

AN INSTRUMENT FOR MEASURING CERTAIN ASPECTS OF INTELLIGENCE IN RELATION TO GROWTH, PRACTICE, FATIGUE, AND OTHER INFLUENCES

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It seems likely that psychologists will wish to make repeated measurements of the same individuals in such aspects of intelligence as tests of the type of the Army Alpha measure. Growth, practice and fatigue, are three notable cases. It is convenient and time-saving in such work to have an instrument represented in many alternative forms which are nearly equal in difficulty and whose differences in difficulty are rather exactly known.

In an earlier paper¹ I presented provisional facts for a thirty-minute test² (which is essentially a harder Alpha, plus four new tests and one of the old Examination A) prepared in fifteen alternative forms. I am now able to state the relative difficulty of these fifteen forms with much greater precision, precision sufficient probably for any purposes to which such tests are likely to be put.

In the earlier paper it was shown that $A = B + 5$, $J = G + 2\frac{1}{2}$ and $J = C - 1$, with high reliability. I have shown elsewhere that when two forms are given after ten minutes' fore-exercise, the median gain from practice is 8. By using the data just mentioned for J , F , G , and C , and the following data for forms given after ten minutes' fore-exercise, we may equate all forms except K .

¹ *Journal of Applied Psychology*, 1920, vol. 4, pp. 283-288.

² Part I of the 1919-1923 Series of the Thorndike Intelligence Examination for High-School Graduates.

| Order of Forms | Difference | Amount of Difference: Allowing 8 for Practice | Number of Individuals |
|----------------|------------|---|-----------------------|
| BC..... | C - B | + 2 | 86 |
| BG..... | G - B | + 4 | 126 |
| CE..... | E - C | - 3 | 254 |
| DC..... | D - C | + 2 | 168 |
| DE..... | E - D | - 3½ | 312 |
| DE..... | E - D | - 4 | 134 |
| DE..... | E - D | - 4½ | 99 |
| ED..... | E - D | - 4 | 72 |
| EH..... | H - E | + 3½ | 222 |
| EM..... | M - E | - 3 | 86 |
| EM..... | M - E | + 1½ | 34 |
| FM..... | M - F | 0 | 28 |
| JM..... | M - J | - 4½ | 28 |
| FN..... | N - F | - 4 | 167 |
| NF..... | N - F | - ½ | 70 |
| GH..... | H - G | - 2 | 105 |
| GH..... | H - G | + 1 | 28 |
| HO..... | O - H | - 8 | 68 |
| IM..... | M - I | - 4½ | 320 |
| IM..... | M - I | - 9 | 69 |
| IM..... | M - I | - 6 | 24 |
| MI..... | M - I | - 4½ | 409 |
| MI..... | M - I | - 3½ | 419 |
| JL..... | L - J | - 2½ | 392 |
| LO..... | O - L | 0 | 273 |
| LO..... | O - L | - 1 | 286 |
| OL..... | O - L | - ½ | 54 |

From the above we have, in comparison with the average,
 - meaning harder and + meaning easier:

A + 2
 B - 3
 C + 2
 D + 3
 E - 1
 F 0
 G - 1½
 H + 1½
 I + 2½
 J + 1
 L - 1½
 M - 2
 N - 2
 O - 2

The computations were made as follows:

$$C = J + 1 \quad F = J - 1 \quad G = J - 2\frac{1}{2} \quad \text{are given.}$$

$$E - C = -3 \quad E - J - 1 = -3 \quad E = J - 2$$

$$L - J = -2\frac{1}{2} \quad L = J - 2\frac{1}{2}$$

$$O - L = -\frac{1}{2} \quad O - J + 2\frac{1}{2} = -\frac{1}{2} \quad O = J - 3$$

$$N - F = -4 \text{ from 167 cases}$$

$$N - F = -\frac{1}{2} \text{ from 70 cases}$$

$$\text{So } N - F = -3 \quad N - J + 1 = -3 \quad N = J - 4$$

But since from other scattered data we have $N = J$, we may estimate N as $J - 3$.

