

harmlessly to the boiler in one case, deposits itself immediately over the fire in the other. Thus the seam of rivets in externally-fired boilers have to contend with the combined influence of tensile strain, the direct action of the fire, and too frequently with an accumulation of incrustation tending to overheating, and even where this does not form a positive coat, it may yet suffice so to thicken the water that the steam lifts it from the surface of the plate, when over-heating unavoidably ensues; added to which, sudden drafts of cold air, on opening the furnace doors, cool the outer laps of the plate at the seams, which thus become subjected to the constant alternations of expansion and contraction.

Under these circumstances it is not surprising that the seams of rivets in under-fired boilers should frequently be found suddenly to give way, for which the surest remedy will prove to be the substitution of internally-fired boilers in their place. Where, however, those externally-fired are still adopted, it is earnestly recommended, in the first place, that good materials and workmanship should be secured; in the second, that every means should be adopted for the prevention of incrustation; and, in the third, that the seams of rivets should be constantly and narrowly watched, so as to detect the first signs of weakness, which should be immediately repaired.

Ready examination is facilitated by setting these boilers, as some of our members are doing, with a single direct flash flue, in which are a series of bridges, one behind the other, for keeping the flame in contact with the boiler; an entrance being made beneath the furnace bars, as well as a small archway through the back bridges, to allow of a communication throughout.

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Rules for selecting the Exponents in Nystrom's Parabolic Construction of Ships, and approximating the Dimensions of a Vessel when its purpose is given. By JOHN W. NYSTROM, C. E.

(Continued from page 107.)

From the preceding formulas, 7 and 9, we have

$$D = \left(\frac{n' 2n''}{(n' + 1)(2n'' + 3n'' + 1)} \right) L B d,$$

in which the factor in the parenthesis is the co-efficient for the length, breadth, and draft of water of the vessel. Let us call this co-efficient = n , we have $D = N L B d$. In the accompanying Table IV., N is calculated for different exponents n' and n'' , as indicated in the columns. The draft of water in the column d , means the draft in proportion to the size of the vessel; when the draft is half the beam, it is considered great, and if only $\frac{1}{4}$ th or $\frac{1}{10}$ th of B , it is small. The purpose of the vessel is noted under the Table. Suppose it is required to build a steamer of light draft for the purpose of carrying freight and passengers, we find in the Table that the exponent for the displacement should be about $n'' = 4$, and that for the cross-section α about $n' = 8$, when the co-efficient $N = 0.632$, and the displacement $D = 0.632 L B d$.

Let us assume certain proportions of the length L , beam B , and draft d ; the displacement given in tons T , we have

$$T = \frac{N L B d}{35}, \text{ or } T = L \left(\frac{N B d}{35} \right)$$

in which the factor in the parenthesis is a co-efficient for L , and may be a function L , so that $T = \frac{L^3}{O^3}$ and $L = O \sqrt[3]{T}$. The factor O is calcu-

lated in the Table V. for different proportions of L , B , and d , as required for the different purposes and conditions. Suppose the size of a vessel is given in $T = 1200$ tons of displacement, to be constructed for freight with moderate speed, and for navigating very shallow water. Required the exponents n' , n'' , and the dimensions, length L , beam B , and draft of water d ? From Table IV. we may select $n' = 10$ and $n'' = 6$ when $N = 0.720$; find the nearest number to this in column N , Table V., which is $N = 0.743$; continuing this line to the three columns, of river steamers of light draft, we may select the co-efficient $O = 36.8$, when the length of the vessel will be $L = 36.8 \sqrt[3]{1200} = 391.1$ feet. Divide this length by the number of the column 82, or

$391.1 : 82 = 4.77$ feet, the draft of water, and the beam

$$B = \frac{35 T}{N L d} = \frac{35 \times 1200}{0.743 \times 391.1 \times 4.77} = 30.3 \text{ feet.}$$

TABLE IV.—To Approximate Size and Shape of Vessels.

d	$\frac{B}{n'}$	Exponents for Displacement n'' .								
		2	2.5	3	3.5	4	5	6	8	10
Draft of Water. Light. Middling. Deep.	2	.356	.397	.429	.453	.494	.500	.528	.558	.577
	2.5	.381	.425	.459	.486	.508	.541	.566	.597	.620
	3	.400	.447	.482	.510	.533	.563	.594	.627	.650
	3.5	.414	.462	.500	.528	.552	.589	.615	.650	.673
	4	.427	.476	.514	.544	.569	.606	.633	.668	.693
	5	.444	.496	.535	.567	.592	.631	.660	.696	.722
	6	.458	.509	.550	.583	.610	.649	.679	.717	.742
	8	.474	.529	.571	.605	.632	.673	.704	.743	.770
	10	.490	.547	.590	.625	.654	.696	.720	.759	.797
Purpose.		Speed and Passengers.			Freight and Passengers.			Freight and Slow Speed.		

