

ARE THE DIMENSIONS OF THE PHYSICAL WORLD ABSOLUTE?

SPACE, GEOMETRIC AND ACTUAL.

I.

IT is a fact of daily experience, that a body can change its position in the space which we inhabit, without undergoing any visible alteration of form. Its displacement is apparently a simple change of place, nothing more. Places, accordingly, are regarded by us as all alike, as indifferent; and we infer from this fact that space is everywhere the same, that is, has everywhere the same constitution—the same capacity of receiving bodies. This is the quality which is improperly called its homogeneity, but which we shall call its isogeneity.

It follows from this that we conceive of space as limitless. If it were circumscribed, if, for example, the vault of heaven were its boundary, space would cease to be everywhere the same; once arrived at the furthestmost bounds, the advance of bodies would be impeded, they would flatten themselves against the barriers and necessarily change their form. We must then conceive of space as infinite and boundless.

And yet closer observation will show us that the isogeneity of space is rather theoretical than real. Liquids assume the shape of the vessel into which they are poured; the vapor of water rising in the air, the floating clouds, change their form constantly. Further, science, working with instruments which are becoming daily more accurate, proves ocularly that even solids vary their shapes in this

manner and assume different forms, according to the mediums in which they are. In the same way as a balloon, to use a rough comparison, in the course of its aerial journey, becomes elongated, flattened, puffed out, or folds over upon itself continually.

Thus, solid bodies, by simply having their position changed on the globe, following either a vertical, a meridian, or a parallel, undergo, by virtue of this change of position, numerous constitutional modifications, which are followed by both interior and exterior alterations of form. Inalterability of form is assured only on the supposition that the properties of space are everywhere invariable. This would imply that temperature, pressure, magnetism, gravity, light are always invariable, a fact which is not, and never can be true. But this is equivalent to the declaration that isogenous space is a hypothetical and imaginary space, quite different from actual space, and inaccessible to experiments, since experiments can only be practised in the sphere of actuality.

Such space may be called geometric space. It is distinguished from actual space by this theoretical property, that a body can change its position in it, without changing its form. There may be terrestrial or even local physics, chemistry, and mineralogy, but there is no terrestrial or local geometry. There is but one geometry, which is a universal geometry—universal in the sense that every intelligent being, no matter who or where he is, who has this conception of space will arrive at the same geometrical propositions.*

But this is not all. Not only is geometrical space isogenous, as we first believed real space was, but it is also homogeneous in the true sense of the word; that is to say, it has the capacity of receiving *similar* bodies, or bodies of the same form but different dimensions.

* I cannot, accordingly, accept the view of M. Poincaré (*Revue générale des sciences*, Dec. 15, 1891, p. 774), that hypothetical "beings, whose minds and senses should be formed like ours, but who lacked all previous education, could receive impressions from some properly chosen outside world such as would lead them to frame a totally different geometry from Euclid's, and to localise the phenomena of such a world in a non-Euclidean space, or even in one of four dimensions." This assertion of M. Poincaré's is a strange one, to say the least, and he himself calls it "slightly paradoxical."

In order clearly to explain the difference between isogeneity and homogeneity, we may compare a plane with the surface of a sphere. The plane is homogeneous, the surface of a sphere isogenous. On the sphere as on a plane, a figure, a triangle for example, can be displaced in any way, yet will still remain the same. But while on a plane one can draw similar figures, that is, such as are the exact counterparts of each other and only differ in size, it is impossible to proceed thus on the sphere. Thus, two equilateral spherical triangles, belonging to the same sphere, if not of the same size, are not similar, because the angles of the one are not equal to those of the other; whilst all equilateral plane triangles are similar and have the same angles. Spherical triangles which are similar belong to spheres of different radii. We may also call attention to the fact, that on the plane, straight parallel lines can be drawn, but not on the sphere.

Geometric space appears to us thus as capable of indefinite extension or indefinite contraction. A geometric figure is one that can be enlarged or reduced at will, can be viewed from the large or the small end of a telescope without change of form, or the gain or loss of any of its properties.

II.

But the mathematician works in physical space; the figures which he fancies he traces in his own space, he really traces in actual space. Are we not, then, endeavoring to establish a useless distinction founded on a wrong interpretation of our experiences? Just as we believe by instinct, almost, that actual space is isogenous and that bodies, especially solids, can be displaced in it at will without alteration of form, might bodies also not possibly be enlarged or contracted without varying their form? The question resolves itself into this: are the dimensions of the world in which we live, constant, and, consequently, are they absolute?

