

THE MEAN FREE PATH OF AN ELECTRON IN A GAS AND ITS MINIMUM IONIZING POTENTIAL.

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INTERESTING conclusions result from a consideration of the gas pressure at which the current through an ionized gas reaches its maximum value. At very low pressures the electrons acquire large velocities in the intervals between collisions with gas molecules and there is great probability of ionization at each collision, but the number of collisions is so small that no considerable increase of the current occurs as the result of ionization. At high pressures, on the other hand, there are many collisions but small probability of ionization. There is evidently some intermediate pressure at which the rate of production of new ions by collision is maximum for any given value of the electric field.

It is possible to calculate the value of this particular pressure required by each of the various theories of ionization by collision which have been proposed and to compare these calculated values with those actually observed. Such a test of the theories has an important advantage over the usual comparison of predicted with observed values of α in that it is independent of the distance between the electrodes in the gas and is therefore not subject to the error in calculating α introduced by the fact that all the electrons leaving the cathode travel a certain distance through the gas before any of them acquire the minimum ionizing velocity.

The current through a gas, when α ionizing collisions are made on the average by each electron in a centimeter path, is given by Townsend's equation

$$i = i_0 e^{\alpha d}, \quad (1)$$

where d is the distance between the electrodes and i_0 is the current which would pass in a perfect vacuum. If V represents the potential difference between the electrodes and V_0 the minimum ionizing potential, it is evident that none of the electrons emitted by the cathode are able to ionize until they have moved a fraction V_0/V of the distance toward the anode. Thus, as Partzsch¹ has shown, the current is more accurately expressed by

$$i = i_0 e^{\alpha d(1 - V_0/V)}. \quad (2)$$

¹ Ann. d. Phys., 40, p. 157, 1913.

Both of these equations neglect ionization due to collisions by the positive ions, which is known to introduce no appreciable error within the range of values of V and d and the pressure p considered in this paper.

For any fixed values of the distance d and the difference of potential V between the electrodes the current is obviously maximum when α is maximum. It is our problem, therefore, to discover the pressure p_m at which α has its greatest value.

According to an equation recently proposed by the writer,¹ α for the ordinary gases in which the collisions are of the inelastic type is given by

$$\alpha = PNp, \quad (3)$$

$$P = e^{-\frac{(1+P)pNV_0}{X}} + (1+P)\frac{pNV_0}{X} Ei\left(-\frac{(1+P)pNV_0}{X}\right). \quad (4)$$

In these equations p represents the pressure in millimeters, N is the average number of collisions made by an electron while moving one centimeter through the gas at one millimeter pressure, P is the probability that an electron will ionize a gas molecule when they collide and X is the electric intensity V/d . $Ei(\)$ represents the exponential integral, values of which are given in Laska's *Sammlung von Formeln*.

From these equations we may find p_m by setting the derivative of α with respect to p equal to zero and solving for p in the equation

$$\frac{d\alpha}{dp} = N\left(P + p\frac{dP}{dp}\right) = 0. \quad (5)$$

In order to differentiate P by p it is convenient to write equation (4) in the form

$$P = e^{-\frac{(1+P)pNV_0}{X}} + (1+P)\frac{pNV_0}{X} \int_{V_0}^{\infty} \frac{1}{V} e^{-\frac{(1+P)pNV}{X}} dV,$$

whence

$$\begin{aligned} p\frac{dP}{dp} = & -\left[(1+P)\frac{pNV_0}{X} + \frac{pNV_0}{X}p\frac{dP}{dp}\right]e^{-\frac{(1+P)pNV_0}{X}} \\ & + \left[(1+P)\frac{pNV_0}{X} + \frac{pNV_0}{X}p\frac{dP}{dp}\right]Ei\left(-\frac{(1+P)pNV_0}{X}\right) \\ & + (1+P)\frac{pNV_0}{X}p\int_{V_0}^{\infty} \frac{1}{V}\left[(1+P)\frac{NV}{X} + \frac{pNV}{X}\frac{dP}{dp}\right]e^{-\frac{(1+P)pNV}{X}} dV. \end{aligned}$$

When the last term is integrated it is found to cancel the first term leaving, after solving for the term of the left member,

$$p\frac{dP}{dp} = \frac{(1+P)\frac{pNV_0}{X}Ei\left(-\frac{(1+P)pNV_0}{X}\right)}{1 - \frac{pNV_0}{X}Ei\left(-\frac{(1+P)pNV_0}{X}\right)}.$$

¹ PHYS. REV., 7, 1916.

