

# NOTE ON A FORMULA ASSOCIATED WITH THE PATH OF A RAY THROUGH A PRISM.

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PROFESSOR SOUTHALL has recently published a trigonometrical formula for the calculation of the angle of incidence of a ray which traverses a prism in a principal section and emerges under a prescribed angle of deviation.<sup>1</sup> With regard to this formula he makes the following comment: "It is possible that it may be arranged in a form more convenient for logarithmic computation." More than seven years ago the present writer published the solution of this problem for the case of a ray traversing a prism obliquely.<sup>2</sup> Accordingly it may be of practical interest to present, in this place, the logarithmic formulæ as simplified for a principal plane.

The angle  $\alpha_0$  is first calculated from the equation

$$\sin \alpha_0 = n \sin \frac{1}{2}\beta,$$

where  $\alpha_0$  and  $\beta$  denote respectively the angle of incidence corresponding to minimum deviation, and the refracting angle of the prism.

An auxiliary angle  $\phi$  is next computed from the equation

$$\tan \phi = \tan c \cdot \sqrt{\left\{ \sin \left[ \frac{1}{2}(\epsilon + \beta) + \alpha_0 \right] \sin \left[ \frac{1}{2}(\epsilon + \beta) - \alpha_0 \right] \right\}},$$

where  $c$  and  $\epsilon$  symbolize respectively the critical angle, and the prescribed angle of deviation. If more convenient,  $\tan c$  may be replaced by

$$1 / \sqrt{(n+1)(n-1)}.$$

The two values of the angle of refraction,  $\alpha'$ , may now be obtained by the aid of the following formula:

$$\tan (\alpha' - \frac{1}{2}\beta) = \pm \sin \phi \cot \frac{1}{2}(\epsilon + \beta).$$

<sup>1</sup>James P. C. Southall: "Note on the path of a ray through a prism in a principal section." *Jour. Opt. Soc.*, 4, p. 283, 1920.

<sup>2</sup>H. S. Uhler: "On the deviation produced by prisms." *Am. Jour. Sci.*, 35, p. 396, 1913.

Finally

$$\sin \alpha = n \sin \alpha'.$$

In the illustrative case, taken by Professor Southall, where  $n = 1.5$ ,  $\beta = 60^\circ$ , and  $\epsilon = 40^\circ$  we find

$$\begin{aligned} \alpha_0 &= 48^\circ 35' 25.4'' \\ \phi &= 7^\circ 56' 28.7'' \\ a' - \frac{1}{2}\beta &= \pm (6^\circ 36' 45.8'') \\ (a')_+ &= 36^\circ 36' 45.8'' \\ (a')_- &= 23^\circ 23' 14.2'' \\ (a)_+ &= 63^\circ 27' 27.7'' \\ (a)_- &= 36^\circ 32' 32.3'' \end{aligned}$$

The numerical values of  $\alpha$  just given confirm those of Professor Southall absolutely.

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Nov. 29, 1920.