

The Proper Motions and Parallaxes of the Tenth Magnitude Stars. By *George C. Comstock.*

In Astr. Nachr. No. 4430 Mr. H. E. Lau presents the results of a discussion of the proper motions derived from a micrometric comparison of 102 faint stars with brighter companions of known proper motion. The material employed consists solely of the observations of Lau and Engelhardt which are separated by an interval of approximately 19 years and, despite the shortness of this interval, the results appear to their author adequate to serve as at least a provisional control upon those of Comstock contained in Publications of the Washburn Observatory, Vol. XII, although the latter are derived from much more extensive data covering an interval from two to three times longer than that employed by Lau. While in the main Comstock's results are confirmed by Lau, they are at two points controverted, viz. a) No dependence of the average amount of the proper motion upon galactic latitude is found to exist in Lau's results. b) The mean parallax of the tenth magnitude stars as found by Lau is less than one-fourth the value assigned by Comstock. As no value of this mean parallax is given in the Vol. XII above cited, it may be presumed that reference is here made to a provisional discussion published by Comstock in Astr. Journ. No. 558.

Lau's discussion of his data is based upon the assumption that there exists between his observations and those of Engelhardt a systematic difference (Auffassungsfehler) whose amount may be determined and eliminated from the observations through a process in substance equivalent to subtracting from each observed proper motion in right ascension or declination the mean of all the observed proper motions in the corresponding coordinate, e. g., the mean values of his proper motions as directly observed are $-0''.009$ and $-0''.026$ in $\alpha \cos \delta$ and δ respectively and, from a least square solution, Lau finds as the correction for systematic error, $+0''.013$ and $+0''.026$. The latter of these corrections appears to the present writer both wrong in principle and opposed to the present state of belief concerning the sun's motion in space. For if the sun's motion relative to the stars in question has a sensible component parallel with the earth's axis (e. g. if the declination of the apex of the solar motion be not 0°) then the effect of this component must be shown in a systematic drift in declination (toward the south if the apex lies north of the equator) and a marked drift of this kind, which exists in his observed proper motions, is eliminated by Lau through the process above described. From quite a different point of view also, the systematic corrections introduced by Lau seem open to question since if they be multiplied by 19 we obtain as the amount of the systematic difference between the observations of Lau and Engelhardt $0''.25$ and $0''.50$ in $\alpha \cos \delta$ and δ respectively. While »Auffassungsfehler« of this magnitude are perhaps not impossible in a ten or twelve-inch telescope, this amount

seems suspiciously large in view of the fact that the radius of the first dark interference fringe in the diffraction pattern produced by a star in such a telescope is a quantity of the same order of magnitude as these alleged errors. Improbable also seems the form attributed to these errors, a constant difference depending only upon the stars' relation to the pole and quite independent of any psychological or physiological effect depending upon the observer's position with respect to the star or to the zenith.

Despite these a priori objections, it has seemed to me proper to investigate the character of these alleged errors by the following methods: The assumed relative error of perception must be the difference of the absolute errors of the observers and, while we possess no well determined standard with reference to which such errors may be defined, common practice adopts as such a standard the mean result obtained by many observers. I have therefore sought to compare the observations of both Lau and Engelhardt with the nearly simultaneous results obtained by other astronomers, in the belief that any persistent absolute error that could give rise to so large a difference between these observers must make itself evident in such a comparison. For Lau's work, the only available data are found in my own nearly simultaneous observations of 93 out of the 99 stars employed by him. The interval separating his epoch from mine ranges from -8.0 to $+3.1$ years and its average value is -0.88 years. In view of this small mean interval separating the two series of observations no serious error need be anticipated if the relative motions of the stars during this period were ignored and the series treated as strictly simultaneous. I have nevertheless in each case taken into account these relative motions, using as a means for bridging the longer intervals the results of least square solutions in part contained in Publications of the Washburn Observatory, Vol. XII, and in part as yet unpublished, while for the shorter intervals, not exceeding two years, I have used a more summary consideration of the rate of change of position angle and distance as derived from a comparison with Engelhardt. The 93 stars thus compared furnish as the mean systematic difference between the rectangular coordinates derived from simultaneous micrometer observation, taken in the sense Comstock—Lau,

$$\text{in } \alpha \cos \delta + 0''.02 \pm 0''.030, \text{ in } \delta - 0''.07 \pm 0''.025.$$

The probable error of a single difference, C—L, derived from the residuals furnished by the comparison, is respectively $\pm 0''.29$ and $\pm 0''.24$. From corresponding observations of 127 stars I have similarly derived for the systematic differences Comstock—Burnham, the values

$$\text{in } \alpha \cos \delta - 0''.03 \pm 0''.014, \text{ in } \delta - 0''.05 \pm 0''.016$$

the probable error of a single difference being respectively $\pm 0''.16$ and $\pm 0''.18$.

