 1	519·54 n
1579 2	529·02 s	1510 1	535·10 s	918 1	601·22 n	1735 5	516·72 n
1584 2	528·60 s	1532 1	533·14 s	953 1	596·17 n	1900·32	504·86 n
1586 1·5	528·43 s	1567 1	530·07 d	986 1	591·77 n	2384 3	476·44 n
1591 5	528·01 s	1594 7	527·75 s	1473 10	538·38 n	3329 7	438·93 n
1602 3	527·17 s	1605·51	526·93 s	1517 10	534·47 n	3730 2	427·35 n
1604 3	527·01 s	1607 1	526·77 s	1556 1	531·01 n	3831 6	424·83 n
1617 5	525·96 s	1619 5	525·79 s	1747 1	515·83 s		
1619 3	525·79 s	1626 5	525·23 s	1843 10	508·83 s		
1622 5	525·55 s	1640 5	524·10 s	2315 8	480·27 s		

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TABLE IV (continued).

MERCURY.			H		λ		H		λ		OSMIUM.		
H	λ										H	λ	
685	1	638.41	n	1270	4	558.15	s	1240.53	561.40	s	641	1	646.70 s
697	1	636.20	n	1357	4	548.96	s	1248	5	560.57 s	741	7	628.43 s
826	7	615.46	n	1366	4	548.05	s	1259.27	558.24	s, d	929	7	599.59 s
863	2	609.63	n	1396	4	545.03	s	1281.1-5	556.96	s	1029	1	581.31 s
1008.1-5	589.01	n	1438.34	541.40	s	1299	1	555.03 s	1093	5	578.37 s		
1019	5	587.59	n	1485	4	537.32	s	1303.1-5	554.60	n	1141	1	572.69 s
1060	3	582.41	n	1548.23	531.69	s	1331.1-5	551.65	n	1264	1	558.91 s	
1074	5	580.68	h	1562	2	530.50	s	1380.1-5	546.63	s	1322	2	552.60 s
1083.58	579.51	n	1658	5	522.68	n	1412	5	543.51 s	1405	5	544.14 s	
1100.58	576.93	n	1773	1	513.89	n	1456	2	539.86	1432.1-5	541.89	s	
1177	2	568.55	n	1909	5	504.27	n	1492	5	536.71 s	1683	5	520.72 s
1252	1	560.13	n	2191	1	487.01	h	1511	5	535.01 s	1859	5	507.71 s
1385	10	546.13	n	2245	1	484.00	h	1548	5	531.73 s	2861	7	454.76 s
1421.1-5	542.81	n	2341	5	478.80	h	1569	3	529.88	3225	2	442.24 s	
1487	5	537.15	n	2497	470.58	h	1617	5	525.96 s	3421	1	436.05 s	
1583	5	528.69	n	2595	1	465.91	n	1622	5	525.55 s	3583	1	431.33 s
1662	3	522.37	h	2613	1	465.09	n	1642.1-5	523.94	s	3645	1	429.61 s
1777	5	513.60	h	2730	1	460.01	n	1674	7	521.42 s	3773	3	426.24 s
2033	1	496.24	h	2739	1	459.64	n	1735	2	516.72 s			
2101	5	492.12	h	2875	454.23	h	1798.1-3	512.05	s				
2263	5	483.02	n	3051	1	447.93	h	1807	1	511.40 s			
3421	8	436.05	n	3435	5	435.62	n	1873	5	506.73 s			
				3619	2	430.32	n	2175.1-5	487.89	s			
				3779	426.09	h	2279.52	482.12	s				
							3065	7	447.45 s				
							3963	4	421.57 s				
TELLURIUM.			PALLADIUM.			TIN.			PLATINUM.				
H	λ		H	λ		H	λ		H	λ			
657.37	643.59	s	685	5	638.40	s			689	5	637.63 s		
693	1	636.93	s	762	2	624.96	s		913	601.95	h		
703	5	635.11	n	837	2	613.72	s		939	598.14	h		
735	2	629.46	s	995	590.64	h			950	596.59	s		
765.33	624.44	n	1005	589.43	h				958	595.47	s		
774	3	623.07	s	1023	3	587.07	s		1041	584.78	s		
894	2	604.84	s	1031.5	586.00	h			1045	584.28	s		
917	1	601.37	s	1056.7	582.90	s			1073	580.80	d		
927	5	599.90	n	1068	581.42	h			1367	547.95	s		
945.36	597.26	s	1084	579.46	h				1459	539.59	s		
971.51	593.66	s	1127	3	574.31	s			1484	537.41	s		
1030.35	586.14	s	1129	1	574.08	s			1561	530.76	s		
1035	2	585.54	s	1185	5	567.65	s		1653	523.08	s		
1111	7	576.21	n	1199	5	566.08	s		1689	520.25	s		
1122	1	574.90	s	1212	5	564.63	s		1879	506.32	s		
1151	7	571.53	n	1219	563.86	h			2857	454.92	s		
1204	6	565.53	n	1233	562.29	h			2936	451.97	s		
1230	1	562.65	s						2999	449.72	s		
									3156	444.45	s		
									3525	432.95	s		

I have stated above that the principal difficulty with which I have had to contend was the identification of a sufficient number of scale lines with lines of which the wave lengths had been measured either by Angström or by Ditscheiner. Upon the correctness of this identification the value of my work depends. For the purpose of verification, before proceeding to determine parabolic formulas for different parts of the scale, I carefully compared Kirchhoff's and Huggins's scales with each other for their whole lengths, selecting the lines about whose correspondance there could be no doubt. When the formulas

had been obtained, I calculated the wave lengths for all these lines upon both scales. The results of this comparison are given in Table V.

TABLE. V.

K	H	K λ	H λ	Δ	E
717.5	619	650.68	650.91	-0.23	Sr
718.7	621.5	650.24	650.44	-0.20	Ba
720.1	625	649.88	649.77	+0.11	Ca
729	637	647.60	647.47	+0.13	Ca
731.7	642	646.93	646.51	+0.42	Ca
736.9	649	645.60	645.17	+0.43	Ca
740.9	658	644.60	644.03	+0.57	Ca
753.8	669	641.38	641.39	-0.01	Sr
756.9	673	640.60	640.64	-0.04	Fe
771.8	696	636.99	637.37	-0.38	Zn
831	772	623.52	623.38	+0.14	Fe
849.7	795	619.60	619.92	-0.32	Fe
860.2	813	617.47	617.52	-0.05	Ca
863.9	818	616.71	616.71	0.00	Ca
880.9	837	613.50	613.73	-0.23	Sb
884.9	843	612.94	612.78	+0.16	Ca
890	847	611.75	612.15	-0.40	Ba
894.9	856	610.80	610.73	+0.07	Ca
958.8	939	598.22	598.14	-0.08	Pt
961.5	943	597.05	597.58	-0.53	Ba
1002.8	1000	590.07	590.02	+0.05	Na
1006.8	1005	589.43	589.42	+0.01	Na
1029.3	1031	585.87	586.05	-0.18	Ca
1031.8	1034	585.53	585.67	-0.14	Ba
1043	1045	583.95	584.28	-0.33	Au
10.0	1057	582.91	582.79	+0.12	Ba
1158	1177	568.34	568.56	-0.22	Hg
1200.6	1225	562.95	563.20	-0.25	Fe
1207.3	1236	561.96	562.00	-0.04	Fe
1217.8	1247	560.75	560.62	+0.13	Fe, Ca
1219.2	1249	560.46	560.46	0.00	Ca
1231.3	1261	559.12	559.14	-0.02	Fe
1235	1265	558.65	558.70	-0.02	Ca
1245.6	1276	557.38	557.49	-0.11	Fe
1274.7	1311	553.92	553.75	+0.17	Sr
1301	1341	550.79	550.61	+0.18	Sr
1337	1383	546.72	546.33	+0.39	Fe
1343.5	1391	545.93	545.53	+0.45	Fe
1352.7	1400	544.94	544.64	+0.30	Fe
1385.7	1439	541.33	541.30	+0.03	Cr
1410.5	1467	538.77	538.90	-0.13	Fe
1421.5	1481	537.80	537.67	+0.17	Fe
1430.1	1491	536.62	536.79	-0.17	Co
1440.2	1501.5	535.61	535.86	-0.25	Co
1443.5	1506	535.25	535.45	-0.20	Ca
1506.3	1582	528.77	528.77	0.00	Fe
1523.7	1599.5	527.42	527.37	+0.05	Fe, Ca
1539	1617	526.00	525.96	+0.04	Sr
1566.5	1642	523.87	523.94	-0.07	Co
1575	1651	523.20	523.22	-0.07	Sr
1577.5	1656	523.08	522.83	+0.25	Sr
1582	1659	522.74	522.60	+0.14	Sr
1596	1670	521.63	521.73	-0.10	Co
1601.5	1677	521.31	521.18	+0.12	Cr
1627.2	1702	519.25	519.24	+0.01	Ca
1650.3	1728	517.56	517.26	+0.30	Fe
1769.5	1843	508.77	508.83	-0.06	Cd
1832.8	1907	504.65	504.39	+0.26	Ca
1867.1	1940.5	502.25	501.86	-0.39	Fe
1961	2036	496.11	496.05	+0.06	Fe
1989.5	2075	493.78	493.67	+0.11	Ba
2058	2172	488.40	488.06	+0.34	Ca

