



## XI. Short electric waves obtained by valves

E.W.B. Gill M.A. B.Sc. & J.H. Horrell M.A.

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XI. *Short Electric Waves obtained by Valves.* By E. W. B. GILL, M.A., B.Sc., *Fellow of Merton College, Oxford,* and J. H. MORRELL, M.A., *Magdalen College, Oxford* \*.

1. **T**HERE have recently been discovered methods for the generation of continuous oscillations of short wave length (of the order of about a metre) by means of three electrode valves. In January 1920, Barkhausen and Kurz † found that with hard valves—i. e., valves at extremely low pressure, if the filament and the plate were approximately the same potential, or, indeed, if the plate were at a potential considerably lower than the filament, provided that the grid was kept at a high potential with regard to them, continuous oscillations could be maintained in a circuit of the Lecher Wire type connected to the grid and plate. The wave length depended primarily on the grid voltage, but also on the emission from the filament and on the plate voltage.

Whiddington ‡ had previously described another method of getting oscillations of lower frequencies using a soft valve, i. e., a valve containing gas at low but appreciable pressure. He employed more usual circuits for a valve, in that the plate was at a high positive potential with regard to the filament and the grid at a few volts above the filament. In this case longer waves were emitted, and he noticed that if  $V$  was the grid potential and  $\lambda$  the wave-length emitted, then  $\lambda^2 V$  was constant §.

There appear to be other arrangements not hitherto recorded which will also give these waves. With a hard valve and with the grid at a positive potential, oscillations can be obtained if the Lecher Wire system is connected across the filament and grid; the plate may be positive, negative, or at the same potential as the filament, or it may be insulated. Further, the third electrode—the plate—is unnecessary, for oscillations can be sustained by means of a valve consisting of a filament and an anode formed as a spiral of wire concentric with the filament, when these two are connected to the Lecher wires. An intermediate arrangement has been worked successfully in which the wave-length of the diode connected as above is modified by a cylinder concentric with

\* Communicated by Prof. J. S. Townsend, F.R.S.

† *Physikalischer Zeitschrift*, Jan. 1920.

‡ Whiddington, 'Radio Review,' Nov. 1919.

§ For a general account of these experiments see 'Radio Review,' June 1920.

the anode, but placed outside the valve and set at various potentials. The best conditions for these cases are still under investigation.

2. Barkhausen and Kurz were apparently unable to give any explanation of the way in which the oscillations were sustained, while Whiddington assumed that the emission of ions from the filament was discontinuous and occurred in bursts. The authors, on the other hand, do not think that any special assumptions are necessary, and that the ordinary conditions for the maintenance of oscillations by continuous emission will account for all the facts they have observed, provided that the time taken by the electrons to pass between the electrodes is taken into consideration, as this time is of the same order as the period of the short waves.

In the present paper only oscillations of the Barkhausen type are considered in detail, but the theory can be extended to cover all the types, and an account of some experiments on the last type (with a diode) will be published later.

It is worth noting that certain writers give the impression that the seat of the oscillations is in the gas or in the electrons in the valve, and that the Lecher wires connected to the valve serve only to demonstrate their existence\*. It appears from our experiments that the wires or conductors attached to the electrodes are a necessary part of the oscillatory system. Even with the Lecher wires removed, there will always be some circuit composed of the connecting wires to the batteries or even the valve leads up from the sockets, which will have natural periods of a suitable order for short wave oscillation. This fact seems to have been overlooked in some recent determinations of ionizing potentials, where large emissions from a heated filament were used as a source of electrons. Oscillations will take place even when the valves contain a small amount of gas, but in all the experiments described in this paper gas-free valves were used.

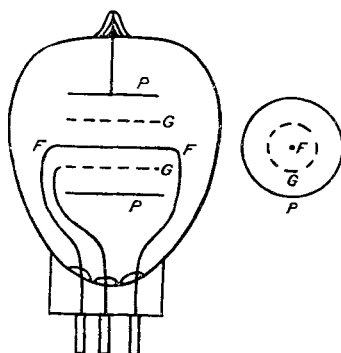
3. It will probably be most convenient first to describe the experiments in detail, and then to set out the theory and apply it to the observed facts.

