Review
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which is a particular case of one arrived at just now. The immediate consequence of this

$$e^{t'X_2'} = e^{tX_1}e^{t'X_2}e^{-tX_1},$$

and the more general $$T' = STS^{-1},$$

are in interestingly close analogy with well-known facts as to the transformation of one substitution by another.

One of the many notes at the end of the volume contains a new proof, not given in full detail, of the difficult second part of Lie's third fundamental theorem.

E. B. ELLIOTT.

A Text-book of Field Astronomy for Engineers. By GEORGE C. COMSTOCK, Director of the Washburn Observatory, Professor of Astronomy in the University of Wisconsin. Pp. x, 202. (John Wiley & Sons, New York.)

This work consists of nine chapters, entitled: Introductory, Coordinates, Time, Corrections to coordinates, Rough determinations, Approximate determinations, Instruments, Accurate determinations, and The transit instrument. As its name implies, the book is intended for engineering students, and the "unconventional views" contained therein have developed during many years' experience in teaching the elements of practical astronomy. The "unconventional views" are excellent things. In the reviewer's experience the conventional view of a transit instrument from a student's stand-point is that three errors are connected with it; these he is quite eager to explain, but he is rather inclined to believe that the instrument is of no manner of use, and he is rather insulted if one asks him what it is for. Therefore, let those who teach and examine in astronomy, and who have no instruments to look after and no observations to make, get this book and acquire unconventional views as speedily as possible. The book will be found interesting and useful to the numerous amateurs who take pleasure in making time observations, etc. We have tried to show that the work will be useful and interesting to persons for whom it was not written, and we wish it to be inferred *a fortiori* that the engineer will find in it all he wants—and he often wants what he cannot find in the ordinary text-book on astronomy.

C. J. JOLY.

Specielle Algebraische und Transcendente Ebene Kurven. By GINO LORIA. Translated into German by FRITZ SCHUTTE. 2 vols., pp. xiv, 744, with 174 figures.

It may be admitted with considerable truth that few exercises are more useful to the student than the tracing of a number of curves. It is imperative that before the pupil is introduced to the general theory of curves he shall be familiarised with asymptotes, nodes, cusps, and the like, by the detailed discussion of numerical equations. The study of curves has its practical value in Mechanics and Physics, and we ought not to ignore its aesthetic value. Beauty and elegance, and even the "wild civility" of the asymmetric curves, have their attractions to the human young. Curves, moreover, appeal to the historic sense as perhaps does no other branch of science. The first real impulse which awakened the study of this department of Mathematics out of the lethargy from which it had suffered since the days of the Greeks was given by Descartes. Then came a period of development in which figure the names of Cavalieri, Wallis, Roberval, Pascal, and Newton. In reading the works of these founders of a new school, one has to be wary. Lack of communication, and the rarity and costliness of literature made it inevitable that there should be many working in the same field who were unknown to each other. Almost the last piece of work done by Pascal was the discussion of a curve to which he had given the name of the roulette. The same curve was dealt with by Roberval under the name of the trochoid; to this generation it is familiar under the title of the cycloid, and indeed it was known to Galileo under that name. The historical sense of the school-boy may in this case be tickled by the knowledge that Pascal was suffering from the toothache and insomnia when the thought of his "roulette" entered his mind. The disappearance of all aches and pains shortly after he had begun to brood over the subject was piously regarded by him as an intimation from above, that the Great Architect of the Universe was viewing with unqualified approval the attack of the problems which had exercised the ingenuity of Galileo when dealing with rolling curves in connection with the construction of the arches of bridges. Most teachers may have noticed that it is comparatively easy to excite an interest in the personality of the inventor of a theorem or a law. Who would not prefer the name "Euler line" to "CONG line" for example? To study a curve in detail, to see how the various properties are brought to light, to realise how the history of the curve has been influenced by the progress of discoveries which perhaps had nothing to do with the art of curve tracing, to see where and why one man failed and his friend and rival, it may be, succeeded—this cannot but prove a stimulus to the young and enquiring mind, and may prove the germ of what in later years will ripen into efflorescence. But for even a short lecture such as is here suggested the teacher cannot rely for information on the ordinary text-books. Take for instance Frost's Curve Tracing. It contains, we verily believe, but two names in all its two hundred pages—Newton and De Gua. Think how much more interesting that admirable work might have been made by even the most elementary references to the history of the art. The teacher has no longer the excuse that he does not know whence to draw his material. This translation into German of Dr. Loria's encyclopaedic work will be far more than is necessary for such a modest programme as is permitted to the teacher when other claims are considered. It is monumental. From cover to cover it teems with interest. A copy should be on the shelves of every College Library, for "the sight of means to do *good* deeds make *good* deeds done." The student who wishes to carry further his researches will find all that he requires in the shape of references from the literature of the earliest days up to the most recent memoirs. The amount of patient labour which these two volumes with their 750 pages represent is colossal.

The Foundations of Geometry, by D. HILBERT. Authorised translation, by E. J. TOWNSEND. Pp. viii, 142. 4s. 6d. net. (Open Court Co.: Kegan Paul.) 1902.

A translation of Hilbert's fascinating *Grundlagen der Geometrie* is heartily welcome in this country, and the volume under notice is further enriched by the author's additions, which appeared in the French translation which M. Laugel published some years ago (Gauthier-Villars). It also contains a summary of a memoir embodying Hilbert's latest researches, which has probably already appeared in the *Math. Ann.*

The first attempt to prove the concurrence in the plane of lines which are not parallel was made by Legendre. He showed that if any one triangle has the sum of its angles equal to two right angles, then the sum of the angles of all triangles will be two right angles; but he failed in his endeavour to prove the existence of one such triangle. Saccheri in 1733, and later Gauss, Lobatchewsky, and Bolyai attacked the same problem, but on different lines. They started with the negation of the axiom of parallels, and to the great surprise and alarm of Saccheri (v. Russell, *Foundations of Geometry*, p. 8) the result was more than one Geometry to the logical basis of which no objection could be found. Their success led to further investigations as to the axioms in general. The conception of space as a manifold of numbers gave Riemann, Helmholtz, and Lie the opportunity of establishing on an analytical basis both the non-Euclidean system of Lobatchewsky, and the system in which Euclid's "straight line" is avoided. In the former the sum of the angles of a triangle is always less, and in the latter always greater, than two right angles. On the other hand, we have the purely geometrical investigations of Veronese and Hilbert. How then are the researches of Hilbert to be placed with reference to the analytical researches of other workers in the same field? Helmholtz showed that Euclid's propositions were in disguise but the laws of motion of solid bodies. The non-Euclidean propositions were in the same manner the laws to which are subject bodies analogous to solid bodies, but with no physical existence. Lie went further. Combining all the possible transformations of a figure he calls the total a group. To each of these groups he attached a geometry; all these geometries have common properties; but the generality of his conclusions is impaired by the fact that all his groups are continuous. His space is a *Zahlenmannigfaltigkeit*. His geometries are subject to the forms of analysis and arithmetic. Now, as M. Poincaré points out (*Bull. des Sciences Mathématiques*, Sept. 1902), this is exactly where Hilbert comes in. His spaces are not *Zahlenmannigfaltigkeiten*. The objects he calls point, line, or plane are purely