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XXV. *On the Rules for Algebraical Multiplication.* *By Mr.* J. Dillon.

To Mr. Tilloch.

Sir,—Having seen in your Magazine for January some remarks by Sir H. Englefield, tending to explain the algebraical theorem, at once so necessary for the young mathematician to master, and yet so difficult for him fully to understand, that a *negative* quantity, multiplied into a *negative* quantity, gives a *positive* result, I beg leave to add a few observations which have occurred to me upon the subject, and which may perhaps in some degree tend to place the matter in a clearer point of view.

The signs + and − appear too generally (at least in elementary works) construed to mean *plus* and *minus;* a sense which, though perhaps always included in, does not appear to constitute the whole of their definition. The sign + signifying, in fact, that the term to which it is prefixed is *positive,* and the sign − that such term is *negative,* that the one should be *plus,* (or the object of addition,) and the other *minus,* (or the object of subtraction,) when addition or subtraction with other quantities is in question; these are rather *consequences* flowing from, than essential parts of the nature of, such signs of + and −.

The fallacy of considering + and − as merely meaning *plus* and *minus,* will plainly appear where multiplication or division is intended, as $-a \times b$, or $\frac{-a}{+b}$, where it is evident neither *plus* nor *minus* can be meant by the signs + and −; and it is in this fallacy, as it appears to me, that all the difficulties of the present question have their origin; for, by always affixing the sense of *positive* and *negative* to these signs, nearly all the obstacles which impede the progress of the learner on this subject will vanish.

I scarcely need previously to observe, that the algebraist is as conversant with the idea of a negative as of a positive quantity. Considerable confusion appears, however, to have arisen from attempts to render this idea familiar to minds not accustomed to abstract reasoning. Thus, therefore, it is frequently represented that $-a$ is not so much the negative quantity a, as it is the positive quantity a with a mark affixed to it, signifying that it is to be subtracted from some other quantity either actually known, or to be discovered; whereas, in fact, it should be considered as strictly a *negative* quantity, capable of destroying or counteracting a positive quantity of equal value, when it comes in contact with such, and existing in the mind in a way perhaps somewhat similar to the ideas of darkness, silence, or vacuum, which

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which may be considered as abstract *negative* ideas, expressive of the absence of light, sound, or matter.

To apply these premises to the subject under consideration, namely, the multiplication of algebraical quantities, under all the varieties of the signs + and —: of this multiplication there are three cases.

1st. When the terms are both positive, $+ \times +$.

2d. When only one of the terms is positive, $- \times +$.

3d. When both the terms are negative, $- \times -$.

For the *first* then, to take an instance, $+ a \times + b$; we have only to remember the well known principle that the multiplier is merely an abstract quantity, expressive of the number of times the multiplicand is to be added within itself, and we shall immediately perceive that the result must be +, as it is merely proposed that $+ a$ should be taken b number of times, without any alteration of the signs, which are indeed expressly *affirmed* by the sign + affixed to b.

For the second case, $- a \times + b$; it is equally evident that $- a$ taken b number of times must on the other hand always remain —, whatever may be the value of b.

For the third case, $- a \times - b$, where the difficulty is supposed to rest, it may be previously remarked that if $- a$ when multiplied (as in the second case) by b or $+ b$ gives a *negative* result, then may it beforehand be expected that this same $- a$ when multiplied by $- b$ will give a contrary, that is a *positive* result.

It is a well known position in logic that two negatives make an affirmative; to say that a thing is *not not* so, is in fact but a more circuitous manner of saying that it *is* so, and exactly this process appears to take place in the case before us. The result of $- a \times - b$ is $+ ab$ for this reason; $a \times b = ab$, and the negative sign of the a is (if I may so express it) itself *negated* by the negative sign of the b. The quantity a had, we must suppose, become negative by some previous process; the *reversal* therefore of this sentence of negation must be as necessarily the consequence of its being multiplied into a *negative* quantity, as the continuing subject to that sentence would have resulted from its being multiplied into an affirmative or *positive* quantity: in other words, the sign — prefixed to the b is, in fact, the *negation* of the sign belonging to a, (the quantity to be multiplied,) whether the sign of such quantity be positive *or* negative

The origin of the error, and the consequent existence of the difficulty in question, appear to be this: that the affirmation of a positive quantity, (as $a \times + b$,) and the negation of a negative quantity (as $- a \times - b$,) are supposed to be the contrast, or antithesis of each other; whereas, in fact, so far from being opposed,

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posed, they are but the same thing stated in other words; or rather they tend, by different methods, to a similar result: the *real* contrast will be found to exist in the affirmation of a positive quantity, (as $a \times + b$,) and in the affirmation also (*not* the negation) of a negative quantity, (as $- a \times + b$;) and this contrast is exemplified in the contrary results of the first and second cases stated above.

Two observations only, in the way of elucidations, further occur to me: it should be remembered that in the multiplication, as in other processes of algebra, the signs only affect the signs, and the quantities the quantities: in the multiplication $- a \times - b$ to perceive the separate effect of the one sign upon the other, let us suppose b equal to unity, and we shall find, according to the foregoing principles, $- a \times - b = + a$; then taking any other value of b, as 10, the result will be $- a \times - b = + 10 a = + ab$. To show that the result of $+ a \times + b$ must be similar to that of $- a \times - b$, it may be observed that as a new result of the multiplication $+ a \times + b$ is produced by the alteration of *one* of the signs, as $- a \times + b$, so the original result will again be equally and indifferently brought about either by the restoration of the original sign belonging to a, when it will stand, as before, $+ a \times + b$; *or* by the further additional reversal of the sign of the *remaining* term ($+ b$,) when the *same* product will be represented by $- a \times - b$: hence $+ a \times + b = - a \times - b$; and hence a *negative* quantity multiplied into a *negative* quantity gives a *positive* result.

Trusting you will excuse a degree of prolixity, and even of tautology, which appeared necessary to elucidate a subject extremely exposed to difficulty and misconstruction,

I remain, Sir,

Your very obedient servant,

Paddington Green, Feb. 20, 1815. JOHN DILLON.

XXVI. *Notices respecting New Books.*

A Treatise on the Construction of Maps; in which the Projections of the Sphere are demonstrated, and their various practical Relations to mathematical Geography deduced and explained, systematically arranged, and scientifically illustrated from Twenty-eight Plates of Diagrams; with an Appendix and copious Notes. By Alex. Jamieson. 8vo. pp. 202.

THE claims of the author are modest: "In a science that has outlived the vicissitudes of two thousand years, and become splendid amidst even the riot of barbarism, originality is hardly to