TABLE V.—Length of Vessels = Tabular number $O \sqrt[3]{T}$.

	$n' \& n''$ N	Proportion of Draft and Length of Vessels.								
		8	12	18	26	36	48	64	82	102
Heavy Ordinary Sailing Freight. Vessels. Yachts.	.356	14.9	19.0	23.7	28.9	34.0	38.9	43.6	47.2	48.0
	.425	13.9	17.7	22.1	26.9	31.7	36.2	40.6	43.9	44.5
	.482	13.3	17.0	21.2	25.8	30.3	34.7	38.9	42.1	42.7
	.528	12.9	16.5	20.5	25.0	29.4	33.7	37.7	40.8	41.3
	.569	12.5	16.0	20.0	24.4	28.7	30.8	36.8	39.8	40.3
	.631	12.1	15.5	19.4	23.5	27.6	31.6	35.4	38.3	38.8
	.679	11.8	15.1	18.8	22.9	26.9	30.8	34.6	37.4	38.0
	.743	11.6	14.8	18.5	22.5	26.5	30.3	34.0	36.8	37.2
	.797	11.2	14.3	17.9	21.8	25.6	29.3	32.9	35.6	36.0
Conditions.		Vessels for Deep Water.			Ordinary Navigation.			River Steamers, Light Draft.		

Should a greater beam be required, select ϕ in the column 102 when the draft of water will be less.

For light draft and speed, the exponents should be selected towards the corner $\cdot 490$, Table IV.; and for freight and light draft towards the corner $\cdot 797$. For heavy freight and light draft, the proportions of the vessel should be selected towards the corner $36\cdot 0$, Table V.; sailing yachts for deep water towards the corner $14\cdot 9$, and ordinary vessels for deep water in the middle of column 12.

The following Tables VI. and VII. are calculated for twenty parabolic exponents.

TABLE VI.

Exponent n or n' .	Sub-ordinates for the Water-line a or Cross-section \overline{W} . $b = 1$.							$a = BL \times$ $\overline{W} = b d$	k .
	1	2	3	4	5	6	7		
2.	2345	4375	6094	7500	8593	9375	9844	6666	1.94
2.25	2595	4766	6627	7897	8899	9558	9834	6923	1.98
2.5	2838	5129	6912	8232	9139	9687	9944	7142	2.00
2.75	3073	5466	7254	8513	9326	9779	9967	7333	1.98
3.	3301	5781	7558	8750	9472	9844	9980	7500	1.94
3.25	3521	6074	7829	8944	9587	9889	9988	7647	1.91
3.5	3733	6346	8070	9116	9677	9922	9993	7777	1.88
3.75	3939	6600	8284	9256	9747	9944	9995	7894	1.85
4.	4138	6836	8474	9375	9802	9961	9997	8000	1.82
4.5	4517	7260	8794	9557	9878	9978	9998	8181	1.76
5.	4871	7627	9046	9687	9926	9990	9999	8333	1.70
5.5	5202	7945	9246	9779	9954	9994	9999	8461	1.64
6.	5512	8220	9404	9843	9972	9997	1.000	8571	1.58
6.5	5802	8459	9528	9889	9983	9998	1.000	8666	1.52
7.	6073	8665	9627	9922	9989	9998	1.000	8750	1.46
8.	6564	8969	9767	9960	9996	9999	1.000	8888	1.34
9.	6993	9249	9854	9985	9998	9999	1.000	9000	1.28
10.	7369	9437	9909	9990	9999	1.000	1.000	9090	1.18
12.	7963	9683	9976	9998	1.000	1.000	1.000	9231	1.08
16.	8819	9991	9999	1.000	1.000	1.000	1.000	9412	1.00

TABLE VII.