TABLE V (continued).

K	H	K λ	H λ	Δ	E
2071.3	2186	487.19	487.29	-0.10	Co
2103.3	2236	484.66	484.48	+0.18	Co
2132.3	2286	482.00	481.80	+0.20	Co
2148.5	2315	480.56	480.27	+0.29	Cd
2157.4	2325	479.76	479.70	+0.06	Co
2626.5	3156	444.61	444.45	+0.16	Pt
2668.5	3239	441.90	441.75	+0.15	Cd
2721.6	3341	438.75	438.55	+0.20	Fe
2822.3	3532	432.82	432.75	+0.07	Fe
2834.2	3561	431.77	431.92	-0.15	Ca
2854.7	3597	431.03	430.92	+0.11	Fe
2864.7	3617	430.79	430.36	+0.43	Ca
2869.7	3628	430.21	430.05	+0.36	Ca

In this table K represents the scale number upon Kirchhoff's scale, H that upon Mr. Huggins's scale, K λ and H λ the corresponding wave lengths, and Δ their difference. An examination of this table will show, I think, upon the whole, a very satisfactory correspondence. For the portion of Mr. Huggins's scale beyond G I have been obliged to rely upon my unassisted judgment for identification, as Kirchhoff's scale does not extend much farther than this point. This is therefore probably the least reliable portion of my work, though as yet I have seen no reason to doubt its accuracy. For the portion of the scale between A and C we possess very few measurements of wave lengths, and I have therefore postponed its discussion until more ample materials should be at hand. To form a correct estimate of the degree of accuracy to be obtained by interpolation in cases like the one before us, it is desirable to compare with each other the measurements of wave lengths made by different observers. To facilitate this comparison as much as possible, I have brought together in tables VI and VII all the measurements of wave lengths which I have been able to find, with the exception of a few isolated cases.

This table contains measurements by Van der Willigen,* Angström† and Ditscheiner,‡ with columns of differences for convenience of reference. Column first contains the numbers denoting the lines measured by Van der Willigen; column second, the corresponding numbers on Kirchhoff's scale; column third, Angström's measurements, reduced to millionths of a millimeter; column fourth, the measurements of Van der Willigen; column fifth, Ditscheiner's measurements, reduced for the value of D_2 , given by Angström; column sixth, Ditscheiner's measurements, as given in his second paper,§ in which the absolute value of the interval between two successive

* Archives du Muséum Teyler, vol. i, p. 1.

† Pogg. Ann., cxxiii, p. 489.

‡ Sitzungsberichte der k. k. Akad. der Wissenschaften, Bd. I, 1864.

§ Band lii. 289.

NOTE.—I have not been able to find in any public library in this country a copy of the Annales Scientifiques de l'école normale supérieure à Paris, vol. iv, containing an extended memoir on wave lengths by Mascart.

TABLE VI.

V.D.W.	K.	Ang. λ .	V.D.W. λ .	Ditsch. λ 1.	Ditsch. λ 2.	A.and V.D.W.	A.andD.1.	A.andD.2.	V.D.W. and D.1.
4 α	593	687.49	687.48	687.81	688.33	+0.01	-0.32	-0.84	-0.33
5	694	656.76	656.56	656.77	657.11	+0.20	-0.01	-0.35	-0.21
6	711.5	-----	651.94	652.09	652.58	---	---	---	-0.15
7	719.5	-----	649.77	650.05	650.54	---	---	---	-0.23
9	850	619.17	619.45	619.72	620.09	-0.28	-0.55	-0.92	-0.27
10	864	616.33	616.49	616.73	617.19	-0.16	-0.40	-0.86	-0.24
11	877	614.31	613.96	614.24	614.70	+0.35	+0.07	-0.39	-0.28
12	885	612.34	612.52	612.76	613.22	-0.18	-0.42	-0.88	-0.24
13	895	610.45	610.52	610.82	611.28	-0.07	-0.37	-0.83	-0.32
14 α	1002.8	590.04	589.86	590.07	590.53	+0.18	-0.03	-0.49	-0.21
14 γ	1006.8	589.43	589.26	589.43	589.89	+0.17	0.00	-0.46	-0.17
15	1200.4	-----	562.70	562.97	563.39	---	---	---	-0.27
16	1207	561.99	561.80	561.98	562.40	+0.19	+0.01	-0.41	-0.18
17	1280	-----	553.19	553.27	553.68	---	---	---	-0.08
18	1324.8	-----	547.86	548.13	548.54	---	---	---	-0.27
19	1343	545.97	545.83	546.05	546.46	+0.14	-0.08	-0.49	-0.23
20	1421.6	-----	537.38	537.52	537.92	---	---	---	-0.14
21	1463	533.16	533.05	533.29	533.69	+0.11	-0.13	-0.53	-0.24
22 α	1523.5	527.32	527.24	527.38	527.83	+0.08	-0.06	-0.51	-0.14
23	1569.8	523.69	523.50	523.74	524.13	+0.19	-0.15	-0.44	-0.24
24	1577.5	-----	522.96	523.09	523.49	---	---	---	-0.13
25	1634	518.79	518.63	518.73	519.12	+0.16	+0.03	-0.33	-0.10
26	1648.8	517.66	517.51	517.70	518.09	+0.11	-0.04	-0.43	-0.19
27 β	1655.6	517.15	517.07	517.15	517.54	+0.08	0.00	-0.39	-0.08
28	1750.4	-----	510.18	510.30	510.68	---	---	---	-0.12
29	1777.4	-----	508.27	508.41	508.79	---	---	---	-0.14
30	1834	-----	504.37	504.55	504.93	---	---	---	-0.18
31	1961	-----	496.01	496.15	496.53	---	---	---	-0.14
32	2041.4	489.50	489.38	489.54	489.90	+0.12	-0.04	-0.40	-0.16
33	2067	487.55	487.46	487.55	487.91	+0.09	0.00	-0.36	-0.09
34	2080.1	486.52	486.39	486.49	486.87	+0.12	+0.03	-0.35	-0.10
35	2309	-----	467.00	467.07	467.42	---	---	---	-0.07
36 α	2489.4	-----	453.75	453.73	454.09	---	---	---	+0.02
37	2721.6	438.63	438.58	438.75	439.08	+0.05	-0.12	-0.45	-0.17
38	2797	434.28	434.28	434.34	434.66	0.00	-0.06	-0.38	-0.06
39	2822.8	432.78	432.74	432.82	433.14	+0.04	-0.04	-0.36	-0.08
40	2854.7	431.03	431.12	431.35	431.70	-0.09	-0.32	-0.67	-0.23
H	-----	397.16	397.13	397.10	397.42	+0.03	+0.06	-0.26	+0.03
H $_1$	-----	393.59	393.76	393.74	394.05	-0.17	-0.15	-0.46	+0.02

