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1917 Trans. Opt. Soc. 18 142

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On the Design and Testing of Telescope Objectives.

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With contributions by T. SMITH, B.A., and S. D. CHALMERS, M.A.

Introduction.

THE function of the objective or object-glass of a telescope is to form as perfect an image as possible of an object area which is usually at a great distance and covers an angle of view rarely exceeding 5° from the optical axis.

A simple positive lens was first used for this purpose, but the performance of a telescope with such an objective was poor and its usefulness restricted by the aberrations of the simple lens; in course of time improvements were effected by the construction of compound objectives, and it is the design and testing of these compound objectives that I propose to discuss in this paper.

The Four Principal Aberrations of the Telescope Objective.

In any lens system when the rays under consideration are restricted to extremely narrow pencils indefinitely close to the axis, Gauss showed that the performance of the system depended on the properties of six points on the axis, the two focal points, the two principal points, and the two nodal points, which, when the refractive indices of the first and last media are the same, coincide with the principal points. As in practically every case, a telescope is so constructed that the media in front of and behind the objective are the same, viz., air, the six points mentioned are reduced to four, the two focal points and the two principal points. The properties of these fundamental points are well known to all students of geometrical optics, and I intend only to deal with such as are necessary for our purpose.

Fig. 1 shows the optical axis of a lens system; F_1 , F_2 are the two focal points, H_1 , H_2 the principal points, and the two planes drawn through H_1 , H_2 perpendicular to the axis are known as the principal planes. The distances H_1F_1 , H_2F_2 are equal, and are known as the anterior and posterior focal lengths, or more shortly, the focal lengths. We have further the well-known geometrical construction that a ray parallel to the axis before refraction behaves as if it were produced to the first prin-

cial plane, proceeds from there to the second principal plane in a direction parallel to the axis, and is then deviated to pass through F_2 , where the image of the infinitely distant object on the axis is formed.

In actual lens systems as contrasted with the ideal system, the four points F_1 , H_1 , H_2 , F_2 are not fixed but subject to changes depending on the particular rays considered; in other words, the system is subject to aberrations.

Variation of the position of F_2 with the wave-length (or colour) of the light forming the pencil is defined as "chromatic aberration of the intersection-distances"* and variation of the length H_2F_2 with the wave-length of the light is called "chromatic aberration of the focal lengths." These two terms will be referred to more shortly as chromatic aberration and the chromatic difference of magnification.

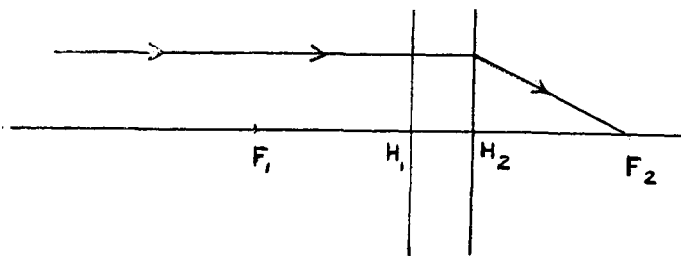


Fig. 1.

The Gauss theory of the geometrical representation of the formation of images illustrated in Fig. 1 involved the condition that the light pencils were extremely narrow and indefinitely close to the axis; the lens system was supposed to be one of indefinitely small aperture, and consequently would only transmit an indefinitely small amount of light. An actual lens system to be of use must form a bright image, i.e., transmit a considerable amount of light, and we must now consider the effect produced by enlarging the aperture of the system.

Consider a narrow pencil of monochromatic light coming from an infinitely distant object on the axis and incident on the system at a considerable distance from the axis. It will be parallel to the axis and will intersect it after refraction at a point near F_2 , if it does not actually pass through F_2 . The variation of this point from the position of F_2 for the corresponding pencil close to the axis is called the longitudinal spherical aberration of the pencil.

* The "intersection-distance" is the distance of F_2 from the vertex of the posterior surface of the lens system.

The spherical aberration will vary with the distance of the incident pencil from the axis, and is a direct consequence of the law of refraction being a relation between the sines of the angles of incidence and refraction, and not the angles themselves.

The whole aperture of the lens system may be regarded as filled with an infinite number of these narrow pencils, which will intersect the axis at various points, giving a circular patch of light (if the aperture be circular), which is called the circle of least confusion. When the spherical aberration of a system is mentioned without defining the distance from the axis of the particular narrow pencil involved it will be understood that the pencil is incident at the edge of the aperture, and is consequently at the maximum distance from the axis.

