The Cosmochrony Research Programme: A Structural Roadmap and Paper Inventory

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Abstract

The Cosmochrony programme develops a pre-geometric framework in which physical structure — spacetime, quantum mechanics, gauge symmetry, and Standard Model observables — arises from a single primitive: the local structure of admissible non-injective transitions between observable states. The corpus comprises 116 papers across three theory branches. Branch I (Foundation track) develops four admissibility axioms and their consequences. A finite S_3 countermodel now proves that the algebraic properties extracted from those axioms in the published carrier-selection argument do not force a central commutator, a finite Heisenberg group, or a Weil representation. The Heisenberg/Weil carrier is therefore a supplied realisation, not an axiomatic theorem. Branch II (O-series) develops the spectral admissibility sub-programme, studies the conjugate-pair fibre structure on a supplied Weil model and measures its capacity exponent δ_{pair}. The native Heisenberg span-growth law proves that the proposed conversion $\delta_{\mathrm{pair}} \to \beta^*$ has no derived carrier on the measurement substrate; its numerical agreement with the charged-lepton hierarchy remains a phenomenological coincidence check rather than a structural derivation. The same branch's former identification of the Standard Model gauge group has been withdrawn in O31 version 2.0; the colour factor and the complete gauge group are open. Branch III (Q-series and companion papers) develops spacetime geometry and gauge dynamics from the Branch I axioms and supplied Branch II spectral data, and states a universal singlet correlator conditional on an explicit, undemonstrated invariance hypothesis (paper Q3; the Born rule remains open), the effective Lorentzian co-metric $g^{\mu\nu} = \mathrm{diag}(-2, 2, 2, 2) \propto \eta^{\mu\nu}$ read off the geometric-emergence papers under two open inputs (the continuum-limit hypothesis [H-L], and the identification of the measured admissible space with the spin-1 module, which paper Q7 version 2.0 records as supplied by no source); and the spectral derivation of the Einstein (a_2) and local Yang–Mills (a_4) sectors from a single spectral-entropy functional (papers Q12–Q13). This paper provides a complete inventory, a logical dependency map, and a structured account of what is proved, structural, heuristic, or open, intended both as an entry point for external readers and as an internal navigation reference. *Interpretive status.* The programme's unifying reading — that physical structure is organised by admissibility and by spectral (Seeley–DeWitt) order rather than by an enlarged symmetry group — is offered as an interpretive outlook, not a theorem; each individual result carries the epistemic label (proved, structural, heuristic, or open) recorded in the inventory below.

Keywords. Non-injective projection; admissibility; Born–Infeld constraint; spectral admissibility; Heisenberg group; Weil representation; emergent spacetime; lepton mass hierarchy; quantum mechanics; gauge structure; programme overview.

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1 Introduction and Purpose

The Cosmochrony programme was initiated with a white paper [19] positing that all physical structure emerges from a non-injective projection $\Pi : \Omega \to \mathcal{O}$ from a pre-geometric substrate χ onto observable space. The initial framework postulated a Born–Infeld (BI) saturation constraint, a relaxation mechanism, and an iterated projection cascade $U_{t+1} = \Pi \circ \sigma(U_t)$ as the engine of dynamics.

Over the course of 2026, the programme has evolved in two parallel directions. On one side, the spectral admissibility sub-programme (O-series) has progressively built the computational machinery for extracting observables from the spectral structure of the fibre. O24 [40] develops the conjugate-pair construction conditionally on a stated fibre structure for Π; the identification of that fibre structure from first principles remains open. The Span-Growth Note [35] proves, however, that the O4–O6 growth-equation closure, square-root frontier, pair observable, and exponent coordinates do not supply a native Heisenberg derivation of the downstream conversion $\delta_{\text{pair}} \to \beta^*$. On the other side, the Foundation paper [6] proposes a reformulation from four axioms (A1–A4), dispensing with an explicit substrate χ and relaxation mechanism. Its carrier-selection step is not derived: HeisenbergStructure [2] (version 2.1) contains a finite countermodel that satisfies the algebraic properties available before centrality while its commutator is non-central. The Q-series then builds quantum mechanics, SU(2) symmetry, and Lorentzian spacetime on top of the Foundation axioms and O-series spectral data.

The corpus comprises 116 papers with complex mutual dependencies. The interactive dependency graph covers all 116 entries of the inventory and distinguishes ordinary dependencies from sub-programme synthesis links. A reader entering the programme faces two difficulties: it is not obvious where to start, and the distinction between a theorem, a structural argument, and an open conjecture is not always transparent from individual paper titles. This roadmap addresses both difficulties.

Conventions on result status. Throughout this paper each result is classified as one of the following: **proved** (formally proved as a theorem within the stated assumptions); **structural** (derived from structural arguments that are internally consistent but not yet fully formalised); **numerical** (the principal contribution is a dataset or an exact finite computation with its reproduction code, and no structural or physical claim is asserted on its basis; a theorem proved about the data is a secondary label); **conditional** (a theorem whose conclusion is read as a claim about the programme, and at least one of whose hypotheses is a programme-level identification or postulate that this inventory lists as open, such as [H-L], [H-F], a carrier selection or a supplied selection rule; the label names that hypothesis; a theorem about a supplied object as such, whose conclusion does not depend on the object being the programme's, is **proved** with the supplied input stated, and the missing identification is recorded as a secondary **open**); **heuristic** (physically motivated and numerically supported but not yet rigorously derived: a physical claim is asserted beyond what the numerical or structural support establishes); **open** (explicitly conjectured or known to be unsettled). A seventh entry, **synthesis**, is reserved for the sub-programme presentation and synthesis notes: these papers consolidate and organise results established in their constituent papers and carry no new result of their own, so their epistemic content is that of the papers they map. Each paper carries one primary label, stated first in the inventory table, in the dependency figures and graph, and first in its synthesis paragraph where that paragraph states a status, followed where useful by the labels of its secondary components; the primary label is the status of the paper's principal contribution, the result named by its title; where the row's prose names an open or conditional item of the paper's own principal result, the status cell records it as a secondary label: a proved no-go or countermodel is **proved** as a negative result, and a withdrawal notice is **open**, any counterexample it proves being recorded as a secondary **proved**. These classifications record the current audited status. They may therefore correct an original paper's self-classification when a published countermodel or obstruction establishes that

a claimed implication does not follow.

2 Conceptual Architecture

The programme is organised in three structural levels.

Level 1 — Axiomatic primitive. The primitive is the local structure of admissible non-injective transitions. No background spacetime, no quantum postulate, and no substrate field are introduced at this level. The four axioms are: local projective admissibility (A1), structural non-injectivity (A2), admissibility as non-premature selection (A3), and projection locking (A4). From A1–A2 alone, irreversibility and the arrow of time follow. Under the published formalisation, A3 supplies irreducibility and a non-trivial commutator. It does not force centrality, class-two nilpotence, or the Weil representation. From A4, the discreteness of quantum transitions follows. Papers at this level: ENI [38], Foundation [6], HeisenbergStructure [2], noscale [36].

Level 2 — Spectral fibre structure. The O-series supplies the Heisenberg group, its irreducible Schrödinger representation π_c on $\mathcal{H}_c = L^2(\mathbb{Z}/q\mathbb{Z})$, and the distinct associated Weil action of the symplectic automorphism group as its mathematical carrier data. Foundation and the earlier HeisenbergStructure versions claimed to derive this carrier, but HeisenbergStructure version 2.1 [2] refutes that selection proof. The O-series computes the spectral properties of the supplied fibre, including the capacity exponent δ_{pair}. Its proposed conversion to the cascade exponent β^* is not native to the Heisenberg substrate: the measured numerical agreement remains a conditional, cross-substrate phenomenological comparison rather than a derived mass-hierarchy mechanism. Papers at this level: SpectralAdmissibility [55], SpectralCapacity [56], SpectralGramRigidity [41], SpectralRelaxation [9], and O1–O33 ([47]–[79]).

Level 3 — Physical observables. Effective spacetime geometry, gauge structure, Standard Model charges and masses, gravitational waves, and cosmological observables are derived as consequences of Levels 1–2. No additional physical postulates are introduced beyond the continuum-limit hypothesis [H-L] of Q5a–Q5b (unestablished; Q5a version 3.2, Q5b version 2.1). Papers at this level: the Q-series (Q1 [30], Q2 [7], Q3 [83], Q5a–Q7), the gravity track (Spectral2 [51], Gravity [18], Thermodynamics, Lorentz [37]), and the companion papers ([14, 12, 3, 17, 63, 4, 72]).

3 Branch I: Axiomatic Foundations

3.1 The White Paper

The white paper [19] ("Cosmochrony: A Non-Injective Projection Theory of Emergent Physics," v3.14) is the programme's primary overview document. It introduces the substrate field χ, the non-injective projection Π, the BI saturation constraint, the relaxation mechanism, and the qualitative derivation of spacetime and quantum structure. The spectral entropy functional $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ appears here as the structural origin of the Einstein equation.

Several structural refinements informed by the O-series and Foundation results are incorporated:

- *Oriented state machine:* the admissible cascade is reformulated as a directed graph on admissible projections; physical histories are directed paths, and the first transition is guaranteed by non-commutativity $[X, \sigma(X)] = Z \neq 0$ [6].
- *Initial conditions dissolved:* the minimal state χ_0 cannot be a global minimum of the cascade; the fibre non-commutativity prevents any admissible projection from having trivial kernel, so the cascade begins as a structural necessity.

