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## THE OPTICS OF LIGHT PROJECTION.

BY VAL. H. MACKINNEY.

*Communicated on May 12th, 1908.*

BEFORE dealing with the subject matter of this article I must mention the fact that I am very much indebted to the "Manual of Military Electric Lighting," and to an article on the "Photometry of Projectors," by M. Fèry,<sup>†</sup> an abstract of which appears in Vol. 23, No. 110, of the 'Journal of the Institution of Electrical Engineers,' for enabling me to deal with the subject in so concise a manner. I wish it to be clearly understood that to M. Fèry and to the writer of the said Manual, all credit is due, all that I have done in this article is to collect various matter together, rearrange it, and endeavour to explain points which to me have seemed at first sight rather ambiguous.

In dealing with this subject it is as well to bear in mind first, that the amount of light falling normally upon a screen (illumination) varies inversely as the square of the distance of the screen from the source.

This may be expressed as  $I = B/d^2$ , where B is the total brilliancy of the luminous source for the particular direction (candle-power, not intrinsic brilliancy). (1)

Secondly, that the amount of light falling obliquely upon a surface varies as the cosine of the angle of turning of the surface from the normal position.

We may, therefore, write  $I = B/d^2 \cos. \theta$ , where  $\theta$  is the angle of turning of the surface from the normal position. (2)

Thirdly, that the quantity of light  $\phi$  received

by a surface is given by the following equation, due to Lambert,

$$\phi = B_1 \frac{A_1 A_2}{d^2} \cos \theta_1 \cos \theta_2, \text{ where } B_1 \text{ is the}$$

mean intrinsic brilliancy (mean brilliancy per unit area) of the luminous source for the particular direction,  $A_1$  and  $A_2$  the areas of the luminous source and receiving screen, respectively (plane surfaces are assumed), and  $\theta_1$  and  $\theta_2$  the angles made by the normals to the luminous and receiving surfaces with the line joining the centres of the two surfaces. (3)

It is as well, too, to bear in mind that these formulæ are only correct providing the dimensions of the luminous source are small as compared with the distance  $d$ .

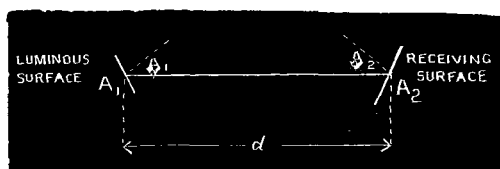


FIG. 1.

In practice we may wish to project light from a luminous source that is in no way under our control—the sun, for instance. Under such circumstances we are practically limited to the use of a plane mirror. A plane mirror mounted for flashlight signalling, and intended in practice to be employed in conjunction with sunlight, is termed a heliograph.

In fig. 2 let  $S$  represent the sun,  $H$  the heliograph, and  $R$  the receiving station. Now, in projecting the sunlight, by means of the heliograph, to the distant receiving station  $R$ , it is of the utmost importance that we should know, first, what the intensity of illumination at the receiving station does, and does not, depend upon; secondly, what the area illuminated at

the receiving station does, and does not, depend upon, and thirdly, what the shape of the area illuminated at the receiving station does, and does not, depend upon.

Let us consider, then, in the first instance, this simple case of the projection of sunlight by the heliograph mirror. We cannot, of course, take into account the absorption of light by the atmosphere, as this is by no means constant; it is as well to bear in mind, too, that in practice the diameter of the heliograph mirror is small in comparison with its distance from the luminous source, even if the luminous source

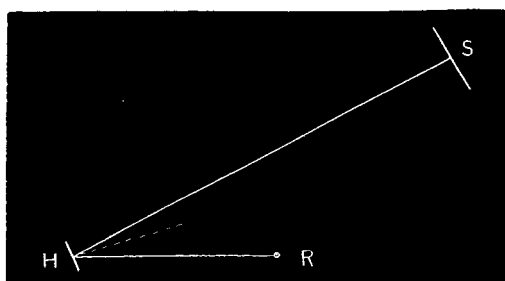


FIG. 2.

is an artificial one. Moreover, the plane of observation, in other words, the receiving station, is always at some considerable distance from the heliograph (signalling station). Assuming such conditions we may summarise our results as follows :—

### **Intensity of Illumination at Receiving Station.**

Varies directly as the mean intrinsic brilliancy of the luminous source.

Varies directly as the area of the plane mirror.

Varies directly as the cosine of the angle of turning of the mirror from its position directly facing the receiving station.

Varies inversely as the square of the distance

of the mirror (heliograph) from the plane of observation (receiving station). Refer remarks made later.