We will now proceed to demonstrate that these dimensions are absolute and constant, and that consequently actual and geometric space are different. But before entering upon this question, it will

be best clearly to observe its nature and bearing. To this end let us simplify the problem.

Here is the earth, upon whose surface, men, intelligent beings, live and move. They have measured the length of a meridian, they have taken a quarter of it and divided it into ten million equal parts, which they have named a metre ; from this metre they have derived the hectare, the litre, the kilogramme ; with these they have measured everything, even themselves and have established, for example, the fact that their average height is 1.60 metres.

Near the earth, and resembling it, though of smaller size, is another globe. To fix our ideas, we will suppose this globe is half the size of ours ; that intelligent beings inhabit it ; that they are half as small again as we, though of exactly the same form ; that these too have obtained a metre by measuring their meridian and dividing its quadrant into ten million parts—a metre which compared with ours is, say, only 50 centimetres. From this metre, they have derived their hectare, litre, etc.

We will now suppose that an inhabitant of our globe finds himself suddenly transported to this other one, and that he also experiences a proportional diminution of size ; will he be aware of his change of *habitat* ? If actual space were identical with geometric space, if it were homogeneous, we should be tempted to answer at once no ; because in geometric space the size of a figure is not absolute ; it increases or diminishes according to the size of the unit adopted.*

* This proposition is demonstrated in mechanics : Laplace formulates it thus (*Exposition du système du monde*, Liv. V, Chap. V, *ad finem*) : " The law of inverse attraction according to the square of the distance is that of all emanations proceeding from a centre. It appears to be the law of all forces whose action is felt at appreciable distances, as in the case of electricity and magnetism. Thus, this law, being found applicable to all phenomena, must be regarded, by reason of its simplicity and its generality, as absolute. One of its most remarkable properties is, that if the dimensions of all the bodies in the universe, their distances and their velocities were proportionately increased or diminished, they would still describe exactly the same curves as they now do, so that were the universe reduced to the smallest space imaginable, they would still appear the same to our eyes. Consequently these appearances are independent of the dimensions of space : as by virtue of the law of the proportionality of force to velocity, they are also independent of any motion that may take place in space. The simplicity of the laws of nature, accordingly, per-

The question here put is not wholly one of geometry, nor one of physics. A psychological element has been introduced ; for a comparison has been instituted between two states of consciousness, one present and one past, presupposing memory and a sense of measurement. This sense is neither sight nor touch. Sight enables us to judge only of the relative dimensions of objects, the only things not affected by distance. The inhabitant of the globe we are speaking about, can derive no help from the sense of touch : for his limbs, his hands particularly, will be reduced to a size proportioned to the objects he has to take hold of. Let us pass over taste and smell. Let us also admit that his acuteness of hearing, that is to say, his faculty of judging distances by the loudness or faintness of sounds, will also be adapted to his new abode.

These senses, then, will be of no practical use to him. But there is one of which we must make an exception, unless we mean to regard him as a being deprived of intelligence ; and that is motility, or the sense of motion, the faculty of moving and of feeling that one does move. In a word, he retains the sensation of effort and fatigue. He will experience this each time he displaces any heavy object or even moves his own body.*

Having premised this, let us now go on and suppose the planet Mars to be reduced to half the size of the earth. In supposing this, we do not depart much from the truth, seeing its density is about 0.95 of the earth's, and its radius about 0.517.

We will regard this as exact in all points and assume that Mars has its old and new continents and possesses cities like New York and Paris, on a small scale. Of course, we must give it a sun, as small again as our own and half as far away. We will suppose a Parisian is transported during his sleep to the Paris in Mars, where he finds himself on awakening by all the objects familiar to him, in-

mits us only to observe and know relations." This proposition can be absolutely true, only if psychical phenomena also depend on the law of attraction.

* In all my writings on philosophy, and particularly in my *Psychologie comme science naturelle* and in *Le sommeil et les rêves*, I have emphasised the importance of this sense, which though not muscular, partakes of that nature, and shown how indispensable it is for the operation of the mind.

cluding his wife and children, his furniture and tools, his neighbors, the shops, theatres, and boulevards. He opens his eyes, beholds each thing in its accustomed place, and feels no surprise. But as soon as he has risen, washed and dressed himself, gone down stairs and come up again, everything seems different. The common characteristic of all these acts is the raising and lowering of weights. Thus, the moment he steps out of bed, or descends the stairs, he lowers his body; when he lifts the water-pitcher, picks up his clothes, when he reascends to his apartment, he raises his body. The sensation of effort here comes into play. Let us see what will happen.