- *Inaccessibility of absolute zero:* the Heisenberg structure of the admissible fibre forbids projective stasis $U_{n+1} = U_n$ at any stage.
- *Algebraic necessity of fluctuations (Remark):* no admissible projection can have trivial kernel, so quantum vacuum fluctuations are a structural necessity, not a contingent feature.
- *Schrödinger as Mosco limit:* the continuous Schrödinger equation is derived as the Mosco limit of the discrete oriented admissibility cascade.
- *Extended numerical scope:* the PTO monotonicity is verified for $q \in \{29, 61, 101, 151, 211\}$, with 274 conjugate pairs and 360 post-burn-in steps.

Role: Programme overview; structural motivation; first formulation of the BI constraint and the relaxation cascade. *Evolution note:* The explicit substrate χ and relaxation mechanism of the white paper are not postulated in the Foundation reformulation: they emerge from axioms A1–A4. The white paper remains the primary narrative reference; Branch I papers are the formal foundation.

3.2 Non-Injectivity as a Structural Necessity of Genuine Emergence

The ENI paper [38] proves the following no-go theorem. Any genuinely emergent (non-isomorphic effective) physical description is necessarily non-injective and intrinsically compressive. The argument is purely structural: given a surjective $\Pi : \Omega \to \mathcal{O}$ such that $\mathcal{O}$ carries no independent microstate-resolving structure beyond Π, non-isomorphism of levels forces non-injectivity. A projection entropy $S_\Pi \geq 0$ is defined, and the equivalence genuine emergence $\Leftrightarrow$ Π non-injective $\Leftrightarrow$ $S_\Pi > 0$ is established (**proved**, Theorem 2.1 and Corollary 3.1).

Role: Foundational justification for structural non-injectivity (Axiom A2). Corollary 6 establishes recursive non-injectivity: any tower of genuinely distinct emergent levels carries one independent non-injective projection per stratum; Section 5 covers towers of emergence and structural colour confinement as projective necessity. A Remark (algebraic route) shows that in frameworks whose fibre carries a Heisenberg algebra $[X, Y] = Z \neq 0$, $S_\Pi > 0$ also follows directly from fibre non-commutativity: a trivial-kernel projection would require simultaneous diagonalisation of X and Y, which is structurally forbidden. The two derivations are complementary: Theorem 2.1 answers *what* is forced (framework-independently); the algebraic route answers *why* in Heisenberg-fibred frameworks. Cited by Foundation, HeisenbergStructure [2], Q1 [30], Bell, O32 [82].

3.3 Projection as Conditional Expectation and the Capacity Ceiling

The ProjCE note [95] (v1.0.0) derives the amplitude-level action of the non-injective projection: under four graded hypotheses — fibre-invariance (forced by the ENI emergence criterion), fixity on resolved content, linearity (the load-bearing structural transfer, at the tier of the ENI pivot lemma), and non-amplification (or a positivity/equivariance module variant) — the assignment of observed values is the conditional expectation onto the sigma-algebra of the distinguishable-history record, by two independent classical characterisations (contractive projections; Moy–Rota–Douglas averaging operators), with the operator invariant under the degraded minimum-distortion selection (hypotheses **structural**; theorems **proved** given the hypotheses). Two consequences follow as mathematics: strict ensemble-power suppression equal to the erased intra-fibre power (law of total variance; the structural answer to the cosmic-variance objection), and the parameter-free capacity ceiling $(N-1)/N$ for a complex Gaussian mode supported on N distinguishable histories (one-shot rate–distortion converse) — identically the Bessel envelope of LowLCapacity [89], upgraded from estimator heuristic to information-theoretic bound.

Role: Closes the entropy-to-power step of LowLCapacity as a conditional theorem chain; supplies the counting-to-power converter consumed by the low-ℓ note, the cosmological branch [3], and any $w(n)$ extraction from full-history compression [97].

3.4 Foundation: Admissible Non-Injective Transitions as the Primitive

The Foundation paper [6] (v2.2) states the axiomatic basis of the programme. From the four axioms A1–A4 it claims: (i) irreversibility and the arrow of time (A1+A2); (ii) the proto-state as a physically real unresolved configuration retaining phase coherence (A3); (iii) a non-trivial commutator between conjugate generators, forced by A3 and the minimality of A1; (iv) the identification of the admissible fibre symmetry group as $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ for a prime q, together with BI parity (A1–A3); (v) the identification $F_n \cong V_\rho$ (Weil representation) as Theorem 5.6; (vi) the discrete character of quantum transitions from A4; (vii) projective incompleteness (Corollary to Theorem 5.4): every admissible projection Π_n has a non-trivial kernel, so no observable description is ever complete — a direct consequence of $[X, \sigma(X)] = Z \neq 0$. The paper also establishes the threefold role of Π_n (Remark 3.5): admissibility filter, generator of temporal order, and revelation operator — three roles that are algebraically inseparable and all sourced in the non-commutativity. v1.10 adds the co-metric result $g^{\mu\nu} = \mathrm{diag}(-2, 2, 2, 2)$ (from Q5b, Q6b, Q8, Q10, Q11, all five now pending revision in the present cascade) to the results table, and a new section placing the four axioms relative to established formalisms (Hamilton–Jacobi, symplectic geometry, WKB, functional renormalisation group). v1.13 revises the entanglement framework: fibres are now defined as *infra-projectable cells* (unsplit configurations below the resolution of Π), the derivation of entanglement entropy is integrated into the foundation text by importing the admissible pair entropy of *Q3* [83] and *ENT-E2* [73], while the proposed Bell non-applicability implication is not available: a non-injective projection can possess deterministic local response functions, so A2 alone does not obstruct Bell factorisation.

Items (iv)–(v) are not established. The finite S_3 countermodel in HeisenbergStructure version 2.1 [2] has a faithful irreducible unitary carrier, a minimal generating pair exchanged by an involution, and a non-trivial commutator, but the commutator is not central. It also shows that the prime-order step reverses Schur's lemma.

Status: **structural** for the axiomatic architecture and the consequences that do not consume carrier selection; the Heisenberg/Weil identification is a postulated realisation; **open** for the continuum limit (Q5a version 3.2 withdraws the Mosco derivation; the spatial limit is the unestablished hypothesis [H-L] of Q5b version 2.1). *Role:* Axiomatic spine. Cited by all Q-series papers, HeisenbergStructure [2] , noscale, and recent companion papers.

3.5 A Finite Obstruction to Heisenberg Carrier Selection from Admissibility Constraints

HeisenbergStructure [2] (version 2.1) records the published proof attempt for Foundation Theorem 5.6, omitted from the Foundation paper for concision, and refutes it by an obstruction theorem. The central structural lemma — *admissible non-factorisability* — is a direct formalisation of Axiom A3. Its next step, "minimality implies central commutator", is false: a two-generator group need not be nilpotent of class two. The later prime-order argument is also invalid because an irreducible complex carrier contains one character of a central subgroup, not one non-zero eigenspace for every available character.

Status: **structural** for the non-factorisability setup; **open** for Heisenberg selection. *Role:* Documents the attempted carrier-selection proof and its conditional final Stone–von Neumann step.

HeisenbergStructure gives the six-element countermodel $G = S_3$, $X = (12)$, $Y = (23)$, with σ given by conjugation by (13). The standard action on the two-dimensional zero-sum subspace of $\mathbb{C}^3$ is faithful, unitary, and irreducible; σ exchanges the minimal generating pair; and $[X, Y]$ is non-trivial but non-central because $Z(S_3) = \{1\}$. The note independently corrects the Schur-lemma step and separates the failed selection edge from downstream mathematics internal to a supplied Heisenberg/Weil carrier (**proved**).

Role: Published evidentiary basis for the Found/Heis status correction.

3.6 No External Dimensional Scale at the Level of Admissibility

The noscale paper [36] establishes a structural rigidity result (Level 1): any deformation of the admissibility constraints that introduces a dimensional parameter independent of c_{BI} breaks at least one of BFS-consistent spectral scaling, structural non-injectivity (A2/ENI), or bounded admissibility closure (A3–A4). Dimensional analysis within the framework is therefore fully constrained by c_{BI} alone (**proved**).

Role: Closes the naturalness question for dimensional parameters.

3.7 Support and Preparation Criteria for Independent Effective Subsystems

EffComp [86] names the transversal contract

$$\mathrm{K4}: \quad \mathsf{Admissibility} \longrightarrow \mathsf{Composable\ effective\ subsystems},$$

whose source carries no supplied subsystem factorisation, and characterises its two prerequisite layers in the finite case. On a substrate Ω with reference admissible set, two effective projections, and cylindrical conditions combined by intersection, the independence blocks — outcome-set pairs that are margin-stable and jointly rectangular — are exactly the nonempty bicliques $S_A \times S_B \subseteq R$ of the projected support relation (**proved**). On a block, the full preparation simplex lifts every joint law, so the absence of a supplied factorisation alone obstructs nothing (**proved**); for the strictly positive cylindrical-conditioning families, closure under products of independently locally conditioned states holds iff the fibre-weight matrix has rank one (**proved**), and the counting countermodel N_3 shows that support rectangularity does not imply preparation independence (**proved**). A steering discipline separates locally prepared from steered conditional states; violating it manufactures spurious witnesses. The contract itself is proposed, not derived from A1–A4 or from an operational theory; the paper consumes no Heisenberg/Weil carrier and no continuum, and the general preparation layer and the states-and-effects layer of K4 remain open (**open**). Q3's bipartite singlet and the entanglement note's conjugate-pair closure are typed as consumers of a supplied composition structure.