Does not depend upon the total brilliancy of the luminous source, in other words, upon the size or shape of the luminous source.

Does not depend upon the obliquity of the luminous source to the central incident path to the mirror.

Does not depend upon the distance of the luminous source from the mirror (neglecting absorption).

Does not depend upon the shape of the plane mirror.

### **Area Illuminated at Receiving Station.**

Varies directly as the area of the source of light.

Varies directly as the cosine of the angle of turning of the luminous source from the plane perpendicular to the central incident path to the mirror.

Varies directly as the square of the distance from the mirror to the receiving station.

Varies inversely as the square of the distance from the luminous source to the mirror.

Does not depend upon the intrinsic brilliancy of the luminous source.

Does not depend upon the shape of the plane mirror.

Does not depend upon the shape of the luminous source.

Does not depend (for all practical purposes) upon the size of the plane mirror.

Does not depend (for all practical purposes) upon the obliquity of the mirror to its position directly facing the receiving station.

### **Shape of Area Illuminated at Receiving Station.**

Depends upon the size of the source of light, and resembles the foreshortened view as would be seen from behind the mirror. It must, therefore, depend also upon the obliquity of the prin-

cipal face of the source of light to the direction of the central incident path to the mirror.

Depends upon the shape of the source of light.

Depends upon the size of the mirror if the distance from the mirror to the source is not negligible.

Depends upon the shape of the mirror.

The shape of the area illuminated at the receiving station does not depend upon anything else.

The above remarks should be of great interest to sight-testing opticians, for in determining objectively the refractive condition of the eye (retinoscopy and ophthalmoscopy) they are

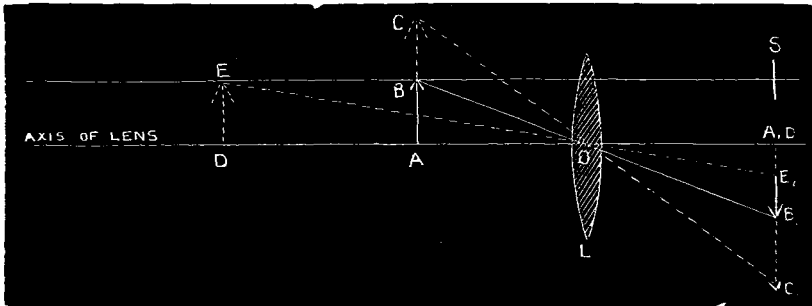


FIG. 3.

working under very similar circumstances, or perhaps I had better say *conditions*.

It has been stated above that the intensity of illumination at the receiving station does not depend upon the size of the luminous source. Now, I have heard it stated that if such was strictly the case it would be impossible for the stars of greater magnitude, having the same mean intrinsic brilliancy as those of lesser magnitude, to appear to us brighter than those of lesser magnitude. But this case is by no means analogous to the one under consideration at the present moment, and for this reason. When we view a luminous point source of light, subtending as it must a very minute angle at the

front nodal point of the eye, we are not even employing the total area of one end-organ of vision (a cone in this case) as the receiving screen for our image. Hence, it is only reasonable to suppose that the nerve-stimuli set up vary directly with the size of the image when the image is smaller than the area of one cone, and, consequently, that the star of greater magnitude appears brighter than the one of lesser magnitude.

From the above figure (3) let us consider this matter more fully. Let  $L$  represent the adjustable crystalline lens of the human eye, and  $S$  the plane of the receiving screen, the retina. Let  $A_1 B_1$  be the image of the luminous object  $A B$  at a distance  $A O$  from the lens. Consider first a luminous object  $A C$  of double the diameter (four times the area) occupying the same position, let the mean intrinsic brilliancy be the same as before. The image will obviously be  $A_1 C_1$ , and its area will be four times the area of the image  $A_1 B_1$ ; hence, the mean intrinsic brilliancy of the image is unaltered, and consequently the nerve-stimuli set in motion for each individual cone is the same in the two cases. Therefore, the intensity of illumination at the receiving station (the retina) does not depend upon the size of the luminous source, providing that the mean intrinsic brilliancy remains unaltered. Secondly, let us consider the luminous object  $A B$  in the position  $E D$ , where  $D O$  equals  $2 A O$ . The image will now be  $D_1 E_1$ , and in area it will be just one quarter that of  $A_1 B_1$ . In the image  $D_1 E_1$  we have a reduction in the mean intrinsic brilliancy owing to the luminous object having been placed further from the lens. If the distance from the lens is doubled, then, from the law of inverse squares, the mean intrinsic brilliancy of the image must be a quarter of its original value, with luminous object in the position  $A B$ . But the area covered by the image is now just one quarter of its original value, therefore, surely, for this

