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Sir MAURICE FITZMAURICE, C.M.G., President,
in the Chair.

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“Experiments on Earth-Pressures.”

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THE supporting power of earth and its pressure against walls must have engaged the attention of many from the earliest time, and even to-day no subject is of more vital importance to engineers, for it is hardly too much to say that of all engineering failures 90 per cent. are due to faulty foundations, while on the other hand it is difficult to estimate the millions that have been expended in making foundations and walls stronger than is necessary, to withstand pressures that can only be approximately determined.

Probably the best-known and most widely-accepted theories of earth-pressures are those of Rankine who treats masses of earth and sand as if they were elastic solids. Now it is a long jump from a block of steel, a typical elastic solid, to a lump of clay, and one would suppose that such a theory could only be accepted after exhaustive experimental verification; but so far as the Author can ascertain, Rankine did not investigate, his theories being based on *a priori* reasoning.

Experiments on a large scale are tedious and expensive, and small-scale laboratory experiments are objected to on the grounds that they are unsatisfactory and may be misleading. This objection is no doubt reasonable, but if theory and experiment cannot be made to agree where all the materials and conditions are under control, little confidence can be felt in applying the theory in actual practice. If, however, experimental results confirm the theory, it may reasonably be accepted and applied, provided the stresses imposed on the materials are kept within the limits to which they have been tested

Since Rankine's time some experiments have been made to test his theories, but it can hardly be said that they are altogether satisfactory, for the results vary within wide limits. Such experiments as have been made do show, however, that the lateral pressure exerted by sand and earth against walls is considerably less than that deduced by Rankine's theory, as it is usually applied.

There is no need to give here the proofs of Rankine's formulas, which are to be found in numerous text-books and are familiar to all engineers, but it may be well to state briefly the principles upon which his reasoning is based. His assumptions are—

- (1) That a mass of earth, sand, or clay consists of an incompressible homogeneous mass, granular and without cohesion, the particles being held together by friction.
- (2) That the top is bounded by a plane surface of indefinite extent and the mass rests on a homogeneous foundation.
- (3) That the whole mass behaves as an elastic solid in a state of strain, and that the principles of the ellipse of stress apply equally to each.
- (4) That if the surface is horizontal, the pressure on any vertical plane through the mass is horizontal, and the centre of pressure is one-third of the depth from the bottom.

From these assumptions is derived the well-known formula for the pressure of earth against a wall with a vertical back—

$$P = \frac{Wh^2}{2} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where P denotes the total pressure against the wall, h its height, W the unit weight of the earth, and ϕ the angle of repose.

In order to test the truth of this formula model walls have been constructed, backed with sand or clay, and measurements have been made of the pressure exerted by the filling against the back of the wall. The values of W , h and P having been determined, that of $\frac{1 - \sin \phi}{1 + \sin \phi}$ is calculated and compared with the value that would be obtained by using for ϕ the natural angle of repose.

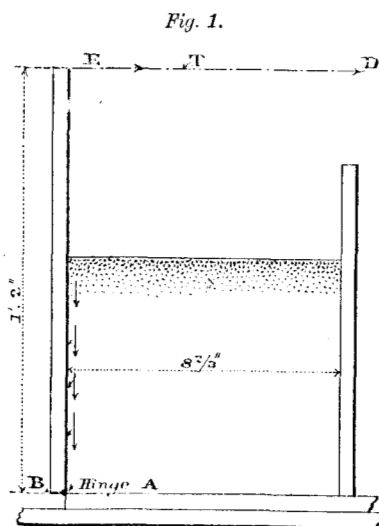
The first and most complete experiments carried out on these lines were communicated to The Institution by the late Sir George H. Darwin in 1883. The experiments consisted of measuring the least force capable of sustaining a hinged door in position against a known height of sand. The door measured 14 inches high and 12 inches wide, and had horizontal hinges at its lower inner edge.

A very large number of experiments were made under different conditions of filling in the sand, of which numerous diagrams are given in the Paper.¹ The observed value of ϕ , the angle of repose, for the sand used was 35° , which gives for $\frac{1 - \sin \phi}{1 + \sin \phi}$, according to Rankine, 0.281. The actual value, however, obtained from the experiments ranged between 0.189 and 0.132. The average of these would give an angle of repose of $46\frac{1}{2}^\circ$. Summing up his conclusions, Darwin says, "The discrepancy from Rankine's value is enormous, and the theory may safely be neglected."

Darwin says, however, that his results agree fairly with Bous-

sinesq's theory, which takes into account the friction between the sand and the back of the wall. This agreement would seem to be rather a coincidence, for under the peculiar conditions of Darwin's experiments the effect of friction between wall and sand is altogether eliminated.

The apparatus used by Darwin is shown in *Fig. 1*. The sand was enclosed in a box with one side hinged at A and kept vertical by the tension in the string ED. The string was kept tight by a spring till the box was full of sand. The tension was then relaxed till the



door was pushed forward by the pressure of the sand. The tension at which this took place was shown on a spring-balance, and from it the pressure of the sand was calculated.

Now an ordinary retaining-wall, if overturned by the filling, will turn round the outer toe B, and the inner side will move relatively to the filling, so that friction will result as shown by the arrows on the sketch. This friction will have a moment round B that will assist the moment of stability of the wall to resist the overturning moment of the sand. In the experiment the turning-

¹ "On the Horizontal Thrust of a Mass of Sand." Minutes of Proceedings Inst. C.E., vol. lxxi, p. 350.

point is the hinge under the back of the wall, and therefore, as the direction of the friction passes through the hinge, its moment round the overturning point vanishes.

Another way of looking at it is that, as the door revolves round the hinge, its motion is everywhere normal to the sand face, so that there can be no friction between the two. There may possibly be cohesion between them, but this would only come into play if an attempt were made to pull the door away from the sand.

As a result of his experiments Darwin concludes that the internal friction of a mass of sand varies from point to point, and points out that merely shaking the box containing the sand increased the density from 1.40 to 1.55 and materially increased the internal friction. He further concludes that the internal friction depends upon the pressure, and also varies from point to point throughout the mass. He also states that the change of density through shaking, which the Author would prefer to call the "aggregation" of the sand, has not been taken into account, and that actual embankments correspond rather to shaken sand in close order than to sand in open order; while the angle of repose of shaken sand is a phrase without meaning.

Though it is not explicitly stated in the Paper, the Author understands Darwin to mean that what governs the stability of a mass of sand is not what is usually termed the "angle of repose," but the "angle of internal friction."

Mr. E. P. Goodrich also carried out some experiments with a wooden retaining-wall backed with moist sand.¹ They were made in a box 6 feet high and 3 feet square. The horizontal pressure of the sand against the side was made by equilibrating it with a known force. The results of his experiments with this model were remarkably consistent, and showed that the horizontal pressure in pounds was one-fifteenth of the vertical pressure plus 15 lbs. If H denotes the depth in feet at which the pressure is measured, W the weight of the sand per cubic foot, and P_1 the horizontal pressure at the depth H , in pounds,

$$P_1 = 0.066 W H + 15.$$

If the wall be 50 feet high, and the sand weigh 100 lbs. per cubic foot,

$$P_1 = 345 \text{ lbs.}$$

¹ "Lateral Earth Pressures and Related Phenomena." Proc. Am. Soc. C.E. vol. xxx, p. 239.

Substituting this value in Rankine's equation,

$$P_1 = W H \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right),$$

$$\frac{1 - \sin \phi}{1 + \sin \phi} = 0.069,$$

and

$$\phi = 60^\circ 37'.$$

Owing to the small scale of the experiment and to the small value of the horizontal pressure obtained, Mr. Goodrich ignored these results, as being of little value, in favour of other experiments to be described.

The problem has been tackled in an entirely different manner by Mr. Goodrich,¹ the late Dr. George Wilson,² and Mr. A. L. Bell.³

Mr. Goodrich enclosed the material to be tested in cylinders, in which pressure was applied by hydraulic power to pistons the full size of the cylinders, so that the material was compressed longitudinally. Calling this the vertical pressure, the resulting horizontal pressure was measured on a cylindrical gauge in the side of the cylinder, by observing the force required to prevent its being pressed out. The researches embraced a large number of substances—sand, gravel, clay, earth, and cinders—at pressures ranging from 2,500 to 10,000 lbs. per square foot.

A number of determinations of $\frac{\text{Horizontal pressure}}{\text{Vertical pressure}}$, or $\frac{1 - \sin \phi}{1 + \sin \phi}$, for different materials, are given in the Paper. For instance, for different sands the values are—

Bank sand	0.10
100-up quick sand	0.20
50 to 100 sand	0.25
Bank sand	0.35
" "	0.40
30 to 50 sand	0.60
20 to 30 "	0.70

These values differ among themselves and would correspond with values of ϕ ranging from 54° to 12° .

¹ "Lateral Earth Pressures and Related Phenomena," Proc. Am. Soc. C.E., vol. xxx, p. 249.

² "Some Experiments on Conjugate Pressures in Fine Sand, and their Variation with the Pressure of Water," Minutes of Proceedings Inst. C.E., vol. cxlix, p. 208.

³ "The Lateral Pressure and Resistance of Clay, and the Supporting Power of Clay Foundations," Minutes of Proceedings Inst. C.E., vol. cxcix, p. 233.

Unfortunately, Mr. Goodrich does not give his own determination of the angles of repose, but merely quotes Rankine's values, namely, 37° to 21° .

In most of his experiments $\frac{1 - \sin \phi}{1 + \sin \phi}$ varies but little with the pressure; but in one specimen of bank sand the value rose from 0.10 at 2,500 lbs. per square foot to 0.25 at 10,000 lbs. per square foot. According to Rankine's theory it should, of course, be constant at all pressures.

In Dr. Wilson's experiments the pressures were obtained in the same way, but were measured by a diaphragm gauge placed in the interior of the sand under experiment, arrangements being made so that the gauge could be placed with its face either vertical or horizontal. By this means either the vertical pressure or the resulting horizontal pressure could be measured. The sand worked with had an angle of repose of 30° , which gives for $\frac{1 - \sin \phi}{1 + \sin \phi}$ the value 0.320. This agrees very well with his determination 0.319. Dr. Wilson also made some determinations of the ratio for sand with different percentages of included water. These are: 6 per cent., 0.221; 12 per cent., 0.212; 17 per cent., 0.280.

Similar experiments with sand and water were made by Mr. Goodrich, but the results were altogether different. In his experiment the ratio in a mixture of sand and water shows one maximum and two minima between dry sand and saturation, a result for which it seems to be difficult to give any satisfactory physical explanation.

Mr. Bell's interesting investigations will be fresh in the memory of most members of The Institution. His apparatus was very similar to Mr. Goodrich's and Dr. Wilson's, the chief differences being the arrangement of the gauge in the side of the cylinder and the fact that, owing to the pressure being applied by hand, Mr. Bell could only reach about 100 lbs. per square inch, whereas Dr. Wilson's pressures reached 1,000 lbs. per square inch, and Mr. Goodrich's 10,000 lbs. per square foot.