When we substitute this expression in equation (5), reduce to a common denominator and simplify, we obtain

$$\frac{P + \frac{pNV_0}{X} Ei \left(- (1 + P) \frac{pNV_0}{X} \right)}{1 - \frac{pNV_0}{X} Ei \left(- (1 + P) \frac{pNV_0}{X} \right)} = 0.$$

Since the denominator of this fraction cannot be infinite the numerator must equal zero, whence

$$P + \frac{pNV_0}{X} Ei \left(- (1 + P) \frac{pNV_0}{X} \right) = 0. \quad (6)$$

In order to solve this equation it was set equal to x and x was plotted as a function of pNV_0/X , the values of x being determined by the aid of equation (4) and tables of values of $Ei(\)$. The intersection of this curve with the axis $x = 0$ gave the solution

$$\frac{pNV_0}{X} = 0.655. \quad (7)$$

We may therefore predict the pressure p_m at which the current through the gas is maximum by the relation

$$p_m = \frac{0.655X}{NV_0}, \quad (8)$$

where the product NV_0 is characteristic of the gas, being known as Stoletow's constant.

By an extensive and apparently very reliable work Partzsch¹ has determined the values of p_m for several gases in electric fields of various intensities. In Tables I.-IV. I have substituted his experimental values

TABLE I.

 N_2 .Mean $NV_0 = 225.4 \pm 0.6$.

V	d	X	p_m	NV_0
124.2	0.208	597	1.70	230
123.4	0.208	593	1.73	225
165.9	0.208	797	2.32	225
165.2	0.208	795	2.30	225
208.3	0.208	1,002	2.80	225
248.5	0.208	1,196	3.45	227
123.9	0.052	2,384	7.00	224
123.8	0.104	1,190	3.58	218
124.1	0.208	597	1.70	230
123.9	0.416	298	0.86	226
123.9	0.832	149	0.43	226

¹ Loc. cit.

of X and p_m in equation (8) to calculate the product NV_0 . At the head of each table is given the mean value of NV_0 and its probable error. The constancy of the product NV_0 deduced from different measurements on the same gas is remarkable.

TABLE II.

*O₂.*Mean $NV_0 = 175.8 \pm 0.7$.

V	d	X	p_m	NV_0
121.7	0.208	585	2.18	176
159.4	0.208	767	2.90	173
157.5	0.208	758	2.75	180
200.0	0.208	963	3.65	173
241.6	0.208	1,161	4.25	179
121.5	0.104	1,170	4.30	178
121.5	0.208	585	2.20	174
121.6	0.416	293	1.11	173

TABLE III.

*Air.*Mean $NV_0 = 223.0 \pm 1.2$.

V	d	X	p_m	NV_0
123.0	0.208	592	1.73	224
163.5	0.208	787	2.30	224
202.9	0.208	976	2.90	221
242.8	0.208	1,167	3.43	222
285.0	0.208	1,371	4.00	224
324.0	0.208	1,557	4.30	237
121.5	0.052	2,339	6.90	222
121.5	0.104	1,169	3.47	221
121.5	0.208	584	1.74	220
121.5	0.416	292	0.89	215

TABLE IV.

*CO₂.*Mean $NV_0 = 236.6 \pm 3.4$.

V	d	X	p_m	NV_0
121.7	0.208	592	1.71	227
162.5	0.208	782	2.13	241
203.5	0.208	979	2.57	249
244.5	0.208	1,175	3.05	252
279.9	0.208	1,347	3.55	248
320.5	0.208	1,542	4.10	247
120.7	0.052	2,320	6.90	220
120.7	0.104	1,160	3.40	223
120.7	0.208	580	1.72	221

Comparison of these results with the values of N and V_0 predicted by the former application of the theory is possible in the case of nitrogen and carbon dioxide. The values of the products NV_0 from the previous papers were: nitrogen, 226; carbon dioxide, 246. When one considers the large range of fields and pressures represented and the fact that the two sets of data were taken under different conditions and for different purposes, this agreement must be taken as a strong confirmation of the essential correctness of the equations.

Experimental determinations of N are being made by Mr. Benade and the writer. The results indicate that N is not a simple function of the mean free path of a gas molecule or of the molecular dimensions as calculated by the kinetic theory, but that N depends on the work done on the electron as it approaches a molecule and on the nearness of its approach. These quantities depend, in turn, on the law of force between electrons and molecules. In view of these results, which will soon be published, it is necessary to modify to some extent the interpretation of the quantity N (or ν) which appears in the expressions for α . The distinction between approximately elastic and inelastic impact still appears to be a valid one, and the new results suggest the physical nature of the two types of collision.

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