Role: Opens the effective-composition track of Branch I; K4 joins carrier selection among the open structural deliverables.

3.8 Finite Criteria for Simplicial States and Local Tomography

STEFT [85] resolves K4's finite, mono-system, and bipartite-composition states-and-effects content, importing EffComp's Theorem 4.4 (rank-one product-closure) unchanged, as an already-proved hypothesis (**proved**). The raw cylindrical-conditioning family is never itself convex: it is finite while its own convex hull is a continuum, unconditionally, for any local outcome set of size at least two (**proved**). An explicit classical-randomisation postulate closes it onto the full probability simplex, locally and jointly, independently of the fibre-weight matrix's rank (**proved**). Affine effects on a simplex correspond bijectively to a cube of vectors, a bookkeeping fact special to simplices and not itself a claim about physical availability (**proved**); a weak coarse-graining axiom already yields the full Boolean effect algebra and the unit effect for free, while the full continuum of effects requires a further, dual randomisation postulate (**proved**). The two randomisation postulates are logically independent, and a common classical control resource reproduces both only under an explicit independence clause between its two interfaces, confirmed by an exact numeric witness (**proved**). Given physical existence of a composed bipartite effect, the pointwise product of two local effects is forced, not postulated (**proved**); local composition reaches only rectangle effects, strictly fewer than all joint sharp effects, and a CHSH-shaped separating witness shows this survives full local randomisation (**proved**). Local tomography holds exactly when each side's effect family linearly spans its ambient space, equivalently when it separates its local states and its linear span contains the unit effect (**proved**). All five governing axioms (classical randomisation of preparations and of effects, a common control resource, coarse-graining, and physical

composition) are explicit imports, not derived from admissibility or from the simplicial geometry alone; no Born rule or quantum state geometry is derived, and the general preparation layer of K4, together with the tensor product as a supplied structure, remain exactly as open as EffComp left them (**open**).

Role: Continues the effective-composition track of Branch I opened by EffComp; resolves K4's finite states-and-effects layer while leaving its general preparation layer, the tensor product, and any phase/modulus law open for a future front outside the classical/simplicial regime.

3.9 Refinement Consistency Obstructs Nonlinear Power Lifts of Finite History Measures

HMPL [99] takes up the phase/modulus law that STEFT leaves open, without deriving it: for any finite branching system, the power-lift family μ^α of its induced valuation is Kolmogorov-consistent, at a branching state reached with positive μ-valuation, if and only if $\alpha = 1$ (**proved**), by an elementary, self-contained comparison of x^α against x on $[0, 1]$ independent of the corpus or of any Cosmochrony-specific structure. A row-renormalised ("escort") lift restores Kolmogorov consistency trivially for every α, but generally departs from the pointwise lift once path-dependent normalisation factors accumulate along the history (**proved**); this situates the result relative to the pre-existing escort-distribution/Rényi literature of nonextensive statistical mechanics, not in isolation. Specialised, conditional on an explicitly named, non-derived maximal-entropy/Parry postulate (H2), to the non-backtracking Heisenberg b-shadow automaton supplied by TrajectoryBranching [97]: the square-root failure is re-derived independently in exact $\mathbb{Q}(\sqrt{2})$ arithmetic (**proved**); the automaton's endpoint and b-shadow quotients are exactly incomparable, with explicit witnesses (**proved**); relabelling $X \leftrightarrow X^{-1}$ transports the Heisenberg endpoint by an exact group automorphism, forcing every uniform-probe phase sum weighted by a real shadow-only valuation, over histories sharing a final b_n, to be exactly real (**proved**); and a bounded exact search over all 13120 non-backtracking words of length up to 8 finds no destructive cancellation among the 64 qualifying classes, 62 of which carry genuine nontrivial relative phase (**proved**, calibrated negative on this range). No Born rule, complex-carrier selection, or coherent-sum construction is established; H2's dependency is confined to the concrete Parry kernel and the search's specific counts, not to the quotient-incomparability or covariance results, which hold for any shadow-only valuation on the same automaton.

Role: Bridges Branch I's effective-composition track (STEFT's open phase/modulus question) and Branch III's trajectory-branching automaton; identifies the exact obstruction any future amplitude-modulus construction on that automaton must confront or explicitly bypass, without constructing one itself.

3.10 Branch I Synthesis: NIF-Note

The papers ENI [38], Foundation [6], HeisenbergStructure [2], and noscale [36] form the *non-injective foundations sub-programme*, whose synthesis paper is the presentation note **NIF-Note** [67] (Presentation Note 5, working-paper version 1.5). NIF-Note separates ENI's framework-independent implication from the conditional representation route: under ENI's surjectivity, informational-completeness and non-isomorphism hypotheses, genuine emergence implies non-injectivity with $S_\Pi > 0$. Foundation proves immediate information loss along non-injective transitions and projective incompleteness under A2; temporal order requires the unestablished acyclicity hypothesis [H-acyc]. Irreducibility requires a representation-theoretic dictionary; non-commutation then requires dimension greater than one and a pair generating the acting group. HeisenbergStructure's finite S_3 countermodel blocks the inference from non-commutation to centrality and Heisenberg carrier selection. Given a supplied Heisenberg group and non-trivial central character, Stone–von Neumann identifies its irreducible Schr"odinger representation; the associated Weil action is separate supplied data. The note records the static substrate and the axiomatic admissibility formulation. Carrier selection, the Born rule in every

sector, the non-trivial spatial limit [H-L] of Q5b, and Level 2 scale determination remain open. Q5a proves a zero-form Mosco limit on its canonical Fourier filtration and a normalisation no-go under [H-depth] and [H-w′].

Role: Synthesis of Branch I. Provides the explicit input/output table of the axiomatic foundations and is the canonical entry point for downstream sub-programmes (Notes 1–4 of the spectral admissibility, emergent geometry, gauge structure, and spectral gravity tracks).

4 Branch II: Spectral Admissibility Sub-Programme (O-Series)

The O-series investigates whether the charged-lepton mass hierarchy can be related to the spectral structure of the admissible fibre. The central quantity is the power-law exponent δ_{pair} of the pair-level capacity observable $\sigma_{\mathrm{pair}}(n) = \sigma_c(n) \cdot \sigma_{q-c}(n)$ along the BFS cascade. The proposed capacity-to-rate chain is

$$c_{\mathrm{BI}} \longrightarrow \delta_{\mathrm{pair}} \dashrightarrow \beta^* \approx 0.126 \longrightarrow m_e : m_\mu : m_\tau. \tag{1}$$

The finite capacity segment is studied within the supplied Weil model, but Weil conjugation is not a derived Born–Infeld fibre identification (O18 [22]); O24 does not remove that open bridge. The dashed segment is not a native theorem: SGN [35] proves that both corpus-defined Heisenberg realisations of the O4 closure fail, the native frontier is $N^{3/4}$ rather than $p^{1/2}$, the block capacity rather than the pair product carries the exact span increment, and the exponent coordinates agree only in the non-native reduced filling model. O25 [59] establishes the numerical agreement, not the dashed implication.

The presentation note **SpAdm-Note** [69] is the synthesis paper of this sub-programme. Version 2.0 maps the five precursor papers, O1 and O3–O33, and the Span-Growth and Critical Coverage companion notes along the chain bounded flux ⇒ admissibility envelope ⇒ Weil-sector capacity ⇒ $\dim_{\mathrm{pair}}$, on a supplied finite Heisenberg carrier and its associated Weil action, while separating proved mathematics, numerical measurements, conditional models, open bridges, and withdrawn claims. It records that the reciprocal prescription $\beta^* \approx 1/(\dim_{\mathrm{pair}} + \frac{1}{2})$ has no carrier on the Heisenberg measurement substrate, so the agreement $\beta^* \approx 0.126$ with the charged-lepton window is a coincidence check and not a derivation; that the SU(2) carrier is supplied conditionally; the O31 withdrawal; O32's numerical-only status; O33's exact finite architecture; and three open deliverables: hypothesis [H-color] (the two levels established in O32 stand), the identification of the spin-$\frac{1}{2}$ sector, and a full-breadth campaign beyond $q = 211$. Their individual contributions follow.

4.1 Precursor Papers: SU(2) Admissibility Sub-Programme

Three papers precede the O-series proper and explore spectral admissibility on Cayley graphs of finite SU(2) subgroups.

