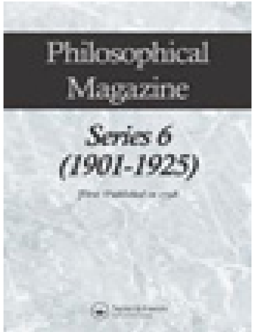

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XLIII. Applications of quaternions to the theory of relativity

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the important factor ($\gamma(u)$) seems to leave no escape. The (superficially) uncoördinated employment of the Lagrange function which proves a salient characteristic of relativity; that is, making a new zero for each frame; as in the elementary case of a lifted body and its weight, would in effect disregard that factor ($\gamma(u)$). But, as we see, to eliminate that factor is detrimental to the full attainment of symmetry. Logarithmic fundamental relations prevent the mere "butt-joint" arrangement that is allowed for vertical intervals and weight. A discontinuity of energy values is avoidable only by some form of agreement that has for corollaries: Invariant transition at the "junction" with a new frame; and then equal activity reckoned for the same time-unit. Relativity contrives to satisfy these conditions however indirectly; at times its perhaps unavowed goal is masked behind an almost opaque veil of four-dimensional mathematics. Yet no just mind would endure cancelling anything of that brilliant achievement of expansion. Once more in physics an inestimable service had its source seemingly in a misapprehended premiss—about Newton's second law and *vis viva*.

The matter is weighty enough, if it resolves a puzzling riddle, to call for immediate publishing in condensed outline of the line of varied attack. A prepared paper discussing the subject less summarily cannot appear for several months.

University of California.
September 12, 1919.

XLIII. *Applications of Quaternions to the Theory of Relativity.*

By H. T. Flint, *M.Sc., Lecturer in Physics, University College, Reading* *.

Introduction.

In this paper it is proposed to express the results of Relativistic Dynamics by means of quaternions. It amounts to expressing the Minkowski four-vector as a quaternion, and bringing about its transformation by a certain operator introduced by Silberstein †. It is shown in the paper, of which the reference is given, that the Lorentz-Einstein transformation is equivalent to

$$q' = Q[q]Q.$$

q' is a quaternion considered in a system of reference S′

* Communicated by Professor W. G. Duffield, D.Sc.
† Phil. Mag. May 1912.

moving with velocity, v, with respect to a system, S, along a direction denoted by the unit vector $\mathbf{v}$.

The vector part of q' is $\mathbf{r}'$, the vector from the origin to a point P′, and the scalar is l' where $l'=it'$, t' is the time in S′ and is measured in units in which the velocity of light is unity. q is similarly $\mathbf{r}+l$, in S.

Q is also a quaternion, and expressed in detail is

$$\frac{1}{\sqrt{2}}\left\{(1+\beta)^{\frac{1}{2}}+(1-\beta)^{\frac{1}{2}}\mathbf{v}\right\}$$

where $\beta=(1-v^2)^{-\frac{1}{2}}$.

It is to be applied in front of and behind the quantity to be transformed and obeys, of course, the rules of multiplication of quaternion analysis.

Expressions like q, which transform in this way, are called physical quaternions, and evidently such quantities, like four-vectors, are capable of expressing the theory of Relativity, in fact, they are just what is required.

We here consider the application of this notation to velocity, force, and momentum.

It will be seen that the well-known results of the Cartesian mode of expression are easily derived, but the results obtained are more general and have no special reference to axes.

An expression for the kinetic energy of a particle, slightly different from that usually accepted, is indicated by the notation. This form has been discussed elsewhere by W. Wilson *.

Application to electric and magnetic forces give quite general transformations, and we again recognize by reference to special directions the Cartesian formulæ resulting. Finally, the problem of the field due to a uniformly moving charge is solved by a very easy application of the general formulæ.

1. *Notation.*

τ will be used in its usual meaning, so that

$$(d\tau)^2=-(dx^2+dy^2+dz^2+dl^2)=(1-u^2)dt^2=\frac{1}{k^2}dt^2 \quad \text{(say)}$$

if
$$u^2=\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2.$$

$d\tau$ is an element of the "proper time" and is invariant in our transformations.

* Proc. Phys. Soc. xxxi. pt. ii. p. 74.