lines on the ruled glass surface employed was determined. The other columns exhibit the differences between the first and second, first and third, first and fourth, and second and third series of measurements.

In table VII, I have given the measurements of the wave lengths of the eleven principal lines of Fraunhofer made by different observers. The differences, as will be remarked, are rather large, but it must be borne in mind that the breadths of the lines themselves is probably one source of error,—different observers measuring different parts of the same line.

In my second discussion of Kirchhoff's scale, I have divided the observations into twelve groups, giving weights to all the measurements of wave lengths, and assigning the probable error in the case of each group as well as the values of the con-

TABLE VII.

	Fraun- hofer.	Ditschei- ner 1.	Ditschei- ner 2.	Angstrom.	v. d. Wil- ligen.	Mascart.	Bernard.	Stefan.
A	-----	-----	-----	761.20	763.36	-----	760.6	759.8
B	687.85	687.81	688.33	687.49	687.48	686.66	686.9	687.2
C	655.63	656.66	657.11	656.77	656.56	656.07	656.1	655.8
D	-----	590.07	590.53	590.04	589.86	589.43	-----	589.4
D	588.77	-----	589.89	589.43	589.26	588.82	588.8	-----
E	526.51	527.42	527.83	527.38	527.24	526.79	526.8	525.3
b	-----	517.70	518.09	517.66	517.51	517.06	-----	518.7
F	485.63	486.49	486.87	486.52	486.39	485.98	485.9	484.3
G	429.60	431.35	431.70	431.03	431.12	430.76	430.6	430.2
H	396.30	397.10	397.42	397.16	397.13	396.72	396.8	-----
H ₁	-----	393.74	394.05	393.59	393.76	-----	-----	-----

stants deduced for each portion of the scale. Unfortunately, several errors have crept into the table of constants ; these I will correct in this place, giving the proper values in table VIII.

TABLE VIII.

	1	2	a	b	c	d	e
1	694.1	877	656.65	-26.325	+1.0490	+0.3618	-----
2	877	1135.1	614.19	-17.670	-5.5610	+4.6196	-0.8913
3	1135.1	1303.5	571.47	-14.008	+1.5466	-0.3707	-----
4	1303.5	1421.5	550.82	-12.842	+2.1860	-0.7367	-----
5	1421.5	1577.6	537.49	-10.063	-0.6920	+0.7960	-----
6	1577.6	1750.4	523.09	-8.125	+0.8911	-0.2463	-----
7	1750.4	1920.2	510.30	-6.792	-0.1661	+0.1180	-----
8	1920.2	2067.1	498.76	-5.029	-4.2235	+1.6624	-----
9	2067.1	2250.0	487.56	-3.7225	-----	-----	-----
10	2250.0	2547.2	471.24	-7.048	-----	-----	-----
11	2547.2	2721.6	449.67	-6.154	-0.4536	+0.2028	-----
12	2721.6	2869.7	438.68	-7.028	+2.1490	+1.5740	-1.6350

By means of these constants the wave lengths of all the metallic lines upon Kirchhoff's chart have been computed. These are given in table IX.*

TABLE IX.

CALCIUM.		K		BARIUM.		K	
K	λ	K	λ	K	λ	K	λ
717.8	650.47 (2b)	1221.6	500.24 (5d)	718.7	650.24 (2)	1269.5	554.52
720.1	649.88 (2c)	1224.9	559.87 (5d)	874.3	614.78 (4b)	1274.7	553.90 (3a)
729	647.60 (2b)	1228.3	559.42 (2d)	890.2	611.75 (1b)	1286	552.57
731.7	646.03 (5b)	1229.6	559.27 (4c)	903.5	609.17	1301	550.83
736.9	645.60 (3b)	1235	558.62 (3d)	903.5	609.17	1317	549.11
740.9	644.60 (5b)	1443.5	535.25 (2b)	934	603.08	1320.6	548.68 (4c)
840	621.81	1522.7	527.40 (6c)	964.5	597.05	1539	525.98
844.5	620.90	1528.7	526.88 (5c)	1031.8	585.51 (2a)	1562	524.18
848	619.95	1530.2	526.75 (4c)	1050	582.88	1575	523.24
849.5	619.54	1532.5	526.55 (4b)	1083	578.51 (2a)	1578.5	522.97
855	618.54	1533.1	526.49 (4b)	1274.2	553.96 (3b)	1582	522.71
857.5	618.03	1627.2	519.24 (5b)	1287.5	552.40 (1c)	2385.5	461.69
860.2	617.50 (3d)	1832.8	504.66 (2a)	1371.4	542.87 (1b)	2386.6	461.62 (2a)
863.9	616.78 (5b)	2058	488.40 (6c)	1980.5	493.78 (6c)	2558.5	431.38 (4a)
884.9	612.94 (4b)	2606.5	445.43 (5c)	2031.1	490.20 (2c)	2800	431.18 (1)
891.9	610.83	2638.5	443.89 (4e)	2461.2	456.36 (6b)		
941.5	602.56	2653	442.96 (1d)	2502.3	453.46 (4c)		
943	602.26	2834.2	433.01 (5c)				
1029.3	585.87 (3c)	2855.2	431.56 (4)				
1217.8	560.69 (5d)	2864.7	430.79 (4b)	STRONTIUM.		MAGNESIUM.	
1219.2	560.53 (3c)	2869.7	430.21 (5c)	K	λ	K	λ
				717.5	650.68	1634.1	518.73 (6g)
				753.8	641.38 (3b)	1649	517.64 (6f)
						1655.9	517.17 (4d)

* In this table the intensities of the lines are denoted on an increasing scale by the numbers 1, 2, 3, 4, 5, 6, and the breadths by the letters a, b, c, d, e, f, g, so that a denotes the least and g the greatest breadth.