SpectralAdmissibility [55] (v2.0.3) carries out the spectral admissibility programme on two parallel chains. For the SU(2) chain $Q_8 \subset 2I \subset \mathrm{PSL}(2, \mathbb{F}_q)$, it derives the admissibility hierarchy $A_n^{\max} = c_{\mathrm{BI}}/\sqrt{\lambda_n}$ in the linearised regime and shows that the non-linear DBI correction preserves it. For the SU(3) chain $\Delta(27) \subset \Sigma(168)$, it establishes via character theory the hierarchy $\lambda_{\mathbf{3}} = \lambda_{\bar{\mathbf{3}}} = 28 < \lambda_{\mathbf{6}} = \lambda_{\mathbf{8}} = 42 < \lambda_{\mathbf{7}} = 48$ on $\mathrm{Cay}(\Sigma(168), C_4)$, proving that the fundamental and anti-fundamental representations are co-admissible and strictly dominate all competitors. The algebraic mechanism is uniform: $\chi_{\mathbf{3}}(g) = 1$ for all $g \in C_4$ plays the same structural role as $\chi_{\rho_4} = 0$ on Q_8 and the golden-ratio identity on $2I$ (**proved**).

SpectralCapacity [56] introduces the capacity functional $\mathcal{C}(G, S)$ and proves the binary-polyhedral maximality conjecture: the adjoint identity $\hat{A}_{\mathrm{adj}} = 2M - I$ (where M is the second-moment tensor of the axis distribution) shows that icosahedral isotropy forces $\mu_{\mathrm{adj}}(2I) = -\frac{1}{3}$, while axis-plane confinement forces $\mu_{\mathrm{adj}}(\mathrm{Dic}_n) \geq 0$ for all $n \geq 3$; combined with a Plancherel uniform bound, this yields a strictly increasing capacity gap $\Delta(\alpha) > 0$ for all $\alpha > 0$ (**proved**).

SpectralGramRigidity [41] shows that the geometry of neutral generating sets on SU(2) is fully encoded in pairwise character values $G_{st} = -\frac{1}{2}\chi_\rho(st)$ (**proved**).

Three Particle Generations [71] derives the three-generation structure from the ADE stratigraphic levels of binary Cayley graphs (**structural**).

4.2 SpectralRelaxation: Expander Graph Dynamics

The SpectralRelaxation paper [9] establishes $\Lambda_{\text{proj}} \asymp \lambda_2$ for LPS expander families (**conditional** on the capacity–isoperimetry closure and spectral-isoperimetric saturation hypotheses), providing the entry point for the O-series dynamics.

4.3 O1–O8: LPS Phase — Cascade Exponent β

O1 [47]: LPS saturation is a fixed-valence artefact; growing valence restores the pre-saturation window (**proved**). (Note: O2 was not written; the O3 derivation path was chosen instead.)

O3 [45]: First quantitative SM connection. The phenomenological window $\beta^* \in (0.09, 0.13)$ is derived from the charged-lepton mass ratios $m_e : m_\mu : m_\tau$ (**structural**).

O4 [13]: Bounded relational flux and Ramanujan expansion do not yet yield a cascade-exponent bound: the former derivation of $\beta \leq 1$ does not follow from the stated argument, and any structural upper bound on β by this route is open (**open**).

O5 [28]: Obstruction theorem. The β gap cannot be closed by any purely representation-theoretic or fixed-representation approach (**proved**).

O6 [34]: Matrix-level dynamic redundancy, $R_n \to 0$ for the Steinberg fingerprint; the confinement of the cascade exponent is a corollary conditional on a power-law Ansatz that the paper's own fingerprint does not satisfy, a genuine power-law regime remaining open (**proved** / **conditional** on the power-law Ansatz / **open**).

O7 [43]: Reformulates the target as a capacity exponent $\delta \in [7.4, 10.6]$ on three-step path fingerprints (**structural**).

O8 [1]: Establishes the geometric obstruction closing the LPS phase. Exponential shell growth $|S_n| \sim p^n$ on LPS graphs compresses the pre-saturation window to $O(\log q)$ BFS steps, on an empirically observed shell-growth premise (**structural** / **numerical**).

4.4 O9–O15: Heisenberg Transition

The geometric obstruction identified in O8 motivates replacing LPS graphs by Cayley graphs of $\text{Heis}_3(\mathbb{Z}/q\mathbb{Z})$.

O9 [44]: Structural transition. Polynomial ball growth $|B_n| \sim n^4$ provides $\Theta(q)$ usable BFS steps (**proved**). This is the key structural transition of the O-series.

O10 [42]: First large-q computation ($q = 101$). Measures $\hat{D} = 3.05$, continuing the convergence toward $D_{\text{hom}} = 4$; a TensorSketch representation obstruction is identified at large q (**numerical**).

O11 [75]: Bidimensional Fourier-character proxy overcomes the TensorSketch obstruction (**numerical**).

O12 [24]: Exact Weil-block fingerprints. The incremental block capacity $\Sigma_n^{(c)}$ depends on the block only through a projective pair (α, β), exactly independent of the central coordinate γ. Finite-q fitted slopes ($q \in \{29, 53, 61\}$) are crossover statistics of the block saturation-depth distribution; the intrinsic unfolded exponent is derived exactly, $\delta_{\text{exact}} = 3$ as $q \to \infty$, $n = o(\sqrt{q})$ (**proved**).

O13 [10]: Extended prime range $q \in \{101, 151, 211\}$. The finite-q exponent sequence $(4.42, 4.80, 4.51, 4.27, 3.59$ at $q \in \{29, 61, 101, 151, 211\})$ does not drift toward the phenomenological target at larger primes, consistent with O12's exact value $\delta_{\text{exact}} = 3$; measurement quality improves systematically with q (**numerical**).

O14 [77]: Diagnoses the observable-class mismatch in the proposed $\delta \mapsto \beta^*$ relation. Fixed-q normalisation changes intercepts, not intra-q slopes; the inter-q estimator layer remains open (**structural**).

O15 [39]: Heisenberg transfer no-go. The native Heisenberg identity is $\Delta r_n = |S_n|\Sigma_n$; the O6/O7 growth-law scalar observable is not equivalent to the exact Weil-block mean $\bar{\Sigma}_n$, so $\beta^* = 1/(\delta + \frac{1}{2})$ does not transfer to the exact-block setting. A checkable aggregation bound, $\alpha_{\mathrm{dyn}} \leq \hat{\delta}_{\mathrm{exact}}$ conditional on a non-negative OLS slope of $\log(1 + V_n)$, holds at two of five tested primes ($q = 151, 211$), open at the other three (**proved** for the non-equivalence theorem; conditional for the aggregation bound).

4.5 O16–O24: Pair Observable and the Fibre-Level Structure Programme

O16 [26]: Fibre admissibility hypotheses (**conditional** on the parity and realisation hypotheses). Conjugate blocks $\{c, q-c\}$ constitute the fibres of Π only under two independent, open premises: that Π carries a parity involution, and that this involution is realised in the Weil representation as $c \mapsto q - c$. If a per-block observable's conjugate-pair ratio is n-independent, the pair-level exponent doubles, $\delta_{\mathrm{pair}} = 2\delta_c$; on the real O12/O13 pipeline this holds trivially, since the exact conjugate-block identity forces the ratio to 1 (**structural** for the fibre hypotheses; algebraic identity for the doubling).

O17 [23]: Exact block-independence, scalar toy model. The raw Gram-Schmidt redundancy count is exactly independent of the character and both initial-condition parameters, for any two blocks whatsoever, not only conjugate ones — too broad a fact to license a same-fibre criterion for conjugate pairs specifically. Scoped to this paper's own toy model; not shown to extend to the real O12/O13 pipeline (**proved** for the block-independence result within the model).

O18 [22]: Born–Infeld parity covariance and the open fibre problem. The action is even, and parity acts on the response family with the character $(-1)^k$ of the derivative order: even-order responses are preserved under the transported-probe comparison, odd-order responses reverse sign (Prop. 2.6). An even countermodel refutes fixed-probe indiscernibility of χ and $-\chi$ (Prop. 2.4), so evenness does not imply $\Pi(\chi) = \Pi(-\chi)$; the fibre identification is a typed open classification problem (Problem 2.8), with (H-rank) one clause of it, and the Weil identification $\chi \mapsto -\chi$ as $c \mapsto q - c$ is an open bridge (**proved** for action parity and response covariance; fibre and Weil identifications: **open**).

O19 [16]: Proposed canonical normalisation. Generic Gram–Schmidt phase invariance and the rank identity are valid, but the factor $r(c, q) \in \{1, 2, 3, 4\}$ is not sourced for the real O12/O13 pipeline, the proposed normalisation is defined through an unspecified reference span, Tests P2/P3 are open, and the pipeline-independence proof is incomplete (**open** for the canonical observable; generic linear algebra: **proved**).

O20 [46]: Conditional threshold-crossing template. Given a constant threshold and an exact decreasing power law, crossing within a fixed cascade window is algebraically equivalent to an exponent interval. The map from rank increments to a fibre amplitude, the inherited Born–Infeld envelope, and the threshold itself are open; the numerical interval $[7.4, 10.6]$ is not derived (**conditional** on the monotone-amplitude and capacity-bound hypotheses).