Various valves were used, but mostly the Marconi M.T.5 valves, which were very kindly given to us by the Marconi Company. These valves consist of a straight filament FF held in the centre of the valve by springy arms. The advantage of the spring is that when the filament is heated and

\* Whiddington's theory is independent of there being any external tuned circuit.

expands, the spring prevents sagging. Surrounding the filament is a cylindrical wire grid, GG, composed of thin wire of square mesh, each square having a side of about 1.5 mm. The lead to the grid goes out at the bottom near the filament leads. A cylindrical plate, PP, surrounds the whole with its lead going out through the top of the bulb. These valves being used for transmitting purposes are very thoroughly "glowed out" and pumped to a very high vacuum. The filament emission is very high when heated with 6 volts direct, and for the low emissions that were generally used it was very constant. As the plate lead passes through the top of the bulb, instead of through the bottom and the sealed

Fig. 1.



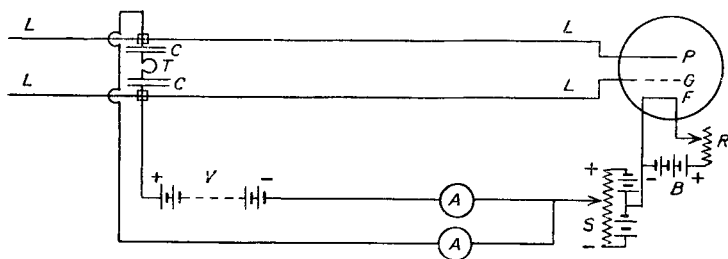
socket, very high insulation is obtained, and, if a strip of tinfoil connected to earth is placed round the outside of the glass, very small anode currents may be measured by an electrometer without any disturbance due to leakage.

It is not necessary for ordinary wireless purposes that the valves should be constructed with the grid and plates either accurately circular in section or accurately centred with regard to the filament; but for the purpose of calculation a symmetrical system of electrodes is necessary and the M.T.5 valve used in most of the experiments was specially selected. All the numerical results to be quoted were obtained from this valve. There is no difficulty in getting the short-wave oscillations with many types of hard valve, the French type produces them quite easily, but the chief reason for selecting the Marconi M.T.5 type was that the electrostatic field between the square-mesh grid and the plate approximates

much more closely to the calculable field between two co-axial cylinders than does the field in the French type, where the grid is a spiral coil of fine wire. The diameter of grid used was 1 cm. and that of the plate was 2.5 cm. to an accuracy of about 5 per cent.

4. The preliminary experiments were made with the apparatus arranged as in fig. 2. The valve is shown diagrammatically: F is the filament, G the grid, P the plate, LL the Lecher wires, which were of copper wire each about 850 cm. long and spaced 5 cm. apart. They were suspended about 200 cm. above the floor from insulators secured to the walls at each end, and from one end were leads about 70 cm. long to the grid and plate respectively. The bridge consisted of two equal condensers, C, C, joined through the heater-coil of a Paul thermo-junction, T. The outer plates were fitted

Fig. 2.



with contacts to slide along the Lecher wires. The capacity of these condensers is unimportant, provided it is large compared with the capacity of the valve. In practice, the capacities were of the order of 1 milli-microfarad. The terminals of the thermo-junction were connected to a galvanometer by two long leads, which are not shown. The sliding contacts were also connected to the negative side of the filament-heating battery B, that on the grid-wire through a high-tension battery V, and that on the plate-wire through a potentiometer S, which could raise the potential of the plate  $\pm 6$  volts above the negative end of the filament.

Two sensitive milliammeters, A, A, gave the steady currents through the valve to the grid and filament respectively. A rheostat, R, controlled the filament-heating. In all cases potentials are measured with regard to the negative end of the filament.

With this arrangement the electrons set free at the

filament move outwards under the positive voltage,  $V$ , of the grid, and a certain number go direct to the grid and are collected there, the remainder pass through the grid, and, if the potential of the plate is just less than that of the filament, they return to, and are finally collected on the grid. If, on the other hand, the plate potential is a little above that of the filament, a certain proportion of those getting through the grid reach the plate. If the plate potentiometer is now adjusted till the plate current is just zero, and the bridge is moved along the wires, it will be found that with the bridge in certain regions a plate current appears. It was the appearance of this plate current which led Barkhausen to the discovery of the short waves. With the present apparatus these oscillations are also made apparent by the deflexion of the galvanometer attached to the thermo-junction. The positions of the bridge at which the galvanometer gave a maximum deflexion were fairly sharply defined, and did not always coincide with the positions for maximum plate current.