Write $$q=\mathbf{a}+\lambda$$

so that $$\mathbf{a}'+\lambda'=Q(\mathbf{a}+\lambda)Q.$$

It follows that

$$\mathbf{a}'=\lambda(1-\beta^2)^{\frac{1}{2}}\mathbf{v}+\mathbf{a}+\mathbf{v}(1-\beta)S\mathbf{a}\mathbf{v} \quad . \quad . \quad . \quad \text{(i.)}$$

and $$\lambda'=\beta\lambda+(1-\beta^2)^{\frac{1}{2}}S\mathbf{a}\mathbf{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

2. From the quaternion q we pass to

$$w=\frac{d}{d\tau}q.$$

We may describe w as the velocity quaternion. It is evidently a physical quaternion, for $d\tau$ is invariant.

This statement, or what is the same thing,

$$w'=QwQ,$$

contains all that need be said about transformation of velocities. It is, however, interesting to derive the well-known formulæ. We have

$$w'=\frac{d}{d\tau}(\mathbf{r}'+l)=Q\frac{d}{d\tau}(\mathbf{r}+l)Q.$$

Thus by (i.) and (ii.)

$$\frac{d\mathbf{r}'}{d\tau}=\frac{dl}{d\tau}(1-\beta^2)^{\frac{1}{2}}\mathbf{v}+\frac{d}{d\tau}\mathbf{r}+\mathbf{v}(1-\beta)S\frac{d}{d\tau}\mathbf{r}\,.\,\mathbf{v},. \quad \text{(iii.)}$$

and

$$\frac{dl'}{d\tau}=\beta\frac{dl}{d\tau}+(1-\beta^2)^{\frac{1}{2}}S\frac{d\mathbf{r}}{d\tau}\cdot\mathbf{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv.)}$$

From (iv.)

$$\frac{1}{(1-u'^2)^{\frac{1}{2}}}=\frac{\beta}{(1-u^2)^{\frac{1}{2}}}+\frac{v\beta}{(1-u^2)^{\frac{1}{2}}}S\frac{d\mathbf{r}}{dt}\,.\,\mathbf{v}, \quad . \quad \text{(v.)}$$

and if $\mathbf{v}$ is taken along Ox this gives

$$\left(\frac{1-u^2}{1-u'^2}\right)^{\frac{1}{2}}=\beta(1-vu_x). \quad . \quad . \quad . \quad . \quad \text{(vi.)}$$

This is the well-known and important transformation of $(1-u'^2)$.

From (iii.) by making use of (vi.)

$$u'_x=\frac{u_x-v}{1-vu_x}.$$

Similarly, we may derive

$$u_y' = \frac{u_y}{\beta(1-vu_x)} \quad \text{and} \quad u_z' = \frac{u_z}{\beta(1-vu_x)}$$

by taking **v** along Oy and Oz respectively.

3. Let m_0 denote the "rest mass" of a particle, and then

$$\mathrm{M} = m_0 w$$

is also a physical quaternion—the momentum quaternion.

We obtain from § 2

$$\frac{m_0}{(1-u'^2)^{\frac{1}{2}}} = \frac{m_0}{(1-u^2)^{\frac{1}{2}}}\beta(1-vu_x), \quad . \quad . \quad . \quad \text{(vii.)}$$

or writing this

$$m' = m\beta(1-vu_x)$$

we obtain the usual transformation for mass.

4. By a second differentiation we pass to acceleration and write:

$$f = \frac{dw}{d\tau}.$$

Transformation of acceleration is completely expressed by

$$f' = \mathrm{Q}f\mathrm{Q}.$$

5. We write $\qquad \mathrm{P} = m_0 f$

and call P the physical force quaternion.

We then have

$$m_0\left(\frac{d^2\mathbf{r}'}{d\tau^2} + \frac{d^2 l'}{d\tau^2}\right) = \mathrm{P}' = \mathrm{Q}m_0\left(\frac{d^2\mathbf{r}}{d\tau^2} + \frac{d^2 l}{d\tau^2}\right)\mathrm{Q}.$$

Thus

$$m_0\frac{d^2\mathbf{r}'}{d\tau^2} = im_0\beta v\mathbf{v}\frac{d^2 l}{d\tau^2} + m_0\frac{d^2\mathbf{r}}{d\tau^2} + \mathbf{v}(1-\beta)m_0\mathrm{S}\frac{d^2\mathbf{r}}{d\tau^2}\cdot\mathbf{v}, \quad \text{(viii.)}$$

and

$$m_0\frac{d^2 l'}{d\tau^2} = m_0\beta\frac{d^2 l}{d\tau^2} + m_0 i\beta v\mathrm{S}\frac{d^2\mathbf{r}}{d\tau^2}\cdot\mathbf{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ix.)}$$