O21 [54]: Local threshold-crossing diagnostic and open transfer. The two-point formula for n_3^{obs} is a valid extrapolated crossing rank under an exact power law and a fixed normalisation. The present registry's audit finds it not invariant under $\sigma \mapsto \lambda\sigma$, since the threshold base carries the factor and only the exponent is ratio-invariant: it scales as $\lambda^{1/\delta}$, so it does not eliminate amplitude or pipeline dependence. The threshold $9/q^2$, shell-alignment condition, and cross-substrate carrier remain external inputs; SGN excludes a native Heisenberg transfer for the corpus-defined objects (**conditional** on the shell-alignment conjecture and the external cross-substrate identification; the local algebra is **proved**; intrinsic rank and transfer **open**).

O22 [53]: BI projection locking forces the O21-defined n_3^{obs} onto a BFS shell within O22's stated model (Theorem 3.2). This does not make the input rank invariant under amplitude

rescaling and therefore does not close O21's intrinsic-definition problem (**structural** within the stated model).

O23 [70]: Conditional adjoint-dimension theorem. For a supplied irreducible two-dimensional unitary carrier $V_\rho \cong \mathbb{C}^2$, the traceless neutral sector is $\mathfrak{su}(2) \cong \operatorname{Im}\mathbb{H}$, of real dimension exactly three; in Q_8 the neutral generating set $\{\pm i, \pm j\}$ spans only two axes, so generator-axis counting cannot produce the three. The carrier selection and the identification of Σ_c with the module dimension are open bridges; the threshold $\Sigma_c(n_3) = 3$ is a supplied selection rule, definitional at the level of O21 (**conditional** on the supplied carrier and selection rule; the adjoint dimension is **proved**; threshold and bridges **open**). Version 2.1 determines what the deposited O12 shell-span filtration supplies on the carrier bridge. Its fingerprint vectors are pure Fourier modes, so every proper level is a Fourier-coordinate subspace whose Weyl support is one line L, and the exact stabiliser of that level in the Weil image is $U \rtimes M_n \subseteq B(L)$ with $M_n = \{s \in \mathbb{F}_q^\times : sF_n = F_n\}$ (**proved**, every odd prime, every generic block, every proper nonzero level). This excludes the *exceptional* binary polyhedral groups $2T$, $2O$, $2I$, including at primes where they exist in the ambient group (**proved**). It does not exclude a spinor carrier: at explicit generic levels at $q = 53$ and $q = 101$ the stabiliser is Dic_q, whose restricted Weil module is multiplicity-free — one character plus four inequivalent two-dimensional irreducibles, exactly two of them faithful and SU(2)-valued (**structural**: exact computation on deposited witnesses with reproduction code, not a universal claim). The carrier bridge therefore splits into two open steps: selecting which prime, block and depth the projection singles out within the generic filtration, which no result addresses; and, inside a level already known to be dicyclic, choosing between the two admissible carriers, for which the candidate "unique odd member of $\{c_\Sigma, q - c_\Sigma\}$" succeeds on all 17 audited instances over 13 distinct subspaces with one step observed and not proved (**open**).

O24 [40]: Capstone of the fibre-level construction. The Verticality Lemma ("$\ker\Pi$ is free, $\operatorname{Im}\Pi$ is constrained") closes the $c_{\mathrm{BI}} \to \delta_{\mathrm{pair}}$ segment with respect to the fibre structure of Π (**conditional** on the Weil realisation and the stated fibre hypotheses). It does not derive the growth-equation conversion to β^*: SGN refutes the native Heisenberg realisation of its O4–O6 closure, frontier, pair-carrier, and exponent-coordinate inputs.

4.6 O25–O30, PTO, and Fibre Erasure: Numerical Validation, Carrier Diagnostics, a Conditional Covariance Model, and Information-Theoretic Axes

O25 [59]: Systematic pair-level numerical campaign, extended to large primes (**numerical**). Computes δ_{pair} exhaustively across all conjugate pairs through $q = 211$; the capped-BFS extension — reproducing the observable exactly — uses 24 representative pairs at $q = 307$ and 12 at each of $q = 401, 601$. The inter-pair concentration establishes a stable fixed-q pair statistic; it does not prove independence from q. Central result of the extension: the *raw* exponent $\delta_{\mathrm{global}}(q)$ descends monotonically into the admissible window $[7.4, 10.6]$ on its own, reaching 7.61 at $q = 601$, with no correction; the O14 finite-size correction, needed at small q, becomes unnecessary and eventually *overcorrects* at large q (sending δ_{corr} below 7.4). The asymptotic value δ_∞ is not pinned ($1/q$ and $1/\sqrt{q}$ laws remain equally viable at $q \le 601$). The reported $\log(n+1)$ convention inflates finite-window slopes relative to the paper-level $\log n$ definition. As a separate numerical comparison the legacy reciprocal image stays in the narrow band 0.108–0.123, near the mass-fitted window. SGN proves that this agreement is not a native capacity-to-rate derivation (**structural** as a phenomenological coincidence check).

O26 [49]: Dictionary between conjugate Weil blocks and rank-1 matrices in $M_2(\mathbb{C})$, connecting the Heisenberg fibre to $2I \subset \mathrm{SU}(2)$ (**structural**). The note takes the pair-sector fibre structure as a structurally motivated hypothesis (O18 states the fibre identification as open): in the pre-saturation regime $\sigma_{\mathrm{pair}}(n)$ has the growth exponent of the Hilbert–Schmidt norm of an associated matrix trajectory (Level I, **proved**); the quotient identification (Level II) is a conjecture whose open point is the n-independence of the proportionality constant, and the canonical

representation-theoretic identification in an isotypic sector of $2I$ (Level III) is conditional on the admissible embedding $\Phi_{q,\rho}$. Version 2.1 proves that under Hypothesis 4.4(a), in a fixed conjugation basis, the embedded products are uu^{T} and span a subspace of $\mathrm{Sym}^2(V_\rho)$: their span has dimension at most $d_\rho(d_\rho+1)/2$, hence at most three for $d_\rho = 2$. The rank-four target is excluded under this contract, not for other embeddings or constructions. Tests 1–2 are implementation checks; the O28–O29 effective ranks are measured in $\mathrm{End}(H_{\mathrm{eff}})$, not $\mathrm{End}(V_\rho)$, and occupy no row of the embedded-covariance decision table. This representation-side result does not supply the missing rate equation.

O27 [5]: Quotient factorisation alone does not imply equivariance (**proved** / **open**). Saturation, the supplied carrier, and parity covariance are explicit inputs. Under the stated equivariance hypotheses, the Schur-level factorisation and multiplicity counts are proved; a canonical equivariant vector lift remains open. The result supplies no derived interpretation of β^*.

O28 [8]: Two measurements from the Q5a-O5 checkpoints (**numerical**). Part A: out of sample (to $q = 601$) the calibrated linear extrapolation $n_1 \sim \hat{\alpha}\, q$ ($q \le 211$) fails, overpredicting n_1 by up to +88%; the finite measurements reject only that extrapolation, and the exact law is the interval theorem of the CC-Note [32], of which the measured depths are the transitional-regime data; the declared finite-size adjustment $\delta_{\mathrm{corr}}(q)$ lies in $[7.4, 10.6]$ over $q \in \{29, 61, 101, 151, 211\}$, and O14 establishes that this term does not correct the intra-q fitted exponent, so its stability is a diagnostic and not a corrected capacity exponent. Part B: formal effective-dimension computation of O26 Criterion 5.4 in $\mathrm{End}(H_{\mathrm{eff}})$, $H_{\mathrm{eff}} = \mathbb{C}^3$ (HEFF_DIM = 3, implementing the supplied selection rule $\Sigma_c(n_3) = 3$; the retained dimension is fixed in advance, so these ranks characterise the truncated carrier, not a measured admissible rank). O28 reported 1% threshold rank $r_{\mathrm{eff}}^{1\%} = 3$ and resolved spectrum $[\lambda_1 : \lambda_2 : \lambda_3] = [1 : \frac{1}{2} : \frac{1}{2}]$ across its stated perimeter. O29's extended recomputation does not reproduce pairwise invariance: rank three occurs for 32/50 pairs at $q = 101$ and 103/105 at $q = 211$. It re-audits the finite-data rank and degeneracy without assigning them an exact axial origin. The gap from $d_\rho^2 = 4$ cannot be inverted to identify V_ρ: O29 distinguishes the measured span in H_{eff}, an auxiliary PCA truncation, and the separately supplied spin-$\frac{1}{2}$ module.

CC-Note [32]: Exact resolution of the saturation-depth asymptotics of O25–O28. Every implemented Weil–BFS fingerprint is a non-zero scalar multiple of a single Fourier character, so the cumulative rank is the sumset cardinality $r_c(n) = |T_c' + C\, I_{n+1}|$, with universal shellwise ceiling $\Delta r_c(n) \le 2|T_c'| \le 18$ and a complete classification of the block resonances that lower it (**proved**). Exact sphere counts of the infinite Heisenberg group give the strict crossing $s_{22} = 16{,}934 < 18{,}000 < s_{23} = 19{,}412$: deterministically $n_1(q) \le 22$ and $5/q^2 \le x_1(q) \le 99{,}689/q^2$ for every prime $q \ge 311$ (**proved**), and under uniform generic block sampling $n_1(q) \to 22$ and $q^2 x_1(q) \to |B_{22}| = 99{,}689$ in probability (**proved**). The measured O28 depths are the transitional resonance regime of this law (effective exponent ≈ -0.76 en route to the exact -2); the coverage-collapse ansatz is degenerate near the threshold front, and the implemented auto-calibrated window closes deterministically for $q > 937{,}024$. In the Q5a dual-window limit the potential coefficient vanishes, $\lambda = \lim x_1/C_{\mathrm{Heis}} = 0$: no oscillator route toward a spatial limit opens from this observable.