It is not necessary for the plate potential to be so adjusted that the plate current is just zero when oscillations are not occurring. The plate may be set at a considerable negative potential, or the plate voltage may be positive. It was found that for a given grid potential there is a certain plate potential at which the oscillating current through the thermo-junction is a maximum. Also as the potential of the plate was increased, for plate potentials only slightly positive, if oscillations commence the plate current increases; at a certain plate potential no change is noticed in the plate current; and at higher potentials the plate current decreases. For the M.T.5 valve this critical potential was about +2 volts, when the voltage drop down the filament due to the heating current was about 4 volts.

In the first experiments with this apparatus the position of the bridge was varied and the current in the thermo-couple observed when the grid voltage  $V$ , the heating current, and the plate potential were all kept constant.

The oscillating circuit consists of a condenser formed by the plate and grid of the valve, the distributed inductance and capacity of the Lecher wires up to the bridge, and the capacities  $C$ ,  $C$  in series with the wires and with the short resistance of the thermal heater which connects them. Hence, if there is an optimum wave-length  $\lambda$  corresponding to the grid voltage  $V$ , and if, starting near the valve, the bridge is pushed along the wires, maximum amplitude of oscillation

should occur when the above circuit is tuned to  $\lambda$ ,  $2\lambda$ ,  $3\lambda$ , etc., these positions being indicated by the deflexions of the galvanometer connected to the thermo-junction. Moreover, the distances measured along the wires between successive positions of maximum oscillations should be equal to  $\frac{\lambda}{2}$ , and

all therefore should be equal. It was soon found that this simplicity was not attained, in certain cases equi-spaced positions were found, but in the majority of cases there were at least two sets of positions forming two series of equal spaces, which, as the spacing distance of the two sets was different, appeared to indicate two optimum wave-lengths.

These effects are due to the different modes of oscillation of the system, and, according to the theory which we give below, a grid voltage  $V$  will, under suitable conditions, sustain oscillations of short wave-length between certain limits. Any mode of oscillation corresponding to a wave-length between these limits will be maintained. It was therefore desirable to arrange the apparatus so as to avoid these complications.

5. The most obvious improvement was to give up the idea of finding the wave-lengths by moving the bridge, and to put the bridge and its leads at the far end of the parallel wires joined to the valve, and to measure the wave-lengths of the oscillations by means of a loosely coupled secondary circuit. The system of wires connected to the valve is thus fixed. A second pair of long Lecher wires were set up with a loop joining one end, and this loop was brought near the valve circuit. When the secondary is in tune with an oscillation in the primary the current in the primary is reduced. The deflexion of the galvanometer connected to the thermo-junction in the primary circuit may be reduced by 50 per cent. when the bridge in the secondary circuit is in the tuned position, and a movement of 0.5 cm. either way will restore the deflexion to its original value. The distances between the successive positions of the bridge on the secondary circuit, for which the deflexions of the galvanometer attached to the primary circuit are a minimum, are the same, and are equal to half the wave-length of the oscillation in the primary circuit. All the wave-lengths quoted were measured on this form of wave-meter and may be taken as accurate to 0.5 per cent. \*

With the condenser bridge and thermo-couple at the far end of the Lecher wires the filament was heated to give an emission of a few milliamperes (this is low heating for an

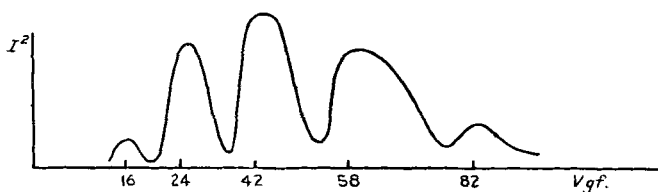
\* Townsend and Morrell, *Phil. Mag.* Aug. 1921, pp. 266-268.

M.T.5 valve) and the grid voltage was raised by two volts at a time by means of batteries of small accumulators from 16 volts to 120 volts, while the plate was kept about 2 volts positive, as this gave large deflexions. The corresponding galvanometer deflexions are shown in fig. 3.

The deflexions are plotted against grid volts; as a thermal detector was being used, the deflexions are proportional to the mean square of the oscillating current.

The curve shows that oscillations are occurring over nearly the whole range, but that there are maxima for certain voltages—viz., 16, 24, 42, 58, 82, 114, approximately. The wave-lengths measured as above give from 16 to just below 24 volts  $\lambda$  586 cm., from 24 to 40 volts  $\lambda$  451, and so on, the wave-lengths for successive portions of the curve being 366, 307, 262, 233. These correspond to the free oscillations of

Fig. 3.



the system, the wires of which were 850 cm. long with leads to the valve about 70 cm. long, with a slight addition for the leads within the valve itself.