If **v** is along Ox we find from (ix.)

$$\frac{dm'}{dt'} = \frac{dm}{dt} - \frac{mv}{(1-vu_x)}\cdot\frac{du_x}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(x.)}$$

Writing $F_x = \frac{d}{dt}(m_0 u_x)$ etc. we find from (viii.)

$$\left.\begin{aligned} F_x' &= \frac{F_x - v\frac{dm}{dt}}{1-vu_x} = F_x - \frac{vu_y}{1-vu_x}F_y - \frac{vu_z}{1-vu_x}F_z \\ &\text{and similarly,} \\ F_y' &= \frac{F_y}{\beta(1-vu_x)}, \quad F_z' = \frac{F_z}{\beta(1-vu_x)} \end{aligned}\right\} \text{.(xi.)}$$

These are Planck's equations for transformation of force.

6. If mass be regarded as a manifestation of contained energy we may, on this view, regard m_0 as a measure of the energy of a body at rest.

The expression for the energy is $\frac{m_0}{(1-u^2)^{\frac{1}{2}}}$. Thus the scalar term $m_0\frac{dl}{d\tau}$ of M is equal to i (energy).

From the definition of $d\tau$ we have

$$(d\tau)^2 = (d\mathbf{r})^2 - (dl)^2. \quad \ldots\ldots \quad \text{(xii.)}$$

Hence

$$\left(\frac{d\mathbf{r}}{d\tau}\right)^2 - \left(\frac{dl}{d\tau}\right)^2 = 1$$

and

$$\frac{d}{d\tau}\left(\frac{d\mathbf{r}}{d\tau}\right)^2 = \frac{d}{d\tau}\left(\frac{dl}{d\tau}\right)^2,$$

or

$$S\frac{d\mathbf{r}}{d\tau}\cdot\frac{d^2\mathbf{r}}{d\tau^2} = \frac{dl}{d\tau}\cdot\frac{d^2l}{d\tau^2}. \quad \ldots\ldots \quad \text{(xiii.)}$$

Multiply throughout by m_0 and the term on the left becomes

$$\frac{1}{(1-u^2)}\frac{d}{dt}\left\{m\frac{d\mathbf{r}}{dt}\right\}.$$

On the right we have

$$-\frac{1}{(1-u^2)}\frac{d}{dt}\left\{\frac{m_0}{(1-u^2)^{\frac{1}{2}}}\right\}.$$

Thus

$$-S\frac{d\mathbf{r}}{dt}\cdot\frac{d}{dt}\cdot\left(m\frac{d\mathbf{r}}{dt}\right) = \frac{d}{dt}\cdot\left\{\frac{m_0}{(1-u^2)^{\frac{1}{2}}}\right\}.$$

This equation represents the principle of conservation of energy, for on the left we have the activity of the force and on the right the rate of change of energy.

If Kw denotes the quaternion conjugate to w and P is the force quaternion, the equivalent of (xiii.) is

$$SP \,.\, Kw = 0. \quad \ldots \ldots \quad \text{(xiv.)}$$

This is the same as the condition for constancy of internal energy given in the 'Theory of Relativity' *.

The quaternion Kw or any physical quaternion of the form

$$q_c = l - \mathbf{r}$$

is transformed to S′ by the operation $Q_c q_c Q_c$, where Q_c is derived from Q by writing $-\mathbf{v}$ instead of $\mathbf{v}$ †.

Transformed to S′, SP . Kw becomes

$$SP'Kw' = SQPQ \,.\, Q_c K w Q_c = SQPKwQ_c.$$

Thus PKw is an "R" quaternion ‡ whence its scalar is invariant. Thus $SP'Kw' = 0$, or the principle of energy is invariant.

7. From (xiii.) we obtain a more general result by regarding the term on the right, viz. $\frac{dl}{d\tau}\frac{d}{d\tau}\left(\frac{dl}{dt}\right)$ prefixed with the negative sign and multiplied by m_0 as the rate of change of energy, *i. e.*

$$\frac{d}{d\tau}\,(\text{energy}) = -\frac{dl}{d\tau}\frac{d}{d\tau}\left(m_0\frac{dl}{d\tau}\right).$$

This leads to the expression $\frac{1}{2}m_0 k^2$ for the energy, omitting an arbitrary constant.

We may denote the kinetic energy by the expression

$$\tfrac{1}{2}m_0(k^2-1)\,\S.$$

It has been pointed out by Jeffreys ‖ that while there is a certain arbitrariness in the choice of the exact form for the kinetic energy there is convenience in the adoption of this form.