If in a lens system the spherical aberration just defined be eliminated, the monochromatic image of an object point on the axis will be another point on the axis. In order that the image of a small area or element

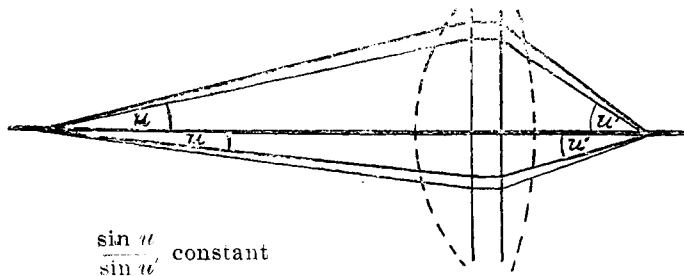


Fig. 2.

of surface on the axis and perpendicular to it shall be reproduced as a corresponding area in the image when the whole aperture of lens system is used, it can be shown that the condition to be fulfilled is that $\frac{\sin u}{\sin u'}$ shall be constant where u and u' are the angles with the axis formed by any narrow pencil before and after refraction in proceeding from the object to the image, as shown in Fig. 2.

For an object at an infinite distance the angle u becomes infinitely small, and the condition reduces to another form. In its new form the condition becomes that the distance measured along the refracted ray from the focus F_2 to its intersection with the unrefracted ray produced shall be constant and equal to the focal length H_2F_2 ; in other words, the points of intersection of the unrefracted ray and refracted rays shall lie on a circle, centre F_2 and radius H_2F_2 . The distances AF_2 , BF_2 ,..... FF_2 in Fig. 3 are equal to H_2F_2 , the focal length of the

axial pencil, and are in reality the focal lengths of the individual narrow pencils forming the complete cone of light transmitted by the lens system.

Since the magnification varies as the focal length, the sine condition expressed in its simplest form is that the magnification due to the lens system shall be constant over the whole aperture.

The four principal aberrations of the telescope objective have now been described, and it will be noticed that two of them, chromatic and spherical aberration, are aberrations of the intersectional distances; while the other two, chromatic difference of magnification and the error against the sine condition, are aberrations of the focal lengths.

Other Aberrations.

Lens systems in general are subject to other aberrations, such as astigmatism of oblique pencils, curvature of field, distortion, chromatic aberrations of higher orders, &c., but as will be seen later, there will not

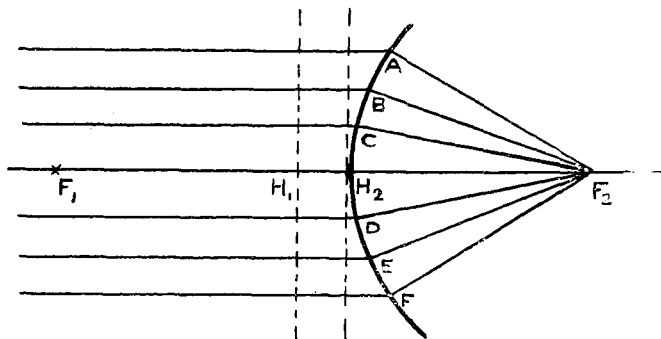


Fig. 3.

be sufficient constructional elements available to eliminate these; and, further, they are never present to such a degree as to injuriously affect the performance of telescope objectives, the field of view being so small that they may be neglected, except in the case of objectives used in the production of photographic charts of the stars. In that case it is sufficient to determine the distortional errors introduced by the objectives, and allow for them in the reduction of the measurements of the negatives.

Residual Aberrations.

Consider an objective from which the four principal aberrations have been removed; it will satisfy the following conditions:—

- (1) The axial pencil will be free from chromatic aberration for two wave-lengths, e.g., the C and F lines of the spectrum.

- (2) The axial pencil will be free from chromatic difference of the focal lengths for the same two wave-lengths.
- (3) The axial and marginal pencils for any one wave-length, say C, will intersect the axis at the same point.
- (4) The focal lengths of these two axial and marginal pencils will be equal.

Condition (1) only considers light of two particular wave-lengths, C and F, and gives no information as to what happens to light of any other wave-length that also passes through the system. In reality it is found that pencils of light of wave-lengths intermediate between C and F, and also for a little distance on either side of that interval come to a focus very close to the common focus of the C and F pencils, but do not coincide with it. There is thus a residual chromatic aberration which may or may not be sufficient to impair the efficiency of the system. If the aberration is removed for light of three different wave-lengths the amount of residual aberration is very much reduced. The residual chromatic aberration when light of two wave-lengths is brought to a common focus is frequently called "secondary spectrum," while the much smaller residual after bringing the foci of three wave-lengths to coincide is called "tertiary spectrum."