The system of wires connected to the valve therefore present a selection of various modes of oscillation with wave-lengths 586, 451, 366, 307, etc., cm., from which the valve chooses the one suitable for the particular voltage  $V$  between the grid and plate—the sharp rises just before the various maxima showing that the system oscillates on the longer wave-lengths by preference. For each particular wave-length there is a certain grid voltage which gives the strongest oscillations when the heating current in the filament and the plate voltage is constant; but the heating current and the potential of the plate relative to the filament both affect the optimum voltage for a given wave-length. Increased emission has the same effect, but this effect depends on the degree of saturation of the emission current.

In the preceding experiments the wave-lengths of the oscillations were measured with a constant heating current



in the filament, but the current from the filament varied with the grid voltage. For the lower voltages all the electrons leaving the filament do not reach the grid space, some returning to the filament. For theoretical reasons it is more convenient to find the grid voltages which give the maximum amplitudes of oscillation on the various wave-lengths when the heating current is so adjusted that the same current flows from the filament to the grid space for all the voltages, the plate voltage being kept constant as before.

The table below gives a set of experiments done under such conditions with an emission current of 6 milliamperes, and the plate at 1.3 volts positive to the filament. In column 1 are given the wave-lengths  $\lambda$  in cms., in column 2 the grid volts  $V$ , which excite these wave-lengths most strongly, and in column 3 the product  $\lambda^2 V$ :—

$\lambda$ .	$V$ .	$\lambda^2 V$ .
208 cm.	156.5	$68 \times 10^5$
233	122.5	66
262	92.5	64
307	68.5	64.5
366	50.5	67.5
451	36.5	74

All these results, with the exception of the last, agree well with the relation  $\lambda^2 V = \text{const.}$

It is not difficult to see why this agreement should be less exact as  $V$  decreases. The electrons concerned are not all moving under similar conditions. Owing to the voltage drop of the heating current down the filament, the field between filament and grid differs by about 4 volts for electrons starting from the extreme ends of the filament. And when  $V$  becomes comparable to this 4 volts a disturbing factor is introduced.

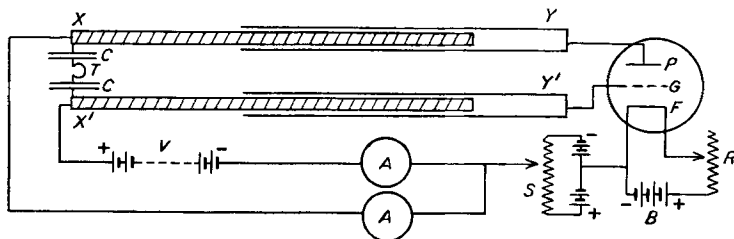
6. These experiments thus give the grid voltages which produce the strongest oscillations on certain definite wave-lengths determined by the particular length of wire used.

To find the range of wave-lengths maintained by a given grid voltage a slightly different apparatus (fig. 4) was used. An adjustable circuit was constructed of two rods, and two telescopic tubes fitted over the rods, so that the effective lengths of the system could be varied by sliding the tubes over the rods.

The condensers and thermo-junctions were attached at the ends  $X$ ,  $X^1$  of the rods, and the ends  $Y$ ,  $Y^1$  of the tubes were connected to the plate and grid of the valve respectively, the other connexions being as before.

For brevity, the adjustable circuit will be referred to as the rods. It is not possible to graduate the rods in wave-lengths as against extension of the arms, as this wave-length

Fig. 4.



depends on the emission and on the plate volts. Thus with a fixed length of the arms and 44 volts between grid and plate:

(1) With plate potential fixed.

Emission 2.2 m.a.	$\lambda = 311$ cm.
6.8	306
9.8	300

(2) With emission constant at 5.2 m.a.

Plate potential 1.2 volts.	$\lambda = 308$ cm.
2.4	314

Hence for a given setting  $\lambda$  decreases as the emission rises, and increases as the plate voltage is increased. This is due to the fact that the plate and grid are not a potential node of the oscillating system, but are a variable distance from it depending on the alternating voltage necessary to sustain the oscillation, and this in turn depends on the emission and plate voltage. It is not, however, necessary to go further into this, as the wave-lengths were always found directly by a secondary circuit as in Paragraph 6, the rods being used as a convenient way of varying continuously the wave-length of the system connected to the valve. With all the other factors fixed, the rods were pulled out a centimetre at a time and the oscillating current and wave-length recorded for each position. In one experiment the emission was 1.5 m.a., the grid potential 44 volts, the plate potential 1.8 volts, and oscillations were maintained from  $\lambda = 320$  cm. to  $\lambda = 451$  cm. with a maximum oscillation about  $\lambda = 323$  cm. It was always found that the maximum oscillation was close to the short-wave end of the range.