This expression, like $m_0(k-1)$, reduces to the ordinary value $\frac{1}{2}mv^2$ for velocities very much smaller than that of light.

* Cunningham, Theory of Rel. p. 167.
† Silberstein, Phil. Mag. May 1912.
‡ Silberstein, *ibid.*
§ *Cf.* W. Wilson, Proc. Phys. Soc. xxxi. pl. ii. p. 74.
‖ H. Jeffreys, Phil. Mag. July 1919.

8. The equation of motion is to be written

$$P = m_0 \frac{d^2 q}{d\tau^2},$$

where $P = \mathbf{F} + A$ and the relation between the scalar and vector parts of P is

$$i\mathrm{K}A = \mathrm{S}\frac{d\mathbf{r}}{d\tau}\mathrm{F},$$

and this is the same as

$$\mathrm{SK}w\mathrm{P} = 0.$$

9. An examination of P shows that it is constructed so that

$$\mathbf{F} = k\mathbf{p} \quad \text{and} \quad A = ik\frac{dw}{dt},$$

where $\mathbf{p}$ is the force as it enters into ordinary mechanics, and $\frac{dw}{dt}$ is the rate of change of energy.

We may easily derive the force in S′ in terms of the S measure. We have merely to transform P′,

$$P' = \mathbf{F}' + A' = Q(\mathbf{F} + A)Q.$$

Equations (i.) and (ii.) give immediately

$$\mathrm{F}' = A(1-\beta^2)v + \mathbf{F} + \mathbf{v}(1-\beta)\mathrm{S}\mathbf{F}\mathbf{v} \quad . \quad . \quad \text{(xiv.)}$$

and

$$A' = \beta A + (1-\beta^2)^{\frac{1}{2}}\mathrm{S}\mathbf{F}\mathbf{v} \quad . \quad . \quad . \quad . \quad \text{(xv.)}$$

$$\therefore \quad k'\mathbf{p}' = -k\beta v\frac{dw}{dt}\,.\,\mathbf{v} + \mathbf{F} + k(1-\beta)\mathbf{v}\mathrm{S}\mathbf{p}\mathbf{v}.$$

Using the ratio $\frac{k}{k'}$ given by (v.)

$$\mathbf{p}' = \frac{\mathbf{p} + \mathbf{v}(1-\beta)\mathrm{S}\mathbf{p}\mathbf{v} - v\beta\mathbf{v}\frac{dw}{dt}}{\beta(1+v\mathrm{S}\mathbf{u}\mathbf{v})}, \quad . \quad . \quad \text{(xvi.)}$$

and in the same way

$$\frac{dw'}{dt'} = \frac{\frac{dw}{dt} + v\mathrm{S}\mathbf{p}\mathbf{v}}{1 + v\mathrm{S}\mathbf{u}\mathbf{v}}\,. \quad . \quad . \quad . \quad . \quad \text{(xvii.)}$$

These two equations represent the general transformation, and there is no particular direction for the vectors occurring in them. Equations (xi.) are particular cases.

As an example, we may apply the transformation to the mechanical force on a moving charge.

Thus if

$$\mathbf{p}=\mathbf{E}+\mathrm{V}\mathbf{u}\mathbf{H}, \quad \frac{dw}{dt}=-\mathrm{S}\mathbf{E}\mathbf{u},$$

and making use of the Principle of Relativity, the physical laws being unaltered by transferring to S′.

We have

$$\mathbf{p}'=\mathbf{E}'+\mathrm{V}(\mathbf{u}'\mathbf{H}'), \quad \frac{dw'}{dt'}=-\mathrm{S}\mathbf{E}'\mathbf{u}'.$$

On making the appropriate substitutions in (xvi.) and (xvii.) and remembering that

$$\mathbf{u}'=\frac{\mathbf{u}+\mathbf{v}(1-\beta)\mathrm{S}\mathbf{u}\mathbf{v}-\beta v\mathbf{v}}{\beta(1+v\mathrm{S}\mathbf{u}\mathbf{v})} \quad . \quad . \quad . \quad . \quad \text{(xviii.)}$$