$$D - C = 2 \quad C = J + 1 \quad D - J - 1 = 2 \quad D = J + 3 \\ n = 168$$

$$E - D = 4 \quad E = J - 2 \quad J - 2 - D = -4 \quad D = J + 2 \\ n = 312 + 134 + 99 + 72$$

$$M - E = -3 \quad M - J + 2 = -3 \quad M = J - 5 \quad n = 86$$

$$M - E = 1\frac{1}{2} \quad M - J + 2 = 1\frac{1}{2} \quad M = J - \frac{1}{2} \quad n = 34$$

$$M - F = 0 \quad M - J + 1 = 0 \quad M = J - 1 \quad n = 28$$

$$M - J = -4\frac{1}{2} \quad M = J - 4\frac{1}{2} \quad n = 28$$

$$\text{So } M = J - 3$$

$$M - I = -4\frac{1}{2} \text{ for } n = 729$$

$$M - I = -3\frac{1}{2} \text{ for } n = 419$$

$$M - I = -9 \text{ for } n = 69$$

$$M - I = -6 \text{ for } n = 24$$

$$\text{So } M - I = -4$$

$$M = J - 3$$

$$J - 3 - I = -4\frac{1}{2}$$

$$I = J + 1\frac{1}{2}$$

$$H - E = 3\frac{1}{2} \quad H - J + 2 = 3\frac{1}{2} \quad H = J + 1\frac{1}{2} \quad n = 222$$

$$H - G = -2 \quad H - J - 2\frac{1}{2} = -2 \quad H = J + \frac{1}{2} \quad n = 105$$

$$H - G = 1 \quad H - J - 2\frac{1}{2} = 1 \quad H = J + 3\frac{1}{2} \quad n = 28$$

$$H - O = -8 \quad H - J - 2\frac{1}{2} = 8 \quad H = J - 5\frac{1}{2} \quad n = 68$$

$$\text{So } H = J + \frac{1}{2}$$

$$C - B = 2 \quad J + 1 - B = 2 \quad B = J - 1 \quad n = 86$$

$$G - B = 4 \quad J - 2\frac{1}{2} - B = 4 \quad B = J - 6\frac{1}{2} \quad n = 126$$

$$\text{So } B = J - 4$$

Replacing J by (Average + 1) in the above, we have A , B , C , etc., in terms of the average, as shown above.

Since the average score for college entrants in these tests after fore-exercise is about 100, the numbers + 2, - 3, + 2, etc., may be considered as percents.

K was equated as $J - 0.3/4$ in an earlier paper on the assumption that the practice effect from a second to a third full trial was 3. If this practice effect is counted as 4, $K = J - 1\frac{3}{4}$. K is thus very near the average. We have called it 0.

Using these values we determine the practice effect from first to second trial when no fore-exercise is given as follows:

| Order | Difference | Raw Difference | Difference Corrected for Difficulty of Forms | Number of Cases |
|------------|------------|-----------------|--|-----------------|
| AF | $F - A$ | 10 | 12 | 160 |
| AG | $G - A$ | 9 | $12\frac{1}{2}$ | 113 |
| AJ | $J - A$ | 12 | 13 | 154 |
| BF | $F - B$ | 15 | 12 | 200 |
| BG | $G - B$ | $9\frac{1}{2}$ | 8 | 20 |
| FG | $G - F$ | 10 | $11\frac{1}{2}$ | 27 |
| FJ | $J - F$ | $13\frac{1}{2}$ | $12\frac{1}{2}$ | 36 |
| GJ | $J - G$ | 15 | $17\frac{1}{2}$ | 33 |
| JL | $L - J$ | 11 | $13\frac{1}{2}$ | 28 |

This practice effect is thus $12\frac{1}{4}$.

Using the same values we determine the practice effect from second to third of three full trials preceded by no fore-exercise as follows:

| Order | Difference | Raw Difference | Difference Corrected for Difficulty of Forms | Number of Cases |
|------------|------------|----------------|--|-----------------|
| GH | $H - G$ | 9 | 6 | 113 |
| FK | $K - F$ | $2\frac{1}{2}$ | $2\frac{1}{2}$ | 351 |
| JK | $K - J$ | 3 | 4 | 144 |
| HL | $L - H$ | - 1 | 2 | 36 |
| JM | $M - J$ | $2\frac{1}{2}$ | $5\frac{1}{2}$ | 40 |
| GJ | $J - G$ | 5 | $\frac{1}{2}$ | 26 |

The practice effect from a second to a third trial thus

seems to be about $3\frac{1}{2}$, but is less reliably determined than the $12\frac{1}{4}$.