Condition (3) requires the axial and marginal pencils to intersect the axis at the same point for light of one wave-length C, and condition (1) also requires the axial pencil for F to pass through the same point; neither condition requires the marginal pencil for F to pass through this point, and in general it will not do so. The conditions laid down do not demand the spherical aberration to be corrected for more than one wave-length, and the residual spherical aberration for light of any other wave-length F is known as the chromatic difference of spherical aberration, and should not be confused with secondary spectrum.

Before passing on further it is important to note that condition (3) only refers to the axial and marginal pencils, and not to the intermediate pencils which also pass through the lens system and are found to intersect the axis at different points for the different concentric zones of the aperture of the system. These give residual spherical aberration which is frequently called zonal aberration, which, in the case when the spherical aberration for the marginal pencils is rigidly corrected, reaches a maximum for the zone distant from the axis $h/\sqrt{2}$, where h is the semi-diameter of the aperture.

Similar residual aberrations to those just mentioned with regard to conditions (1) and (3) also exist for conditions (2) and (4), but as they are of much smaller importance, they have not received the dignity of special names, nor need they be specially considered.

The Six Conditions in Order of General Importance.

Before passing on to the consideration of the possibilities offered by various types of lens systems it will be useful to state shortly the six conditions which it is important to fulfil as far as practicable in the actual work of designing telescope objectives. Although special circumstances will require special treatment, the conditions in order of their general importance are :—

- (a) The objective shall have a given aperture and a given focal length.
- (b) The spherical aberration shall be corrected for a specified wave-length.
- (c) The chromatic aberration shall be corrected for two wave-lengths chosen with regard to the purpose for which the system is to be used.
- (d) The sine condition shall be fulfilled for the wave-length specified in condition (b).
- (e) The chromatic aberration shall be corrected for a third specified wave-length.
- (f) The spherical aberration shall be corrected for a second specified wave-length (sometimes known as the Gauss condition).

The above six conditions will generally be sufficient, but attention should be paid to the chromatic difference of magnification, which will usually be negligible, although it is possible to construct a system in which this aberration would be so large as to seriously impair the performance of the objective.

Main Types of Telescope Objectives and the Possibilities they offer the Designer.

Cemented Doublets.

This type of objective is probably that in most general use to-day, and is universally used in the construction of prism binoculars. The constructional elements available are the three radii, choice of two glasses, and to an extremely limited extent the choice of two thicknesses.

The suitable choice of the three radii enables the designer to fulfil conditions (a), (b), and (c). Alteration of the two thicknesses within practical limits allows of a slight variation in the aberrations of the focal lengths, but the resulting change is so small that it is not usually appreciable. There still remains one more possibility; by a suitable choice

of the pairs of glasses it is possible either to reduce the secondary spectrum or to satisfy the sine condition, and in the case of prismatic binocular objectives this latter condition should always be fulfilled.

H. Harting has investigated the problem very fully, and has published tables* by means of which it is possible to tell in a few minutes whether any given pair of glasses will satisfy not only conditions (a), (b), and (d), but also condition (c). If the pair of glasses is suitable, *i.e.*, will satisfy condition (c), the radii and thickness can be found from the tables, so that in about half-an-hour by means of these tables and a glass catalogue it is possible to obtain the approximate radii and thicknesses of the system.

Smith & Cheshire† have recently published tables, at the request of the Ministry of Munitions, showing the constructional data of cemented doublets for certain types of glasses and giving the approximate amount of spherical aberration and relative amount of coma remaining uncorrected.

Uncemented Doublets.

No telescope objective over 3 ins. in diameter, and also no objective where the utmost obtainable freedom from spherical aberration is required, as in collimator objectives, should be cemented on account of the strains set up owing to the difference in the co-efficients of expansion of the glasses.

In an uncemented doublet there are four radii available which will always allow of conditions (a), (b), (c), and (d) being satisfied without unduly restricting the choice of glass pairs. A suitable choice of glass pairs will then enable the designer either to reduce the secondary spectrum, as is frequently done, or to satisfy some other condition such as (f), although I am not aware of this latter condition having been fulfilled simultaneously with the sine condition (d) in any actual example of an objective.

Another possibility in this case is to choose a pair of glasses giving the flattest curves obtainable, thus reducing manufacturing costs, and also tending to reduce the residual zonal aberration.

Triplet Objectives.

Triplet objectives are of two types, being either doubly cemented or with one cemented contact and one air space.

The doubly cemented type of triplet offers the advantage of a smaller residual zonal aberration for the same pair of glasses and the same

* *Zeitschrift für Instrumentenkunde* 1898, Heft 12; also *Gleichen, Lehrbuch der Geometrischen Optik*, p. 324.