Span-Growth Note [35]: Native span growth and the failure of the Heisenberg capacity-to-rate transfer. The exact increment identity $\Delta r = \sigma_c\, |S_n|$ and the polynomial spheres ($D_{\mathrm{hom}} = 4$, Bass–Guivarc'h) yield a three-branch classification for abstract real-valued weights: growth, logarithmic growth, and saturation according as $\delta_c < D$, $\delta_c = D$, or $\delta_c > D$ (**proved**). Since the measured rank increments are integers, the pointwise hypothesis becomes unrealizable when $\delta_c > D - 1$; the production measurements therefore support only an exact finite-window formulation with two-sided bounds, not an asymptotic rank law. SGN proves a three-legged no-go for the proposed native transfer: both corpus-defined valence–exploration closures fail; the Heisenberg frontier is $N^{3/4}$ rather than $p^{1/2}$ and no Born–Infeld feedback carrier is defined in the corpus; the native block increment is neither the pair product nor expressed in the reduced-model exponent coordinates. Thus the $1/(\delta_{\mathrm{pair}} + \frac{1}{2})$ comparison is a cross-substrate phenomenological prescription, not a native derivation. The shell-growth factor $D - 1$ is recovered additively, while the

inter-q estimator correction remains open; fitting against $\log(n+1)$ instead of $\log n$ inflates the fitted exponent by 8–13% on the stated production windows. The window-effective exponents make contact near $q = 211$ and separate strictly for $q \geq 307$ ($\delta_c^{\text{eff}} - D_{\text{eff}} = -0.21$ to -0.49 at 6.7–14.5 inter-pair bootstrap standard errors, sign stable across all window-shift and knee-trimming variants and both logarithmic conventions): the contact is a transitional crossing in the accessible data, no current indication forces $\delta_c = D$, and the asymptotic value of δ_c remains open. The note's repository embeds the block-level data actually used (with frozen windows and per-checkpoint provenance), a single reproduction script, and the crossing figure: every quoted number is reproducible from the published deposit alone.

O29 [74]: Veronese collapse in the measured trajectory span (**numerical**). The exact Weil-block identity $\rho_{q-c} = \bar{\rho}_c$ (an O17 theorem recalled in O18) motivates but does not prove the projected-coordinate relation $\pi_{q-c}(v) = \overline{\pi_c(v)}$, which the stored data satisfy only approximately; under it every outer product $M_j = \pi_c(v_j) \otimes \pi_{q-c}(v_j)^*$ is complex symmetric. Rank three is modal in $\mathrm{End}(H_{\text{eff}})$, with seed-sensitive single-sample exceptions of four to nine at $q = 101$ and five at $q = 211$. None of these measurements is an embedded-covariance rank in $\mathrm{End}(V_\rho)$. On the O25/Q5a checkpoints the admissible trajectory spans all three complex dimensions of H_{eff}. At $q \in \{29, 61\}$ the symmetric-square evaluation matrix has numerical rank 3 at the declared tolerance; fitted constraint commutators have numerical rank 3 in a chosen $\mathfrak{so}(3)$ basis and the computed commutant is one-dimensional. These finite-precision diagnostics are compatible with the adjoint module supplied conditionally by O23, but do not derive or identify it. Numerically: $r_{\text{eff}} = 3$ universally for $q = 61$ (30/30 pairs) and $q = 151$ (75/75 pairs) with zero inter-pair variance; at $q = 211$, the single-sample run gives $r_{\text{eff}} = 3$ for 103/105 pairs; at $q = 101$, modal $r_{\text{eff}} = 3$ is confirmed by a dedicated multi-sample analysis ($M = 5$ resamples per anomalous pair, 76.7% of (pair, sample) cells satisfy $r_{\text{eff}} = 3$, with modal $r_{\text{eff}} = 3$ for all 18 pairs), without establishing that the remaining dispersion is an artefact. The two-dimensional PCA space $U_c^{(2)} \subset H_{\text{eff}}$ is auxiliary and has no canonical representation status; rank 4 in $\mathrm{End}(U_c^{(2)})$ after broken locking identifies neither V_ρ nor $d_\rho = 2$. The latter remains an O26 minimality selection, not an O29 data result.

O30 [78]: Conditional 3×3 covariance model on $\mathrm{Sym}^2(V_\rho)$ (**conditional** on hypotheses (H0)–(H3)). Under the supplied hypotheses (H0)–(H3), including Veronese data, an inter-block identification, weight exchange, and a fixed projected symmetric square, a lift satisfies $\beta_j = \varepsilon_j \bar{\alpha}_j$ with $\varepsilon_j \in \{\pm 1\}$. In the equatorial regime, the central component $\sqrt{2}\,\alpha_j \beta_j$ of $\mathrm{vec}(M_j)$ is phase-independent (the phases cancel exactly), while the diagonal components α_j^2 and β_j^2 oscillate as $e^{\pm 2i\phi_j}$. The covariance diagonalises only under the two weighted conditions $\langle \varepsilon_j r_j^4 e^{2i\phi_j} \rangle = 0$ and $\langle r_j^4 e^{4i\phi_j} \rangle = 0$, yielding $\mathcal{C}_c = \langle r_j^4 \rangle \cdot \mathrm{diag}(1/4, 1/2, 1/4)$ and spectral ratio $[1 : \frac{1}{2} : \frac{1}{2}]$ exactly. The factor of 2 is the squared symmetric-tensor normalisation coefficient in $e_0 = (v_+ \otimes v_- + v_- \otimes v_+)/\sqrt{2}$, isolated by the equatorial constraint as the sole source of the unequal diagonal entries. The Carnot grading labels the direction that carries the factor; it is not its algebraic cause. O30's 3×3 object is distinct from O28's measured 9×9 covariance and does not derive its spectrum; the O28 ratio remains empirical.

O31 [115]: Withdrawal notice for unsupported colour-triplet co-admissibility and gauge-group claims (**open** / **proved** for the three counterexamples). Version 2.0 withdraws eight result families: uniqueness of SU(3); irreducibility of the proposed fibre-preserving group; equality of the three individual capacity profiles; the arithmetic criterion $q \equiv 1 \pmod 3$ for triplet existence; the metaplectic step that changes central character while using a centre-fixing automorphism; the $\Delta(27)$ spectral proposition; the mistyped rank-eight test in $\mathrm{End}(\mathbb{C}^3)$; and every derivation of a gauge factor or of the complete Standard Model gauge group. The notice gives three explicit counterexamples: the proper irreducible subgroup $\mathrm{SO}(3) \subset \mathrm{SU}(3)$ refutes uniqueness; the diagonal torus $T^2 \subset \mathrm{SU}(3)$ satisfies the stated constraints while acting reducibly; and $\{1, 2, q-3\}$ is an admissible triplet for every prime $q \geq 7$, refuting the arithmetic criterion. The remaining scalar, arithmetic and discrete-group material is under re-audit. O31 supplies no positive conclusion about SU(3), a gauge factor, or the Standard Model gauge group. **O32** [82]: tests [H-color]

numerically on $q \in \{61, 151, 211, 307\}$ (test, $q \equiv 1 \pmod 3$) and $q \in \{29, 101\}$ (controls). Noise floor `ctrl_ref` $= 1.657 \times 10^{-3}$. Normalised ratios: $R = 2.1$ ($q = 61$, 1/4 triplets valid), $R = 0.4$ ($q = 151$), $R = 1.0$ ($q = 211$), $R = 0.7$ ($q = 307$). All four pass $R \leq 3$; $\delta_{\text{tri}} \approx 3\delta_c$ at the percent level (0.7% at $q = 151$, 1.3 to 2.2% elsewhere); C_{color} numerical rank 1 at $q \in \{61, 151, 211\}$. The rank and pointwise levels of [H-color] rested on the superseded O31 and are open; the averaged level ($O(q^{-1})$, O32 Proposition 1) and the effective level stand as O32's own results. The O32 numerical values remain reported measurements; they do not establish individual-profile co-admissibility or identify a gauge group. Status: **numerical**for the reported measurements; their former O31-based structural interpretation is withdrawn and under re-audit.