For some reason Mr. Bell's instrument failed to give any consistent results with sand. The results are described in the Paper as altogether anomalous and irregular: the readings on some of the gauges could be made to fluctuate widely by tapping the gauges, while other gauges failed to record anything.

For clay consistent results were obtained which gave for $\frac{1 - \sin \phi}{1 + \sin \phi}$

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values ranging from 0.89 to 1.0, corresponding respectively to $\phi = 4^\circ$ and $\phi = 0^\circ$.

Mr. Bell also investigated the coefficient of internal friction of sands and clays by shearing. He found that sand did obey Rankine's law in so much as the resistance to shearing was strictly proportional to the pressure, but that the angle of internal friction was much less than the angle of repose usually assigned in practice. With clay, however, he found that this simple law was not followed. If q denotes the resistance to shearing, P the unit-pressure, α the internal angle of friction in the sand or clay, and k a constant, then the following expressions show how clay and sand resist shearing—

$$\text{(Sand)} \quad q = P \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{(Clay)} \quad q = K + P \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Put into words, these expressions mean that in sand the resistance to shearing is proportional to the internal coefficient of friction, while in clay it is proportional to the same plus a constant, which possibly represents cohesion. This interpretation, however, cannot be accepted as final. Other forces must come into action, and in the case of damp sand or clay there certainly is surface tension, which between small particles may be very great—perhaps exceeding the internal friction—and will tend to increase the stability of the mass. On the other hand, when clay or sand is completely saturated, there may be fluid friction that would be independent of the pressure and which would reduce the stability, the water acting as a lubricant between the particles. That water has some such action is shown by Dr. Wilson's experiments on moist sand.

A great deal of information on the subject will be found in Sir Benjamin Baker's Paper,¹ where many calculations are given. These, however, are not derived from experiment, but by calculation from existing retaining-walls. The lateral pressures found by Sir Benjamin Baker are much below what would be obtained by Rankine's formulas using the angles of repose generally adopted.

An altogether different line of investigation was adopted by Mr. C. J. Meem.² He enclosed sand in a long box, open at the top and with a false bottom. This box was filled with sand, tightly rammed, and when the sand was consolidated as completely as

¹ "The Actual Lateral Pressure of Earthwork." Minutes of Proceedings Inst. C.E., vol. lxxv, p. 140.

² "Notes and Experiments on Earth Pressures," Proc. Engineers' Club, Philadelphia, vol. xxix, p. 114.

possible the false bottom was removed. It was then found not only that the sand remained in position, supported only by the friction against the sides of the box, but also that it could carry in addition a considerable surface load. Mr. Meem's conclusion was that there is an arching effect on the sand whereby the load is transmitted to the sides and the bottom is relieved of weight. This is an important point in connection with the design of grain-bins, for the result is that the pressure on the bottom of a bin reaches a maximum at a certain depth of the grain, and beyond that depth no increase of pressure on the bottom results from increasing the depth of the grain.

This arching does not affect the supporting power, or pressure of earth against walls, but, as will be shown later, it has an important bearing on some of the experiments mentioned.

The Author believes there can be no doubt that of the methods of investigation already described, the experiments against walls have most value, for they give direct values of $\frac{1 - \sin \phi}{1 + \sin \phi}$, the ratio of lateral to vertical pressure. Unfortunately, on small models the pressures are infinitesimal compared with those to which the materials are subjected in practice. For example, in Darwin's experiments the wall was only 12 inches high, and the vertical pressure less than 100 lbs. per square foot, while in Goodrich's wall the pressure was only about 600 lbs. per square foot, whereas at the foot of a wall, say, 40 feet high, the pressure due to the weight of earth would be about 4,000 lbs. per square foot, while under the wall itself it might be 4 or 5 tons per square foot. Also there is great difficulty in measuring the pressure against a model wall of any size, for it is not practicable to balance it as Darwin did by the tension in a string holding back the top. Consequently the pressure has been estimated by balancing the thrust against one of the horizontal boards forming the wall by a known weight, or against a gauge of known size in the face of the wall. Both of these methods are open to the objection that the true pressure may be altogether masked by the arching effect proved by Mr. Meem's experiments. This arching effect probably accounts for Mr. Bell being unable to get any indications on his gauges when sand was used in his cylinder, and for the erratic way in which they behaved when the apparatus was tapped, the taps at times being sufficient to break down more or less the sand arches.

Turning now to the experiments on sand enclosed in cylinders, it seems difficult to see how they could either prove or disprove

Rankine's theory. The latter supposes that the sand has an indefinite free surface, and that the only internal force acting in it is the friction between the particles, which except for this are free to move. Now the object of the experiments is to measure this internal friction, and it appears doubtful whether it is possible to do so by compressing a mass of sand or earth in a cylinder whose rigid sides prevent all movement of the particles.

There is, however, another way of determining the value of $\frac{1 - \sin \phi}{1 + \sin \phi}$ which is not open to these objections.

From equation (1) Rankine develops a theory for the "safe depth of foundations." This theory assumes that when a building is subsiding the earth underneath is squeezed out laterally, the material displaced tending to move in the direction of least resistance, that is, upwards, and the building subsides till the downward pressure of the building is balanced by the resistance to movement of the upheaving earth. It should be noted that this so-called "safe" depth provides for no factor of safety, but is merely a state of exact equilibrium between the weight on the foundations and the displaced earth.

Rankine's formula for this safe depth is—

$$d = \frac{P}{W} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \quad \dots \quad (4)$$

where d denotes the safe depth in feet, P the pressure on the foundations in pounds per square foot, W the weight of the soil in pounds per cubic foot, and ϕ the angle of repose.

It is a simple matter to put this formula to the test of experiment by weighting a plunger of known diameter, measuring the penetration, d , and weighing the earth. Then, substituting those values in (4),

$$\frac{1 - \sin \phi}{1 + \sin \phi} = \left(\frac{d \times W}{P} \right)^{\frac{1}{2}} = r$$

and if the value of ϕ , the internal angle of friction, is required

$$\phi = \sin^{-1} \frac{1 - r}{1 + r}$$

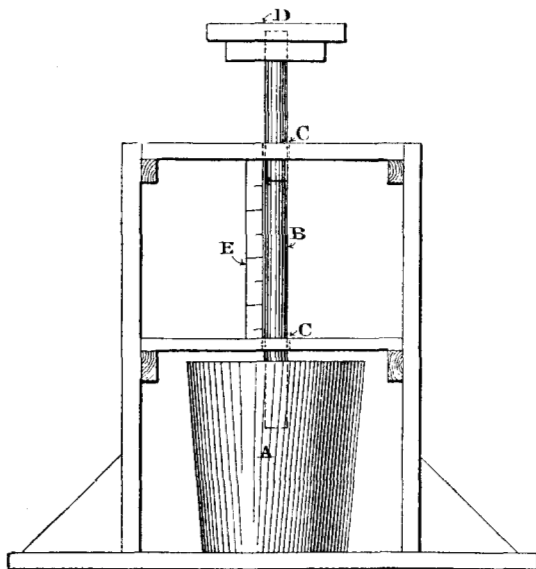
Now, if Rankine's theory is correct, (1), ϕ so deduced should be the same as the angle of repose obtained by observation of the natural slope, and (2) the penetration of the plunger should be directly proportional to the intensity of pressure,

In order to test the theory a large number of experiments have been made on sand, earth, cinders and ashes, and clay.

The apparatus is shown in *Fig. 2*. The material to be tested is held in the strong steel bucket *A*, 11 inches in diameter and 12 inches high. The plunger *B* is free to move vertically through the guides *C, C*. On the top of *B* is a Table, *D*, to carry weights.

When making an experiment the material is filled into the bucket and the surface is struck off even with the top. The bucket is weighed and the weight per cubic foot of the material is calculated. It is then placed in position, and the apparatus is levelled so that *B*

Fig. 2.



is truly vertical. A glass plate about 5 inches square is placed on the surface of the material, and the reading of a mark on the plunger is taken on the scale *E*. The thickness of the glass added to the scale reading gives what the reading would have been had the plunger rested on the material. The glass plate is then removed, weights are then piled on *D*, and readings are taken when the plunger has come to rest after each additional load. When this state of rest is reached it is evident that the downward pressure due to the weight on the plunger is balanced by the resistance of the surrounding earth, and that the penetration of the plunger is Rankine's safe depth.

Probably it will be objected that material contained in a small bucket cannot reasonably be compared with a mass of an indefinite extent, with an unlimited free surface. This objection is no doubt valid if the experiments are pushed beyond a certain point. If, however, they are stopped before the plunger reaches a depth such that a line drawn from its base at an angle with the horizontal, equal to the angle of repose, cuts the surface of the material inside the bucket, the material may, in the Author's opinion, be considered as indefinite in extent.

Fig. 3.

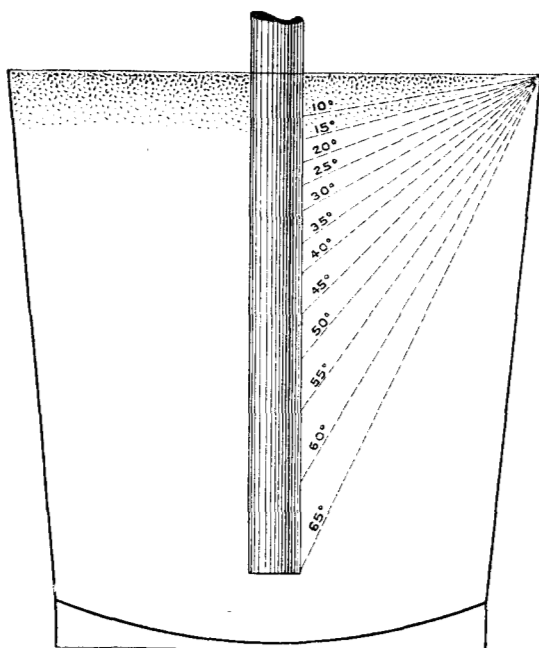


Fig. 3 shows the depth to which the experiments can be carried, keeping them within these limits, for materials having angles of repose up to 65° ; but in dealing with anything except the softest materials the limit is seldom reached.

Care must also be taken to stop the experiments before the plunger reaches too near the bottom of the vessel, as this would interfere with the free lateral escape of the material owing to that underneath the plunger becoming consolidated.

In order to draw a comparison between the experiments and Rankine's theory, it is necessary to make true measurement of the angle of repose, and few experimenters give any information to indicate how their measurements were made. One authority says he made a heap of the sand on a table and measured the surface slope after tapping. This procedure seems to the Author to be altogether inadmissible, for by judicious tapping the slope could be flattened down to any angle required. What is really wanted is the steepest slope that the sand can possibly be made to stand at; for if Mosley's principle of least resistance is true, that is the angle of friction that will be developed in the mass in order to make the internal movement a minimum. The Author has found it very difficult to obtain this maximum angle of slope with such materials as dry sand, for the slightest shaking of the table, or roughly adding sand to the pile, will cause a slip that will flatten the slope several degrees. With damp materials there is not the same difficulty, for they have sufficient cohesion or friction to stick together and withstand slight jars.

