



## XLV. Petrovitch's apparatus for integrating differential equations of the first order

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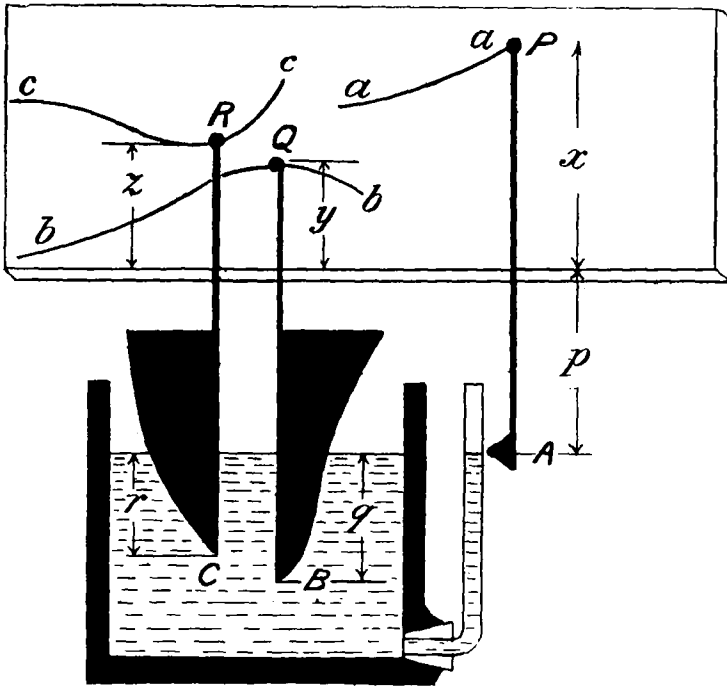
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*XLV. Petrovitch's Apparatus for integrating Differential Equations of the First Order.* By W. A. PRICE\*.

**M.** MICHEL PETROVITCH, Professor of Mathematics at Belgrade, has described in a recent number of the *American Journal of Mathematics* (vol. xxii. No. 1) an interesting instrument for integrating differential equations of the form  $Xdx + Ydy = 0$ .

Two curved templates, B, C (see fig. 1), made from thick plank are plunged into a rectangular tank of water. To B is attached a pointer Q, and to C a pointer R. A pencil P is

Fig. 1.



attached by a vertical rod to an index A, which is placed at the surface of the water. AP, BQ, CR can be moved vertically but not horizontally. The lengths AP, BQ, CR are drawn for convenience of the same length,  $l$ , though this is not essential. A board is moved horizontally behind PQR, and the templates B, C are moved in and out of the water by keeping Q, R upon curves previously drawn on the board.

\* Communicated by the Author.

The index A is continuously adjusted so as to be always at the water surface. The pencil P describes a new curve on the board. The ordinates of P, Q, R measured from a horizontal axis drawn on the board are  $x, y, z$  respectively. The meanings of the quantities  $p, q, r$  are seen from the figure.

If the sectional area at the water level of the B template be  $\phi(q)$ , and of the C template  $\psi(r)$ , and if the sectional area of the tank is M, we have for any small displacements of the templates

$$\phi(q)dq + \psi(r)dr + Mdp = 0.$$

Since

$$p = l - x,$$

$$q = x - y,$$

$$r = x - z,$$

the differential equation becomes

$$\phi(x-y)d(x-y) + \psi(x-z)d(x-z) = Mdx.$$

If  $t$  be the horizontal distance of the board from any fixed point the equations of the curves  $b$  and  $c$  may be written

$$y = f_1(t), \quad z = f_2(t);$$

and we have

$$\begin{aligned} & [\phi\{x-f_1(t)\} + \psi\{x-f_2(t)\}]dx \\ & - [\phi\{x-f_1(t)\}f_1'(t) + \psi\{x-f_2(t)\}f_2'(t)]dt = Mdx. \end{aligned}$$

a differential equation connecting the coordinates of P of which the curve  $aa$  is a particular solution. A series of such particular solutions is obtained by changing the quantity of water in the tank.

In the particular form in which the apparatus has been constructed by Prof. Petrovitch, the template C is fixed, so that  $f_2(t)$  is constant, and  $f_2'(t) = 0$ . The construction is seen in the figures 2 & 3. The paper is attached to a drum turned by an endless screw. The tank is narrow and the flat templates are placed one behind the other. The level of the water is observed by means of a vertical gauge-glass, on the opposite side of the large wooden block on the top of which the drum is mounted. The vertical rods are the guides for the moving templates, and for the rod which carries the pencil.

M. Petrovitch shows how the instrument may be used in a number of ways :

- (1) As an integrator.
- (2) As an integragraph.

- (3) For the solution of equations of the form  $Xdx + Ydy = 0$ ,  
or which may be reduced to that form.
- (4) To describe curves of the form  $\phi\{f(x)\}$  where  $\phi$  and  $f$  are  
known functions.