K		λ		CADMIUM.		K		λ		K		λ	
2414.8	459.62 (2b)			730.5	647.22	1337	546.72 (4d)			2013.5	492.73		
2565	448.57 (6c)			740.9	644.59 (5b)	1343.5	545.98 (6c)			2018	491.34		
2568	448.39 (3b)			1453.7	531.27	1351.1	545.12 (5d)			2136	489.88 (5b)		
				1769.5	509.00	1352.7	544.93 (5b)			2240	472.25 (3a)		
				2148	480.56	1362.9	543.80 (5b)			2291.8	468.29 (2g)		
				2294.5	463.10 (2b)	1367	543.34 (6d)						
				2667.7	441.94	1372.6	542.74 (5b)						
						1380.5	541.89 (4c)						
						1384.7	541.44 (4c)						
						1389.4	540.93 (6c)						
						1390.9	540.78 (5d)						
						1397.5	540.07 (5c)						
						1401.6	539.63 (4c)						
						1410.5	538.08 (4c)						
						1421.5	537.51 (6c)						
						1423	537.32 (5b)						
						1425.4	537.11 (5b)						
						1428.2	536.81 (5b)						
						1450.8	534.50 (5c)						
						1451.8	534.40 (5b)						
						1462.8	533.26 (5c)						
						1463.3	533.21 (5c)						
						1466.8	532.90 (5c)						
						1473.9	532.13 (5b)						
						1487.7	530.75 (5b)						
						1506.3	528.93 (5c)						
						1508.6	528.76 (5b)						
						1522.7	527.41 (6c)						
						1523.7	527.32 (6c)						
						1527.7	526.98 (5c)						
						1569.6	523.63 (5c)						
						1577.2	523.07 (5c)						
						1622.3	519.62 (5c)						
						1623.4	519.53 (5b)						
						1630.3	517.55 (6b)						
						1633.7	517.31 (6b)						
						1655.6	517.16 (6c)						
						1662.8	516.67 (5b)						
						1693.8	514.47 (6c)						
						1701.8	513.57 (5c)						

TABLE IX (continued).

NICKEL.		K		K		K		K	
K	λ	K	λ	K	λ	K	λ	K	λ
856.8	618.19 (2a)	1573.0	523.39 2	1302	550.74 1	977.4	594.60		
857	618.10 (2a)	1623.1	519.56 1	1303.4	550.60 2	977.7	594.54		
887.7	612.23 (2a)	1629.2	519.10 } 2	1317.6	549.05 1	982.0	593.75		
891.7	611.47 (2a)	1630.4	519.01 } 2	1345.4	544.62 1	982.3	593.70		
1005	589.73 (2b)	1683.1	515.20 1	1486.8	530.84 } 2	988.9	592.51		
1029.3	585.87 (3c)	1725.5	512.18 1	1489.2	530.60 } 2	989.2	592.45		
1324.8	548.17 (4d)	1777.5	508.45 2	1622.3	519.62 1	989.6	592.38		
1643	518.03 (1b)	1782.4	508.12 } 1	1623.3	519.55 1	993.1	592.77		
1653.7	517.30 (6b)	1784.5	507.97 } 1	1716.6	512.82 2	993.4	592.73		
1672.2	515.97 (4a)	1938.8	497.68 2	1728.8	511.95 2	998.1	590.90		
1684	515.25 (4a)	2052.3	483.56 1	1894.5	500.52 } 2	999.2	590.70		
1690	514.71 (5b)	2221.5	474.22 1	1895.2	500.48 } 2	1000	590.57		
1697	514.21 (5c)			1903	499.97 1	1001.4	590.33		
1727.3	512.05 (3b)			1940.2	497.60 1	1005.8	589.60		
1748.9	510.48 (3c)	DIDYMIUM.		1988.6	493.86 } 1	1008.3	589.17		
1749.6	510.43 (2d)	K	λ	1989.5	493.78 } 1	1009.2	589.02		
1775.8	508.57 (3b)	1225	559.83 2	2003.8	492.56 2	1010.5	588.81		
1776.5	508.52 (3c)	1230	559.22 1	2004.7	492.49 2	1013.9	588.26		
1842.2	504.02 (4b)	1364.5	544.62 } 1	2031	490.25 2	1015.1	588.07		
1868.4	502.25 (5b)	1365.2	544.55 } 1	2081	486.49 2	1016.4	587.86		
1920.2	498.76 (4b)	1431.9	536.44 1	2121.4	483.02 1	1017.7	587.65		
1925.8	498.50 (4b)	1471.1	532.42 1	2208.2	475.44 2	1018.2	587.57		
1987.5	493.87 (3a)	1518.6	527.78 } 1	2214.5	474.87 2				
2008.1	492.19 (1b)	1519.4	527.71 } 1	2217.8	474.57 2				
2025.7	490.59 (4a)	1536	526.24 1	2646.2	443.39				
2064.7	487.66 (2c)	1541.4	525.79 1			PALLADIUM.			
2073.5	487.03 (3b)	1548.9	525.18 2			K	λ		
2086.9	485.97 (3b)	1567.5	523.79 1	ATMOSPHERIC LINES.		1114.7	574.36 1		
2112.7	483.82 (3b)	1709.2	513.35 2	K	λ	1146.2	569.94 2		
2115	483.56 (3a)			711.4	652.13	1164.9	567.42 2		
2164	479.16 (4a)	LANTHANUM.		948	600.27	1185.6	564.66 1		
2199.2	476.21 (3a)	K	λ	949.4	599.99	1264.6	555.10 2		
2249.7	471.26 (6a)	1411.6	538.56 } 2	949.8	599.91	1269	554.58 2		
2334.1	465.31 (2d)	1412.8	538.43 } 2	951.7	599.54	1279.1	553.39 1		
		1416.8	538.00 2	954.2	599.05	1400	539.80 } 2		
		1451	534.48 1	958.8	598.15	1407	539.06 } 2		
		1606.8	520.80 2	959.6	598.00	1430.1	536.62 1		
		1627.9	519.20 2	961.9	597.56	1447.0	534.83 1		
		1634.8	518.69 2	963.7	597.20	1477.0	531.82 1		
		2136.8	481.59 1	964.4	597.06	1495.2	530.01 3		
				965.7	596.81	1540	525.90 1		
				967.7	596.22	1566.5	523.86 } 2		
				969	596.18	1567.1	523.82 } 2		
				969.6	596.06	1601.4	521.21 1		
				970.5	595.90	1660	516.85 } 3		
				972.1	595.60	1660.7	516.80 } 3		
				974.3	595.18	1732.9	511.65 2		
				975	595.05	1801.9	506.78 1		
				975.7	594.93	2062	487.76 2		
				976.1	594.85	2128.6	482.81 2		
						2163	479.33 2		

As it is important to be able to judge of the probable error in the determination of each of these lines, I reprint, for the sake of convenience, the table of data employed and differences obtained given in my paper on the determination of wave lengths by the method of comparison. In this reprint, two or three trifling errors have been corrected.

I have already called attention to the very large probable error (± 0.66) in the tenth group of observations discussed. That it may not be supposed that I have too hastily adopted a straight line as most nearly representing the data assumed, I will here give the successive orders of differences corresponding to parabolas of the 2d, 3d, 4th and 5th degrees, which I have endeavored to draw through the same points.