We find

$$\mathbf{E}'+\mathbf{v}(1-\beta)\mathrm{S}\mathbf{E}'\mathbf{v}=\beta(\mathbf{E}-v\mathrm{V}\mathbf{H}\mathbf{v}), \quad . \quad . \quad \text{(xix.)}$$

and this contains the well-known formulæ

$$\mathrm{E}_x'=\mathrm{E}_x, \quad \mathrm{E}_y'=\beta(\mathrm{E}_y-v\mathrm{H}_z), \quad \mathrm{E}_z'=\beta(\mathrm{E}_z+v\mathrm{H}_y).$$

On application to the expression $(\mathbf{H}-\mathrm{V}\mathbf{u}\mathbf{E})$ we obtain in the same way

$$\mathrm{H}_x'=\mathrm{H}_x, \quad \mathrm{H}_y'=\beta(\mathrm{H}_y+v\mathrm{E}_z), \quad \mathrm{H}_z'=\beta(\mathrm{H}_z+v\mathrm{E}_z),$$

or

$$\mathbf{H}'+\mathbf{v}(1-\beta)\mathrm{S}\mathbf{H}'\mathbf{v}=\beta(\mathbf{H}+v\mathrm{V}\mathbf{E}\mathbf{v}). \quad . \quad . \quad \text{(xx.)}$$

10. *The Field due to a uniformly moving electron.*

The case of the uniformly moving electric charge can be easily dealt with by means of equations (xvi.) and (xvii.).

The problem is to determine the field at a point in system S due to a charge moving with velocity $v\mathbf{v}$. If the system S′ moves with this velocity the charge is at rest in that system, and from the point of view of S′ observers the case is electrostatic.

Consider a charge, e, at rest in S′ and suppose there is a unit charge at a point P′ moving with velocity $\mathbf{u}'$. We shall ultimately write $\mathbf{u}'=-v\mathbf{v}$, so that P′ is at rest in S.

The force on P′ is

$$\mathbf{p}'=\frac{e}{r'^2}\cdot\mathbf{r}_1'=\frac{e}{r'^3}\cdot\mathbf{r}',$$

where $\mathbf{r}_1'$ is the unit vector in the direction from e to P′.

Let e be situated at the origin for convenience. Also

$$\frac{dw'}{dt'}=-\frac{e}{r'^3}\cdot\mathrm{S}\mathbf{u}'\mathbf{r}',$$

where r'^3 means the cube of the tensor of $\mathbf{r}'$.

Thus from (xvi.) by applying the transformation from S′ to S, *i. e.*, writing $-\mathbf{v}$ instead of $\mathbf{v}$ in the formula

$$\mathbf{p}=\frac{1}{\beta(1-v\mathrm{S}\mathbf{u}'\mathbf{v})}\frac{e}{r'^3}\cdot\left\{\mathbf{r}'+\mathbf{v}(1-\beta)\mathrm{S}\mathbf{r}'\mathbf{v}-v\beta\mathbf{v}\mathrm{S}\mathbf{u}'\mathbf{r}'\right\},$$

or writing $\mathbf{u}'=-v\mathbf{v}$ and after a simple modification

$$\mathbf{p}=\frac{\beta e}{r'^3}\left\{\mathbf{r}'+\mathbf{v}\mathrm{S}\mathbf{r}'\mathbf{v}\left(1-\frac{1}{\beta}\right)\right\}. \quad . \quad . \quad . \quad \text{(xxi.)}$$

It is, of course, natural to measure from the instantaneous position of the moving charge e, as it is viewed by observers at rest in S. It is easy to take this new point of reference.

For let the instant in S′ be zero, *i. e.*, $t'=0$. From (i.)

$$\mathbf{r}'=-v\beta\mathbf{v}t+\mathbf{r}+\mathbf{v}(1-\beta)\mathrm{S}\mathbf{r}\mathbf{v},$$

and from (ii.)

$$t'=\beta(t+v\mathrm{S}\mathbf{r}\mathbf{v}).$$

These are merely the Lorentz-Einstein formulæ.

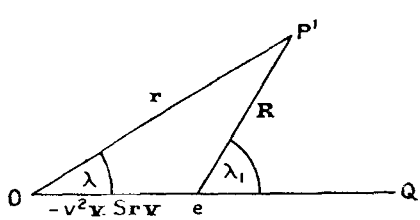
If $t'=0$,

$$t=-v\mathrm{S}\mathbf{r}\mathbf{v}$$

$$\therefore \quad \mathbf{r}'=\mathbf{r}+\mathbf{v}\mathrm{S}\mathbf{r}\mathbf{v}\left(1-\frac{1}{\beta}\right). \quad . \quad . \quad . \quad . \quad \text{(xxii.)}$$

If O is the initial position of the electron, *i. e.*, its position at time $t=0$, in the interval $-v\mathrm{S}\mathbf{r}\mathbf{v}$ it will have moved to e where Oe is $-v^2\mathbf{v}\mathrm{S}\mathbf{r}\mathbf{v}$.