reason the mean intrinsic brilliancy of the image must be four times its previous value. In other words, the two effects exactly counterbalance one another. Therefore, again, we may say that the mean intrinsic brilliancy of the image remains unaltered, and, consequently, the nerve-stimuli set in motion for each individual cone is the same as before. When, however, we are dealing with images whose size enables them to be received well within the area of one cone we must realise that the nerve-stimuli set in motion for the one cone is different to the maximum value, and, consequently, that under such circumstances we cannot say that the intensity of illumination at the receiving station, although still constant, produces the same visual effect.

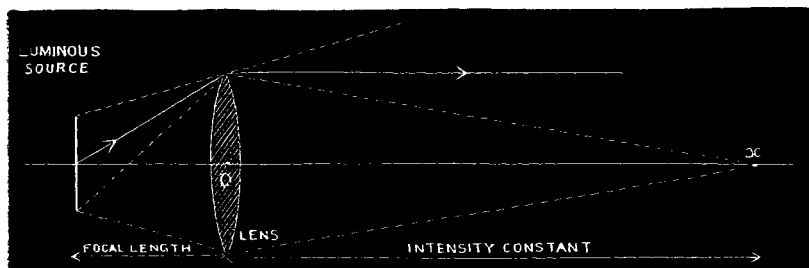


FIG. 4.

In the case of the projection of sunlight, or some other powerful distant luminous source, by the heliograph mirror, it is to be noticed that to an eye at the receiving station the light received by it, although actually diverging from the distant point, might be considered as a parallel beam (pencil). If the mean intrinsic brilliancy of the luminous source is very great and the aperture of the pupil of the eye very small, in comparison with the size of the luminous source, then, neglecting absorption, we might almost say that the beam of light is of constant intensity. The point that it is desired



a better result—the beam passing to the eye-piece, under the same assumptions as above—with a relative strength of 25 per cent. This is not easily realised in the case of a “transparent” reflector covering the whole aperture of the objective, but it can be approximately realised by removing part of the glass disc and by silvering the remaining part. Such an arrangement has the further advantage that the light forming the images does not pass through a thin sheet of glass in its passage up the tube; the presence of such a thin sheet should, theoretically, make very little difference to the resulting images, but in practice such thin sheets are never flat or parallel, since they are merely cut from blown sheets and are not optically worked; consequently, there is always a very decided deterioration of the image when the glass-disc reflector is used. On the other hand, the clear glass reflector is the only means of obtaining strictly normal illumination of our surfaces, since by its means alone can a truly axial beam be passed down the microscope. For certain purposes of observation, therefore, the glass disc remains essential, but it is not convenient, or desirable, for other purposes. A very close approximation to truly axial illumination can be obtained by the use of opaque reflectors, but for this purpose it is essential that the precise position, both vertically and horizontally, of the reflector itself should be readily regulated, as well as its angle to the incoming beam of light. Now, the ordinary form of “vertical illuminator,” as sold with most metallurgical microscopes, does not provide for these adjustments, and this appliance has the further disadvantage of being a detachable fitting, which is interposed between microscope tube and objective—just at that point in the microscope where the greatest rigidity and directness of attachment are required. Both of these objections are completely met by the arrangement of the instrument which I have designed for this work, and examples of which are on the table. I do not propose to describe this instrument in detail here, since a full account of it has already appeared, so that I will merely draw attention to those features which are connected with the question of the interpretation of images. Among these the adjustments of the illuminator are probably the most important, since they enable the operator to alter the lighting from

truly axial to very slightly oblique by the interchange of one form of reflector for another without affecting the focus-setting of the most powerful lenses. This enables the operator to make accurate comparisons between appearances under slightly different lighting which would be impossible with the older forms of illuminator.

In addition, by utilising the wide angular apertures of high-power objectives, it is possible to send oblique beams upon specimens while examining them under immersion objectives, where the introduction of oblique light has hitherto been impossible. An example of oblique illumination under an immersion objective obtained in this way is shown on the table.