Combining the above expressions we have for the constant difference between Lau and the mean of Burnham and Comstock, $\frac{1}{2}(\beta + C) - L$,

$$\text{in } \alpha \cos \delta + 0''.04 \pm 0''.024, \text{ in } \delta - 0''.04 \pm 0''.020.$$

Without laying too great stress upon the particular numbers thus found we may summarize their import in the statement that within a very few hundredths of a second of arc any constant systematic error affecting Lau's observations is in the mean shared by Burnham and Comstock. In view of the great differences between the telescopes employed by these observers, ranging from an aperture of 10 inches for Lau to 40 inches for Burnham, it is difficult to regard any errors of perception of the given type and common to the three observers as being of serious magnitude, and therefore substantially the whole systematic difference alleged to exist between Lau and Engelhardt must be attributed to the peculiarities of the latter observer. To determine the amount of this error I have compared Engelhardt's observations with the results of the least square solutions given in Publications of the Washburn Observatory, Vol. XII, and with similar solutions as yet unpublished. The mean system of differential coordinates determined by these solutions rests fundamentally upon the observations of W. Struve, Otto Struve, Dembowski, Burnham and Comstock with considerable supplement, however, from Dubiago, Hall, Schiaparelli, and H. Struve. From 47 stars I find for Engelhardt's relation to the system thus defined in the sense, Mean - E.

$$\text{in } \alpha \cos \delta - 0''.18 \pm 0''.029, \text{ in } \delta - 0''.15 \pm 0''.029.$$

The probable error of a single difference is in each coordinate $\pm 0''.20$. In each coordinate more than 75 per cent of the differences are negative quantities and I consider the mean differences above found to be real peculiarities of Engelhardt's observing. They immediately raise the question whether such peculiarities are to be regarded as of common occurrence or as abnormal characteristics of a single astronomer. As a partial response to this query, I have compared with the mean system above defined the observations of all those astronomers for whom it seems feasible to obtain corrections comparable with those above found for Engelhardt and obtain thus the results shown in the following table:

Observer	Mean - Observer	
	in $\alpha \cos \delta$	in δ
Otto Struve	$-0''.04 \pm 0''.01$	$+0''.04 \pm 0''.01$
Dembowski	$+0.02 \pm 0.01$	-0.03 ± 0.01
Dubiago	$+0.05 \pm 0.04$	$+0.01 \pm 0.02$
Hall	-0.05 ± 0.01	-0.02 ± 0.01
Hermann Struve	-0.05 ± 0.02	-0.01 ± 0.02
Schiaparelli	-0.04 ± 0.02	-0.04 ± 0.02
Engelhardt	-0.18 ± 0.03	-0.15 ± 0.03

It may be inferred from the preceding table and from the comparison between Lau, Burnham and Comstock that Engelhardt's peculiarities, of the form with which we are here concerned, are of quite exceptional magnitude, and identifying provisionally the mean of Comstock and Burnham's observations with the mean system above employed for comparison, I obtain for the systematic difference between Lau and Engelhardt, $L - E$

$$\text{in } \alpha \cos \delta - 0''.22 \pm 0''.036, \text{ in } \delta - 0''.11 \pm 0''.035$$

and dividing these quantities by 19 find in lieu of Lau's systematic corrections to the observed proper motions the values,

$$\text{in } \alpha \cos \delta + 0''.0115 \pm 0''.0019, \text{ in } \delta + 0''.0058 \pm 0''.0018.$$

It will be observed that the first of these quantities is in very fair agreement with Lau's value, to which the theoretical objection above raised in connection with the sun's motion does not apply. In declination the case is quite different and the substantial variance between the correction above found and that derived by Lau is presented as evidence that the latter correction is inadmissible.