III.

The radii of Mars and the earth being as one to two, their volumes are as one to eight; and as their densities are the same, their masses also are in this proportion. It follows thence that the weight of any object on the surface of Mars is one-half what it is on the surface of the earth. With respect to mass alone, it would be one-eighth; but as bodies on the surface of Mars are one-half nearer the centre of mass than bodies on the surface of the earth, and as the force of gravity is inversely as the square of the distance, the weight of bodies on Mars, being the combined result of the mass and the distance, is only one-half as great as it is on the earth.

Our Parisian there will be reduced in height, from, say, 1.60 metres to 80 centimetres. From this it will follow that he will have lost seven-eighths of his mass, and, moreover, as he is living in Mars, where the attraction of gravity is only one-half as great as it is on the earth, his weight will be only one-sixteenth of what it was, so that, if on the earth he weighed 80 kilogrammes, in Mars he will weigh but 5. When on the earth, he was obliged to make a vigorous effort to raise his body to a given height. We will suppose he could jump half as high as himself, say about 0.80 metre by terrestrial computation. If he retain the same muscular energy when on Mars, where his weight is only one-sixteenth of what it was, he could jump, with the same effort, sixteen times as high in terrestrial measure. But it is reasonable to suppose that his energy has suffered the

same diminution as his mass, consequently he will only be able to jump twice as high, that is, 1.60 metres in terrestrial measure, or 3.20 metres in that of Mars.

On the earth, with a given effort, his jump was equal to one-half of his height; on Mars, with the same effort, it will be twice as great,—that is, proportionately, four times greater. When our Parisian gets out of bed, when he descends the stairs, and, more particularly, when he reascends them, he will feel four times lighter. He will go upstairs four steps at a time. His water-pitcher, his clothes, will seem four times lighter. In reality, in proportion to his strength, these objects are only half as heavy as they were, but as he only lifts them half the distance to obtain the same result, his efforts are additionally lessened in the same ratio. If it took him one hour to make the ascent of the Eiffel Tower, it will only take him a quarter of an hour to accomplish the same journey in the Martian Paris. Probably he will not attribute this peculiarity to its real cause, but he will at least be aware of it.

It is hardly worth while to consider the contrary effect upon an inhabitant of Mars who would be transported to our earth. To him, everything, including his own body, will appear four times heavier; the steps by which he ascends to his room, three times higher. If he wishes to make the ascent of the Eiffel Tower, before he has gone a quarter of the distance, he will be out of breath, and will probably wonder what can be the cause.

We have taken for granted that the density of Mars was the same as that of the earth. This is the most rational supposition. But let us go still farther. Let us suppose Mars to be the earth in a condensed state and that, in consequence of this, its mass is exactly the same.* The force of gravity on its surface would then be four times greater than that on the earth, the distance from its centre being only half as great. Our Parisian would then weigh four times as much on Mars as he did on the earth. True, he will have

* Such a supposition is incompatible with Newton's theorem, just mentioned in the words of Laplace. That theorem assumes that in the reduced scale, the density remains the same at every homologous point. I only touch this hypothesis to forestall any objection that may possibly arise in the mind of the reader.

retained the same mass, and, consequently, the same muscular strength. But the height of his jump will be none the less diminished in the proportion of four to one, that is to say, he will only jump 20 centimetres (terrestrial measure), equal to 40 centimetres (Martian measure). And all objects will appear to him twice as heavy. In reality, they will be four times as heavy; only, as by reason of his height, he need not lift them more than half as high, he gains the difference.

There is only one way to dupe our man, and that is to suppose that as soon as he is transported to Mars (the Mars of our first supposition*), instead of shrinking to half his former size, his former size is doubled. Then, his volume and mass will be increased in the proportion of 8 to 1, as will be his strength. On the earth, he will continue to jump to the height of 80 centimetres, one-fourth of his new height, while on Mars, where the force of gravity is one-half, he will jump twice as high, that is, 1·60 metres terrestrial measure, or 3·20 metres Martian measure, which is just half as high as his increased stature. Thus he will be cognisant of no change.

But, on the other hand, we see that Mars will have ceased to be a geometrically reduced counterpart of the earth. The houses of this Martian Paris, too, will have to be twice as high, and deep, and broad; and this imaginary city will occupy four times the actual space covered by the original one, and sixteen times as much, if we remember the relative measures of Mars. In a word, the new supposition is a direct contradiction of the conditions of our problem.