A further point of some optical interest arises in connection with these illuminators, and it serves to illustrate the necessity for having a metallurgical microscope provided with a completely rotating stage. When an opaque reflector is used, the closing of half the aperture of the objective, which is covered by the reflector, affects the formation of the image. The resolving power of the objective, in fact, is considerably smaller in that direction in which the aperture is only one-half of its full value. Now, in alloys we have frequently to deal with finely laminated structures, and sometimes the question as to whether a given structure is visibly laminated or is "amorphous" or non-descript in structure affects the conclusions to be arrived at as to the past history and properties of the metal. Now, I have frequently observed that when a specimen showing fine lamination which can just be resolved in the most favourable position is turned round through a right angle, the laminations disappear and an indefinite grey or mottled appearance is seen. This observation shows not only the importance of using lenses of the highest resolving power obtainable, but also the need of examining a specimen in all positions by the aid of a rotating stage. This question of resolving power which I have just touched upon is, I believe, at the bottom of a good deal of the controversy which has raged around certain questions in connection with the micro-structure of steel. Some of the more minute structures which result from rapid cooling are undoubtedly at, or just beyond, the limits of resolution by means of the microscope—and the opinion formed as to the existence or non-

tus, it is usual to employ the crater of an electric arc as the luminous source. The luminous source, therefore, is comparatively small, and, consequently, the point  $x$  in such cases is at a considerable distance from the reflecting or refracting system.

In order, then, to determine, experimentally, the intensity of the beam, we should have to place our photometer considerably beyond the point  $x$  in order to employ the law of inverse squares with accuracy. Anywhere between the reflecting or refracting system, and

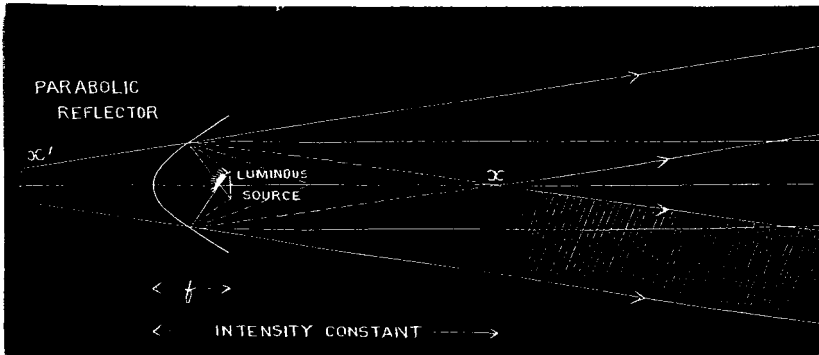


FIG. 7.

the point  $x$ , the illumination on the photometer screen will be constant, for from equation (3) we have (fig. 8),

Quantity of light falling on photometer screen  $P_1$  is given by

$$\phi_1 = B \frac{A_1 P_1}{x_1^2}, \text{ and } [A = \text{Area}]$$

quantity of light falling on photometer screen  $P_2$  is given by,

$$\phi_2 = B \frac{A_2 P_2}{x_2^2}.$$

Now, if  $x_2 = 2x$ , then, owing to increased distance, illumination is one quarter, according to the law of inverse squares, but if the diameter of aperture  $a_2 = 2a_1$  then the area of the aperture  $A_2$ , will equal four times the area of the aperture  $A_1$ . Consequently, the illumination on  $P_2$  will equal the illumination on  $P_1$ , providing  $a_2$  does not exceed half the aperture of the reflecting or refracting system.

In other words, because the photometric screen is small (and is enclosed in a box usually) in comparison with the aperture of the reflecting or refracting system, it will only receive light from a certain definite area of the mirror or lens surface when it occupies any position inside the point  $x$ . The area of the active surface will be proportional to the square of the distance of the photometric screen from the surface

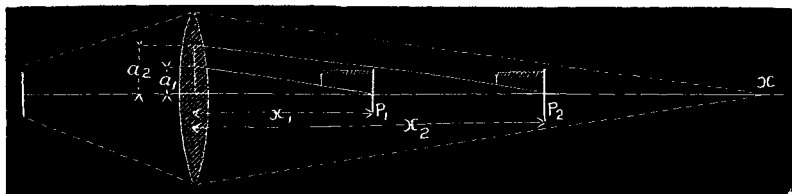


FIG. 8.

(mirror or lens), so that up to the point  $x$  the illumination on the photometer screen will be constant. Some objection might reasonably (at first sight) be raised to the idea of applying the law of inverse squares for light between the mirror or lens surface and the point  $x$ , but the experimental results of M. Fèry definitely show that we can, with accuracy, and, therefore, that the reason why the illumination remains constant up to the point  $x$  is because  $\frac{\text{active surface}}{\text{distance squared}}$  remains constant.