TABLE X.*

N	W	K	λ	Δ	ϵ	N	W	K	λ	Δ	ϵ
1	9	694.1	656.65	0.00		55	4	1750.4	510.28	-0.02	
2	2	711.4	652.07	-0.06		56	6	1777.5	508.39	-0.02	
3	2	719.6	650.03	+0.02		57	5	1799.0	506.89	-0.04	
4	1	783.8	634.23	+0.08		58	13	1834.3	504.54	+0.03	
5	4	831.0	623.59	+0.08		59	5	1854.9	503.24	+0.12	
6	4	849.7	619.60	0.00		60	5	1867.1	502.25	-0.05	
7	7	860.2	617.48	0.00		61	2	1873.4	501.66	-0.23	
8	8	863.9	616.71	-0.04		62	7	1885.8	501.05	-0.01	
9	6	874.3	614.72	0.00		63	4	1908.5	499.70	+0.12	
10	9	877.0	614.23	0.00	± 0.04	64	6	1920.2	498.76	-0.07	± 0.08
10	9	877.0	614.23	+0.05		64	6	1920.2	498.76	0.00	
11	7	884.9	612.75	0.00		65	17	1960.8	496.14	+0.01	
12	7	894.9	610.80	-0.06		66	2	1975.7	495.03	+0.08	
13	2	958.8	598.12	-0.02		67	7	1983.3	494.33	0.00	
14	11	1002.8	590.07	-0.05		68	6	1989.5	493.75	-0.05	
15	18	1006.8	589.43	-0.02		69	7	2005.2	492.31	-0.15	
16	4	1019.3	586.25	+0.35		70	3	2018.5	491.39	+0.07	
17	3	1096.1	576.71	-0.13		71	9	2041.3	489.52	+0.09	
18	3	1102.9	575.77	-0.18		72	2	2058.0	488.16	0.00	
19	8	1135.1	571.49	+0.06	0.13	73	9	2067.1	487.53	0.00	± 0.06
19	8	1135.1	571.49	0.00		73	9	2067.1	487.53	-0.03	
20	2	1155.7	568.69	+0.02		74	21	2080.0	486.49	+0.06	
21	10	1174.2	566.32	+0.09		75	1	2103.3	484.62	+0.32	
22	5	1200.6	562.95	+0.07		76	8	2119.8	482.80	-0.16	
23	16	1207.3	561.96	-0.08		77	1	2148.9	480.54	+0.23	
24	9	1217.8	560.75	0.00		78	1	2157.4	479.53	-0.03	
25	8	1231.3	559.12	0.00		79	3	2160.6	479.21	-0.19	
26	7	1242.6	557.76	0.00		80	6	2187.1	476.92	-0.18	
27	10	1280.0	553.25	-0.06		81	3	2201.9	475.89	+0.09	
28	1	1303.5	551.11	+0.59	± 0.16	82	2	2221.7	474.33	+0.22	
28	1	1303.5	551.11	+0.28		83	5	2233.5	473.35	+0.30	
29	2	1306.7	550.68	+0.26		84	2	2250.0	471.24	-0.38	± 0.14
30	12	1324.8	548.11	-0.07		84	2	2250.0	471.24	+0.02	
31	3	1337.0	546.76	+0.02		85	4	2264.3	470.69	+0.05	
32	8	1343.5	546.03	+0.04		86	12	2309.0	467.06	-0.02	
33	5	1351.1	545.06	-0.07		87	5	2416.0	460.61	+1.06	
34	10	1367.0	543.40	+0.04		88	5	2436.5	458.65	+0.34	
35	11	1389.4	540.90	-0.04		89	3	2457.5	456.80	+0.18	
36	1	1410.5	538.77	+0.09		90	6	2467.6	455.70	-0.20	
37	9	1421.5	537.50	0.00	± 0.12	91	8	2489.4	453.71	-0.66	
37	9	1421.5	537.50	0.00		92	2	2537.1	450.54	-0.46	
38	9	1450.8	534.49	-0.02		93	1	2547.2	450.12	-0.17	$+ 0.66$
39	18	1463.3	533.27	+0.04		93	1	2547.2	450.12	+0.44	
40	6	1492.4	530.21	-0.09		94	9	2566.3	448.45	-0.05	
41	6	1506.3	528.77	-0.18		95	7	2606.6	445.99	+0.05	
42	7	1515.5	527.99	+0.10		96	4	2627.0	444.63	0.00	
43	9	1523.7	527.42	+0.07		97	7	2638.6	443.83	-0.05	
44	2	1541.9	525.97	+0.20		98	8	2670.0	441.87	-0.01	
45	8	1569.6	523.72	+0.05		99	7	2686.6	440.86	+0.01	
46	8	1577.6	523.08	-0.05	± 0.08	100	12	2721.6	438.74	0.00	± 0.15
46	8	1577.6	523.08	-0.01		100	12	2721.6	438.74	+0.15	
47	1	1589.1	521.96	-0.21		101	7	2734.9	437.79	+0.09	
48	6	1601.7	521.31	+0.12		102	8	2775.7	435.66	+0.14	
49	4	1622.3	519.65	+0.03		103	11	2797.0	434.32	-0.33	
50	18	1634.1	518.71	-0.03		104	11	2822.8	433.80	+0.23	
51	14	1648.8	517.68	+0.01		105	17	2854.7	431.34	-0.25	
52	13	1655.6	517.13	-0.05		106	7	2869.7	430.36	+0.26	± 0.24
53	6	1693.8	514.63	+0.16							
54	4	1737.7	511.41	+0.05							
55	4	1750.4	510.28	-0.16	± 0.10						

* In this table the column W gives the weights for the different wave lengths.

$\Delta^2\lambda$	$\Delta^3\lambda$	$\Delta^4\lambda$	$\Delta^5\lambda$
+0.02	+0.48	-0.46	-1.04
+0.05	+0.34	-0.13	-0.24
-0.02	-0.19	-0.12	+0.25
+1.06	+0.69	+0.57	+0.19
+0.34	+0.04	-0.17	-0.39
+0.18	+0.04	-0.21	-0.13
-0.20	-0.25	-0.47	-0.27
-0.66	-0.48	-0.51	-0.12
-0.46	+0.38	+1.58	+1.27
-0.17	-0.85	+2.41	+1.46

These differences are sufficient to show that a straight line gives, upon the whole, the best representation of the observations, but that in all probability the form of the function to be assumed for the purpose of interpolation is not parabolic. The same remark applies, though in a much less degree, to the ninth group. In this case the successive differences are as follows:—

$\Delta^2\lambda$	$\Delta^3\lambda$	$\Delta^4\lambda$	$\Delta^5\lambda$
-0.03	+0.04	+0.13	+0.25
+0.06	+0.08	+0.07	0.00
+0.32	+0.25	+0.17	+0.12
-0.16	-0.27	-0.36	-0.29
+0.23	+0.09	+0.08	+0.25
-0.03	-0.16	-0.15	-0.01
-0.19	-0.32	-0.30	-0.17
-0.18	-0.27	-0.19	-0.27
+0.09	+0.06	+0.14	-0.09
+0.22	+0.28	+0.30	+0.10
+0.30	+0.43	+0.37	+0.31
-0.38	-0.15	-0.38	+0.20

In computing the wave lengths of the lines on Kirchhoff's scale between the scale numbers 2067.1 and 2250.0, a parabola of the 5th order was employed, because this distributes the errors rather more evenly than the straight line the constants for which are given for group 9 in table VIII. The probable error is about the same in both cases.

To complete my work, it remains for me to show to what extent the wave lengths calculated by the two wholly independent processes of interpolation, with different data, agree with each other. Table XI contains all the lines for which the wave lengths can be determined for both scales.

In this table columns K and H give the scale numbers upon Kirchhoff's and Huggins's scales, respectively; columns $K\lambda$ and $H\lambda$ the corresponding wave lengths, and column Δ , their differences. An impartial examination of this table—which of course includes table V—will show, I think, as close an agreement as can be reasonably expected. The differences between the wave

TABLE XI.