IV.

Nevertheless, nature shows us tigers and cats, crocodiles and lizards, pythons and eels, rats and mice, lobsters and crabs. Moreover, we have been able to breed large and small dogs, large and small hens. Many living creatures, fish, for instance, grow all their lives and yet retain the same form. We have ourselves invented the art of drawing, which is based upon the principle of similitude. Thus, writers of fairy stories and humorous authors, like Swift, have fami-

* Radius, one-half; density, unity; weight at surface, one-half.

liarised us with the idea that there may exist cities of dwarfs and of giants, the exact counterparts of ourselves, though of different dimensions. If Gulliver, on coming to Lilliput, or Brobdignag, had been increased or diminished in size, so that his stature harmonised with the stature of the inhabitants of these countries he would not have noticed any abnormality in their appearance.

To imagine this, is, nevertheless, wholly incompatible with the known results of science. The cat is not a tiger *in petto*, nor is the Lilliputian a reduced image of a Brobdignagian, nor a tiny crystal of alum the precise copy of a large one. If this were so, we should no longer be in need of atoms, molecules, or cells. Mathematically speaking, the cell, molecule, or atom, are worlds capable of assuming within their limits all kinds of shapes; but from a chemical or physiological point of view they are absolute quantities not susceptible of change.

This consideration refutes at once an objection which here presents itself: namely, that we commit an error in not applying the reduction of dimensions to atoms and their distances, and, consequently, to the molecule, the cell, and the other natural units. It would result from this, such objectors say, that the combustion of a reduced molecule of carbon, that is to say, its precipitation on oxygen, would produce only a reduced living power, reduced, namely, in the proportion of the square, (equivalent in our hypothesis to a reduction of one-fourth,) and that thus under the same volume the Martian would possess a muscular energy only one-fourth of that of an inhabitant of the earth.

But this objection is only specious. It involves (1) an error of fact, and (2) an error of doctrine.

An error of fact: for observation discloses that carbon, hydrogen, and all chemical substances manifest the same properties at the most distant points of space. An error of doctrine: for the bodies which we call carbon, hydrogen, and so forth, are defined by their atomic weights; and a pretended molecule of carbon, composed of four atoms having a volume and a weight eight times less, and placed at distances half as great, would not be a molecule of carbon. In fact in the enunciation of his theorem, Laplace formally excluded

phenomena whose actions were manifested only at inappreciable distances.

Consequently, if there is water on Mars, that water is exactly like the water of the earth, and its molecule has the same form. Consequently, also, on our fictitious Mars, iron is iron, wood is wood; and it must be admitted that their resistance is the same as the iron and wood of the earth.

Geometrical units, properly so called, can be reduced without the reduction affecting other units, such as the second, the atomic weights, densities, cells.

One thing more. The brain of the Martian is, both in volume and weight, one-eighth of that of an inhabitant of the earth; should we say, therefore, that the ideas of the people of Mars are only one-eighth of the ideas of the people of the earth, and that their judgments and conclusions possess only one-eighth the validity of ours.

But I must abandon these transcendental observations, to regain the *terra firma* of our argument. Let us see whether our inhabitant of Mars, who is like us, could have houses like ours, that is to say, having the same proportions in all its parts. We will proceed by reducing the problem to its simplest form: A board resting on two supports capable of sustaining the weight of a man.

Let P be the weight, l , b , and h the length, width, and thickness of the board, and R the resistance of the material employed. According to a well-known formula we shall have:

$$\frac{Pl}{4} = \frac{Rbh^2}{6}, \text{ or } P = \frac{2}{3}R \frac{bh^2}{l}.$$

On Mars l , b , and h become $l/2$, $b/2$, and $h/2$, so that this structure, reduced one-half, will be capable of supporting a weight

$$P = \frac{2}{3}R \frac{bh^2}{4l},$$

that is to say, a weight four times less than P . But, as we have already seen, the weight of a man on Mars is $P/16$; hence, this structure will be four times as strong as it need be. We should, accordingly, have to employ materials of four times less resistance, or push the supports four times further apart, or let the board be four times lighter or else half as thick. In a word, the structure re-

quired on Mars must be four times lighter than that which pure geometry would call for.

Further back we noticed that in Mars the steps ought to be relatively four times higher than they are on the earth. Besides, their burglars and thieves must be so much more agile than ours. Their fences, accordingly, must be four times higher, and their windows barricaded up to the second and third stories.