† "Constructional Data of Small Telescope Objectives," by T. Smith, B.A., and R. W. Cheshire, B.A.

relative aperture than the doublets, and consequently is frequently employed when a relatively large aperture is required in conjunction with a comparatively short focal length. Owing to the designer having four radii available, it is possible to satisfy all four conditions (a), (b), (c), and (d), as in the case of an uncemented doublet, without the restrictions as to choice of glass pairs required to satisfy conditions (d) in the case of a cemented doublet.

If three glass types are used instead of two, the chromatic corrections may be improved by uniting three wave-lengths instead of two, condition (e), but this will usually be at the expense of satisfying the sine condition (d).

It is easy to satisfy all the five conditions, (a) to (e), if the designer makes use of the second type of triplet having one cemented contact and one air space, thus obtaining another radius, which enables the fifth condition to be fulfilled.

Other Types of Objectives.

Other types of lens systems have been used as telescope objectives for special purposes ; they are really other types of instruments modified for the particular purpose for which they are required, and need not be further considered.

Choice of Type.

It is not intended to lay down any hard and fast rules as to choice of type, but the following short table may be of some use as a general guide :—

	Cemented doublet	.. Small objectives generally.
	Uncemented doublet	.. Large objectives for astronomical use, collimators and general use where the utmost perfection of image is demanded, provided that ordinary achromatism suffices.
Ordinary glass pairs.	Doubly cemented triplet	Small objectives where large aperture is required in conjunction with short focal length. The relative aperture may be pushed as far as $f/4$ or $f/6$ according to size.

Special choice of glass pairs to satisfy condition (<i>d</i>).	Cemented doublet .. Small objectives generally, specially suitable for prismatic binoculars.
Special choice of glass pairs to improve the chromatic corrections.	Uncemented doublet.. Large objectives for astronomical use, collimators and general use where the utmost perfection of image is required together with the best possible chromatic corrections. Secondary spectrum eliminated. Relative apertures up to about $f/18$. Cemented triplet with one air space Objectives where the uncemented doublet fails owing to the aperture being relatively small. The aperture of this type may be made as large as $f/10$.

Actual Calculation of the Objective.

Having decided upon the type of objective required and the particular glasses it is intended to use, the approximate radii are first obtained from tables* wherever possible, past experience in similar cases, or, in default of any other source, by calculation from the known formulæ† which lead to an approximate solution. This latter method is very rarely necessary, and so only the references are given.

The approximate radii and thicknesses are now known, and of course the aperture and focal length. In order to determine the actual aberrations, it is required to trace trigonometrically through the system, for each wave-length to be considered, a ray parallel to the axis and indefinitely

* (1) "Constructional Data of Small Telescope Objectives," T. Smith, B.A., and R. W. Cheshire, B.A. (2) "Additional Data for the Construction of Small Telescope Objectives," T. Smith, B.A., and R. W. Cheshire, B.A. (3) H. Harting, *Zeitschrift für Instrumentenkunde*, 1898, Heft 12; also Gleichen, *Lehrbuch d. Geom. Optik*, p. 323.

† Moser, *Zeitschrift für Instrumentenkunde*, 1887, Bd. 7, pp. 225 and 308; also Gleichen, *Lehrbuch d. Geom. Optik*, p. 315; H. Harting, *Zeitschrift für Instrumentenkunde*, 1900, Heft 8; also Gleichen, *Lehrbuch d. Geom. Optik*, p. 320.

close to it, called the axial ray, and another ray also parallel to the axis but incident on the objective at the extreme edge of its aperture, the edge ray.

The formulæ of L. von Seidel are very well adapted to this end, and are given below as far as they are required for the purpose of this paper. The notation used is very simple and if further details than those given are desired, reference should be made to Appendix III. of Professor Silvanus Thompson's translation of Lummer's "Photographic Optics."

The quantities used are :—

μ the index of refraction.

R the radius of the surface, positive when convex to the direction of the incident light.

A the distance from the vertex of the surface to the point of intersection with the axis of the refracted ray with regard to that surface, positive in the direction of the incident light.

H the height of the perpendicular to the axis from the point of incidence on any surface, positive when the point of incidence is above the axis.

F the focal length or distance from the 2nd principal point to the intersection of the ray with the axis, positive in the direction of the incident light.

D the thickness or separation between two consecutive surfaces, always positive.

ϕ the angle of incidence.

ψ the angle of refraction.

$\delta = \phi - \psi$ the angle of deviation.

λ the angle at which the refracted ray meets the axis.

The signs of the angles are such that if A and H are positive, then λ is positive; also ϕ is positive for a ray parallel to the axis incident on a surface of which R is positive, if H is positive.