I adopt as the best values at present available for the systematic corrections to Lau's centennial proper motions, $+1''.2$ and $+0''.6$ in $\alpha \cos \delta$ and δ respectively and in what follows I assume the proper motions printed in Astr. Nachr. No. 4430 to be multiplied by 100 and increased by these quantities. Of the proper motions thus obtained 27 pertain to stars whose motions are determined in Publications of the Washburn Observatory Vol. XII, from data absolutely independent of that employed by Lau. A small systematic difference should exist between the proper motions thus diversely determined since in the one case they are referred to the equinox of 1850 and to Newcomb's proper motions while in the other they are referred to 1900 and to Boss. Ignoring this difference I have compared the proper motions common to Lau and Comstock and find for the mean difference of the centennial motions, in the sense $C - L$

$$\text{in } \alpha \cos \delta - 0''.16 \pm 0''.25, \text{ in } \delta - 0''.11 \pm 0''.22.$$

These differences are insignificant and Lau's results are therefore in general agreement with Comstock's and in particular they confirm the marked southerly drift of the tenth magnitude stars found by Comstock. This implies a higher declination, viz. a greater galactic longitude, for the apex of the solar motion than is furnished by the brighter stars, a result entirely conformable to previous experience and one which indicates the need for a rediscussion of Lau's data, introducing the coordinates of the solar apex as unknown quantities. It is not intended here to enter upon such a discussion beyond what is requisite in connection with the two points designated a) and b), at which Lau's results are alleged to be inconsistent with those found by Comstock.

a) Relation of Proper Motion to the Galaxy. For each of Lau's stars, with exception of the three excluded by him as abnormal, I have derived the total proper motion along a great circle, μ , and have found mean values of these motions within selected zones of galactic latitude as follows:

Limits	Mean β	**	Centennial Mean μ
$-60^\circ, -20^\circ$	-36°	18	4".1
$-20, 0$	-7	16	3.8
$0, +20$	$+10$	28	3.6
$+20, +45$	$+35$	19	3.9
$+45, +90$	$+64$	18	5.6

It is sufficiently evident from inspection of the last column of this table that Lau's results agree with those of Comstock in furnishing a minimum proper motion in the

galaxy instead of showing no dependence upon galactic latitude as affirmed by him.

b) Mean Parallax of the Tenth Magnitude Stars. The small value found by Lau for the mean parallax of the tenth magnitude stars will be in considerable measure increased by restoring to the proper motions in declination the difference between his systematic correction and that found above. It may also be questioned whether Campbell's value of the solar motion can properly be combined with proper motions similar to those of Lau and Comstock if the differences in the position of the solar apex that are found in the several discussions are to be regarded as real quantities. Without attempting a solution of the question thus suggested, I have sought to derive the mean parallax of Lau's stars by a method which, at least partially, avoids them, adopting for this purpose a relation that is supposed, in principle, to be due to Kleiber*), viz.

$$\varrho_0 \pi_0 = k \mu_0 \quad (\text{Equ. 1})$$

where π_0 denotes the mean parallax of a group of stars and ϱ_0 , μ_0 are their mean radial velocity and mean proper motion respectively. When μ_0 is expressed in seconds of arc of a great circle per century, and ϱ_0 is given in kilometers per second the proportionality factor becomes, in seconds of arc,

$$k = \frac{K}{50 \pi T} = 0.0302$$

where T denotes the number of seconds in a sidereal year and K is the number of kilometers in the earth's mean distance from the sun. I adopt as the best available approximation to the mean radial velocity of the stars relative to the sun the value found by Campbell in 1901 for the brighter stars, viz., $\varrho_0 = 34.1$, and for the mean proper motion as determined by Lau, without reference to galactic latitude, $\mu_0 = 4''.15$, and find from these data

$$\pi_0 = 0''.0037.$$

Although not greatly different from the result $\pi_0 = 0''.0045$, derived by Comstock in Astr. Journ. No. 558, these two results should not be compared, since the latter value is the result of a provisional discussion based upon approximately one-third of the material now available. I have therefore discussed this larger body of material by the method above applied to Lau's data, rejecting all stars of magnitude brighter than $8^m.5$, and find as the mean centennial proper motion of 164 stars whose mean magnitude is $10^m.65$ on the Harvard scale, $\mu_0 = 3''.37$, $\pi_0 = 0''.0030$. The mean magnitude of Lau's stars is $9^m.84$ and corresponding to this difference of magnitude the value of π_0 last given should be increased by $0''.0002$ to render it comparable with Lau's value. It hardly need be said that such a correction is purely formal; the quantities themselves can profess to be little more than an index of the order of magnitude of the parallaxes involved and from this point of view their numerical difference is insignificant. The difference between Comstock's provisional result and that here

found is perhaps significant of a systematic difference between the results furnished by an assumed secular motion of the sun based upon spectroscopic data, and the use of radial velocities of stars without reference to the solar motion.