The Author does not know any means for determining the natural slope of clay. Some writers say that the angle that should be taken is the slope that an embankment would stand at permanently, after exposure to the weather for a considerable time. This assumption may be reasonable when applied to an embankment where the slope will be determined by the disintegrating effects of frosts, of alternate drying and wetting, and of water running over its surface, but it does not seem reasonably to apply to the backing of a wall which is altogether protected from atmospheric influence; and it is known that to protect embankments from this surface action they are usually sodded, and sometimes stone-pitched. There would therefore be no reason for applying this surface angle of repose when calculating the horizontal thrust of earthworks against walls, for this must depend on the internal friction and cohesion alone.

Many writers seem to differentiate materially between friction and cohesion, but as both tend to prevent movement in the mass it is not clear why they should not be considered together, unless they can be shown to obey different laws. For instance, it is well known that damp sand will stand at a much steeper slope than dry, and this is usually attributed to increased cohesion, which enables it up to a certain height to stand vertical. Rankine, however, does not seem to make this hard and fast distinction between cohesion and friction, for he says that "moisture in earth to an extent just sufficient to expel air from its crevices seems to increase the coefficient

slightly," and he gives different coefficients of friction for dry, damp, and wet clays.

In his experiments, which it is hoped may be of some interest to members of The Institution, the Author has only been able to deal with a few examples, but he has endeavoured to select such as will be typical of widely different classes of materials, and to make the experiments on each as complete as possible, rather than to deal incompletely with a wider range of materials. From the little experience he has gained, he is convinced there is a large amount of ground to be covered in detail, if any real advance is to be made on the theory dealing with clay pressures, which appear, from the experiments to be described, to obey laws totally different from those that govern the pressures of sands and other granular masses.

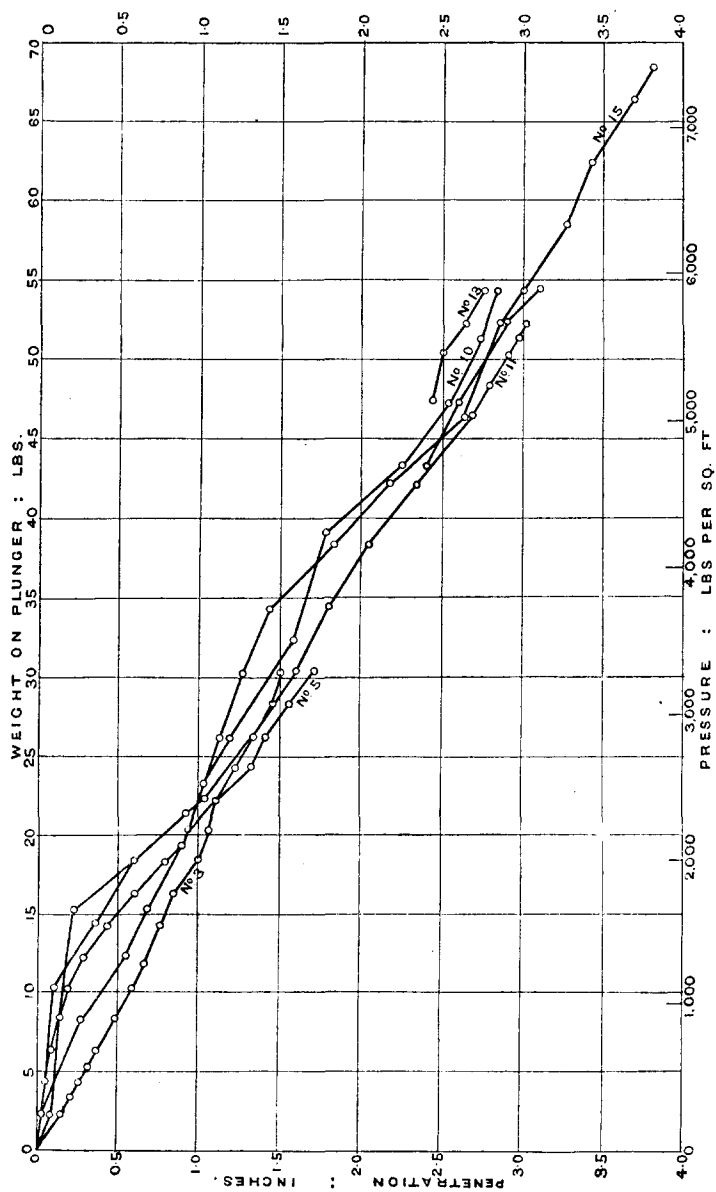
The experiments will be described as nearly as possible in the order in which they were made, following the programme which the Author mapped out before commencing the work.

SAND.

The first experiments, repeated as nearly as possible under the same conditions, were made to see if consistent results could be obtained. The material was Leighton Buzzard sand, used somewhat damp, as received from the pit. The mean of five measurements gave for the angle of repose $52^{\circ} 28'$.

The sand was well shaken into the bucket and in this condition weighed 93 to 98 lbs. per cubic foot. The results of the experiments are shown in *Fig. 4*, upon which the penetrations are plotted against the weights on the plunger, the pressure in pounds per square foot being shown on the scale at the bottom of the diagram. They proved that consistent results can be obtained, indeed, far more so than the Author expected. In order to bring out the differences between the various experiments the penetrations are plotted to a larger scale than in the other diagrams, so that the discrepancies between the different experiments are exaggerated. It is evident that it is practically impossible to ensure that the sand will be equally compacted and shaken together throughout, and it was found that merely striking off the top to get an even surface affected it for nearly an inch down, as is shown by the humps on the lines for experiments Nos. 5, 10, and 11. Except for the humps at the beginning of these curves, it will be seen that a straight line can be drawn that will fairly represent all the observations within the limits of experimental error that may reasonably be expected,

Fig. 4.



and this should be the case if the penetrations follow Rankine's law.

Calculating $\frac{1 - \sin \phi}{1 + \sin \phi}$ and ϕ from the observed penetrations and pressures, the values given in Table I are obtained.

TABLE I.

Experiment No.	Weight of Sand.	Pressure.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .
	Lbs. per Cu. Ft.	Lbs. per Sq. Ft.	Inches.		
3	98.35	3,282	1.51	0.0617	62° 6'
4	95.8	3,282	1.98	0.0671	61° 0'
5	95.8	3,282	1.72	0.0648	61° 16'
10	93.0	5,136	2.85	0.0656	61° 34'
11	93.0	5,657	3.02	0.0642	61° 34'
13	93.0	2,400	1.15	0.0611	62° 12'
14	93.0	2,400	1.25	0.0636	61° 42'
15	93.0	7,566	3.81	0.0626	61° 54'

Experiments Nos. 13 and 14 were made on a plunger of 3.25 square inches area, while the remainder were made with the ordinary plunger 1.33 square inch in area.

The mean of the eight experiments gives, for $\frac{1 - \sin \phi}{1 + \sin \phi}$, 0.0636 from which the maximum and minimum values differ by 50 per cent. and 4 per cent. respectively. The mean calculated value of ϕ is 61° 38', with maximum and minimum errors of 38'.

This differs from the observed value for the angle of repose 52° 28' by 9° 10', but is in marked agreement with Mr. Goodrich's experiments on his retaining-wall, which give 60° 37' for ϕ . In these experiments the weight of sand in the bucket varied only by about 5 lbs. per cubic foot, and a further set of experiments was made with the same damp sand to see to what extent the values of the ratio and ϕ would vary with different degrees of aggregation of the sand in the bucket. The results are shown in *Fig. 5*, in which the curves for the damp sand are shown by dotted lines, and the calculated results are given in Table II. The weights in these experiments ranged from 79.8 to 101.1 lbs. per cubic foot.

Fig. 5.

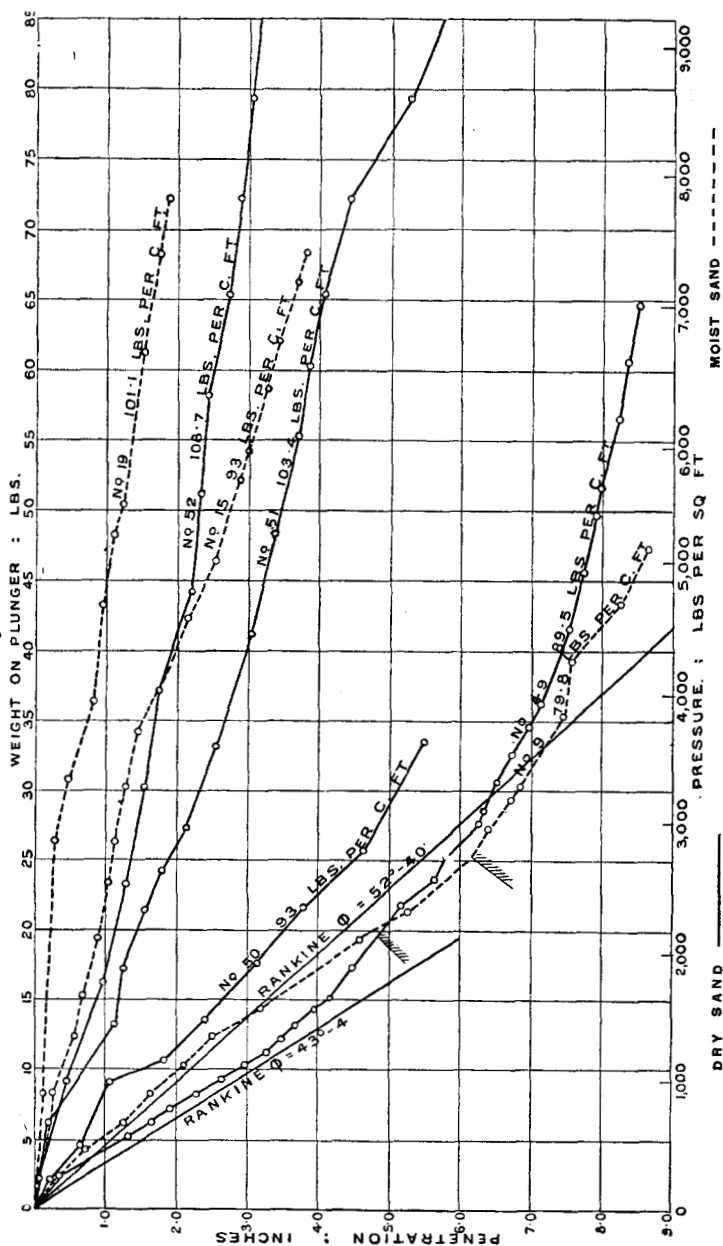


TABLE II.