Using $12\frac{1}{4}$ and $3\frac{1}{2}$, we may check the determinations of the relative difficulty of the forms from the cases where no fore-exercise was given.

Thus $J - A = 12 - 12\frac{1}{4}$ or $-\frac{1}{4}$ for $n = 154$. We had $J - A = -1$, a difference of $\frac{3}{4}$. $F - A = 10 - 12\frac{1}{4}$ or $-2\frac{1}{4}$ for $n = 160$. We had $F - A = -2$, a difference of $\frac{1}{4}$. $F - B = 15 - 12\frac{1}{4}$ or $2\frac{3}{4}$ for $n = 200$. We had $F - B = 3$, a difference of $\frac{1}{4}$. $G - A = 9 - 12\frac{1}{4}$ or $-3\frac{1}{4}$ for $n = 113$. We had $-3\frac{1}{2}$, a difference of $\frac{1}{4}$. $K - F = 2\frac{1}{2} - 3\frac{1}{2}$ or -1 for $n = 351$. We had $K - F = 0$, a difference of 1 . $K - J = 3 - 3\frac{1}{2}$ or $-\frac{1}{2}$ for $n = 144$. We had $K - J = -1$, a difference of $\frac{1}{2}$. The average discrepancy between these differential determinations by the two methods is thus $\frac{1}{2}$.¹ The determinations are thus sufficiently reliable for any use that is likely to be made of the tests, since the median difference between two trials by the same individual is about fifteen times the probable error of the estimate of difficulty.

¹ I have checked further with recently obtained data from tests given after fore-exercise as follows:

| | | |
|--------------------------|-------------------------|--------------------------|
| The order being Practice | <i>I L</i> , the median | $L - I = 8$, $n = 44$ |
| " " " " | <i>I M</i> , " " | $M - I = 6$, $n = 40$ |
| " " " " | <i>I N</i> , " " | $N - I = 5.5$, $n = 41$ |
| " " " " | <i>J L</i> , " " | $L - J = 8$, $n = 41$ |
| " " " " | <i>J M</i> , " " | $M - J = 7.5$, $n = 41$ |
| " " " " | <i>J N</i> , " " | $N - J = 8.5$, $n = 45$ |

Equating *I* with *J* by their differences from *L M* and *N* practice we have $I - J = 1.8$. By our standard values $I - J$ was 1.5 .

| | | |
|--------------------------|-------------------------|-------------------------------------|
| The order being Practice | <i>J E</i> , the median | $E - J$ was 5.5 , $n = 73$ |
| " " " " | <i>J I</i> , " " | $I - J$ " 8 , $n = 64$ |
| " " " " | <i>J L</i> , " " | $L - J$ " 5 , $n = 63\frac{1}{2}$ |
| " " " " | <i>J O</i> , " " | $O - J$ " 3.5 , $n = 56$ |

Using the standard values for *E*, *I*, *L* and *O*, and reversing the 8 allowance for practice, we have $J = +2$. By our standard values $J = +1$.

| | | |
|--------------------------|-------------------------|------------------------------|
| The order being Practice | <i>E H</i> , the median | $H - E$ was 8.5 , $n = 32$ |
| " " " " | <i>E M</i> , " " | $M - E$ " 9.5 , $n = 34$ |
| " " " " | <i>E O</i> , " " | $O - E$ " 6 , $n = 33$ |

Using the standard values for *H*, *M* and *O*, and reversing the 8 allowance for practice, $E = -0.8$. By our standard it was -1 . The discrepancies in these cases are about the same as before.

These differences in difficulty are of course from medians and will fit fairly well the sort of person who scores about 100 in the test. It cannot be in any way guaranteed that, say, Form *B* is 3 per cent. harder than the average form for very low scorers and for very high scorers. I have data from which the differences in difficulty between forms could be computed, for high scorers and low scorers separately, but the matter does not at present seem important enough to justify the great expense of time required.