Exponent n'' .	Ordinate Cross-sections \overline{O} . for Displacement $\overline{W} = 1$.							Displacement.	
	1	2	3	4	5	6	7	$D = \overline{W} L \times$	$T = \overline{W} L \times$
2.	0545	1914	3713	5625	7384	8909	9688	5333	0152
2.25	0673	1795	4260	6236	7919	9135	9671	5663	0162
2.5	0805	2512	4772	6777	8352	9383	9889	5952	0170
2.75	0944	2987	5262	7247	8697	9563	9934	6204	0177
3.	1090	3342	5713	7667	8972	9691	9960	6429	0184
3.25	1239	3689	6129	7999	9191	9779	9976	6629	0189
3.5	1394	4027	6512	8310	9365	9845	9986	6806	0194
3.75	1551	4356	6862	8567	9500	9888	9990	6968	0199
4.	1712	4673	7181	8789	9608	9922	9994	7111	0203
4.5	2039	5270	7736	9146	9751	9956	9996	7364	0211
5.	2373	5817	8183	9384	9853	9980	9998	7575	0216
5.5	2706	6312	8548	9563	9908	9988	9998	7755	0221
6.	3038	6757	8844	9688	9944	9995	9999	7924	0227
6.5	3366	7155	9078	9780	9966	9996	9999	8047	0230
7.	3688	7562	9268	9845	9978	9996	9999	8170	0233
8.	4309	8080	9540	9920	9992	9998	1.000	8366	0239
9.	4890	8554	9710	9970	9996	9998	1.000	8521	0244
10.	5430	8906	9819	9980	9998	9999	1.000	8656	0247
12.	6341	9375	9934	9996	9999	9999	1.000	8861	0253
16.	7777	9991	9998	9999	9999	1.000	1.000	9126	0261

In this article it is intended to show the process by which some of the formulas are obtained. The whole system is based on the simple parabolic formulas $y = \sqrt[n]{px}$, $x = \frac{y^n}{p}$, $p = \frac{y^n}{x}$, and $n = \frac{\log.p + \log.x}{\log.y}$

in which the parameter p is the gauge for the parabolas. In order to apply the formulas as direct as possible to the subject in question, it is best to make a gauge that will be ready at hand, by limiting the parabolas within the size of the vessel, when the limit $x = b$, and $y = l$; see fig. 1, Plate I.

$$p = \frac{y^n}{x} = \frac{l^n}{b}, \quad y = l \sqrt[n]{\frac{x}{b}} \quad \text{and} \quad x = \frac{b y^n}{l^n},$$

in which the length and breadth of the vessel is the gauge for the parabolas.

Let us first find the formula 5, page 99, for the area of the load water-line.

We are now obliged to refer to the calculus, from which we know that the increment of the area of any plan figure bounded by a curved line and rectangular co-ordinates, is equal to the increment of the abscissa multiplied by the ordinate; which two latter are obtained from the formula of the curve.

$$x = \frac{b y^n}{l^n}$$

$$\text{Differential } dx = \frac{n b y^{n-1} dy}{l^n},$$

multiplying this by the ordinate y , we obtain the differential of the area a ,

$$da = y dx = \frac{n b y^n dy}{l^n},$$

of which the integral is the area a .

$$\int da = a = \int \frac{n b y^n dy}{l^n} = \frac{n b}{l^n} \int y^n dy,$$

$$a = \frac{n b}{l^n} \cdot \frac{y^{n+1}}{n+1},$$

but when we limit the area to the length $y = l$, we have

$$a = \frac{n b}{l^n} \cdot \frac{l^{n+1}}{n+1} = \frac{n b l}{n+1},$$

which gives one-quarter of the area of the water-line, but by taking the whole beam B and length L , the whole area will be

$$a = \frac{n B L}{n+1}.$$

The process is precisely the same for finding the formula 7, for the cross-section \mathfrak{A} , where d is inserted for l .

Let us now find the formula 9, for the displacement.

The least possible resistance to a vessel of a given displacement bounded within the given length, breadth, and depth, is when the square root of the cross-sections \varnothing are ordinates in a parabola of the exponent n'' ; in this case the ordinates are taken at right angle to the centre line, and the abscissa from \mathfrak{X} , or the ordinates should be $\sqrt{\varnothing} = b - x$, in which it should be clearly understood that the exponent for the parabola of the water-line n need not be the same as n'' for the ordinate cross-sections \varnothing , but for simplicity of notation the two dots will be omitted in the following formulas, where n means n'' .

From the calculus of cubature we know that any solid bounded by an irregular surface, its increment of solidity is equal to the increment of the abscissa, multiplied by the area of the ordinate cross-section. In this case y will be the abscissa.

$$\text{Formula 4. } \varnothing = \mathfrak{X} \left(1 - \frac{x}{b}\right)^2$$

From the formula of a parabola we have

$$\text{Differential } dx = \frac{n b y^{n-1} dy}{l^n},$$

$$\text{and } dy = \frac{dx l^n}{n b y^{n-1}},$$

and for the displacement D we have

$$\text{Differential } dD = dy \varnothing = \frac{\mathfrak{X} l^n \left(1 - \frac{x}{b}\right)^2 dx}{n b y^{n-1}}.$$