O33 [79]: arithmetic Mackey–Hecke structure of the O12 three-character filtration (exact core, **proved**). The cumulative Fourier mode set is proved to be the cyclic sumset $K_n = c[-(n+1), n+1] + \{0, \pm(c_2 + c_3)\} + \{0, \pm c_3\}$. This yields an exact cyclic-spacing criterion for boundary porosity and reduces the full Weil stabiliser to the multiplicative symmetry group $A_K = \{a : aK = K\}$. Weyl quantisation realises the induced Mackey carrier internally in $\text{End}(V_\rho) \cong \mathbf{1} \oplus \mathcal{D}_+ \oplus \mathcal{D}_-$. Generic principal-series pairs carry canonical $M_2(\mathbb{C})$ commutants; Harper dynamics gives the normalised split $1 \pm q^{-1/2}$. The eliminated residue supplies a second axis exactly when the short boundary character sum $C_\theta(K)$ is non-zero: a Galois separation theorem proves $\Delta_{\theta,c}(K) \neq 0 \iff C_\theta(K) \neq 0$ for every generic character, the hole pairs are geometrically bounded by four, and the zeros of $C_\theta(K)$ are completely classified into antipodal-pair, cube-triple, and double-pair mechanisms. The deposited parity-even operator algebra has zero projection onto $\mathcal{D}_-$, proving an odd-sector protection theorem. The stabiliser order divides $\gcd(q-1, |K|-1)$, and two uniform constructions give exactly C_4 from a square root of -1 and C_6 from a primitive cubic root. In the degenerate regime the depth-zero square extends to an infinite uniform Gaussian-box family $Q_r(i)$ with $C_4 \subseteq A_K$, realised at depths $r-1$ for $r \leq 4$, so the family statements are mechanisms rather than an exhaustion; an exhaustive audit through $q = 151$ classifies all 75 exceptional sets into Gaussian boxes, free four-point complement orbits, and Eisenstein C_6 sets, with no order eight at $q = 193, 241, 257$; completeness of the refined mechanisms remains conjectural. The exact stabiliser value is proved inside two of the three mechanisms. A fourth-moment invariant, resting on an exact Gaussian collision dichotomy and an explicit $\mathbb{Z}[i]$ division certificate, gives $A_{Q_r(i)} = \langle i \rangle \cong C_4$ for every proper box, direct or wrapped; a free four-point complement orbit likewise has stabiliser exactly C_4, by the free action of A_K on a four-element set. Completeness of the refined mechanisms, and a uniform proof of the Eisenstein top autocorrelation level, remain open; the unit groups of the Gaussian and Eisenstein integers are recorded as an organising principle for that residual question, not as a completeness theorem. The asymptotic porosity law is proved: the porous depths of a uniform admissible block converge to an explicit limiting point process, with count law $(3/20, 7/24, 9/40, 5/24, 1/8)$, mean $28/15$, piecewise affine intensity, and conditional mean normalised depth $491/10080$, machine-certified by an archived exact rational-polyhedral script. An exact audit of 31,166 porous cases through $q = 311$ confirms the zero classification with no exception; no Standard-Model or Dirac-mass identification is claimed.

PTO [48]: The cumulative projected capacity $I(n) = \sigma_{\text{pair}}(0) - \sigma_{\text{pair}}(n)$ is strictly non-decreasing and hence compatible with an arrow of time. Confirmed numerically (**numerical**) for $q \in \{29, 61, 101, 151, 211\}$. The admissible cascade is shown to carry the structure of an oriented state machine: nodes are admissible projections, edges are admissible transitions, and branching weights play the role of quantum probabilities. The first transition $U_0 \to U_1$ is guaranteed by projective incompleteness (Foundation Corollary to Theorem 5.4): temporal ordering starts as a structural necessity, not a postulate. Open: whether the characteristic shell $n^*(q)$ coincides with the locking depth $n_c(q)$ at which BFS coverage transitions from locally to universally dominated — which would connect the spectral ordering to the effective geometry results of Q10 [80].

Fibre Erasure [96]: Hub paper between ENI [38] and the BFS spectral cascade (**conditional** on [H-suff] and [H-sg]). Defines a residual fibre-information functional $I(c; \sigma(\ell))$ on the BFS coarse-graining axis ℓ, distinct from and orthogonal to the temporal axis n of PTO. Conditional on the structural sufficiency hypothesis [H-suff], the data-processing inequality implies

that $I(c;\sigma(\ell))$ is non-increasing in ℓ: fibre information is erased, not created, under admissible coarse-graining (Thm 4.1). [H-suff] is proved unconditionally at the BFS rank level (Prop. 3.6, label-blindness from Schur's lemma); for the full Born–Infeld capacity it is numerically supported for $q \in \{61, 151, 211\}$. Strictly not a c-theorem: no counting of effective degrees of freedom is claimed (listed as open problem [O-2]). Together with PTO, constitutes the information-theoretic architecture of the admissibility cascade: resolution axis ℓ (Fibre Erasure) and temporal axis n (PTO) are structurally orthogonal.

5 Branch III: Q-Series and Physical Applications

5.1 Q-Series: Quantum Mechanics and Spacetime Emergence

The Q-series develops quantum-mechanical and geometric structure from the Foundation axioms, taking as supplied input the Heisenberg/Schrödinger carrier, its distinct associated Weil action, and selected O-series results; additional targets denoted V_ρ are typed separately in the papers that use them.

Q1 [30]: Proves an exact, unconditional Fourier-support rigidity theorem for the associated finite Weil action on the supplied Heisenberg/Schrödinger carrier (**proved**), and a precisely scoped no-go corollary for support-, rank-, and Gram-based diagnostics. Does not derive phase coherence, a singlet correlator, a Tsirelson bound, or the Born rule (**open**).

Q2 [7]: Proves an exact finite-group fact (**proved**): on the binary icosahedral group $2I$, the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sectors share the Laplacian eigenvalue $\lambda_{1/2} = \lambda_{3/2} = 18$. Does not derive phase coherence, a Casimir/isotropy normalisation, the Born rule, a Tsirelson-type bound, or an SU(2) fixed-point/sector-selection argument (**open**).

Q3 [83]: Proves a theorem conditional on an explicit, undemonstrated hypothesis (**conditional** on [H-inv] and the supplied composition): given a supplied bipartite carrier $V_{\chi_{2j+1}} \otimes V_{\chi_{2j+1}}$ and the hypothesis that the joint state is invariant under the diagonal $2I$-action — *not* derived from admissibility or from Born–Infeld indiscernibility — Schur's lemma applied to $\mathrm{Hom}_{2I}(V^*_{\chi_{2j+1}}, V_{\chi_{2j+1}})$ uniquely identifies the state as the singlet $|\Omega_j\rangle$, with an explicit Casimir correlator $E(\hat{a},\hat{b}) = -j(j+1)/3 \cdot (\hat{a}\cdot\hat{b})$, for all five admissible sectors $j \in \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$ of $2I$. Does not derive the invariance hypothesis itself, phase coherence, the Born rule, or a CHSH/Tsirelson-type bound (**open**).

The papers Q1, Q2, Q3, and the Bell paper [14] form the *quantum structure sub-programme*, whose synthesis paper is the presentation note **QS-Note** [68] (Presentation Note 7). QS-Note states plainly the current, narrow scope of each constituent: Q1 proves an unconditional Fourier-support rigidity theorem unrelated to phase coherence or the Born rule; Q2 proves an exact finite-group eigenvalue degeneracy on $2I$; Q3 proves a theorem conditional on an explicit, undemonstrated diagonal-invariance hypothesis. No chain from Born–Infeld indiscernibility to phase coherence, a singlet correlator, a Tsirelson bound, the Born rule, or an SU(2) fixed point is currently established by these three papers. Bell [14] is published in *Quantum Reports*, but its claimed implication from non-injectivity to failure of Bell factorisation is disproved by a finite Bell-local countermodel. Its PR-box construction also has empty support. The general no-go theorem and the correct obstruction are proved in [91]. The Bell bridge is open: the programme does not derive an obstruction to a setting-independent global coupling. The note records the proved/structural/open status of every constituent result and states the open deliverables: the Born rule, phase coherence beyond Q1's narrow rigidity statement, and a derivation (rather than a supplied hypothesis) of Q3's diagonal-invariance premise.

The papers Q5a, Q5a-O2, Q5b, Q6b, Q7–Q11, U1, W1, and H2 form the *emergent geometry sub-programme*, whose synthesis paper is the presentation note **EG-Note** [61]. It maps these twelve papers into the chain $\Pi_q \to L_{\text{eff}} \to g^{\mu\nu} = 2\eta^{\mu\nu}$, deriving the effective four-dimensional Lorentzian metric from the admissibility filter acting on the Weil representation, with all four metric coefficients read off the $\mathfrak{su}(2)$-Casimir on the spin-1 module $\mathrm{Sym}^2(V_\rho)$. The chain now

carries two distinct gaps. At its first link, Q5a version 3.2 withdraws the Mosco derivation of the spatial limit operator, which is now the explicit, unestablished hypothesis [H-L] (Q5b version 2.1). At the Q7 step, Q7 version 2.0 records the identification of the measured space with that spin-1 module as supplied by no source, so the carrier of those four coefficients is itself open. The geometric convergence results are independent of both. The algebraic rigidities are independent of [H-L] but not of the identification: they are statements about the spin-1 module, and what is open is that the measured space is it. Their individual contributions follow.

Q5a [15]: Version 3.2 (*Canonical Fourier Filtration and the Obstruction to a Spatial Continuum Limit*) works on a supplied finite Heisenberg group and non-trivial central character; it does not select that carrier or its associated Weil action. It identifies the canonical filtration exactly — the stage is precisely the toric Fourier window $\mathrm{span}\{e^{2\pi ibx/q} : |b| \le n\}$ — proves that the published admissibility form converges to the zero form on it (rate q^{-2}), and proves a normalisation no-go: no common scalar normalisation yields a non-trivial toric differential operator; the only non-trivial rescaled limit is a conditional Dirichlet form in the rescaled *frequency* variable. The normalisation no-go is proved under [H-depth] and the measured positive finite weight limits; the dual-window limit is conditional on a subsequential critical-coverage limit. The former Mosco derivation of $L_\Pi = -A\partial_x^2$ (versions up to 2.1) is withdrawn; the spatial limit operator is now the hypothesis [H-L] and Q5 is open (**proved** for the negative results; **open** for the spatial continuum).