Supplementary to the above discussion it is of some interest to compare the proper motion results of both Lau and Comstock with such values of the mean proper motions as may be otherwise obtained. For this purpose I adopt the results of Auwers, AG Zone $+15^\circ - +25^\circ$, Berlin A (p. 142) and give in the first part of the following table the adjusted values of the product, magnitude \times mean centennial proper motion, as found loc. cit.

m	$m \mu_0$	**
1.0	26".3	
2.0	27.5	
3.0	28.8	
4.0	30.2	
5.0	31.6	
6.0	33.1	
7.0	34.6	
8.0	36.3	
8.4	36.0	30
9.8	34.6	178
11.3	35.0	83

The second part of the table gives similar data from the combined proper motions of Comstock and Lau, including all of Lau's observations in the line for which $m = 9^m.8$. In forming this table all values of μ considered to be abnormally large have been excluded, the criterion adopted for this purpose being that of Auwers', viz. $m \mu > 120''$. In general the values shown in the second part of the table may be regarded as in excellent agreement with those of the first part, i. e. the mean proper motions of the faint stars as found by Comstock and Lau are of the order of magnitude that should be expected by extrapolation from the brighter stars. They do not, however, confirm the progressive increase in the value of the product $m \mu_0$ found by Auwers, but rather imply a constant value at least for the telescopic stars. It seems worth inquiry if the smaller values of $m \mu_0$ found for the brighter stars are not due, at least in part, to the circumstance that the enumeration of their proper motions is complete, every proper motion however small being included in the mean, while for the fainter stars an undue preponderance of large proper motions may be expected since these are more readily detected than are the smaller ones. Such prejudicial selection can hardly exist in the observing lists of Lau and Comstock since the stars to be observed were chosen solely with relation to conditions other than amount of proper motion, the amount of this motion being quite unknown when the stars were placed upon the observing list. However, there are among my own observations a considerable number of cases in which it is impossible to determine whether the resulting motion of a faint star is a proper motion or is orbital with reference to its

*) See Kobold, Fixsternsystem, p. 211. I have been unable to consult Kleiber's original paper, but it is evident that Equ. 1 involves the assumption that a star's parallax, its linear velocity and the angle between the direction of that velocity and the direction from star to earth are quantities independent of each other.

bright companion star. All of these doubtful cases have been omitted from the data employed above and since, for the most part, these doubtful motions are small, it is possible that many, or even all of them, are real proper motions, whose inclusion in the mean would have somewhat reduced the value above found. Similarly the values for the fainter lucid stars may ultimately be reduced by including in the mean the motions of all such stars. I have therefore re-discussed my own data including in the discussion all of these doubtful cases and find thus from 235 stars of the average magnitude $10^m 3$, $m\mu_0 = 35''.8$. The close agreement of this number with those above found tends to show their freedom from any selective effect tending materially to alter the mean amount of the proper motions and I conclude, provisionally, that $m\mu_0 = 36''$ is a fair approximation to the mean value of this product for stars included between the eighth and twelfth magnitudes. The corresponding parallax of the tenth magnitude stars as furnished by Equ. 1 is $\pi_0 = 0''.0032$.

A somewhat remarkable result of the constancy of the product $m\mu$ and the assumed relation above represented by Equ. 1, is that among the telescopic stars in question mean distance is proportional to numerical magnitudes, e. g. the average twelfth magnitude star is only twice as distant as the average star of the sixth magnitude. While this result accords well with the view that appears to be gaining credence among astronomers that the stars of different apparent

Washburn Observatory, Nov. 20, 1910.

magnitudes are more thoroughly intermixed than has been hitherto supposed, it presents a somewhat extreme form of that doctrine which, although indicated by the existing data, will require more evidence in its support.