By integrating we have

$$\text{Displacement } \int dD = \int \frac{\mathfrak{X} l^n \left(1 - \frac{x}{b}\right)^2 dx}{n b y^{n-1}},$$

$$\text{but } y = l \sqrt[n]{\frac{x}{b}} = \frac{l x^{\frac{1}{n}}}{b^{\frac{1}{n}}} \text{ and } y^{n-1} = \frac{l^{n-1} x^{1-\frac{1}{n}}}{b^{1-\frac{1}{n}}},$$

this value of y^{n-1} inserted in the denominator of the integral, will be

$$D = \int \frac{\mathfrak{X} l^n b^{1-\frac{1}{n}} \left(1 - \frac{x}{b}\right)^2 dx}{n b l^{n-1} x^{1-\frac{1}{n}}} = \int \frac{\mathfrak{X} l \left(1 - \frac{x}{b}\right)^2 dx}{n b^{\frac{1}{n}} x^{1-\frac{1}{n}}},$$

$$\text{of which } D = \frac{\mathfrak{X} l}{n b^{\frac{1}{n}}} \int \frac{\left(1 - \frac{x}{b}\right)^2 dx}{x^{1-\frac{1}{n}}}, \quad . \quad . \quad . \quad a$$

$$\left(1 - \frac{x}{b}\right)^2 = 1 - \frac{2x}{b} + \frac{x^2}{b^2}, \text{ and}$$

$$\int \frac{\left(1 - \frac{x}{b}\right)^2 dx}{x^{1-\frac{1}{n}}} = \int \left(\frac{dx}{x^{1-\frac{1}{n}}} - \frac{2x dx}{b x^{1-\frac{1}{n}}} + \frac{x^2 dx}{b^2 x^{1-\frac{1}{n}}} \right).$$

$$\int \frac{dx}{x^{1-\frac{1}{n}}} = \int x^{\frac{1}{n}-1} dx = nx^{\frac{1}{n}},$$

$$\int -\frac{2x dx}{bx^{1-\frac{1}{n}}} = \int -\frac{2x^{\frac{1}{n}} dx}{b} = -\frac{2x^{1+\frac{1}{n}}}{b(1+\frac{1}{n})},$$

$$\int \frac{x^2 dx}{b^2 x^{1-\frac{1}{n}}} = \int \frac{x^{1+\frac{1}{n}} dx}{b^2} = \frac{x^{2+\frac{1}{n}}}{b^2(2+\frac{1}{n})},$$

By inserting these integrals in the formula *a*, we have the displacement

$$D = \frac{\mathcal{M} l}{n b^{1-\frac{1}{n}}} \left(nx^{\frac{1}{n}} - \frac{2x^{1+\frac{1}{n}}}{b(1+\frac{1}{n})} + \frac{x^{2+\frac{1}{n}}}{b^2(2+\frac{1}{n})} \right).$$

It is required to find a formula expressing the whole displacement from \mathcal{M} to the stern or stem of the vessel, for which we can in the preceding formula make $x = b$, when

$$D = \frac{\mathcal{M} l}{n b^{\frac{1}{n}}} \left(n b^{\frac{1}{n}} - \frac{2 b^{1+\frac{1}{n}}}{b(1+\frac{1}{n})} + \frac{b^{2+\frac{1}{n}}}{b^2(2+\frac{1}{n})} \right),$$

$$D = \frac{\mathcal{M} l}{n b^{\frac{1}{n}}} \left(n b^{\frac{1}{n}} - \frac{2 b^{\frac{1}{n}}}{1+\frac{1}{n}} + \frac{b^{\frac{1}{n}}}{2+\frac{1}{n}} \right).$$

Rejecting the factor $b^{\frac{1}{n}}$, we have

$$D = \frac{\mathcal{M} l}{n} \left(n - \frac{2}{1+\frac{1}{n}} + \frac{1}{2+\frac{1}{n}} \right).$$

The factor in the parenthesis will be

$$\frac{2n^2}{2n^2 + 3n + 1},$$

and we arrive to the final formula for the displacement, when taking the whole length *L*, and resuming the dots on the exponent n'' ,

$$D = \mathcal{M} L \frac{2n^{2''}}{2n^{2''} + 3n^{1''} + 1}.$$

This is the formula as it appears in "Nystrom's Treatise on Ship Building and Marine Engineering," which gives the same result as that in this *Journal*, page 99, or

Formula 9.
$$D = \frac{n^{1''} \mathcal{M} L}{(n^{1''} + 1)(1 + \frac{1}{2n^{1''}})}.$$

Such an operation is necessary for many formulas, which are apparently very simple when finished; the formulas, 11, 12, 13, and others, required each a more extensive operation, which shall be left for the reader to work out.