Experiment No.	Weight of Sand.	Pressure.	Penetration.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Remarks.
	Lbs. per Cu. Ft.	Lbs. per Sq. Ft.	Inches.			
9	79.8	2,739	6.18	0.1224	51° 27'	{3.25 sq. in. plunger used
12	79.3	2,400	6.37	0.1287	50° 32'	
15	93	7,526	3.81	0.0626	61° 54'	
19	101.1	7,829	1.86	0.0436	67° 45'	

The first of these determinations agrees within 1° of the actual value obtained by direct measurement of the angle of repose, and this was obtained when the sand was in the loosest possible state of aggregation, just the condition it would be in when the angle of repose is taken on a table. Referring to Table II and *Fig. 5* it will be seen that for experiment No. 9 the calculation has been made for the point on the curve where the pressure is 2,739 lbs. per square inch. The reason for selecting this particular point, which, as a matter of fact, gives about the worst agreement with Rankine's line that could have been selected, is that it is the lowest penetration of the plunger from which a line drawn from its base at an angle to the horizontal equal to the angle of repose cuts the surface of the sand inside the bucket; and it will be noticed that below this point the penetration curve flattens off, showing that beyond this depth the sides of the bucket are beginning to affect the results by an amount that increases with the depth. Where the penetration passes this point it is marked on the diagrams by the shaded sloping lines, and in almost all cases it will be seen that there is a decided flattening off of the curves if the experiments are pushed beyond it. A further series of experiments are shown by full lines on the same diagram. They were made with the same sand when thoroughly dry in order to see what effect the moisture had. Table III gives the results.

The measured angle of repose was 43° 4', which is 7° 7' less than the angle found by experiment and calculation.

With this dry sand it was found impossible to fill the bucket with sand weighing less than 89½ lbs. per cubic foot. This is curious, for one would expect the damp sand to be the heavier, whereas in experiments 9 and 12 it actually weighed 10 lbs. per cubic foot less. The reason appears to be that the damp sand sticks together in little nodules, between which there are compara-

TABLE III.

Experiment No.	Weight of Sand.	Pressure.	Penetration.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ
	Lbs. per Cu. Ft.	Lbs. per Sq. Ft.	Inches.		
49	89·5	2,013	4·82	0·1317	50° 11'
50	93·0	3,607	5·44	0·1077	53° 38'
51	101·1	9,363	5·83	0·0731	59° 44'
52	108·7	16,116	4·46	0·0500	64° 47'

tively large air-spaces, while the dry sand is so mobile that it runs together, leaving few open spaces, and readily assumes what Darwin calls close order. In order to get 101 lbs. per cubic foot of damp sand into the bucket it had to be well pounded in, but with the dry sand 108 lbs. per cubic foot could be got in merely by shaking. The ease with which the dry sand, owing to its mobility, becomes aggregated would seem to account for the large discrepancy of more than 7° between the angle of repose and the calculated value of ϕ .

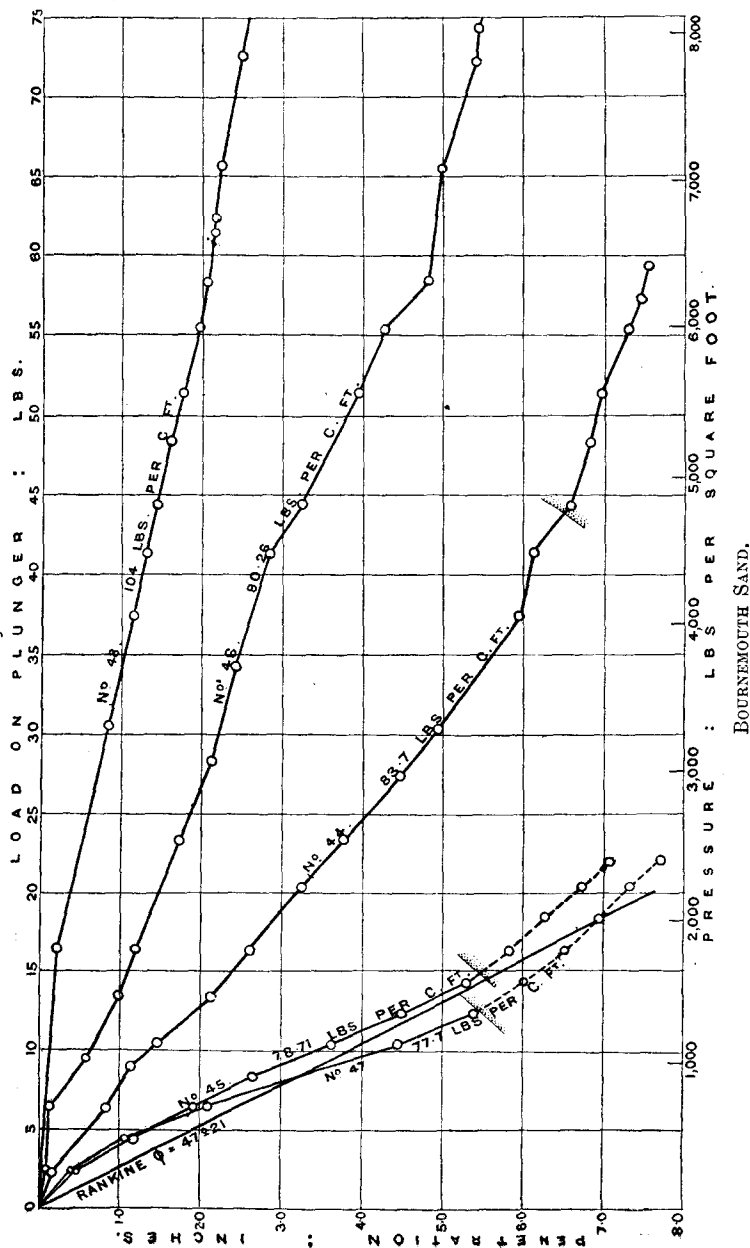
Fig. 6 and *Table IV* show the results of a set of experiments on a fine sand obtained from the cliffs at Bournemouth, which in its damp state had an angle of repose of 47° 21'.

TABLE IV.

Experiment No.	Weight.	Pressure.	Penetration.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Remarks.
	Lbs. per Cu. Ft.	Lbs. per Sq. Ft.	Inches.			
45	78·71	1,449	5·5	0·1449	47° 34'	} mean 46° 51'
47	77·77	1,353	5·5	0·1621	46° 8'	
44	83·7	4,023	5·93	0·100	54° 54'	
46	90·26	8,307	6·64	0·0773	58° 56'	
48	104·0	15,408	3·92	0·0470	65° 32'	

In experiments Nos. 45 and 47 the sand was in its loosest possible condition ; it will be seen that the value of the angle of repose and the calculated value of ϕ agree to within 1°, while, as in the other experiments, the value of ϕ increases regularly with the density, or rather the aggregation, of the sand, till it reaches a maximum value of 65° 32', practically the same as the maximum for Leighton Buzzard sand, 64° 47' dry, and 67° 48' damp.

Fig. 6.

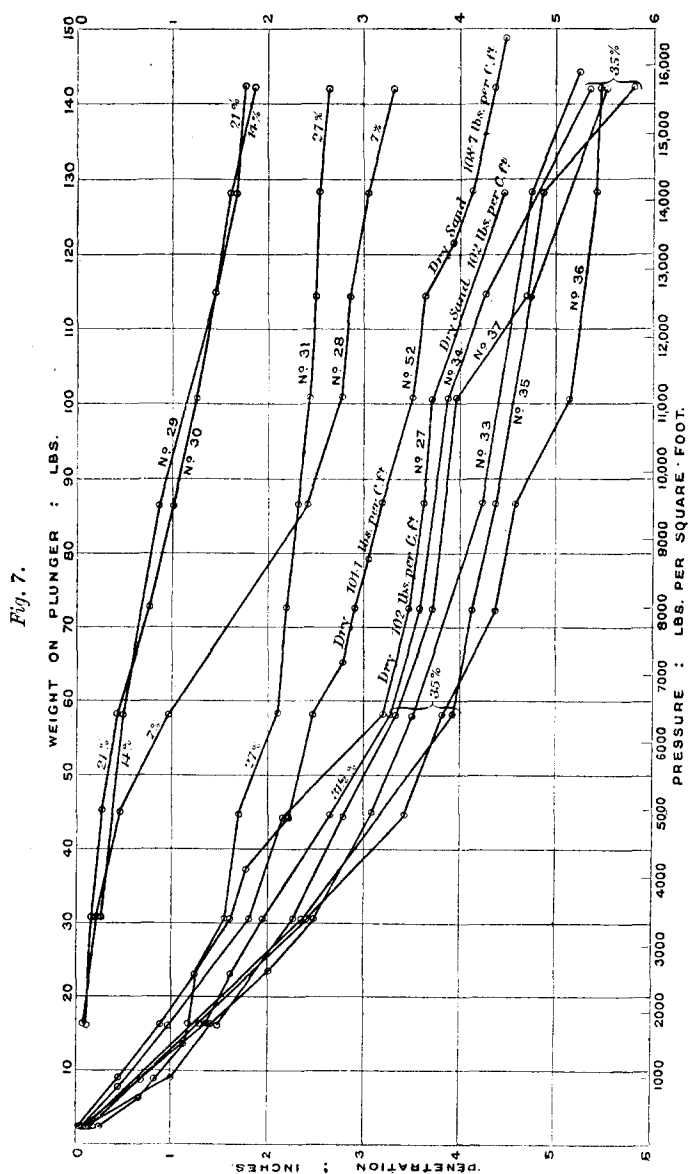


In order to investigate the effect of water on sand, a number of experiments were made with sand diluted with different percentages of water. Slightly damp Leighton Buzzard sand (experiment No. 27) was well shaken into the bucket, and measured quantities of water were sprinkled over the surface with a fine-rosed watering-pot. The penetrations were observed first with the moist sand. The sand was then shaken up, and the first dose of water was given, the penetration being again measured, and so on after each dose of water, till the sand contained $31\frac{1}{2}$ per cent. of water by bulk, when it would take no more. As the Author did not feel quite certain that water sprinkled over the surface would completely permeate the mass, the contents of the bucket were turned out into a deeper vessel and were thoroughly stirred with an excess of water.

The wet sand was then filled into the bucket, and weighed 121.42 lbs. per cubic foot. A subsequent experiment in a graduated flask showed that in this completely saturated state it contained 35 per cent. of water. The results of these experiments are shown in *Fig. 7* and Table V.

TABLE V.