CALCIUM.									
K	H	Kλ	Hλ	Δ	K	H	Kλ	Hλ	Δ
717·8	622	650·47	650·34	+0·13	1539	1617	525·98	525·95	+0·03
720·1	625	649·88	649·77	+0·11	1562	1638	524·18	524·26	-0·08
729	637	647·60	647·47	+0·13	1575	1651	523·24	523·23	+0·01
731·7	642	946·93	646·51	+0·42	1578·5	1656	522·97	522·83	+0·14
736·9	649	645·60	645·17	+0·43	1582	1659	522·71	522·60	+0·11
740·9	655	644·60	944·03	+0·57	SILVER.				
860·2	813	617·50	617·51	-0·01	K	H	Kλ	Hλ	Δ
863·9	818	616·78	616·67	+0·11	1330	1372	547·55	547·44	+0·11
884·9	843	612·94	612·78	+0·16	1335	1380	546·96	546·63	+0·33
894·9	859	610·83	611·06	-0·23	1600	1675	521·32	521·34	-0·02
1029·3	1031	585·87	586·05	-0·18	GOLD.				
1219·2	1247	560·53	560·65	-0·12	K	H	Kλ	Hλ	Δ
1221·6	1249	560·24	560·39	-0·15	967	951	596·57	596·45	+0·12
1228·3	1258·5	559·42	559·42	0·00	969·5	956	596·08	595·75	+0·33
1229·5	1260	559·27	559·27	0·00	1042·5	1045	583·95	583·38	+0·57
1235	1265	558·62	558·59	-0·03	1572	1647	523·46	523·56	-0·10
1443·5	1506	535·25	535·45	-0·20	2151	2326	479·76	479·64	+0·12
1522·7	1599·5	527·40	527·36	+0·04	ANTIMONY.				
1528·7	1605	526·88	526·93	-0·05	K	H	Kλ	Hλ	Δ
1532·5	1609	526·55	526·61	-0·06	798	729	630·84	630·49	+0·35
1627·2	1702	519·24	519·24	0·00	881	837	613·59	613·73	-0·23
1832·8	1907	504·66	504·40	+0·26	957·5	937·5	598·41	598·72	-0·31
2606·5	3124?	445·93	455·49	+0·44	994	988·5	591·61	591·45	+0·16
2638·5	3181	443·89	443·64	+0·25	1004·8	1005	589·76	589·76	0·00
2653	3212	442·96	442·66	+0·30	1187	1214	564·54	564·41	+0·13
2855·2	3561?	431·56	431·91	-0·38	1247	1279	557·19	557·18	+0·01
2864·7	3602·5	430·79	430·79	0·00	1338	1383	546·61	546·33	+0·28
2869·7	3617	430·21	430·36	-0·15	2252	2488	471·10	471·03	+0·07
BARIUM.					ARSENIC.				
K	H	Kλ	Hλ	Δ	K	H	Kλ	Hλ	Δ
718·7	621·5	650·24	650·44	-0·20	860	812	617·54	617·67	-0·13
890·2	847	611·75	612·15	-0·40	890·5	850	611·69	611·67	+0·02
934	908	603·08	602·70	+0·38	1179·5	1090	578·95	578·73	+0·22
964·5	943	597·05	597·58	+0·53	1460·5	1529	533·55	533·41	+0·14
1031·8	1034	585·51	585·67	-0·16	CADMIUM.				
1050	1067	582·88	582·77	+0·11	K	H	Kλ	Hλ	Δ
1083	1096	578·51	578·00	+0·51	730·5	639	647·22	647·08	+0·14
1274·2	1308	553·95	554·06	-0·11	1453	1517	534·27	534·47	-0·20
1287·5	1327	552·40	552·06	+0·34	1769·5	1843	509·00	508·83	+0·17
1989·5	2075	493·78	493·57	+0·21	2148	2315	480·56	480·27	+0·29
2031·1	2133	490·20	490·23	-0·03	2667·7	3239	441·94	441·81	+0·13
MERCURY.					TIN.				
K	H	Kλ	Hλ	Δ	K	H	Kλ	Hλ	Δ
1157	1177	568·47	568·55	-0·08	736	648	645·83	645·27	+0·56
1340·5	1385	546·33	546·13	+0·20	1748	1821	510·55	510·40	+0·15
1372	1421	542·80	542·80	0·00	IRON.				
STRONTIUM.					K	H	Kλ	Hλ	Δ
K	H	Kλ	Hλ	Δ	756·9	673	640·60	640·64	-0·04
763·8	669	641·38	641·39	-0·01	831	772	623·52	623·38	+0·14
1274·7	1311	553·90	553·74	+0·16	849·7	795	619·60	619·92	-0·32
1286	1324	552·57	552·38	+0·19	1207·3	1236	561·99	561·93	+0·06
1301	1341	550·83	550·61	+0·22	1217·8	1247	560·71	560·69	+0·02
1317	1349?	549·11	549·78	-0·67					
1320·6	1359	548·68	548·75	-0·07					

TABLE XI (continued).

K	H	K λ	H λ	Δ				
1231.3	1261	559.06	559.14	-0.08				
1239.9	1276.7	558.01	557.49	-0.48				
1352.7	1400	544.93	544.64	+0.29				
1367	1413	543.34	543.37	-0.03				
1372.6	1421.5	542.74	542.78	-0.04				
1380.5	1434	541.89	541.73	+0.16				
1384.7	1438	541.44	541.39	+0.05				
1390.9	1445	540.78	540.80	-0.02				
1397.5	1456	540.07	539.86	+0.21				
1401.6	1459.5	539.63	539.55	+0.08				
1421.5	1481	537.51	537.67	-0.16				
1423	1485.3	537.32	537.28	+0.04				
1425.4	1486.5	537.11	537.18	-0.07				
1527.7	1603	526.98	527.09	-0.11				
1569.6	1645	523.63	523.70	-0.07				
1577.2	1653	523.07	523.09	-0.02				
1622.3	1696	519.62	519.70	-0.08				
1623.4	1698	519.53	519.55	-0.02				
1653.7	1728	517.31	517.25	+0.06				
1655.6	1731	517.16	517.03	+0.13				
1693.3	1767	514.47	514.33	+0.14				
1701.8	1775	513.87	513.74	+0.13				
1961	2036	496.11	496.05	+0.06				
2001.6	2092	492.76	492.65	+0.11				
2007.2	2098	492.27	492.30	-0.03				
2041.3	2147	489.42	489.47	-0.05				
2886.4	3272	440.85	440.81	+0.04				
2822.3	3341	438.61	438.55	+0.06				

					COBALT.			
K	H	K λ	H λ	Δ	K	H	K λ	H λ
889.9	844	612.75	612.62	+0.13	1354.5	1401	544.74	544.54
1424	1483	537.25	537.50	-0.25	1429.8	1491	536.65	536.79
1433	1496	536.32	536.34	-0.02	1438.9	1500.5	535.72	535.75
1440.2	1501.5	535.59	535.66	-0.07	1448.7	1514	534.71	534.74
1510.3	1584	528.55	528.60	-0.05	1525	1602	527.20	527.17
1527.7	1604	526.96	527.01	-0.05	1566.5	1642	523.86	523.94
1572	1650	523.46	523.31	+0.15	1596	1670	521.63	521.73
2071.3	2186	487.19	487.29	-0.10	2103	2236	484.66	484.48
2132.3	2286	482.00	481.80	+0.20	2157	2325	479.76	479.70
					ZINC.			
K	H	K λ	H λ	Δ	K	H	K λ	H λ
771.8	696	636.99	637.37	-0.38	896	855	610.64	610.89
1004	1001	589.90	589.90	0.00	2240	2469	472.25	471.98
					LEAD.			
K	H	K λ	H λ	Δ	K	H	K λ	H λ
1213	1240	561.29	561.46	-0.17	1419.5	1479	537.71	537.85
2716	329	439.07	438.93	+0.14				