Let us now see how a workman of Mars proceeds to put up a scaffolding. He has a hammer, a nail, and a plank. If our reasoning is to be governed by geometry, the volume of his hammer must be eight times less than one of ours, while its weight will be sixteen times less; and to sum up, since he only swings it half as far as a workman here does, the energy of its action must also be thirty-two times less. What can he do with such a tiny tool? What must be the resistance offered by the timber? What the force of penetration of the nails, lessened as they are in size in the ratio of 4 to 1? What kind of rivets will they use?

These few observations clearly show us that the inhabitants of Mars do not resemble us in all respects, and also that their industry is not a miniature of ours. In their Stone Age they could have made no possible use of flints, which were one thirty-second as powerful as those which our ancestors used, unless indeed everything else, tools and material, were proportionately weaker. But we have departed widely in this from the geometrical condition of similarity.

Suppose they have to build a pyramid, erect a cathedral or any other edifice; they will work four times as fast as we do; the materials which they use being sixteen times lighter and easier to move and it only being necessary to lift them one-half the distance. This reduces the work to be done in the proportion of 32 to 1, while the strength is only reduced in the ratio of 8 to 1.

v.

Let us now consider another aspect of the question. We have seen that the Martian measures are reduced models of our own. The reductions vary, according as one, two, three, or four factors

are used, or as a mathematician would say, according as they have one, two, three, or four dimensions. Thus, as we have already seen, their metre is but one-half of ours ; consequently, their hectare is but one-fourth ; their litre, one-eighth ; and their kilogramme one-sixteenth : the reason of this last being, that a litre of distilled water on Mars, besides being one-eighth of the volume of ours is attracted only one-half as much.

At first glance there would seem to be in this nothing that would affect a Martian's life. But that is not the case, as we shall see.

First let us take linear and square measures. If an inhabitant of the earth requires six square metres of cloth to make a complete costume, it will take the same amount for the Martian, since the surface of the body he must cover is reduced in the same ratio as the square metre. But, insomuch as his sun does not emit any more heat than ours, the warmth and consequently the thickness of the garments he wears, must be *at least* equal to that of ours. Consequently, if he makes up the cloth himself, or if his wife knits his shirts and socks, they will doubtless be astonished at the quantity of material used and the amount of work necessary.

We have said, *at least* as warm and thick as what we use on the earth ; for our supposititious inhabitants of Mars will soon become aware that bodies are heated and cooled much more quickly than formerly, and this for a reason which they will not be able to understand, namely, because there has been a greater reduction of volume than diminution of surface. If they put their hands upon a cold object, they will be more rapidly chilled ; if on a warm one they will be more rapidly warmed. Changes of temperature being more sudden, they will have to introduce changes into their manner of living, of cooking, and of clothing themselves. We concede, of course, in order not to complicate the problem and to keep within the strict lines of geometrical reduction, that water continues to boil at $100^{\circ}\text{C}.$, and that the degrees of temperature have not varied.

Consequently, their hectare of land, sown with hemp or flax, will not produce as much linen as before. If they use the same piece of land as a sheep pasture, they will be obliged to reduce the number of animals. They will also have to alter the number and

the quality of their meals. The food provided will have to be such as will repair the waste of the muscles and keep up the heat of the body.

For, on Mars, be it remembered, heat is quickly dissipated, the surface to be cooled being larger in proportion to the mass. On this account the inhabitants will eat more. But there is another reason for this. The surface of their lungs being larger in proportion to the quantity of blood and the combustion more active, one kilogramme of meat or of peas will not satisfy hunger there as it would on the earth.

Their pastures also have become insufficient. If, for example, they have heretofore lived on the product of one cow, the hectare now will be inadequate for her support and, as we have just said, it will be too scanty for the same number of sheep.

This applies to liquids as well. Evaporation is more active. A litre of any fluid on Mars will not allay thirst, as a litre will on the earth.

Thus, all measures, the metre, the hectare, the litre, and the kilogramme, though they will have remained alike, that is similar, from a geometrical point of view, from a practical standpoint will have become very different ; and will be almost always totally inadequate for their similar Martian requirements.

Of luminous and acoustic waves we will not speak ; it would be difficult to reduce them geometrically. We will stop our comparisons here.

VI.

The fact is thus established, that actual and geometric space bear no resemblance to each other ; *that the former is not susceptible of geometrical reduction as is the latter* ; that it is not homogeneous ; and that it does not admit of similar figures undistinguishable by the mind.

Homogeneity remains thus the exclusive and characteristic property of geometric space, although this property is incompatible with reality, and cannot even be conceived as realised.