Fig. 1.



We require our formula in terms of $\mathbf{R}$ and possibly the angle P′eQ; OQ is the direction of motion of the electron.

On substitution for $\mathbf{r}'$ in (xxi.)

$$\mathbf{p}=(\mathbf{r}+v^2\mathbf{v}\mathrm{S}\mathbf{r}\mathbf{v})\frac{\beta e}{r'^3}=\frac{\beta e}{r'^3}\cdot\mathbf{R}, \quad . \quad . \quad \text{(xxiii.)}$$

as is easily seen from the figure.

But

$$\mathbf{p}=\mathbf{E}+\mathrm{V}\mathbf{u}\mathbf{H},$$

and since

$$\mathbf{u}=0, \quad \mathbf{p}=\mathbf{E},$$

E is the electrical intensity at P and the formula (xxiii.) shows that it is directed along **R**, the line joining P to the instantaneous position of the electron.

From (xx.) since $\mathbf{H}'=0$ the magnetic intensity due to the charge is

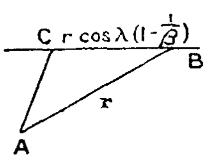
$$\mathbf{H}=-v\mathrm{V}\mathbf{E}\mathbf{v}=v\mathrm{V}\mathbf{v}\mathbf{E}.$$

This immediately shows that **H** is perpendicular to **v** and **E**, and in such a direction that a right-handed screw placed along the direction would rotate **v** into **E**.

It is immediately seen from the figure that the magnitude of **H** is $v\mathrm{E}\sin\lambda_1$.

It remains to put r'^3 in terms of R, these quantities denoting the magnitudes OP′ and eP′.

Fig. 2.



λ denotes the angle between **r** and the direction **v**. Thus

$$\mathbf{r}'=\mathbf{r}-\mathbf{v}r\cos\lambda\left(1-\frac{1}{\beta}\right).$$

Thus

$$\mathbf{r}'=\mathrm{AC}.$$

It immediately follows that

$$r'^2=r^2(1-v^2\cos^2\lambda).$$

Again from fig. 1

$$\mathrm{R}^2=r^2(1-2v^2\cos^2\lambda+v^4\cos^2\lambda)$$

and
$$\frac{\mathrm{R}}{\sin\lambda}=\frac{r}{\sin\lambda'}.$$

From these equations we derive

$$r'^2=\beta^2\mathrm{R}^2(1-v^2\sin^2\lambda_1).$$

Thus

$$\mathbf{E}=\frac{e\mathbf{R}_1}{\beta^2 R^2(1-v^2\sin^2\lambda_1)^{\frac{3}{2}}},$$

$\mathbf{R}_1$ is the unit vector along $e\mathrm{P}'$, and

$$\mathrm{H}=v\mathrm{E}\sin\lambda_1.$$

These results are of course well known, but I think it will be admitted that the above is a particularly easy way of obtaining them. By extension of the principle described to quaternionic operators it is evident that the whole of the theory of Relativity can be very conveniently expressed in this notation.

In conclusion I should like to express my thanks to Dr. Silberstein for reading my paper and for his interest in it.

XLIV. *The Constitution of Atmospheric Neon.* *By* F. W. Aston, *M.A.*, *D.Sc.*, *Clerk Maxwell Student of the University of Cambridge* *.

[Plates VIII. & IX.]

In periodic tables of the elements arranged in order of their atomic weights the part lying between Fluorine on the one hand and Sodium on the other is of considerable interest.

Soon after the discovery of argon and while the monatomic nature of its molecule was still under discussion, Emerson Reynolds, in a letter to 'Nature' (March 21, 1895), described a particular periodic diagram which he had used with advantage. In this letter, referring to the occurrence of the groups Fe, Ni, Co: Ru, Rh, Pd: and Os, Ir, Pt, the following passage occurs:—

"....the distribution of the triplets throughout the whole of the best known elements is so nearly regular that it is difficult to avoid the inference that three elements should also be found in the symmetrical position between 19 and 23, *i. e.* between F and Na, ... of which argon may be one...."

In 1898 neon was isolated from the atmosphere, in which it occurs to the extent of ·00123 per cent. by volume, by

* Communicated by the Author.