M. Fèry obtained the following results with a lens 7 cm. in diameter and of 78 cm. focal

length. It was sensibly deprived of chromatic and spherical aberration for parallel rays in the direction of the axis. He attributes the constant difference between the experimental and calculated values to reflection from the four faces of the two lenses constituting the achromatic objective.

Distance of Photometer from source of Light.	Illumination observed—Candle-meters.	Illumination calculated—Candle-meters.
78 cm.	3.30	3.70
120 „	3.19	3.70
178 „	3.30	3.70
278 „	3.25	3.70
378 „	3.19	3.70
428 „	3.30	3.70
528 „	1.88	2.76
578 „	1.56	2.24
678 „	1.18	1.57
778 „	0.735	1.15
1,078 „	0.255	0.56
1,578 „	0.111	0.24

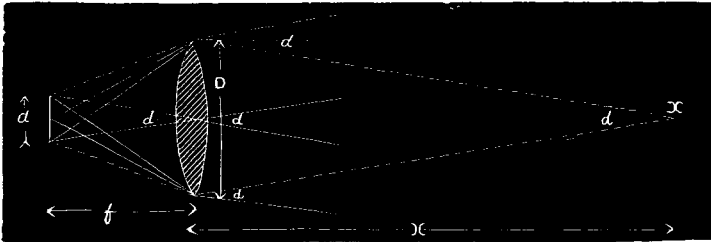


FIG. 9.

M. Fèry employed, as a luminous source, an aperture 1.4 cm. diameter illuminated by a Bengal jet, when making the above observations.

The calculated values were obtained by M. Fèry in the following manner:—

From the above diagram (similar to the one

given by him) we have  $\tan \alpha$  or  $\alpha = d/f = D/x$ .

$$\therefore x = \frac{Df}{d} = \frac{7 \times 78}{1.4} = 39 \text{ meters for the}$$

case in hand.

At the point  $x$  the lens appears uniformly illuminated, and may be considered as a true source of light when viewed along the optic axis. Outside the point  $x$  the illumination varies according to the ordinary law, viz., inversely as the square of the distance. Obviously, then, since the illumination between the lens and the point  $x$  is constant, the multiplying power of the system is  $x^2$ . The reflecting or refracting system may, consequently, be said to pass the light through a distance  $x$ , without any loss in illu-

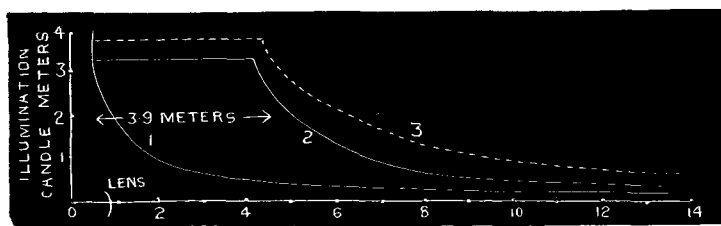


FIG. 10. (THE HORIZONTAL MEASUREMENTS PRESENT DISTANCE FROM SOURCE IN METERS.)

mination per unit area taking place upon the receiving screen.

The illumination per unit area of the lens surface in the above case is given by,

$$I = \frac{d \times B}{f^2}$$

where  $d$  represents the area of the luminous source in square mm., and  $B$  the brilliancy per unit area, or number of candles per square mm., of luminous surface. This, therefore, will give the illumination at the point  $x$  too.

M. Fèry's results are shown graphically in the following diagram, which is reproduced from his article (fig. 10).

The first curve refers to the naked arc, the second to the observed results given above, and the third to the calculated results given above.

M. Fèry refers to a case in which the factor  $x^2$  was wrongly applied.

It was at Chicago, the projector under consideration consisted of a lens of 1 meter diameter and 1.5 meters focal length, illuminated by an arc crater of 20 square mm. area. The value of  $x^2$  in this case would, therefore, be 90,000 ( $x=300$  meters). If the illumination at the point  $x$ , due to the naked arc, is obtained, then the product of this result and the factor ( $x^2$ ) will give the illumination due to the projector at the point  $x$ . At Chicago they obtained their result, 270 million candles, by employing the factor  $x^2$  while obtaining the illumination due to the naked arc at a distance of 1 meter! As M. Fèry points out, if the photometer were placed quite close to the lens, an illumination of 1,333 candle-meters should be obtained with the above conditions.

$$\text{For } I = \frac{K}{d^2} = \frac{20 \times 150}{(1.5)^2} \text{ candle-meters, taking}$$

a value of 150 candles of intensity per square mm. of crater. This illumination will be maintained up to a distance of 300 meters from the lens in this case.