The effect of (A) varying the emission current keeping the grid and plate voltages constant, and (B) varying the plate voltage keeping the emission current and potential between grid and plate constant was investigated with this apparatus.

In (A) increased emission broadened the range and decreased the wave-length of maximum oscillation.

For example, with  $V_{gp}=44$  volts and  $V_{pf}=1.2$  volts the wave-length for best oscillation with total emission 7.0 m.a. was 295 cm. With total emission 10.6 m.a. it was 274 cm.

In (B) increase of plate voltage increased the wave-length and also broadened the range. Thus with  $V_{gp}=44$  volts, and total emission 3.8 m.a. with  $V_{pf}=1.2$  volts  $\lambda=298$  cm., and for  $V_{pf}=3.0$  volts  $\lambda=321$  cm. This last observation must not be confused with the case in which the potential of the plate is increased and that of the grid kept constant. In that case also, increase of plate potential increases the length of the strongest wave, as was observed by Barkhausen, who attributed all the difference in wave-length to the alteration in potential difference between plate and grid. This cannot be the whole of the explanation, for, as stated above, similar results may be obtained by raising both plate and grid equally with respect to the filament.

7. A simple theory to account for the maintenance of the oscillations can be worked out by making some simplifying assumptions; but a general theory will not be attempted, partly because the resistance of the oscillating circuits used was unknown and partly because if the assumptions are not made the calculations become extremely complicated.

These assumptions are :—

- A. That the grid and plate can be regarded as forming a parallel plate condenser.
- B. That, of the electrons which leave the filament, a fixed small proportion pass through the grid in a uniform stream, and that each electron on passing through the grid has the same velocity.
- C. That the electrons which return to the grid from the plate side are nearly all collected directly on it, *i. e.*, only a few pass through on the return journey.
- D. That the oscillating potential differences are small compared with the fixed potential differences employed.

It is also assumed that the pressure of the gas inside the valve is so low that the number of collisions between electrons and gas molecules is negligible—this is certainly true for the valves used.

With these assumptions we shall only attempt to show that an oscillation can be maintained of about the right order of wave-length.

The principle involved is the following :—

Suppose the filament and plate are at zero potential and the grid at  $+V$  ; then the electrons from the filament which pass through the grid with a velocity  $v$  due to the potential  $V$  come to rest at the surface of the plate and return to the grid, which they again reach with velocity  $v$ . In the space between the grid and the plate the total work done by the fixed potential  $V$  on the electrons which move in this space is zero, all the work having been done between the filament and grid.

If now superposed on the fixed potentials there is an alternating potential  $V_0 \sin pt$  between grid and plate due to oscillations, the work done by the potential  $V_0 \sin pt$  on the electrons is not necessarily zero. If the work is positive the electrons are abstracting energy from the oscillating system, and the average velocity with which the electrons hit the grid is increased; the oscillations cannot in this case be sustained by the movement of the electrons. But if the work is negative the electrons are giving energy to the oscillating system, and if the rate at which this energy is given is at least equal to the rate of dissipation of energy in the oscillatory circuit by resistance, radiation, or dielectric loss, the valve will maintain the oscillations. The average velocity with which the electrons hit the grid is in this case less than the velocity  $v$  due to the potential  $V$  which they acquire between filament and grid, and hence the energy put into the system from the battery  $V$  is not all used in heating the grid but part is turned into energy of oscillations.

The above argument is not affected if, in consequence of the oscillation, some of the electrons are collected on the plate. In all cases, provided the total work done per oscillation by the alternating field is negative, an oscillation can be sustained if the dissipation of energy in the oscillatory circuit is not large.

8. The particular case in which the filament and the plate are at the same potential when there are no oscillations may be considered first. Let  $V$  be the potential above the plate of the grid and  $d$  the distance between them.

When there are no oscillations the electron passes the grid with velocity  $v \left[ = \sqrt{\frac{2e}{m} V} \right]$  and is then subject to a constant retardation  $f$ , which brings it to rest just at the plate. If  $T$

is the time the electron takes to pass from grid to plate,  $v=fT$ . A further interval of time  $T$  brings the electron back to the grid with velocity  $v$ .