Each surface, or medium, has a suffix which is to be attached to all quantities relating to the surface, and these suffixes increase numerically with the direction of the incident light, commencing with -1 for the medium on the object side of the first surface, which itself has a suffix 0 ; thus all quantities referring to media such as thicknesses and refractive indices have odd suffixes, while all radii, lengths and quantities referring to surfaces have even suffixes.

The actual formulæ for an object on the axis at an infinite distance are :—

AXIAL RAYS.	EDGE RAYS.	
$\phi_0 = \frac{H_0}{R_0}$	$\sin \phi_0 = \frac{H_0}{R_0}$	} 1st surface.
$\psi_0 = \frac{\mu-1}{\mu+1} \phi_0$	$\sin \psi_0 = \frac{\mu-1}{\mu} \sin \phi_0$	
$\lambda_0 = \delta_0 = \phi_0 - \psi_0$	$\lambda_0 = \delta_0 = \phi_0 - \psi_0$	
$A_0 - R_0 = R_0 \frac{\psi_0}{\lambda_0}$	$A_0 - R_0 = R_0 \frac{\sin \psi_0}{\sin \lambda_0}$	
$\phi_2 = \frac{(A_0 - R_2 - D_1) \lambda_0}{R_2}$	$\sin \phi_2 = \frac{(A_0 - R_2 - D_1) \sin \lambda_0}{R_2}$	} 2nd and subsequent surfaces.
$\psi_2 = \frac{\mu+1}{\mu+3} \phi_2$	$\sin \psi_2 = \frac{\mu+1}{\mu+3} \sin \phi_2$	
$\delta_2 = \phi_2 - \psi_2$	$\delta_2 = \phi_2 - \psi_2$	
$\lambda_2 = \delta_2 + \lambda_0$	$\lambda_2 = \delta_2 + \lambda_0$	
$A_2 - R_2 = R_2 \frac{\psi_2}{\lambda_2}$	$A_2 - R_2 = R_2 \frac{\sin \psi_2}{\sin \lambda_2}$	

for the last or 2nth surface,

$F_{2n} = \frac{H_0}{\lambda_{2n}}$	$F_{2n} = \frac{H_0}{\sin \lambda_{2n}}$
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The practical computer will at once notice that A is determined from $A - R$, and will not therefore be found very accurately if R is very large; in such a case it is preferable to proceed as below.

If R_{2r} is large, we have—

AXIAL RAYS.	EDGE RAYS.
$H_{2r-2} = R_{2r-2} (\psi_{2r-2} + \lambda_{2r-2})$	$H_{2r-2} = R_{2r-2} \sin (\psi_{2r-2} + \lambda_{2r-2})$
$\phi_{2r} = \frac{(A_{2r-2} - R_{2r} - D_{2r-1}) \lambda_{2r-2}}{R_{2r}} \sin \phi_{2r}$	$\sin \phi_{2r} = \frac{(A_{2r-2} - R_{2r} - D_{2r-1}) \sin \lambda_{2r-2}}{R_{2r}}$
$\psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \phi_{2r}$	$\psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \sin \phi_{2r}$
$\delta_{2r} = \phi_{2r} - \psi_{2r}$	$\delta_{2r} = \phi_{2r} - \psi_{2r}$
$\lambda_{2r} = \delta_{2r} + \lambda_{2r-2}$	$\lambda_{2r} = \delta_{2r} + \lambda_{2r-2}$
$H_{2r} = H_{2r-2} - D_{2r-1} \times \lambda_{2r-2}$	$H_{2r} = H_{2r-2} - (D_{2r-1} - \Delta D_{2r-2} + \Delta D_{2r}) \tan \lambda_{2r-2}$
	where $\Delta D_{2r-2} = \frac{2 R_{2r-2} \sin^2 \psi_{2r-2} + \lambda_{2r-2}}{2}$
	and $\Delta D_{2r} = \frac{2 R_{2r} \sin^2 \psi_{2r} + \lambda_{2r}}{2}$
$A_{2r} = \frac{H_{2r}}{\lambda_{2r}}$	$A_{2r} = \frac{H_{2r}}{\tan \lambda_{2r}} + \frac{2 R_{2r} \sin^2 \psi_{2r} + \lambda_{2r}}{2}$

The above formulæ will serve for $R_{2r} = \infty$ by using the artifice of substituting $R_{2r} = 100000$ for $R_{2r} = \infty$, which optically would make no appreciable difference to the system, but the following direct method can also be employed :—