lengths on the two scales are due to several causes, which it may be well to analyze. In the first place, the errors for any one line may be in opposite directions, so that the difference Δ , gives their sum. For the same reason, a close coincidence in the values of the wave lengths of the same line may sometimes be due to errors which are in the same direction and which nearly balance each other. Too much importance, therefore, must not be attached either to absolute coincidence in value, or to large differences. Secondly, the measurements will, other things being equal, be most accurate in the case of the lines which are narrowest and most sharply defined, while those which are broad, or nebulous, will give comparatively large errors from the difficulty of determining their true positions upon the scale. Finally, it is to be remarked that a large number of the lines, the wave lengths of which have been determined by interpolation from Kirchhoff's scale, have been, from necessity, taken from the chart, as they are not entered in the accompanying tables. A new and by no means insignificant source of error is thus introduced. In estimating the degree of reliance to be placed upon wave

lengths determined by interpolation, it must be borne in mind that each wave length is calculated by means of a formula, the constants of which are determined from a number of measured wave lengths, which is of course a material advantage. Part of the irregularity of certain portions of Kirchhoff's scale is undoubtedly due to variations in the positions of the prisms. In addition, however, it is possible that the lines of which the wave lengths were measured by Ditscheiner were not always correctly identified with lines upon Kirchhoff's scale. After the most careful study of the whole subject, I am disposed to give the preference to the values of the wave lengths as determined by my discussion of Mr. Huggins's measurements in connection with those of Angström. With the materials obtained as above I have endeavored to determine whether the distribution of the lines corresponding to any one element is subject to any definite law. The solution of this problem was first attempted by Mr. Hinrichs,* who, from data in my judgment far too limited in number, drew the conclusion that the spectral lines in the case of any element are distributed in groups, the lines belonging to any one group being equidistant. This would probably bring the whole subject within the reach of Wrede's theory of absorption,† which is a special application of the principle of interferences. My own study of the subject does not justify the conclusion to which Mr. Hinrichs arrived. Even a cursory examination of the tables of wave lengths which I have given above will serve to show that, in very numerous instances, the distances between two successive spectral lines of the same element is less than the probable error in measuring the wave lengths. As the differences in wave length measure the distances between the lines, the element of uncertainty becomes much too large to permit us to reason with safety upon the data now at our disposal. In the mean time it cannot, I think, be denied that the success with which Wrede's theory explains the absorption bands in nitrous acid and other vapors, gives a certain probability to Mr. Hinrichs's views.

My grateful acknowledgments are again due to Mr. S. P. Sharples, by whom I have been assisted in the whole of my laborious work, with a zeal and skill which demand the fullest recognition.

Cambridge, Dec. 23d, 1868.

P. S. In a memoir presented to the Royal Society, March 2d, 1867, Mr. Airy has given a complete reduction of the numbers upon Kirchhoff's scale to the corresponding values in wave lengths. The high and well deserved reputation of the Astron-

* This Journal, II, vol. xlii, p. 350.

† Pogg. Annalen, xxxiii.

omer Royal, lends to whatever comes from his pen a more than ordinary interest and value. Those therefore who like myself have felt convinced that an explanation of the physical cause of the spectral lines of the elements could only be sought in an accurate knowledge of the wave lengths of the lines themselves, will eagerly greet the appearance of Mr. Airy's paper. Under these circumstances it becomes desirable to submit the memoir in question to a critical examination and to determine to what extent it furnishes materials for further investigation.

Mr. Airy's method is simple. Taking the original measures of Fraunhofer for the lines C, D, E, F and G, as the basis of computation, and assuming that the relation between the wave lengths and the numbers upon Kirchhoff's scale can be expressed for the whole scale from C to G by a single function of the form*

$$f_1 = a + bk + ck^2 + dk^3 + ek^4$$

he forms five equations, the solution of which by the method of least squares gives the numerical values of the constants a , b , c , d and e . By means of these constants Mr. Airy constructs a table giving the values of the scale-numbers for every ten units of Kirchhoff's scale, from A to G. The portion of this table between A and C is constructed by extrapolation, the data for A and B not being introduced. I shall not dispute the legitimacy of this process with so great an authority as Mr. Airy; it seems to me unsound in principle and therefore unsafe in practice. After the completion of this part of his work, Mr. Airy became acquainted with Angström's memoir, and with the two papers of Ditscheiner on the wave lengths of the spectral lines upon Kirchhoff's chart. He then decided to base his reduction upon the second series of wave lengths as given by Ditscheiner. In place however of beginning *ab initio* with the new data, Mr. Airy preferred to introduce corrections into the results already obtained, adding also a value for the wave length of Fraunhofer's line B. For the method of making these corrections I must refer to the original paper. They are applied directly and only to the lines B, C, D, E, F and G. The corrected values of the wave lengths of these six lines being obtained, the wave lengths in the table above referred to were then corrected. With these corrections a curve was drawn and the corrections for every 0.01 of k were obtained graphically. Finally, the corrections for all the individual lines were interpolated numerically and applied separately to each computed wave length.

From this it appears that out of the 107 absolute measurements of wave lengths made by Ditscheiner, and by him refer-

* $k = \frac{1}{10000} K_1$ and $f_1 = 100000000 \times \text{Frl.}$

red to lines upon Kirchhoff's scale, Mr. Airy, in his interpolation, employed only six, the intervals between these amounting respectively to 101·4, 308·7, 520·9, 556·3, 774·7, scale divisions. Through these six points Mr. Airy endeavors to draw a parabola of the 5th order. He then employs the 101 other lines measured by Ditscheiner only to compare with the numerical results given by his single formula.

As Mr. Airy has quietly ignored the existence of my reduction of Kirchhoff's scale read before the National Academy of Sciences in August, 1866, and published in abstract in this Journal for January, 1867, I naturally inferred that his work would prove so much more perfect than my own as wholly and justly to supersede it. The following comparative tables of differences will show, I think, that this is not the case. For the sake of completeness I add a column of differences obtained in my second reduction of Kirchhoff's scale, in which Cauchy's method of interpolation was used, and weights were given to the observations.

TABLE XII.

Airy.	Gibbs 1.	Gibbs 2.	Airy.	Gibbs 1.	
+0·25	+0·03	—0·06	+2·01	+0·01	—0·05
+0·43	+0·24	+0·02	+1·87	—0·18	—0·23
+0·69	—0·02	+0·08	+2·12	+0·02	—0·01
+0·90	—0·08	+0·08	+2·32	+0·07	+0·12
+0·94	—0·15	0·00	+2·13	—0·11	—0·07
+1·03	+0·06	0·00	+2·21	+0·11	+0·01
+1·02	—0·02	—0·04	+2·04	+0·04	+0·08
+1·15	—0·06	0·00	+1·82	—0·14	0·00
+1·22	—0·04	+0·05	+1·62	+0·04	—0·05
+1·31	+0·04	0·00	+1·00	—0·59	—0·15
+1·38	+0·09	—0·06	+1·10	+0·04	+0·07
+0·58	—0·26	—0·02	+0·67	+0·04	+0·09
0·00	+0·05	—0·05	+0·34	+0·02	0·00
+0·01	+0·10	—0·02	+0·27	+0·03	—0·03
+0·44	+0·09	+0·35	—0·02	+0·06	+0·06
+1·00	—0·06	—0·13	—0·53	+0·05	+0·32
+1·04	—0·04	—0·13	—1·39	+0·01	—0·16
+1·25	+0·20	+0·06	—1·90	—0·04	+0·23
+1·23	+0·17	+0·02	—2·46	0·00	—0·03
+1·26	+0·06	+0·09	—2·60	0·00	—0·19
+1·29	+0·15	+0·07	—3·52	+0·49	—0·18
+1·11	—0·01	—0·08	—3·57	+0·12	+0·09
+1·22	+0·10	0·00	—4·01	—0·03	+0·22
+1·22	+0·32	0·00	—4·32	—0·01	+0·30
+1·23	+0·03	0·00	—5·51	—0·26	—0·38
+1·09	—0·03	—0·06	—5·32	—0·08	+0·05
+1·66	+0·25	+0·59	—6·43	—0·20	—0·02
+1·53	+0·09	+0·26	—6·83	+0·01	+1·06
+0·97	—0·05	—0·07	—7·59	—0·85	+0·34
+0·93	+0·05	+0·02	—8·23	+0·10	+0·18
+0·90	+0·09	+0·04	—8·66	—0·14	—0·20
+0·72	+0·10	—0·07	—9·30	+0·38	—0·66
+0·73	0·00	+0·04	—9·53	—0·04	—0·46
+0·56	+0·02	—0·04	—9·30	+0·20	—0·17
+0·57	+0·01	+0·09	—9·65	—0·18	—0·05

TABLE XII (continued).