Assume now that superposed on the fixed potentials is an alternating potential  $V_0 \sin pt$  between plate and grid; the electric force due to this in the space between plate and grid is  $\frac{V_0}{d} \sin pt$ , and if  $-e$  is the charge on an electron the corresponding force on it is  $\frac{eV_0}{d} \sin pt$  towards the plate.

Since  $V_0$  is taken to be very small compared with  $V$  the motion of the electron may to a first approximation be taken as determined solely by  $V$ , i. e., its time across is  $T$  and retardation  $f$ .

The work done by  $V_0 \sin pt$  depends upon the time  $t_0$  at which the electron passes the grid, and for a particular value of  $t_0$  the work is equal to  $\int_0^d \frac{eV_0}{d} \sin pt \, dx$ . The axis of  $x$  being perpendicular to plate and grid and  $x=0$  being on the grid.

But the velocity at time  $t$  is

$$\frac{dx}{dt} = v - f(t - t_0) = fT - f(t - t_0),$$

and the above work reduces to

$$\int_{t_0}^{t_0+T} f e \frac{V_0}{d} (T + t_0 - t) \sin pt \, dt,$$

which finally gives :

Work on electron going from grid to plate =

$$\frac{2eV_0}{T^2} \left( \frac{T \cos pt_0}{p} + \frac{\sin pt_0 - \sin p(T + t_0)}{p^2} \right); \quad . \quad . \quad (1)$$

similarly, the work done on the same electron as it returns from plate to grid comes out as

$$\frac{2eV_0}{T^2} \left( \frac{T \cos p(T + t_0)}{p} + \frac{\sin p(T + t_0) - \sin pt_0}{p^2} \right). \quad . \quad (2)$$

Thus the velocities of the electrons on their arrival at the plate or on their return to the grid depends on  $t_0$ , that is, on the value of  $V_0 \sin pt$  at the instant they pass through the grid. Assuming a constant stream of electrons through the grid, it is easily seen by integrating (1) for values of  $t_0$  between 0 and  $\frac{2\pi}{p}$  that the total work done per period is 0

and similarly for (2). Hence, if all the electrons returned to the grid an oscillation would not be maintained. The possibility of a maintained oscillation depends in this case on the fact that in each oscillation a certain group of the electrons are collected on the plate and the integral of (2) does not in consequence include all the values of  $t_0$  between 0 and  $\frac{2\pi}{p}$  and its value is not therefore zero, but may be negative.

The first step is, therefore, to find which electrons reach the plate. When there are no oscillations the electrons have sufficient energy on passing the grid to just take them to the plate against the potential  $V$ , and if therefore any extra work is done on them they will be collected on the plate, but if the work is negative they will fall short of the plate and return to the grid. Expression (1) shows that all the electrons which pass through the grid at times  $t_0$ , such that  $T \cos pt_0 + \frac{\sin pt_0 - \sin p(T + t_0)}{p^2}$  is positive, will reach the plate, while those for which it is negative just fail to reach the plate and return to the grid.

Of the electrons then which pass the grid half go on to the plate and half return to the grid, the electrons running to the plate for a time equal to  $\pi/p$  (half the periodic time of the oscillation) and then running back to the grid for time  $\pi/p$  and so on. But the total work done by the oscillating potential on the two halves as they go from grid to plate is zero; and therefore the net work done is the work done on the return journey on the half which returns to the grid.

To find therefore if an oscillation whose periodic time is  $2\pi/p$  and amplitude  $V_0$  can be sustained by a grid voltage  $V$  it is necessary first to find the time  $T$  which the electrons take to pass from the grid to the plate under the field due to  $V$  alone, next to find from equation (1) the values of  $t_0$  for those electrons which return to the grid when the system is oscillating, and, finally, by taking the mean value of expression (2) for these values of  $t_0$  and, knowing the emission current, to find the total work done per second by the oscillating potential. If this work is negative and at least equal to the dissipation loss per second, the oscillation will be maintained.