Experi- ment No.	Weight.	Pressure.	Penetra- tion.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Remarks.
	Lbs. per Cubic Foot.	Lbs. per Square Foot.	Inches.			
52	108.7	16,166	4.46	0.0500	64 47	Sand quite dry. Slightly damp. 7 per cent. water.
27	102.3	13,892	4.44	0.0522	64 16	
28	108.3	15,408	3.31	0.0440	66 18	
29	113.8	15,408	1.86	0.0338	69 8	14 " "
30	119.26	15,408	1.76	0.0336	69 13	21 " "
31	124.7	15,408	2.63	0.0413	68 0	27 " "
32	127.7	15,408	5.33	0.0601	62 27	$31\frac{1}{2}$ " "
33	121.42	15,408	5.28	0.0588	62 51	35 " "
34	121.42	15,408	4.90	0.0567	63 28	35 " "
35	121.42	15,408	5.82	0.0617	62 6	35 " "
36	121.42	15,408	5.51	0.0602	62 12	35 " "
37	121.42	15,408	5.47	0.0601	62 27	35 " "



These observations confirm Dr. Wilson's conclusions that water added to sand decreases the value of $\frac{1 - \sin \phi}{1 + \sin \phi}$ till a minimum value is reached, and that further additions increase it again till finally it reaches a maximum value with complete saturation.

It will be noticed that in these experiments the curves approximate less to straight lines than do those for dry and damp sand. The reason for this appears to be that it is impossible to keep the percentage of saturation uniform throughout the mass, there being always a tendency, unless the sand is completely saturated, for the water to sink to the bottom. This is even the case with slightly damp sand, for the Author has found that if the bucket is filled with very slightly damp sand, and left standing for some days, the upper layers of the sand become almost completely dry while that at the bottom is quite wet.

At the suggestion of a friend some experiments were made to see the effect of putting small piles round the plunger. The results of these experiments are shown in *Fig. 8*, the sand used being saturated and flooded with water. The piles were $\frac{1}{2}$ inch square, driven in 4-inch and 6-inch circles round the plunger. The diagrams explain themselves and show that the piles greatly increase the bearing-capacity. A pressure of about 15,500 lbs. per square foot produced a penetration of $5\frac{1}{2}$ inches in the unpiled sand, which was reduced to less than 2 inches by sixteen piles driven in a circle 4 inches in diameter.

GARDEN EARTH.

The following experiments were made with garden earth, sifted free from stones, that stood in its damp state at an angle of $46^{\circ} 12'$. The results are shown by the full-line curves in *Fig. 9* and in Table VI.

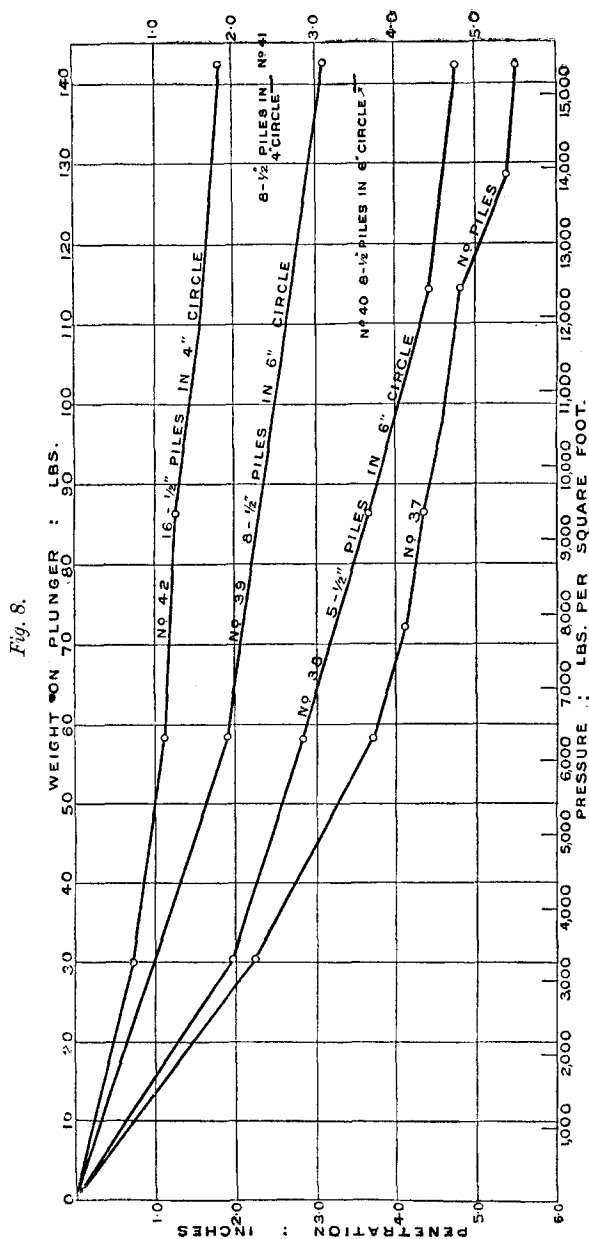
TABLE VI.

Experiment No.	Weight.	Pressure.	Penetration.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .
	Lbs. per Cu. Ft.	Lbs. per Sq. Ft.	Inches.		
25	56	1,008	7.62	0.1878	$43^{\circ} 8'$
26	79.3	6,963	6.92	0.0810	$58^{\circ} 15'$
26A	101.7	13,892.	8.05	0.0696	$60^{\circ} 51'$

In this case the discrepancy between the measured angle of repose and the calculated value of ϕ was $3^{\circ} 4'$.

[THE INST. C.E. VOL. CCIII.]

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ASHES AND CINDERS.

In order to try another class of substance a number of tests were made with ordinary coal-ashes and cinders. For the first experiments the cinders were sifted through a sieve with openings about $\frac{1}{3}$ inch square, an ordinary house cinder-sifter. The results are shown by the dotted lines in *Fig. 9*. It will be seen that these lines are much more irregular than those for sands and earth. For low pressure they have first a flat slope which then becomes very steep and then, when the experiments are prolonged, becomes flat again. They were a great puzzle for some time till it occurred to the Author that it might possibly be due to the very wide range in size of the particles, which ranged from $\frac{1}{3}$ inch in diameter to the finest impalpable dust. It seemed likely that, as the plunger descended, it carried with it the larger particles and formed a sort of foundation for itself: and that when the pressure was increased beyond a certain point this shallow bed would be burst through and the plunger would descend rapidly till another collection of large particles was sorted out, when the curve would again become flat. To test this the ashes were sifted through a 30 by 30 sieve. The material was then shaken into the bucket to correspond as nearly as possible with the weights in the first experiments. The experiments of the second set are shown by a dot-and-dash line on the diagram, and it will be seen that now the lines are much more regular, though not quite so straight as those for sand, probably due to the large percentage of extremely fine ash in the mixture, which renders it less homogeneous.

The angle of repose of the coarse mixture was $53^{\circ} 48'$ and of the fine $47^{\circ} 30'$.

Table VII gives the results. In experiments Nos. 21 and 60 the calculated values of ϕ differ by less than 1° from the observed angles of repose.

TABLE VII.

Experiment No.	Weight.	Pressure.	Penetration.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Remarks.
	Lbs. per Cubic Foot.	Lbs. per Square Foot.	Inches.		" "	
21	31.19	2,307	8.17	0.101	54 15	Ashes and cinders through $\frac{1}{3}$ inch sieve.
23	33.37	2,307	7.89	0.097	55 24	
20	36.15	6,426	7.69	0.06	62 29	
22	40.2	7,829	5.15	0.0468	65 38	
59	41.3	10,861	5.59	0.0328	69 28	
43	37.3	15,408	1.18	0.0224	72 59	Ashes and cinders through 30 by 30 sieve.
60	27.3	576	5.58	0.1482	47 53	
62	31.63	1,876	6.87	0.0982	55 12	
64	35.15	4,796	6.05	0.06	62 29	
63	36.16	4,797	5.60	0.0598	62 31	
61	41.3	15,408	6.18	0.0373	68 8	

There was a curious difference in the behaviour of sand and cinders, for the cinders took hours to come to rest after each additional load, while with dry or slightly damp sand the plunger came to rest almost immediately. The cinders behaved somewhat like a viscous solid.

CLAY.

The results of the experiments on clay are shown in *Fig. 10*. Mere inspection of the Figure will show that clay behaves altogether differently from the other substances, for the penetration, instead of varying directly as the pressure, increases enormously with the pressure. The clay used was a good brick earth. When received from the brick-field it was at once pounded into the bucket and experiment No. 23A was made. In this condition the clay was very stiff, weighing 109 lbs. per cubic foot, and sufficiently dry to show no water on the surface when punned. A load of nearly 14,000 lbs. per square foot produced on the plunger a penetration of only 0.45 inch. The clay was then taken out of the bucket, cut into lumps, and left for about 2 months exposed to the air. Then, having become almost dry, it was tamped with water till wet and again pounded into the bucket for experiment No. 55. It again weighed 109 lbs. per cubic foot, and the penetration with a pressure of 3,282 lbs. per square foot was 7.7 inches.

The other experiments were then made with varying amounts of water, with the results shown. The weights of the clay in Nos. 23A and 55 seem anomalous, for though in the former the clay was very hard and stiff and in the latter almost sloppy, the weights are the same. This is owing to the clay in the first experiment being so stiff that it was almost impossible to pun it into the bucket so completely as to exclude air-spaces, while in No. 55 it was so wet that it readily ran together. In the remaining experiments the weights fall off regularly with increased quantities of water. As these experiments were made under highly artificial conditions, the Author was fortunate in being able to obtain the results of very interesting experiments on clay and mud tested in situ, made by Messrs. Coodé, Matthews, Fitzmaurice and Wilson, which he has kindly been permitted to use. The clay experiments were made on a surface 1 square foot in area and the pressures were carried up to 12 tons on the square foot. Those on mud were made on an area of 16 square feet, with pressures of $1\frac{1}{4}$ ton per square foot. The pressure-curves for the clay are lettered A and B in *Fig. 11* and that for the mud C. Curve D is taken from an experiment made in New York by Mr. W. J. McAlpine and given in his Paper on

