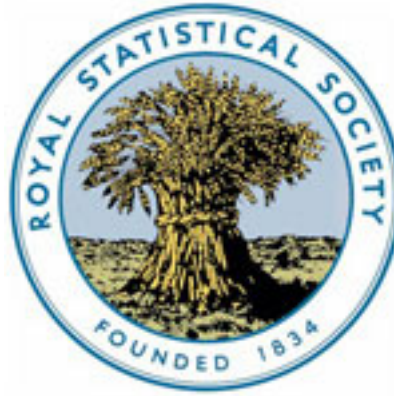


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NOTES UPON MR. KING'S SHORT METHOD OF CONSTRUCTING
ABRIDGED LIFE TABLES.

By C. H. FORSYTH, Ph.D.

IN Part I of the Supplement of the Seventy-fifth Annual Report of the Registrar-General (1914), Mr. King outlines a short method of constructing abridged life tables. Some of the work can be shortened as follows:

Instead of computing graduated quinquennial pivotal values both of the population and of the deaths, these pivotal values may be found of the values of p_x alone and thus shorten the work.

If L_{x+2} refers to the population at the middle of the year represented by the age $x+2$, and $d'_{x+2} = 1/2 d_{x+2}$ where d_{x+2} refers to the number of deaths at age $x+2$, then

$$p_{x+2} = \frac{L_{x+2} - d'_{x+2}}{L_{x+2} + d'_{x+2}}$$

and if we use W_x to refer to the quinquennial groups of population w_x to one-half of those of the deaths, then from Mr. King's formula

$$u_7 = \cdot 2 w_5 - \cdot 008 \Delta^2 w_0$$

$$p_{x+2} = \frac{L_{x+2} - d'_{x+2}}{L_{x+2} + d'_{x+2}} = \frac{\cdot 2 (W_x - w_x) - \cdot 008 (\Delta^2 W_{x-5} - \Delta^2 w_{x-5})}{\cdot 2 (W_x + w_x) - \cdot 008 (\Delta^2 W_{x-5} + \Delta^2 w_{x-5})}.$$

If we let $D_x = W_x - w_x$ and $S_x = W_x + w_x$
and notice that $\Delta^2 W_{x-5} - \Delta^2 w_{x-5} = \Delta^2 (W_{x-5} - w_{x-5}) = \Delta^2 D_{x-5}$
and $\Delta^2 W_{x-5} + \Delta^2 w_{x-5} = \Delta^2 (W_{x-5} + w_{x-5}) = \Delta^2 S_{x-5}$

$$p_{x+2} = \frac{D_x - \cdot 04 \Delta^2 D_{x-5}}{S_x - \cdot 04 \Delta^2 S_{x-5}}. \quad (1)$$

Hence, to find the graduated quinquennial pivotal values of p_x , we first halve each quinquennial group of deaths, then form columns of D_x and S_x by subtracting and adding respectively these groups to the corresponding groups of the population. The columns of D_x and S_x are then differenced twice and corresponding values of p_{x+2} computed by the formula above.

Also, Mr. King's formula

$$u_7 = \cdot 2 w_5 - \cdot 008 \Delta^2 w_0$$

may be generalised, for

$$\text{if } y_x = \sum_{t=0}^x u_x \quad \text{or} \quad \Delta y_x = u_x$$

then

$$\begin{aligned} u_x &= \frac{y_{x+1}}{t} - \frac{y_x}{t} \\ &= \frac{w_0}{t} + (x-2) \frac{\Delta w_0}{t^2} + \frac{x^2 - 9x + 12}{2!} \frac{\Delta^2 w_0}{t^3} + \frac{x^3 - 21x^2 + 116x - 126}{3!} \frac{\Delta^3 w_0}{t^4} \\ &\quad + \frac{x^4 - 38x^3 + 467x^2 - 2014x + 1915.2}{4!} \frac{\Delta^4 w_0}{t^5} + \dots \end{aligned}$$

where

$$w_x = \frac{u_x}{t} + \frac{u_{x+1}}{t} + \dots + \frac{u_{x+t-1}}{t}$$

and

$$\begin{aligned} w_0 &= \Delta y_0 \\ \Delta w_0 &= \Delta^2 y_0, \text{ \&c.} \end{aligned}$$

If it is given the usual value of 5, we have

$$\begin{aligned} u_x &= \cdot 2w_0 + \cdot 04(x-2)\Delta w_0 + \cdot 004(x^2 - 9x + 12)\Delta^2 w_0 \\ &\quad + \cdot 0008(x^3 - 21x^2 + 116x - 126)\frac{\Delta^3 w_0}{3} \\ &\quad + \cdot 00004(x^4 - 38x^3 + 467x^2 - 2014x + 1915.2)\frac{\Delta^4 w_0}{3} + \dots \quad (2) \end{aligned}$$

where the subscript of u has been changed for convenience to unit form.

If x is allowed to take the values $5/5, 6/5, \dots, 10/5$ in (2) up to and including second differences, and the results differenced, we obtain useful leading term and differences for breaking up into its component values the group w_1 of the three quinquennial groups w_0, w_1 and w_2 . The leading term and differences are:

$$\begin{aligned} u_5 &= \cdot 2w_0 + \cdot 12\Delta w_0 - \cdot 032\Delta^2 w_0 \\ \Delta u_5 &= \cdot 04 \text{ ,, } + \cdot 008 \text{ ,,} \\ \Delta^2 u_5 &= \cdot 008 \text{ ,,} \\ \Delta^3 u_5 = \Delta^4 u_5 = \Delta^5 u_5 &= 0 \end{aligned}$$

The leading term and differences for breaking up w_0 (or w_2) of w_0, w_1 and w_2 are:

$$\begin{aligned} u_0 &= \cdot 2w_0 - \cdot 08\Delta w_0 + \cdot 048\Delta^2 w_0 \\ \Delta u_0 &= \cdot 04 \text{ ,, } - \cdot 032 \text{ ,,} \\ \Delta^2 u_0 &= \cdot 008 \text{ ,,} \\ \Delta^3 u_0 = \Delta^4 u_0 = \Delta^5 u_0 &= 0 \end{aligned}$$

If $x=7$ in formula (2) where only second differences are used, we obtain Mr. King's particular formula.