A table of calculated approximate results is given below for various values of the ratio  $T:1/p$ . The second column gives the values of  $pt_0$  for the electrons which return

to the grid in each oscillation, the third gives the total work done per second by the oscillating potential in arbitrary units for a fixed value of  $V_0$  and of emission, and the fourth gives the corresponding wave-lengths for the particular value,  $V=44$ , for which  $T=4.3 \times 10^{-9}$  second (see next paragraph):—

	Values of $p\tau_0$ for electrons which return to grid.	Work.	$V=44$ .
$p = \frac{\pi}{4T}$	$165^\circ$ to $345^\circ$	negligible.	1040 cm.
$p = \frac{\pi}{2T}$	$150^\circ$ „ $330^\circ$	—·47	520 „
$p = \frac{3\pi}{4T}$	$135^\circ$ „ $315^\circ$	—·85	347 „
$p = \frac{\pi}{T}$	$120^\circ$ „ $300^\circ$	—·36	260 „
$p = \frac{2\pi}{T}$	$90^\circ$ „ $180^\circ$	—·32	130 „

In all these cases the work is negative and oscillations can be theoretically sustained, though in practice the dissipation losses in the oscillatory circuits are such that generally only those wave-lengths corresponding to the larger values of the work exist.

For all values of the ratio  $T:1/p$  outside the range of the table the work is small. The larger the work the greater the amplitude of oscillation that will be sustained, and the periodic time  $2\pi/p$  of the oscillation of maximum amplitude for a given value of  $T$  is seen to be in the region of  $\frac{8}{3}T$ . In general, the wave-length corresponding to this will be sustained, and also a certain range of wave-lengths on both sides of it, the limits of the range being determined by the dissipation losses in the oscillatory circuit. The theoretical result is in good agreement with several of the experimental results of paragraph 6.

In the particular case recorded there for  $V=44$  the range of wave-lengths sustained was from 320 to 451 cm., with a maximum amplitude for 323 cm. Increase of emission broadened the range of wave-lengths sustained, which is in accordance with the fact that for a given  $V_0$  the work put into the oscillatory system is proportional to the emission current.

The fact that the wave-length of maximum amplitude of oscillation was near to the short wave-length end of the range also agrees with the calculated fact that the

work done falls off much more rapidly on the short-wave side of the maximum than on the long-wave side.

9. To calculate the time  $T$  an electron from the filament takes to go from the grid to the plate when the grid potential is  $V$  volts above both filament and plate it is not necessary to assume the grid and plate to be parallel, but they may be taken, as they actually are, to be concentric cylinders of radii  $a, b$ .

The retarding force on the electron when it is at distance  $r$  from the axis is  $\frac{k}{r}$ , where  $k = V/\log_e \frac{b}{a}$ .

The equation of motion is therefore

$$m \frac{d^2 r}{dt^2} = -\frac{ek}{r},$$

which gives when integrated twice, remembering that  $\frac{dr}{dt} = 0$ , when  $r = b$ ,

$$T = b \sqrt{\frac{2m}{eV} \log_e \frac{b}{a}} \int_0^{\sqrt{\log_e \frac{b}{a}}} \frac{dx}{e^{-x^2}}.$$

In the actual valve used  $a = .5$  cm.,  $b = 1.25$  cm., and, taking  $\frac{e}{m} = 5.3 \times 10^{17}$  E.S. units and measuring  $V$  in volts,

$$T = \frac{2.87 \times 10^{-8}}{\sqrt{V}} \text{ second,}$$

the accuracy of this being limited by the accuracy to which  $a$  and  $b$  are known and probably from 5 to 10 per cent.

The wave-length for any relation between  $T$  and  $1/p$  can now be at once calculated. If  $p = n\pi/T$ , the time of one oscillation is  $2\pi/p$  or  $2T/n$  and the wave-length in cm. is  $6 \times 10^{10} T/n$ .

The simple theory shows that for the oscillations of maximum amplitude  $pT$  has a certain value about  $3\pi/4$ . But

$T \propto \frac{1}{\sqrt{V}}$  and  $\lambda \propto \frac{1}{p}$ , and hence the connexion between the grid voltage  $V$  and the wave-length  $\lambda$  of maximum oscillation is  $\lambda^2 V = \text{constant}$ .

10. The theory is thus in good general agreement with the experimental results, but there is one fact unaccounted for—that being the variation in the wave-length of the



oscillation of maximum amplitude, for a fixed potential between grid and plate, when either the emission is altered or the plate potential is slightly altered with respect to the filament.

There is also a special case, which is forming the subject of a separate investigation, in which, when the plate is very negative (40 volts or so) with respect to the filament, oscillations can still be produced, but without any current reaching the plate at all. These oscillations are, however, very much weaker and more difficult to produce than those dealt with in our experiments.

The simple theory which depends on the collection on the plate in each oscillation of a group of electrons will obviously not account for this special case.