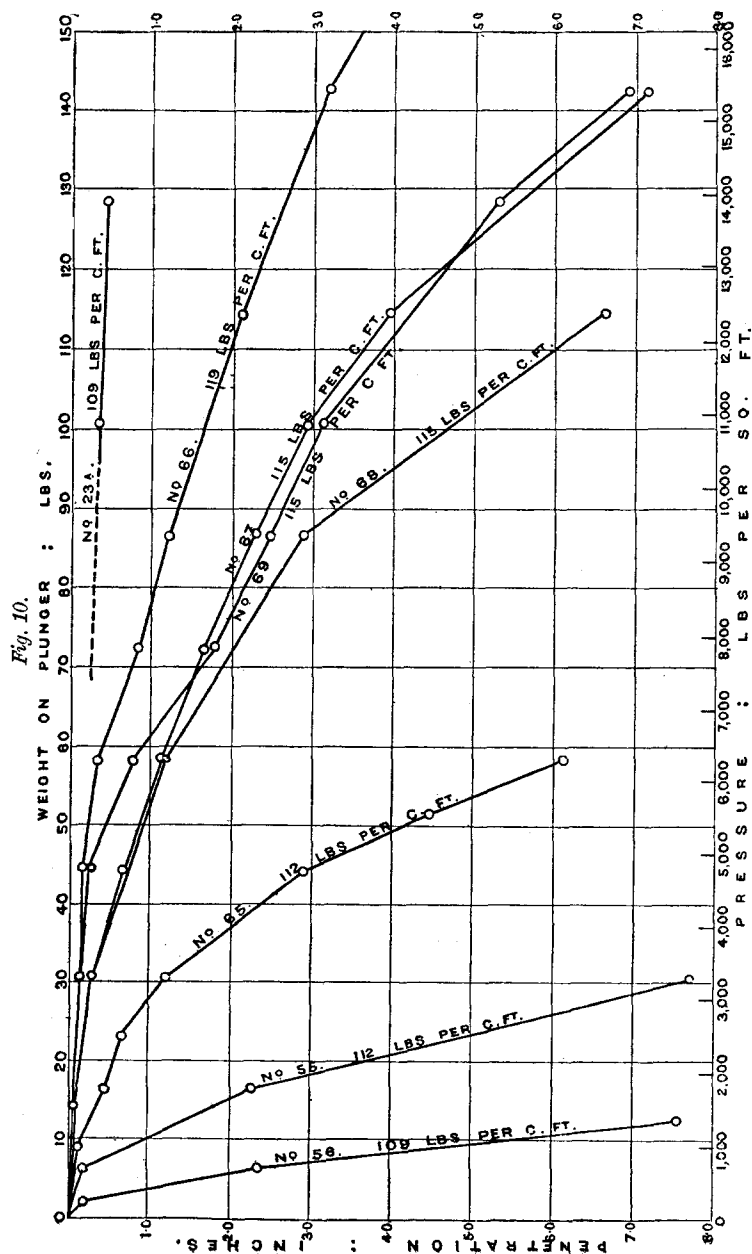
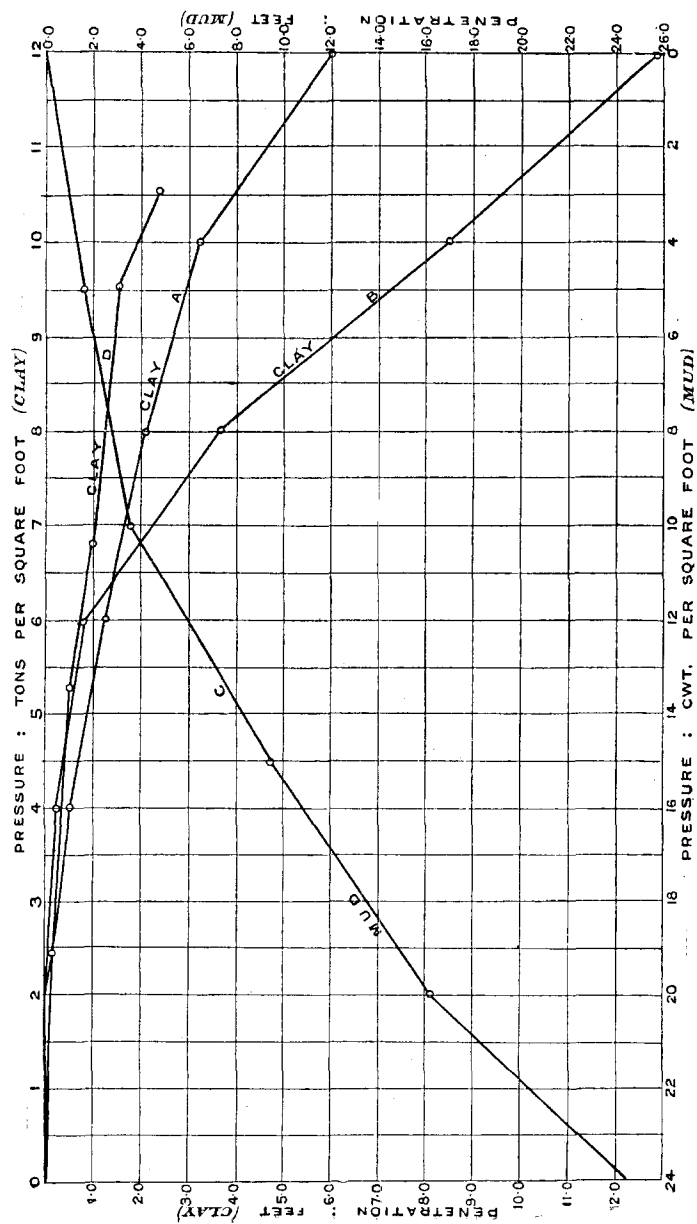


Fig. 11.

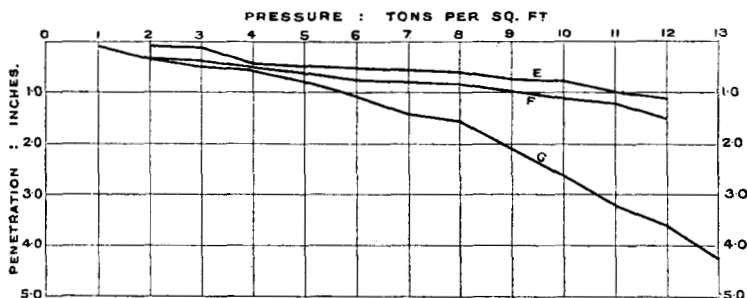


"The Foundations of the New Capitol at Albany, New York."¹ All these experiments were made in natural undisturbed ground and they completely support the Author's in showing that the internal coefficient of friction falls off rapidly with increase of pressure.

A similar set of experiments is recorded by Mr. T. B. Hunter in the Correspondence on Mr. A. L. Bell's Paper, and the results, re-plotted from Mr. Hunter's diagram, are given in *Fig. 12*.

These curves are of special interest, for E and F are for strong boulder clays upon which a pressure of 12 tons per square foot only produced a penetration of 1 inch and $1\frac{1}{4}$ inch respectively. Now the penetration curves for these and that for the Author's experi-

Fig. 12.



ment No. 23A, in which the penetration for 14,000 lbs. per square foot was only 0.45 inch, are practically straight lines, while G, the curve for a weak puddle clay shows the same characteristic as the other weak clays, namely, that equal increments of pressure produce increasing penetrations. It would therefore seem to be suggested that the strong clays approximate more or less to granular masses in obeying Rankine's law, but that the weaker clays do not, the penetration curves deviating from straight lines. It is worth noting that there are two extreme cases in which clay should obey the straight-line law. First, if it is sufficiently stiff to behave as an elastic solid and when obviously the penetration must be proportionate to the load, and secondly when the clay is sufficiently wet to behave as a heavy liquid, for now the volume of the liquid displaced and therefore the penetrations are directly proportional to the load. The curves in *Fig. 10*, though not altogether conclusive, tend to show that clay does behave somewhat in this way, for the

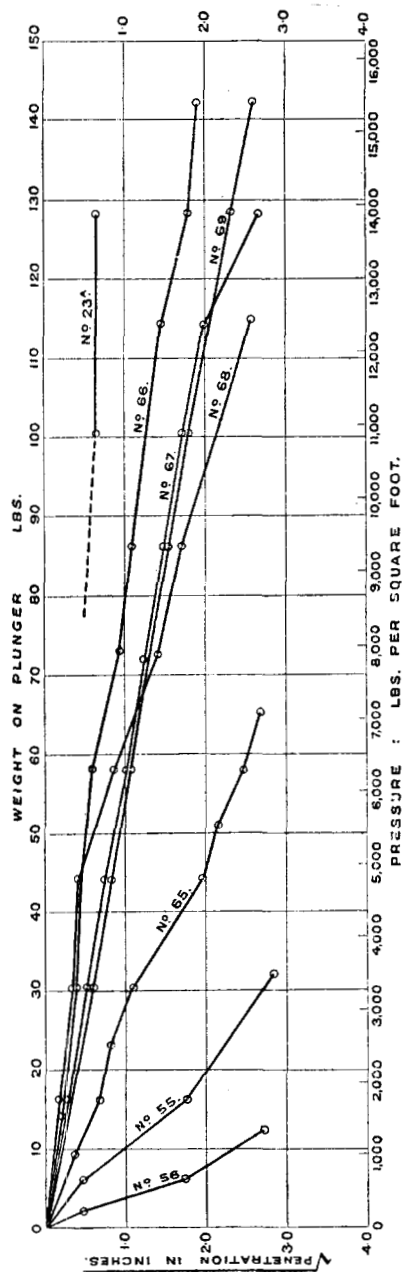
¹ Minutes of Proceedings Inst. C.E., vol. lvii, p. 198.

curve for experiment No. 53A is practically a straight line.

It would not be safe, however, to assume that the stiff clays do obey Rankine's law, for it is quite possible that if the experiments had been carried on to higher pressure the penetration might cease to be proportional to the load; all that can be said is that under the conditions of the experiments, and within the limits of the pressures imposed, they appear to obey the law.

An analysis of the clay curves was kindly undertaken by Dr. Herbert Lapworth, who has found that in all cases they are simply parabolas, the penetration varying with the square of the load per unit area. This is clearly shown in *Fig. 13*, on which the Author's clay experiments (*Fig. 10*) are re-plotted, plotting the loads against the square root of the penetration. The resulting curves are now, allowing for experimental errors, straight lines, so that the original curves, if not actual parabolas, must approximate closely to them.

Fig. 13.



In order to show how the values of ϕ and $\frac{1 - \sin \phi}{1 + \sin \phi}$ vary with the pressure, calculations have been made for the experiments in *Fig. 12*, and the results are given in Table IX. The calculations have been made by substituting in equation (4) the penetration due to each load. This will not give the correct values for the depth of penetration reached, but an approximate average value of ϕ down to that point.

For the upper part the value will be too large, and for the lower part too small. The correct value could be found by using instead of the total pressures and penetrations for each point the increment of pressure and increment of penetration. If, however, the plunger had not come to rest before each additional load was put on, the errors introduced, working thus from point to point, might be very large, and in these experiments it is doubtful if the plunger had come completely to rest, at least for the intermediate loadings.

TABLE VIII.