AXIAL RAYS.	$R_{2r} = \infty$	EDGE RAYS.
$\phi_{2r} = \lambda_{2r-2}$		$\phi_{2r} = \lambda_{2r-2}$
$\lambda_{2r} = \psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \phi_{2r}$		$\sin \lambda_{2r} = \sin \psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \sin \phi_{2r}$
$A_{2r} = \frac{(A_{2r-2} - D_{2r-1}) \lambda_{2r-2}}{\lambda_{2r}}$		$A_{2r} = \frac{(A_{2r-2} - D_{2r-1}) \tan \lambda_{2r-2}}{\tan \lambda_{2r}}$

As an example of determining the actual aberrations with a view to their subsequent correction, I propose to take the case of a cemented doublet. This cemented doublet was made from Chance Med. Bar. Cr. 1493 and Chance Heavy Flt. 1797, the original data being as follows :—

$H_0 = + 15$	$\mu_{-1}^c = 1.00000$	$\mu_{-1}^F = 1.00000$
$R_0 = + 192.64$		
$D_1 = 8.75$	$\mu_{+1}^c = 1.57425$	$\mu_{+1}^F = 1.58439$
$R_2 = - 124.46$		
$D^3 = 4.40$	$\mu_{+3}^c = 1.61105$	$\mu_{+3}^F = 1.62779$
$R^4 = - 2379.3$		

All lengths measured in millimetres.

The aperture was thus 30 mm. and the focal length desired was 350 mm. Carried out according to the above data, the focal length was too short and the definition very poor.

It was decided to reduce the thicknesses, which were unnecessarily large, to 4 mm. and 2 mm. respectively and to satisfy conditions (a), (b), and (c).

Altering the thicknesses, the axis ray for C was computed stopping short at the second surface; to make the focal length 350 mm. it is clear that the last radius must be altered, the new R_4 being found by means of the following :—

$$H_0 = 15 \quad F_4 = 350 \quad \therefore \lambda_4 = \frac{15}{350} \quad \delta_4 = \lambda_4 - \lambda_2$$

$$\phi_4 = \frac{\delta_4}{1 - \frac{\mu_3}{\mu_5}} \quad R_4 = \frac{(A_2 - D_3) \lambda_2}{\lambda_2 + \phi_4}$$

Putting in the new value for R_4 the calculation was completed, and the focal length found to be 349.992 mm. As five-figure logarithms

were used, this result was within the accuracy obtainable by the means employed.

The axis ray for F and the edge ray for C were next computed, the results found being

AXIAL RAYS.				EDGE RAY.			
A ^C	-	-	-	346.169			
A ^F	-	-	-	345.542	A ^F	-	-
							345.500

(N.B.—The formulæ for long radii were used, but the correction terms omitted as being too small to take into account at this stage of the work.)

It will be seen that the chromatic aberration of the axis rays amounts to .63 mm. and is uncompensated, while the spherical aberration is small.

At this stage it is necessary to decide which elements are available to produce alterations in the aberrations and how to allot them to the best advantage.

In this case the only elements available are the two radii R_0 and R_2 , as R_4 has already been disposed of in making the focal length 350 mm. It will be best to use R_2 to alter the chromatic aberration, reserving R_0 to deal with the spherical aberration if necessary.

As the chromatic aberration is under corrected, R_2 must be shortened, and it is necessary to determine by what amount.

It can easily be shown that to alter the chromatic aberration by .60 mm., the alteration to be made to λ_2 when computed into air (not into the flint component) is $.60 \frac{H}{l^2} \frac{\nu\nu^1}{\nu-\nu^1}$ * where ν , ν^1 are the conventional dispersive constants† of the crown and flint glasses. Applying the formulæ on page 152 we are in a position to find the new $R_2 = -113.467$.

Substituting the new value of R_2 , revising the old calculation, and adding the edge ray for C we now find the intersectional distances for the objective having

$$\begin{aligned} R_0 &= +192.64 \\ R_2 &= -113.467 \\ R_4 &= -30.17 \end{aligned}$$

to be

AXIAL RAYS.				EDGE RAYS.			
A ^C	-	-	-	346.200	A ^C	-	-
A ^F	-	-	-	346.042	A ^F	-	-
							346.169
							346.177

and this time the correction terms have been included.

* This is an approximate formula and disregards the thickness of the lenses ; for derivation see Appendix, p. 158.

† $\nu = \frac{\mu_D - 1}{\mu_F - \mu_C}$

The aberrations are :—

CHROMATIC ABERRATION.
 Axis - .16 mm. uncompensated.
 Edge - .01 mm. uncompensated.

SPHERICAL ABERRATION.
 C. - .02 mm. uncompensated.
 F. - .13 mm. overcompensated.