The explanation of the above considerations is to be looked for in the assumptions made in the simple theory. The first assumption that the grid and plate could be regarded as parallel is not important, as the field between cylinders of the size of the grid and plate used is not far from uniform. (It will be noted that the value of  $T$  was calculated for the valve used by taking the field between cylinders.) The only difference between cylinders and parallel plates on the simple theory would be to make the ratio  $T$  to  $1/p$  for maximum oscillation slightly different. But the second assumption that the electrons pass the grid in a constant stream all having the same velocity requires more careful examination. This velocity is not actually the same for two reasons :

- A. Because there will be alternating potentials between the filament and the grid which set up a velocity distribution at the grid.
- B. Because of the voltage drop of the heating current down the filament.

In the usual methods of producing oscillations by means of valves the alternating potential of (A) is most important, as it controls the whole action of the valve, but in our experiments it is only of secondary importance. The alternating potentials induced between filament and grid are smaller than those between grid and plate, and the major part of the work done by the alternating field on the electrons, which is what determines whether the electrons reach the plate or not, is done between grid and plate, and it is therefore nearly correct to say that all the electrons passing the grid at times  $t_0$ , such that expression (1) is positive, reach the plate.

In the extreme case, however, when the plate is so

negative that the system is unable to oscillate by the method of driving groups of electrons on to the plate, the oscillation is almost certainly due to a velocity distribution at the grid, as this means that the electrons do not pass the grid in a uniform stream, and allows the integral of expression (2) to be finite and not zero, as it normally is when all the electrons return to the grid.

The comparative weakness of the oscillations in this case shows that the electrons have all nearly the same velocity when passing the grid.

The simple theory should therefore be in agreement with the observed facts, as it is when the oscillations are mainly due to the collection by the plate, but as the plate is made more negative with respect to the filament the velocity distribution at the grid becomes more important and the simple theory is less accurate.

The velocity distribution at the grid will also be affected by the emission, as this varies the space charge round the filament—this affecting the time the electrons take to pass from the filament to grid,—and this in turn varies the small effect of the alternating field in this space.

The effect of the voltage drop of about 4 volts down the filament is that, instead of dealing with one stream in the field due to the grid being charged to  $V$  volts, there are a series of streams moving under potentials varying from  $V$  to  $V-4$  ( $V$  being the potential difference between the grid and the negative end of the filament). The number of electrons in the various streams varies from a maximum number corresponding to  $V-2$ , the middle of the filament being the hottest. The emission falls off equally on both sides of this middle point.

In the general case, when the plate is slightly positive with regard to the negative end of the filament when there are no oscillations, some of the streams reach the plate and the remainder approach it closely, but to varying distances.

If oscillations commence some of these latter streams are periodically diverted to the plate, while in the other half oscillations some of the former are diverted off.

Thus all the streams concerned maintain the oscillation as in the simple theory, and unless  $V$  is small the wave-lengths they each maintain best are nearly the same, so that the combined effect differs little from that of a single stream moving under potential  $V$ .

The question of whether the mean plate current rises or falls when oscillations begin depends on whether the average

density of the streams diverted to the plate exceeds or is less than that of those diverted from it.

If the plate is at the same potential as the centre of the filament, no change should therefore occur. If it is above this the current should drop, and if below the current should rise, which is in agreement with the results in paragraph 4.

In conclusion, we should like to express our thanks to Professor Townsend, who has assisted us with much valuable advice and criticism.

*Note on the Determination of Ionizing Potentials.*

In the experiments above described, the electric fields in the spaces between the grid, filament, and plate are similar to those used in experiments on the determination of critical potentials when a small quantity of gas is introduced.

In the latter experiments variations in the plate current are observed as the grid potential is raised, and at certain potentials of the grid abnormal variations in the plate current are observed which are interpreted as indicating certain critical potentials, characteristic of the molecules of the gas.

The experiments which are here described show that abnormal variations in the plate current are to be expected, due to oscillations which may be maintained where large currents of the order of a milliamperes flow from the filament towards the grid when the gas is at a very low pressure. The effect of varying the potential of the grid is clearly shown by the curve of fig. 3.

In all the ordinary methods of wiring the valve to the cells and galvanometers or electrometers, the system seems to be as often in a state of oscillation as not.

Even if the plate is so negative that the oscillations are of the weaker type which do not affect the plate current, the difficulty arises that the oscillations superimpose an unknown potential difference between filament and grid, and the electrons are not moving under the fixed field alone.

It is necessary therefore, in order to obtain reliable values of the critical potential, to take precautions to prevent oscillation, which is best done by using emission currents much less than a milliamperes.