Airy.	Gibbs 1.	Gibbs 2.	Airy.	Gibbs 1.	Gibbs 2.
+0.43	-0.05	0.00	-9.22	+0.03	+0.05
+0.28	-0.07	-0.02	-9.10	+0.01	0.00
+0.24	0.00	+0.04	-9.01	0.00	-0.05
-0.08	-0.06	-0.09	-8.52	+0.54	-0.01
-0.22	-0.16	-0.18	-8.12	+0.12	+0.01
-0.14	-0.06	+0.10	-7.14	+0.04	0.00
+0.01	+0.08	+0.07	-6.82	-0.22	+0.09
+0.20	+0.18	+0.20	-4.77	+0.11	+0.14
+0.38	+0.26	+0.05	-3.91	-0.04	-0.33
+0.41	+0.21	-0.05	-2.50	-0.04	+0.23
+0.28	-0.05	-0.21	+0.10	-0.57	-0.25
+0.70	+0.25	+0.12	+1.06	+0.21	+0.26
+0.81	+0.09	+0.03			
+0.80	-0.03	-0.03			
+1.04	-0.03	+0.01			
+0.07	-0.11	-0.05			
+1.60	+0.16	+0.16			
+1.78	+0.15	+0.05			
+1.62	-0.12	-0.02			
+1.50	-0.01	-0.03			
+1.83	+0.04	-0.04			
+1.98	+0.05	+0.03			
+2.14	+0.14	+0.12			

In computing the differences between observation and calculation given in this table, it must be borne in mind that Mr. Airy's results are compared with Ditscheiner's second series of wave lengths, while my own are compared with the first, as stated in my paper above referred to.* The greatest difference given by Mr. Airy's formula amounts to not less than 9.65 units, his unit being made to correspond with mine, by placing the decimal point six figures to the right. Now, in the part of Kirchhoff's scale in which this difference occurs, the uncertainty in the position on the scale of the line in question becomes, for a difference of 9.65, not less than 150 scale divisions. One other remark will be sufficient. In all processes of interpolation which are equivalent to drawing curves through points, a certain uniformity of distribution of the + and - signs, or in other words of the excesses and deficiencies given by the formula, is necessary. Mr. Airy's formula, as will be seen by a single glance at the signs in the column of differences, violates even this elementary principle.

It is not an agreeable task to point out defects in the work of another, but in the interest of science I am obliged to declare, that for all the purposes of physical investigation, Mr. Airy's paper, in its present form, has no value whatever.

In the proceedings of the Royal Society, vol. xvii, p. 1, Mr. G. J. Stoney has given, in connection with a very interesting and suggestive paper on the physical constitution of the sun and stars, a table of wave lengths for certain lines upon Kirchhoff's scale. This table is so arranged as to give the scale

* See Table VI.

numbers corresponding to equal intervals expressed in wave lengths, and is based upon Angström's measurements alone. For the sake of completeness, I give this table in full, together with the wave lengths for the same lines, as found by myself by Cauchy's method of interpolation, and the differences. For the sake of comparison I have changed the position of the decimal point in the wave lengths determined by Mr. Stoney, so as to correspond with my own results.

TABLE XIII.

K	SA	GA	Δ	K	SA	GA	Δ
719.1 ?	650	650.14	—14	1527.9	527	526.94	+06
758.5 ?	640	640.22	—22	1540.2	526	525.88	+12
800 ?	630	630.39	—39	1552.8	525	524.87	+13
845.5	620	620.71	—71	1565.5	524	523.96	+04
855	619	618.54	+46	1578.3	523	522.97	+03
860.6	617	617.42	—42	1591.2	522	522.01	—01
865.7	616	616.50	—50	1604.3	521	520.99	+01
870.9	615	615.40	—40	1617.5	520	519.98	+02
876.1	614	614.44	—44	1630.8	519	518.99	+01
881.3	613	613.46	—46	1644.3	518	517.99	+01
886.6	612	612.44	—44	1658.3	517	516.97	+03
891.9	611	611.43	—43	1673	516	515.92	+08
897.2	610	609.61	+39	1762 ?	510	509.52	+48
948 ?	600	600.26	—26	1913 ?	500	499.30	+70
1003	590	590.07	—07	1950.8	497	496.87	+13
1009.7	589	589.00	00	1962.5	496	496.00	00
1070 ?	580	580.20	—20	1974.3	495	495.05	—05
1144 ?	570	570.25	—25	1986.1	494	494.07	—07
1199.3	563	562.98	+02	1998	493	493.06	—06
1207.4	562	561.98	+02	2010.2	492	492.01	—01
1215.6	561	560.97	+03	2022.6	491	490.95	+05
1223.8	560	559.97	+03	2035.2	490	489.91	+09
1232	559	558.98	+02	2047.9	489	488.89	+11
1240.2	558	558.00	00	2060.8	488	487.95	+05
1248.4	557	557.01	—01	2073.7	487	487.03	—03
1309	550	550.13	—13	2086.7	486	487.15	—15
1334.7	547	547	00	2099.8	485	485.04	—04
1343.3	546	546	00	2164 ?	480	479.34	+66
1352.1	545	545.01	—01	2292.5 ?	470	468.25	+175
1361.2	544	544	00	2422 ?	460	459.12	+88
1370.5	543	542.97	+03	2553.2 ?	450	449.72 ?	+28
1379.7	542	541.97	+03	2651.5	443	443.06	—06
1389	541	540.98	+02	2667.6	442	442.03	—03
1398.3	540	539.98	+02	2683.7	441	441.02	—02
1407.7	539	538.98	+02	2699.6	440	440	00
1417.2	538	537.96	+04	2715.7	439	439.07	—07
1426.6	537	536.98	+02	2732	438	437.95	+05
1436.1	536	536.01	—01	2748.8	437	436.89	+11
1445.6	535	535.14	—14	2766	436	435.95	+05
1454.4	534	534.13	—13	2783.4	435	435.17	—17
1464	533	533.14	—14	2801.1	434	434.46	—46
1473.7	532	532.15	—15	2819	433	433.77	—77
1483.8	531	531.13	—13	2837	432	432.90	—90
1494.1	530	530.12	—12	2855	431	431.59	—59
1504.8	529	529.12	—12	2873.1	430	429.87	+13
1516.1	528	528.02	—02				

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Mr. Stoney does not state what method he employed in interpolating ; the correspondence between his values and my own is in general very close, though I employed the data obtained by Ditscheiner, in place of those of Angström.*

Cambridge, Jan. 1st, 1869.