The values of the focal lengths are :—

AXIAL RAYS.
 F^C - - - 350.000
 F^F - - - 349.875

EDGE RAYS.
 F^C - - - 349.861
 F^F - - - 349.861

their aberrations being :—

CHROMATIC ABERRATION OF THE FOCAL LENGTH.
 Axis - - - .12 mm. uncompensated.
 Edge - - - .00 mm.

ERROR AGAINST THE SINE CONDITION.
 C - - - .14 mm. uncompensated.
 F - - - .01 mm. uncompensated.

From the above it is seen that there is still a little uncorrected chromatic aberration, while the spherical aberration is on the whole slightly over corrected. The sine condition is not quite satisfied, but the objective would be good enough for the purpose for which it was intended.

If it were desired to alter the spherical aberration, R_0 would be altered by a small amount, R_2 being also altered to keep the value of λ , computed into air, constant, while R_4 would be redetermined to keep the focal length of the whole system 350 mm.

In the case of a triplet system the surfaces would be apportioned according to what aberrations were to be removed, the last radius always being utilised to keep the focal length to its correct value; in the case of a triplet with an air space the surface immediately following the air space would be allotted to the spherical aberration, and the first surface to the fulfilment of the sine condition.

The Testing of Objectives.

Focal Length.

In testing an objective the focal length is first determined. This may be done by one or other of the methods of focal length measurement in general use. It is usually more convenient to measure the intersectional distance, which can be very easily done, and to compare the observed with the computed value already found; if these agree, the focal length will also agree with its computed value.

Spherical Aberration, Centering, Figure.

The objective should be mounted as a telescope with an astronomical eyepiece giving a considerably higher power than that with which it is to be used regularly, and an artificial star at a distance examined, pushing the eyepiece inside and outside the focus.

In both cases the patch of light should be circular (or exactly the same shape as the aperture of the objective) and of uniform brightness. Any want of symmetry in the brightness of the patch of light, either inside or outside the focus, reveals an error in the centering of the objective, while any symmetrical lack of uniformity in the brightness of the patches is caused by spherical aberration.

The appearances caused by undercorrected and overcorrected spherical aberration differ, the former appearing as a hard bright edge to the patch of light inside the focus and a bright centre to the corresponding patch outside the focus, while with the latter the positions are reversed.

Defective figuring of any surface will cause the shape of the patches of light just inside and outside focus to differ from the shape of the aperture.

Coma.

Any really serious departure from the sine condition in an objective of relatively large aperture shows itself as coma, provided the spherical aberration is properly corrected. In looking for coma it is necessary to make sure that the eyepiece is not producing the effect and that the observed coma is really caused by the objective.

Chromatic Aberration.

In testing for chromatic aberration the white artificial star should be replaced by a coloured one, in which the colour is pure and also of known wave-length. Such a coloured artificial star is easily constructed by connecting two cadmium electrodes to the coatings of a small Leyden jar fed from the secondary of an induction coil, and also placing a pinhole just in front of the spark gap. The principal lines of the spark spectrum of cadmium are very bright and well distributed over the whole spectrum; their wave-lengths are well known, and the spacing is such that any line may be recognised rapidly and with certainty.

The coloured star is now viewed through a telescope consisting of the objective to be tested and a high-power eyepiece, the component lenses of which are themselves achromatic.

When a direct vision prism of high dispersion is placed immediately behind the eyepiece and the two together moved inside and outside the focus, the observer can at once see which bands of colour are in focus

simultaneously, and can roughly plot the colour curve of the objective, especially if the objective is of considerable focal length, and the focussing tube is provided with a vernier.

Chromatic Aberration of the Focal Lengths.

This aberration very rarely occurs in such magnitude as to be of any importance. Its effect when present to any serious extent in an objective that is properly corrected for chromatic aberration of the intersectional distances is to cause objects of finite area to have images of different sizes, thus producing colour fringes towards the edge of the field of view. It is, of course, essential that the eyepiece used should be thoroughly achromatised as already mentioned in the case of chromatic aberration.

The Hartmann System of Testing.

When extreme accuracy in testing is desired, the Hartmann method of extra-focal measurements should be adopted. This method of testing is so sensitive that there is no telescope objective in existence that will not show aberration when subjected to the following tests.

Spherical Aberration.

A special diaphragm is first prepared, consisting of a card or metal disc arranged to cover up the whole of the aperture and which can be accurately placed over the objective. In this diaphragm a number of small circular holes are made in a series of concentric circles, so that the light passing is restricted to a number of small pencils evenly spaced round a number of zones concentric with the axis of the objective. These pencils should intersect in one point, the focus, if the system is entirely free from spherical aberration over the whole aperture, but in practice they will not do so owing to the residual aberration always present.