Fig. 2.

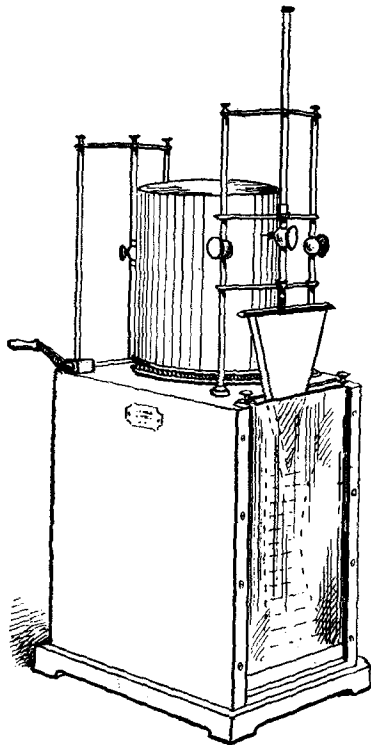
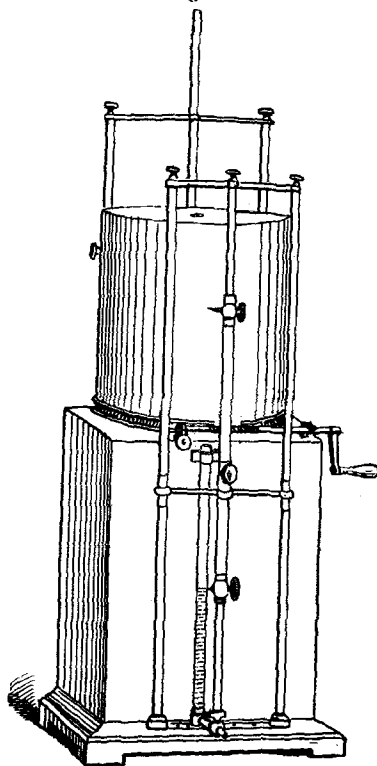


Fig. 3.



As examples of the possible uses of the instrument the following may be given :—

(1) The templates B, C (fig. 1) are triangular, having angles at B, C respectively, so that  $\phi(q) = mq$ ,  $\psi(r) = nr$ . The curves  $bb$ ,  $cc$  are straight lines, so that  $f_1(t) = bt$ ,  $f_2(t) = ct$ .

The differential equation becomes

$$(m+n)x dx - (mb+nc)(xdt+tdx) + (mb^2+nc^2)tdt = Mdx.$$

P describes a series of similar and coaxial ellipses.

In the particular case when the triangular templates are

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similar, and the lines  $bb, cc$  make angles of  $45^\circ$  with the vertical,  $P$  describes a series of concentric circles.

(2) Similar to (1) except that the template  $U$  is inverted, and the lines  $bb, cc$  slope in opposite directions.

Now  $\phi(q) = mq$ ,  $\psi(r) = k - nr$ ,  $f_1(t) = bt$ ,  $f_2(t) = -ct$ .

The differential equation becomes

$$(m-n)x dx - (mb+nc)(x dt + t dx) + (mb^2 - nc^2)t dt + (k-M)dx = 0.$$

$P$  describes a series of similar and coaxial hyperbolas.

(3) An harmonic synthesizer or tide predictor may be constructed of a number of rectangular or cylindrical plungers of sectional areas corresponding to the amplitudes of the several components, lifted and lowered in the tank by eccentrics of the proper periods. The level of the water will give the synthesis required.

(4) Suppose that in Petrovitch's instrument the curve described by the pointer attached to the moving template is a straight line  $y=t$ , and that the curve drawn by the pencil which follows the water-level is the curve  $x=\phi(t)$ , so that  $x=\phi(y)$ . Now cause the pointer to follow the curve

$y = \frac{1}{n} \sin^{-1} t$ , so that the new curve drawn by the pencil is

$$x = \phi\left(\frac{1}{n} \sin^{-1} t\right).$$

The area of the last curve is

$$\int \phi\left(\frac{1}{n} \sin^{-1} t\right) dt = \int \phi(\theta) \cos n\theta d\theta,$$

where  $t = \sin n\theta$ . Thus the Fourier components of the curve  $x=\phi(t)$  can be obtained if the form of the templates can be discovered which will develop the curve itself from a straight-line course for the pointer.

A great number of forms in which instruments can be made on the same general principle will readily suggest themselves. One consists of several vessels of different shapes containing water which may be transferred between the vessels by means of taps, or by a ladle. If the heights of the water-levels in the several vessels are  $x, y, z, \dots$  and the areas of the water-surfaces  $X, Y, Z, \dots$ ; then  $Xdx + Ydy + Zdz + \dots = 0$  and curves or surfaces drawn by plotting the values of  $x, y, z$ , or by a mechanism attached to floats give solutions of the equation.