I cannot conclude this paper without expressing my appreciation of the value of Lau's proper motions. Although based upon the work of only two observers separated by a very moderate time interval, they suffice to furnish a valuable control and confirmation to my own results that involve much more extensive data. The 27 stars for which proper motions have been determined by both Lau and Comstock furnish 54 differences between the results of independent determinations of the motion in a single coordinate. Ignoring the small and doubtful systematic differences, C—L, above found I obtain as the probable magnitude of a single C—L in either coordinate $\pm 1''.25$. The major portion of this probable error must be attributed to Lau's observations on account of the much shorter time interval involved in them, but even if the entire quantity be attributed to the Lau proper motions it still appears that micrometer observations covering a period of only twenty years suffice to determine the proper motions of the tenth magnitude stars with probable errors that in the individual case are less than half of the average value of the motion investigated and that in the mean may be expected to furnish a reliable determination of these motions.

George C. Comstock.

Beobachtungen von Sternschnuppen.

Sternschnuppenbeobachtungen 1910 Mai 17–20.

Bei Gelegenheit des erwarteten Durchganges der Erde durch den Schweif des Halleyschen Kometen haben die Unterzeichneten in den Nächten 1910 Mai 17 bis 20 Sternschnuppenbeobachtungen angestellt. Das Hauptaugenmerk wurde gegen den östlichen Horizont gerichtet. Leider wurden die Beobachtungen sehr durch den Mondschein gestört.

In Prag (Astron. Institut der k. k. böhm. Universität; Beobachter: *Kavdn*) waren auch die lokalen Beobachtungsumstände ungünstig infolge der unmittelbaren Nähe der Stadt. Außerdem lagerten am östlichen Horizonte fortwährend dichte Wolkenmassen. Die Ausbeute war deshalb sehr dürftig.

Am 17. Mai wurde zwischen $12^h 9^m$ M. E. Z. bis $13^h 39^m$ bloß eine Sternschnuppe 3. Größe gesehen.

Am 18. Mai wurde zwischen $13^h 30^m$ bis $14^h 52^m$ folgende drei Sternschnuppen in die Rohrbachschen Karten eingezeichnet:

Nr.	M. E. Z.	Anfang δ	Ende δ	Gr.
1910 Mai 18.				
1	$13^h 34^m 5$	$288^\circ 0' + 29^\circ 4'$	$295^\circ 4' + 31^\circ 5'$	2 ^m
2	$13 37.1$	$313.4 + 45.8$	$313.0 + 41.4$	2.5
3	$14 14.7$	$311.1 + 45.7$	$307.6 + 41.2$	3

Das Ablesen der äquatorialen Koordinaten der Anfangs- und Endpunkte geschah mittels eines passend hergestellten Gradnetzes.

In Zlonín (geogr., der Generalstabskarte entnommene Position: Länge = $32^\circ 10' 26''$ östlich von Ferro, Breite = $+50^\circ 13' 9''$; Beobachter: *Hrase*) waren die Umstände ein wenig günstiger. Trotzdem wurde gerade am 18. Mai die Beobachtung bloß durch Wolkenlücken und zwar erst gegen Morgen möglich. Das Ablesen einer Taschenuhr besorgte auf Anrufen Herr *J. Klejzar*. Zur Bestimmung der Uhrkorrektur diente ein Chronodeik. Bei der Reduktion der Zeitangaben wurde die Längendifferenz Paris-Ferro = 20° angenommen.

Nr.	M. E. Z.	Anfang δ	Ende δ	Gr.
1910 Mai 17.				
1	$12^h 30^m 1$	$81^\circ 5' + 80^\circ 8'$	$49^\circ 2' + 76^\circ 4'$	1 ^m
2	$12 49.5$	$333.7 + 42.6$	$331.9 + 34.8$	1
3	$12 50.0$	$318.6 + 41.6$	$319.2 + 35.5$	3
4	$13 13.7$	$2.8 + 59.4$	$359.5 + 55.6$	2
5	$13 23.1$	$48.5 + 45.2$	$58.0 + 47.9$	3
6	$13 25.5$	$339.1 + 22.3$	$343.4 + 25.2$	2
7	$13 37.1$	$326.8 + 33.8$	$336.4 + 25.6$	2.5
8	$13 50.1$	$347.9 + 26.7$	$352.1 + 27.6$	3
9	$14 7.4$	$307.8 + 64.8$	$311.5 + 71.0$	1
10	$14 29.0$	$313.2 + 30.7$	$315.1 + 31.9$	3
11	$14 34.7$	$307.6 + 29.9$	$306.4 + 32.4$	3
Mai 19.				
12	$13 20.6$	$338.1 + 65.0$	$344.9 + 65.6$	3