Pres- sure.	Penetra- tion.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Pres- sure.	Penetra- tion.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .
Tons per Sq. Ft.	Ft. Ins.			Tons per Sq. Ft.	Ft. Ins.		
<i>Experiment (A).¹</i>				<i>Experiment (B).²</i>			
2	0			2	0		
4	0 6½	0·1008	54° 54'	4	0 3	0·0627	62° 3'
6	1 4	0·1134	52° 47'	6	0 10	0·1081	53° 56'
8	2 1¾	0·1295	50° 25'	8	3 8	0·1692	45° 16'
10	3 7	0·1521	47° 22'	10	8 6	0·2311	38° 29'
				12	11 10	0·248	37° 4'
<i>Experiment (C).³</i>				<i>Experiment (D).⁴</i>			
0·25	1 8	0·5389	17° 15'	1·23	0 0·9	0·0522	64° 10'
0·50	3 8	0·566	15° 56'	2·47	0 1·438	0·0461	65° 22'
0·75	9 6	0·7446	8° 25'	5·18	0 6·18	0·066	61° 17'
1·0	16 3	0·839	5° 1'	6·73	1 2·844	0·077	58° 56'
1·25	24 7	0·928	2° 10'	9·36	1 6·972	0·0868	57° 10'
				10·6	2 5·2	0·101	54° 44'

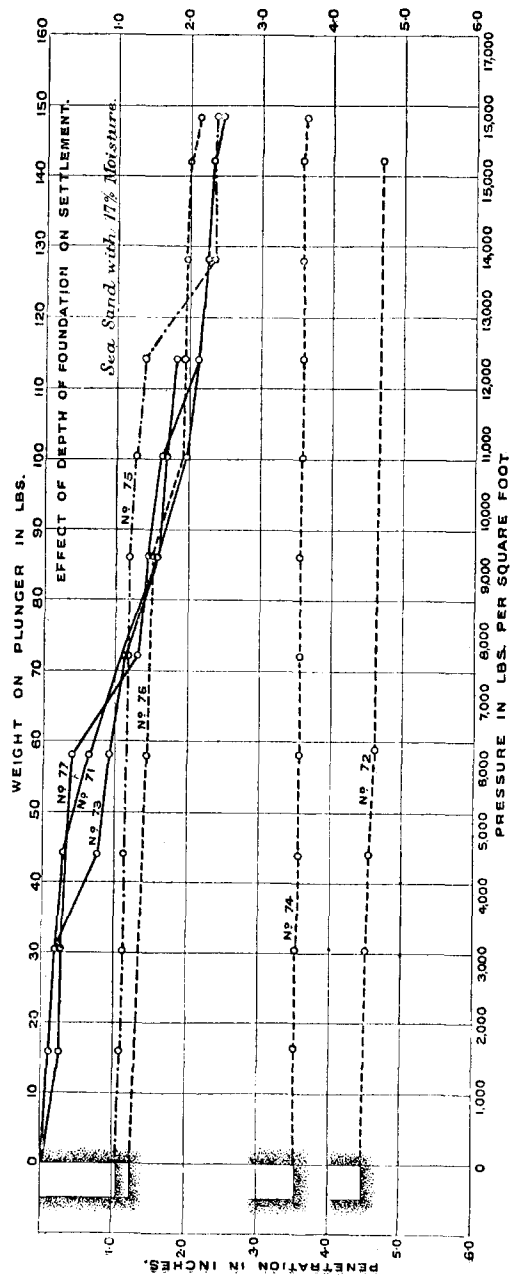
¹ 140 lbs. per cubic foot,² 140 „ „³ 98 lbs. per cubic foot.⁴ 81½ to 101½ lbs. per cubic foot.

DEPTH OF FOUNDATIONS.

The usual way to apply Rankine's formula for the safe depth of foundations is to calculate the safe depth, to excavate and commence the footings at this or some greater depth, and to assume that there will then be no settlement, provided the load used in the calculation is not exceeded. Now, viewing the Author's experiments, it does not seem self-evident that this assumption is justified, for in the case of sand a given load produces a given settlement, or penetration, at whatever depth it is applied, otherwise the load-penetration curves would not plot out as straight lines. To test this a number of experiments that are shown in *Fig. 14* were made on damp sand well pounded into the bucket. Experiment No. 71 was first made, and a load of 12,400 lbs. per square foot gave a penetration of 1.95 inch. A cardboard cylinder was then sunk into the sand to a depth of $4\frac{1}{2}$ inches, well below the safe depth. The sand was then excavated to the bottom of the cylinder, and experiment No. 72 was made with the plunger resting on the sand at the bottom of the hole. It was then found that a load of 15,900 lbs. per square foot only gave a penetration of 0.25 inch. As the Author thought it possible that driving down the cylinder might have compacted the sand, and that the small penetration was due to this, the sand was turned out of the bucket, and experiment No. 73 was made, starting the plunger as in No. 71 at the surface, the penetration with the same load of 15,900 lbs. per square foot being now 2.45 inches. A hole was then scooped out in the sand to a depth of $3\frac{1}{2}$ inches, and experiment No. 74 was made, with the result that the penetration was now 0.13 inch, less than before. These experiments showed that there was practically no settlement when the foundations were kept well below the safe depth.

Experiments were then made to see what would result from starting the foundation above the safe depth. In experiment No. 75 the bottom of the hole was 1.1 inch deep or about 1 inch above the penetration reached in experiments Nos. 71 and 73. From the curve it will be seen that there was little settlement till the load reached 12,400 lbs. per square foot, but the next load produced a penetration of nearly 1.27 inch, while at 13,900 lbs. per square foot the penetration was 1.29 inch, and at 15,900 lbs. per square foot the penetration was 1.31 inch. Measured from the surface of the sand the depth reached by the subsiding load was 2.37 inches. Experiment No. 76, which was started in a hole 1.22 inch deep, gave

Fig. 14.



practically no penetration till the pressure reached 9,350 lbs. per square foot, from which point the penetration regularly increases till at 15,900 lbs. per square foot the plunger came to rest at a depth of 2.15 inches below the surface. From *Fig. 14* it will be seen that the penetrations, measured from the surface of the sand, for the extreme loads on the plunger in experiments Nos. 75, 76, and 77, are approximately the same and independent of the depth to which the hole was sunk and at which the loading of the plunger was commenced.

These experiments prove conclusively, at least so far as laboratory experiments can, that the accepted method of applying Rankine's formula is correct, though as a matter of fact they were made by the Author with the expectation of their proving the reverse. And indeed it is very surprising that a few inches of sand above the base of the plunger should, as in experiments Nos. 72 and 74, enable the sand to carry a load of 16,000 lbs. per square foot with practically no settlement; which leads to the somewhat strange conclusion that in sand that which carries the weight is the sand above and not that below the foundations. Otherwise the penetration would be the same whether the weight was applied on or at a point below the surface.

Had the Author had more faith in theory he should have seen that his experiments on sand had conclusively confirmed Rankine's theory, which takes into account nothing actually below the foundations. But looking at it from a practical point of view, it seems unreasonable to suppose that what is below is not carrying the weight. The experiments, however, speak for themselves, and may, the Author thinks, be almost regarded as crucial tests of the truth of Rankine's theory.

Reviewing the experiments, the Author concludes that for sands and granular materials Rankine's theory holds, provided the proper angle of internal friction be taken and not the angle of repose. The angle of internal friction is not, however, constant for any one material, but varies with the aggregation of the particles and is the same as the angle of repose only where the material is tested for penetration in the same loose state of aggregation, in which it is alone possible to measure the angle of repose. Now it may be said that Rankine's theory is wrong, inasmuch as he assumed that the angle of repose should be used in working out the ratio of the horizontal thrust to the vertical pressure, $\frac{1 - \sin \phi}{1 + \sin \phi}$. It is interesting to see how this works out in practice. In the case of the Bournemouth

sand (*Fig. 6* and Table IV) the angle of repose was measured to be $47^{\circ} 21'$, which gives for $\frac{1 - \sin \phi}{1 + \sin \phi}$ the value 0.1524, practically the same as the values obtained in experiments Nos. 45 and 47, the mean of which is 0.1535. Now if a wall to retain this sand were under consideration, this value would be used to obtain the thrust on the wall, if Rankine were followed. It is well known, however, that sand could never remain in this loose condition for any length of time, but would probably approximate to the condition of the sand

in experiment No. 48. In that experiment $\frac{1 - \sin \phi}{1 + \sin \phi}$ comes out at

0.047, and it may be said that the wall, if designed according to Rankine, is three times too strong. But is this so? If the wall is designed for the ratio 0.047 it will be theoretically strong enough, but without any factor of safety, and in dealing with foundations or retaining-walls it is doubtful whether any engineer would be content with a factor of safety of less than 3. If 0.047 be multiplied by 3, the result is 0.141, which is very nearly the value of $\frac{1 - \sin \phi}{1 + \sin \phi}$ got by using Rankine's formula. So far as

the Author knows, no engineer takes Rankine's value for the thrust and multiplies it by, say, 3 or 4 for safety; and this means that he accepts as a matter of fact that Rankine's calculation provides a factor of safety. The same reasoning applies to all the other granular materials dealt with, namely that Rankine's method applied with the angle of repose provides for a factor of safety ranging from $2\frac{1}{2}$ to 4, and it seems to the Author that, although Rankine does not say so, he saw this and took the angle of repose as the worst condition that need possibly be provided for, giving an ample factor of safety for ordinary working conditions.

This, however, does not seem to be a very satisfactory way to treat the problem, though in the instances cited it may give the correct results. It would seem preferable to determine by experiment the angle of internal friction of the material, and to calculate from it the actual thrust, and then to provide for safety by a suitable factor.

One objection has been raised to Rankine's theory, to which the Author would like to draw attention, for he thinks it is altogether without justification. It has been said that, because in one cutting half-a-dozen natural slopes can be seen, the theory cannot apply. This is no reason for rejecting the theory, for it only means that in that particular place there are different classes of earth, and, naturally, the same section of wall could not suit each. It has also been objected that the theory will not apply to earth inter-

stratified with beds of sand or gypsum that cause the earth to slip in masses. It is obvious that no general theory could deal with such cases. It would be just as reasonable to expect a formula that would give the strength of cast-iron girders containing an unknown number of blow-holes.

In order to compare the results of his experiments with actual practice the Author has collected examples of a number of structures founded on yielding subsoils of which the subsidence has been recorded, and has calculated, from such data as are available, the values of $\frac{1 - \sin \phi}{1 + \sin \phi}$ and ϕ . Unfortunately as in no case is the weight of the subsoil given, he has been obliged to assume and use a weight that he considers reasonable. The calculations are given in Table IX. The values obtained agree reasonably well

TABLE IX.

	Assumed Weight.	Pressure.	Settle- ment.	$\frac{1 - \sin \phi}{1 + \sin \phi}$	ϕ .	Material.
	Lbs. per Cub. Ft.	Lbs. per Sq. Ft.	Ins.		° ' "	
Gable Wall, Hull ¹	115	1,624	3	0·133	50 1	{ Clay 4 feet thick over silty clay.
Beverley Road Baths, Hull ¹	115	2,184	5	0·1478	47 57	{ Clay 4 feet thick over silty clay.
Anlaby Road, Gable Wall, Hull ¹	115	1,444	3	0·1626	45 47	{ Clay 1 foot thick over very soft silt.
Monodnock Building, Chicago ²	110	3,750	5	0·110	53 8	{ Quicksand over clay.
Blackfriars Bridge, London ²	120	17,920	3	0·041	67 5	London clay.
Masonic Temple, Chicago ²	100	3,200	14½	0·1947	43 2	{ Hard clay 6 to 8 feet over soft clay 90 feet to rock.
Walney Bridge ²	110	13,000	½	0·0188	74 23	..
Church Tower, New York ²	120	10,340	⅓	0·011	77 59	{ Stiff gravelly clay.
Church Tower, New York ²	120	15,240	2⅓	0·049	65 11	{ Stiff gravelly clay.