A pair of photographic plates are now exposed, one on each side of the focus, and at some distance from it, the resulting negatives after development giving the positions in which the separate pencils of light intersected the two planes in which the exposures were made. If these negatives are measured by means of a travelling microscope, knowing the separation between the two planes, it is easy to calculate the exact points of intersection of the rays of each zone, and so to chart the spherical aberration of the objective over the whole aperture.

Chromatic Aberration.

When the colour curve of an objective is required a modification of the above method can be employed. A diaphragm is again fitted to the objective but with only two apertures diametrically opposite and each

distant from the axis by two-thirds of the semi-aperture. A prism is now placed in front of the objective with its refracting edge parallel to the line joining the centres of the two apertures.

Two negatives are taken as before, using the cadmium spark as a light source, and the negatives measured. The numerical reduction proceeds on similar lines, and gives the differences of the intersectional distances for the wave-lengths of the spectral lines utilised, or, in other words, the colour curve of the objective.

Conclusion.

Owing to the necessity of restricting the length of this paper, it has only been possible to give a brief outline of this interesting branch of optics. Its purpose is not to be a complete exposition of the subject, but to serve as a starting point from which those who would like to follow up the mathematical side of the problem, but are as yet unacquainted with it, may be led to a larger interest in the subject. The two most valuable books dealing with the telescope objective are Steinheil and Voit's "Handbuch der Angewandten Optik" and Gleichen's "Lehrbuch der Geometrischen Optik." Unfortunately these two books are in the German language, but I believe that they are in course of being translated into English. With these two books, the tables published by Messrs. Smith & Cheshire, a glass catalogue, and a table of logarithms, the designer has all the material required to work out the final curves, substances and residual aberrations of any telescope objective he may desire.

APPENDIX.

A method of finding the alteration required to vary the chromatic aberration by any desired small amount.

Let F_C , F_D , F_F be the focal lengths of the combination for the C, D and F lines of the spectrum; also let ϕ be the power of the crown component, K and K' the total curvatures of the crown and flint components, n and n' with suitable suffixes their refractive indices.

Consider now an objective suffering from positive chromatic aberration (longitudinal) A.

We have

$$\frac{1}{F_F} = K (n_F - 1) + K' (n'_F - 1)$$

$$\frac{1}{F_C} = K (n_C - 1) + K' (n'_C - 1).$$

By subtraction $\frac{1}{F_F} - \frac{1}{F_C} = K \Delta + K' \Delta'$, where $\Delta = n_F - n_C$ and $\Delta' = n'_F - n'_C$
 also $\frac{1}{F_F} - \frac{1}{F_C} = \frac{F_C - F_F}{F_F F_C} = \frac{A}{F_F F_C} = \frac{A}{F^2}$ very nearly.

Now add increases of curvature δK and $\delta K'$ such that

$$0 = \delta K (n_D - 1) + \delta K' (n'_D - 1) \text{ and } -\frac{A}{F^2} = \Delta \delta K + \Delta' \delta K'$$

i.e., we have increased the powers of the lenses without altering the focal length of the combination and have at the same time introduced a chromatic aberration equal to A but of opposite sign.

We have immediately

$$\delta K' = -\frac{\delta K (n_D - 1)}{n'_D - 1}$$

and consequently

$$-\frac{A}{F^2} = \Delta \delta K - \Delta' \delta K \frac{n_D - 1}{n'_D - 1}$$

But

$$\delta K = \frac{\delta \phi}{n_D - 1}$$

$$\therefore -\frac{A}{F^2} = \Delta \frac{\delta \phi}{n_D - 1} - \Delta' \frac{\delta \phi}{n'_D - 1}$$

Now

$$\Delta = \frac{n_D - 1}{\nu} \text{ and } \Delta' = \frac{n'_D - 1}{\nu'}$$

$$\therefore -\frac{A}{F^2} = \frac{\delta \phi}{\nu} - \frac{\delta \phi}{\nu'} = \delta \phi \left(\frac{1}{\nu} - \frac{1}{\nu'} \right) = -\delta \phi \frac{\nu' - \nu}{\nu \nu'}$$

$$\therefore \delta \phi = \frac{A}{F^2} \frac{\nu \nu'}{\nu' - \nu}$$

By the axial trigonometrical formulæ $\delta \beta = H \delta \phi$.

\therefore the alteration to the β of the crown lens to correct a chromatic aberration of the whole system of the amount A is

$$A \cdot \frac{H}{F^2} \cdot \frac{\nu \nu'}{\nu' - \nu}$$

P. F. E.