XX. Note on class of differential equations

George Boole Esq.

XX. *Note on a Class of Differential Equations.*
By George Boole, *Esq.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

THE following may be deemed a proper supplement to the remarks, offered in common by Mr. Hargreave and myself, in the January Number of the Philosophical Magazine on the subject of the Rev. B. Bronwin's differential equations.

In the greater number of those equations, the independent variable x only entered in the first degree. All linear equations which possess this character, whether in differentials or in finite differences, admit of a complete symbolical solution, from which the particular solutions assigned in the papers referred to, may be regarded as deductions.

Every equation, indeed, to which this characteristic belongs, may be expressed in the form

$$x \, \varphi \, (\mathrm{D})u + \psi(\mathrm{D})u = \mathrm{X} ; \quad \ldots \quad (1.)$$

D standing for $\dfrac{d}{d\,x}$, X being a function of x, and φ and ψ denoting arbitrary functions or combinations of the symbol to which they are affixed.

The complete solution of (1.) is

$$u = \{\varphi(\mathrm{D})\}^{-1} \, \varepsilon^{\chi(\mathrm{D})} \, x^{-1} \, \varepsilon^{-\chi(\mathrm{D})} \, \mathrm{X}, \quad \ldots \quad (2.)$$

the form of $\chi(\mathrm{D})$ being given by the equation

$$\chi(t) = \int \frac{\psi(t)}{\varphi(t)} \, d\,t.$$

The analogy which exists between the above solution and that of the linear differential equation of the first order, it is scarcely necessary to notice. It belongs to a class of subjects which have been considered in a paper on the Theory of Developments, published in the Cambridge Mathematical Journal, vol. iv. p. 214.

As a particular illustration of (2.), let us suppose that the given equation is of the form

$$x \, \varphi(\mathrm{D})u + m \, \varphi'(\mathrm{D})u = \mathrm{X}.$$

We have

$$\chi(t) = \int \frac{m \, \varphi'(t)}{\varphi(t)} \, d\,t = m \log \varphi(t),$$

$$\varepsilon^{\chi(\mathrm{D})} = \{\varphi(\mathrm{D})\}^{m}. \quad . \, \varepsilon^{-\chi(t)} = \{\varphi(\mathrm{D})\}^{-m};$$

$$\therefore u = \{\varphi(\mathrm{D})\}^{m-1} \, x^{-1} \{\varphi(\mathrm{D})\}^{-m} \, \mathrm{X},$$

from which the solution of most of the equations considered by Mr. Bronwin will obviously follow. There are however various other cases in which the general solution is interpretable.

<div align="center">

I remain, Gentlemen,

Your obedient Servant,

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Lincoln, Jan. 1, 1847. GEORGE BOOLE.

XXI. *Observations and Experiments respecting the Origin of the Voltaic Current.* By Professor GIANALESSANDRO MAJOCCHI*.

AFTER the discussion which took place in the last sittings, and principally in that of yesterday, respecting the origin of voltaic electricity, among some of our distinguished physicists, and particularly between the illustrious president of the section Prof. Orioli and Prof. Botto, I have been induced to extract the following paper from an essay of mine on the same subject.

At the Turin Congress in 1840, the question of the two theories, the *chemical* and that of *contact,* was agitated, with a view to explain the origin of voltaic electricity. There is no doubt that where a chemical action takes place, there is also a development of electricity; but the fluid which becomes free requires certain conditions to form a current. In like manner it is certain that, on placing two heterogeneous bodies in contact, there is a development of electricity; and in this precisely consist the fundamental facts which led Volta to the discovery of his pile; and on the same principle is founded the ingenious apparatus of insulating plates, armed with leaves of heterogeneous metals, which our colleague Prof. Marianini exhibited at the Congress of Turin, and which, variously arranged, he yesterday showed to this assembly†. The chemico-electricians maintain that the fundamental facts of Volta depend on the chemical action upon the metal, produced by the moisture of the hand, of the air, &c.; but the careful experiments instituted by Marianini, Pfaff, Belli, Peltier, and some other physicists, appear, in the development of electri-

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* Translated from the *Annali di Fisica, Chimica e Matematiche,* and communicated by the Author.

The chief portion of this paper was read by the author at the Congress of Milan, Sept. 27, 1844. See *Annali,* t. xvi. p. 120; and *Atti della sesta Riunione degli Scienziati Italiani,* p. 118–119.

† It is mentioned in his work *Memorie di Fisica sperimentale,* Modena, 1838, and in another work printed at Modena under the same title in 1841, in which is contained the *Memorie sui coibenti armati che si caricano da sè col mutuo contatto d' un' armatura coll' altra.*