¹ Communicated to the Author by Mr. F. W. Bricknell, M. Inst. C.E., City Engineer, Hull.

² Buckley's "Irrigation Handbook," p. 349 *et seq.*

with those derived from the experiments, but the values of ϕ have no resemblance to the angles of repose given in text-books. For instance for "quicksand" the angle is $53^{\circ} 8'$, while the angle of repose usually assigned to it would be between 20° and 30° .

There is one conclusion of Rankine's that the experiments on saturated sand appear to negative. He says, "The cleanest sand, however, may be made completely unstable and reduced to the state of a 'quicksand' if it is contained in a basin of water-holding materials, so that the water mixed amongst its particles cannot be drained off."¹

The completely saturated sand, in experiments Nos. 33 to 37, carried nearly 7 tons per square foot with a penetration of 4.9 to 5.8 inches, so that it certainly was very far from having reached a state of complete instability.

The behaviour of clay under increasing pressures is to the Author a complete mystery, and he can offer no physical explanation as to why the settlement should vary as the square of the pressure, but must leave it to the physicists. If true, the law must be capable of some physical explanation, and no doubt investigations on different classes of clays would help to elucidate the matter. If the experiments are accepted as correct they upset all theories of earth-pressure as applied to clay, for so far as the Author understands them they all assume that the angle of internal friction is the same as the angle of repose,² and is independent of the pressure. The subject seems worthy of further investigation, but it could hardly be undertaken by a private individual, for the work is tedious, each experiment taking 24 to 48 hours, and if the investigation is to be properly carried out, physical and chemical analyses of the clays will be required, which could only be made in a well-equipped physical laboratory, and should be made by a trained physicist.

In connection with the earth-slides experienced at the Panama Canal, it has been suggested that in clay cuttings there is a critical depth below which the sides will not stand, and the Author's experiments on the softer clays clearly show that for these this must be the case. Where the value of ϕ is independent of the pressure the depth of the cutting cannot affect the stability, and the slope will stand at the angle of repose to any height; but where the angle of repose decreases with the pressure, it is evident that eventually a depth and pressure will be reached beyond which the sides of the cutting

¹ "Civil Engineering" (1885), p. 317.

² In Mr. Bell's theory his angle α is less than the angle of repose, but has a constant value, independent of the pressure.—P. M. C.

will not stand. Experiment C on mud, Table IX, shows this falling value of ϕ very clearly, for at the surface its calculated value at a pressure of 0·25 ton per square foot is $17^{\circ} 15'$, which gradually falls to $2^{\circ} 10'$ at a pressure of 1·25 ton per square foot, when the material was little better than a liquid.

In conclusion the Author would like to express his thanks to Messrs. Coode, Matthews, Fitzmaurice and Wilson, M.M. Inst. C.E., for permission to use the experiments cited in the Paper, and also to Dr. A. W. Brightmore and Dr. Herbert Lapworth, M.M. Inst. C.E., for the valuable advice they have given him while carrying out the experiments.

The Paper is accompanied by fourteen drawings from which the Figures in the text have been prepared, and the following Addenda have been furnished by the Author since it was placed before The Institution.

[ADDENDA.

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[THE INST. C.E. VOL. CCIII.]

ADDENDUM I.

THE following experiments relative to the arching effect in sand were made by Professor S. M. Dixon at the City and Guilds Engineering Laboratory after the Paper was presented. The Author had hoped to describe them when the Paper was under discussion, but was unfortunately unable to do so.

1. A strong box was made 12 inches by 9 inches, and 12 inches high. A hole 3 inches in diameter was made in the bottom which could be closed with a tight-fitting plug. The plug was placed in position and the box was filled with clean, coarse, damp sand, well rammed in. The plug was then removed and the box was put in the testing machine. A thick piece of wood, stiffened with cross

Fig. 15.

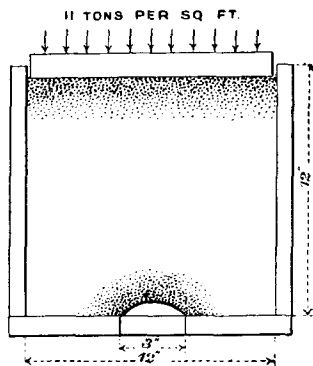
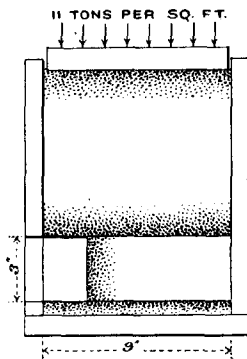


Fig. 16.



pieces, was placed on the surface of the sand (*Fig. 15*) and pressure was applied. At 11.9 tons per square foot the sides of the box showed signs of failure and there was a slight settlement at the surface of the sand, but not a grain of sand came out of the hole in the bottom.

2. The sand was taken out of the box and a 3-inch hole was made in the side. The experiment was then repeated with the side hole open and that in the bottom closed.

Pressure was then applied, and with a load of 11 tons per square foot there was a very slight movement of the exposed vertical face of the sand forward about $\frac{1}{4}$ inch.

3. The pressure was then taken off, and a tunnel of the same

size as the hole was driven right through the sand to the other side of the box as shown in *Fig. 16*.

It was found that the sand could carry 7 tons per square foot, but if this load was exceeded the tunnel gradually caved in.

4. The box was again filled with sand and another tunnel was driven, this time only $2\frac{1}{2}$ inches long, as shown in *Fig. 16*. The load was applied and gradually increased to 10 tons per square foot without anything happening. At 11 tons per square foot the top of the tunnel came away and a new arch formed. The load was again increased and $16\frac{1}{2}$ tons per square foot was carried without further signs of failure. The box, which had been strengthened after the first experiment, then showed signs of bursting and the experiment was stopped.

ADDENDUM II.

Referring to Table VIII, it will be seen that in Experiments A and B for loads of 2 tons per square foot there was no penetration. This is an impossible result unless the clays were absolutely rigid solids, and was a great puzzle to the Author until he discovered that these experiments were made at the bottom of cylinders sunk through the mud tested in Experiment C. In A the depth of mud superincumbent on the clay was 76 feet, and in B 89 feet. Now it is evident that this mud had a certain supporting capacity that would prevent settlement of the clay up to a certain point. It was therefore interesting to see what load 76 feet and 89 feet of the mud would carry, assuming that the mud curve C—*Fig. 11*—continues a parabola down to these depths.

The equation connecting the penetration in inches (d) and pressure in tons per square foot (P) derived from the experiments is

$$d = 188.8 P^2.$$

By substituting in this equation the loads required to give penetration of 76 feet (912 inches) and 89 feet (1,008 inches) in the mud can be found.

In Table XI are shown the depths calculated from this formula compared with the observed depths and the calculated loads for depths of 76 feet and 89 feet.

It will be seen from this Table that the calculated and observed penetrations agree fairly well, except for the first load of 0.25 ton per square foot. The Author is informed that the surface of the mud was very wet and sloppy, which accounts for the large penetration.

TABLE XI.

Tons per Square Foot.	Penetration in Inches.	
	Observed.	Calculated.
0.25	20	11.79
0.50	44	47.2
0.75	114	106.2
1.0	195	188.8
1.25	295	295
2.20	912	..
2.31	1,008	..

It will also be seen that the calculated loads required to produce penetrations of 76 feet and 84 feet are 2.21 and 2.34 tons respectively. It is thus suggested that if the penetration curve remains a parabola to these depths, it should be capable of carrying the above loads, and the underlying clay should be relieved to corresponding amounts. This would account for there being no penetration in Experiments A and B for loads of 2 tons per square foot, just as in Experiment 76—*Fig. 14*—there was practically no penetration until the safe depth was exceeded.

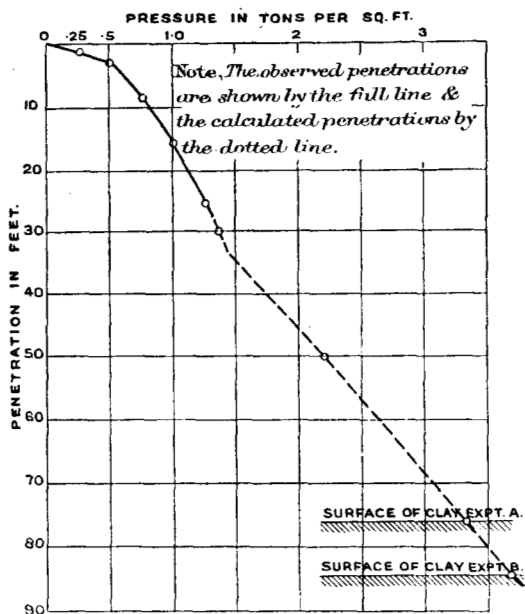
There are, however, good grounds for believing that the parabolic curve does not extend beyond a depth of about 33 feet, and that from this depth on the load-penetration curve is a straight line.

The calculated load to produce a penetration of 33 feet is 1.4 ton per square foot, and this is the weight of 33 cubic feet of the mud. This means that with this load the plunger would be in hydrostatic equilibrium with the displaced mud, or floating. From this point on it is evident that the displacement of the mud should be proportional to the load, and the load-penetration curve will be as shown in *Fig. 17*, the upper part of the curve being a parabola down to the critical depth of 33 feet, when the mud becomes practically a liquid, and from this depth on a straight line, showing flotation equilibrium.

Inspection of the curve shows that for the added loads between $\frac{1}{2}$ ton and $1\frac{1}{4}$ ton per square foot the penetrations were actually greater than they would have been had the plunger been floating in a liquid of the same specific gravity as the mud. This is an unexpected conclusion, but it is the result of actual observation.

If the penetration line is as shown on *Fig. 17*, the bearing capacity of the mud at the surface of the clay in Experiment A should be about 3.55 tons, and at the clay in Experiment B 3.65 tons per square foot. These results would also account for

Fig. 17.



there being no observed penetration in experiments A and B with loads of 2 tons per square foot.

It is unfortunate that this experiment on mud was not carried further, for it would have been most interesting to see what happened when the critical depth of 33 feet was passed.

ADDENDUM III.

The explanation given in the Paper of there being two cases in which clay penetration should follow the straight line law is not strictly correct, though it covers the case which the Author had in mind of a light plunger being loaded with added weights.

The statement should be that the penetration follows the law in two ideal cases:—

First, when the clay is sufficiently stiff to behave as a perfectly rigid solid, for then the penetration for any load, however great, is zero, and the parabola will coincide with the horizontal axis; and secondly, when the clay is so wet that it behaves as a liquid, for then with any load, however small, the penetration is infinite, and the parabola will coincide with the vertical axis.

This latter conclusion does not at first sight seem obvious, but the reasons for it are these. Suppose a plunger is used whose specific gravity is infinitesimally less than the clay. It is evident that this plunger, however long, will float with its upper surface just above the surface of the clay. If now the weight of the plunger be increased so that it is just heavier than the liquid clay, it will sink to the bottom of the liquid whatever the depth may be. That is to say, for an infinitely small load the penetration is infinite.