Q5b [11]: The BFS shell stratification of $\mathrm{Heis}_3(\mathbb{R})$ produces a four-dimensional Lorentzian spacetime, with the temporal direction identified with the BFS depth (**conditional** on [H-L]). Q9 [111] proves a kinetic-limit theorem under separately supplied kinetic Mosco convergence [K] and compatible recovery [R]. It does not derive [H-lift] from the frozen, unestablished [H-F]. The spatial limit input is formalised as the explicit, unestablished hypothesis [H-L], on a supplied Heisenberg carrier (version 2.1); the Carnot/$D_{\mathrm{hom}} = 4$ results are independent of it (**structural** for the geometry; conditional on [H-L], and on the identification Q7 version 2.0 records as supplied by no source, for the metric reading, which is pending revision).

Q6a [31]: Gauge charges are the invariants of the projection-fibre equivalence classes (**open** for the colour factor and the complete gauge group). U(1) and SU(2) are identified as fixed points of the admissibility flow. Its former derivation of $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ used O31 and is unsupported after the O31 withdrawal; the colour factor and complete group are open.

Q6b [20]: Effective Lorentzian geometry from the admissibility filter Π_q; its metric reading consumes the unestablished [H-L] and [H-lift]. Q9 supplies a conditional kinetic-limit criterion, not a discharge of the latter. Derives the effective metric $g^{\mu\nu}(x) \propto A^{\mu\nu}(x)$ of Lorentzian signature $(-,+,+,+)$ from the principal symbol of $\mathcal{L}_{\mathrm{eff}}$; with Q8, Q10, Q11 the co-metric reads $g^{\mu\nu} = \mathrm{diag}(-2,2,2,2)$, which Q7 version 2.0 records as also requiring the identification of the measured admissible space with the spin-1 module, supplied by no source. Q6b's own remarks call that co-metric fully explicit and fully determined, so those conclusions are pending revision. Proves the Schwarzschild metric as the unique stationary exterior solution under spherical symmetry and flux conservation; the horizon appears as a principal-symbol degeneracy. Establishes the Hamilton–Jacobi equation $g^{\mu\nu}\partial_\mu S\,\partial_\nu S = 0$ as the effective massless propagation structure. The Einstein equations emerge as consistency conditions via the spectral entropy functional. Presents the geometric chain $\Pi_q \xrightarrow{\text{Q5a}} \mathcal{L}_\Pi \xrightarrow{\text{Q5b}} g^{\mu\nu} \xrightarrow{\text{Gravity}} G_{\mu\nu}$ as closed (**conditional** on the Q5a Mosco hypotheses and [H-L]); that closure additionally requires the identification, and is pending revision.

Q7 [112]: Version 2.0 states what an equivariant bridge between the supplied admissible dimension $\dim H_{\mathrm{eff}} = 3$ (the O23 selection rule, implemented as HEFF_DIM = 3 in the O25–O29 pipeline) and the three spatial directions of Q5b would require, and proves the requirements that follow from representation theory on a supplied carrier. Proves $[\mathfrak{su}(2)] \ncong \mathfrak{heis}_3$ unconditionally (the bridge cannot be a Lie algebra isomorphism), and, as unconditional representation theory, that $\mathrm{Sym}^2(V_\rho)$ is up to isomorphism the unique irreducible three-dimensional complex $\mathfrak{su}(2)$-module, isomorphic to the complexified adjoint. *Under an explicit identification hypothesis [ID]*

that no source supplies, the measured space inherits that structure and an equivariant map to a three-dimensional adjoint module is either zero or unique up to one non-zero complex scalar (reality and positivity being a separate requirement and then a convention). That the real spatial sector *is* such a module is the separate hypothesis [ACT] below. Non-zero intertwiners exist once the two isomorphic modules are supplied. Their geometric compatibility is separate: real rotation invariance [INV] yields horizontal isotropy, and the coefficient transport additionally requires an adapted unitary intertwiner [ADAPT], not implied by Schur or [ACT]. Q5b's spatial principal symbol has rank two everywhere and cannot equal a positive rank-three Casimir form under invertible transport. The revised conjecture instead asks for a new positive spatial target with a specified operator and compatible action; lower-order terms cannot change the existing principal rank. Proves analytically $[F_c, L_{\mathrm{Weil}}] = 0$; version 2.0 withdraws the application of its corollary to these data on two counts: the three stored rows are not F_c-eigenvectors, so the corollary's hypothesis is not met, and its conclusion would in any case concern a compression of the discrete operator rather than the cross terms of $\sigma_2(L_{\mathrm{eff}})$, whose identification is missing. Its numerical section is re-typed: the three stored basis rows of the O25 checkpoints are pure Fourier modes, so every *diagonal* entry of the compression *in the stored basis* is the Rayleigh quotient $4 - 2\cos(2\pi m/q)$ of $L_{\mathrm{Weil}} = 4I - W_a - W_a^\dagger - W_b - W_b^\dagger$, while an off-diagonal entry of modulus one appears exactly when two stored indices differ by $\pm c$ mod q, which is the paper's replacement account of the two pairs previously reported as anomalies. The entry $\tilde{L}_{00} = 4 - 2 - 0 = 2$, historically named A_z, is thus the flat-mode value. The pipeline reports it in that slot at eleven of the twelve pairs, the flat mode being ordered last at $(101, 3)$. Q7 version 2.0 records that this value follows from the $4I$ normalisation and is not normalisation-invariant, so its agreement with the Casimir carries no evidential weight; A_z itself is left an unevaluated parameter. The difference between the two distinct entries, $4\sin^2(\pi k/q)$, is an identity, so the vanishing of the gap is equivalent to $\mathrm{dist}(k/q, \mathbb{Z}) \to 0$, a property of the pipeline's mode-selection rule that the corpus does not derive; the published fit $\approx 0.364\, q^{-0.52}$ tracks that rule rather than a spectral rate. What the pipeline reports is the compression after conjugation by the covariance eigenbasis, which as a matrix coincides with the stored-basis one at nine of the twelve pairs, agreeing in modulus but not in phase at a tenth and differing outright at the other two; the invariant content is its spectrum. Version 2.0 accordingly withdraws, as results and as supports, the Casimir reading, the isotropy fit, and the downstream values imported from Q8, Q10, U1 and Q11. The exact Q5b principal-symbol identification is ruled out by rank preservation. The broader bridge to a newly supplied positive target remains a missing construction. Separately, Q5a supplies a conditional obstruction under its depth and weight hypotheses and for common scalar normalisations on toric targets. (**proved** for the representation theory and scoped rank obstruction; **open** for the measured-space identification and for a geometric construction supplying a new target with [ACT], [INV] and [ADAPT].)

Q8–Q11, U1, W1, H2 (Geometric emergence cluster). Q8 [58] establishes $A_Z = C_{\mathfrak{su}(2)} = 2$ via Casimir rigidity, within the frozen Q5a framework [H-F] (**structural**). This entry is pending revision in the present cascade: Q7 version 2.0 withdraws the identification and the numerical reading on which that argument draws. Q9 [111] proves that nonnegative modulation energy leaves a kinetic Mosco limit unchanged when the kinetic forms converge under [K] and the same recovery sequences satisfy [R]. Its bound $\mathcal{V}_q(f_q) \le 8\pi^2 r_q a_q(X)\|k f_q\|^2/q^2$ retains the energy normalisation. The free kinetic form neither selects a nonzero Schrödinger character nor equals the full oscillator form. A relative moment bound gives an inequality for each qualifying vector, not a closed coercive form on a derived linear domain. Q5b-O1, [H-lift] and bridge non-obstruction remain open (**conditional** on [K] and [R]; version 2.0). Q10 [80] asserts $A_H \to 2$ under spectral universality [U]: character-independence forces $\mathfrak{su}(2)$-isotropy of the effective form, and the unique such form is read as the Casimir with value 2 (**conditional** on [U]). Reading that value as a coefficient requires [ID], and Q7 version 2.0 no longer reads it as settling its bridge condition; this entry is pending revision. U1 version 2.0 [113] withdraws its proof of [U] and the claimed $O(q^{-1/2})$ rate. Its unconditional result is the exact equidistance of the finite Heisenberg

multiplication generators at distinct central characters. A separate adjacent-character example excludes a uniform modulus for their summed operator. Neither result proves or refutes [U], which remains open (**proved** for the obstruction; **open** for [U]). W1 [114] proves Hypothesis [H-w] (convergence $a_q(s) \to A > 0$ of the admissibility weights) as a structural consequence of the fibre-invariance of admissible observables ($\mathcal{O} = \operatorname{Im}\Pi$); its quantitative step imports the rate now withdrawn by U1 and awaits W1's own revision (**conditional** on [H1]; pending). After W1 and H2, the open hypothesis of the former Q5a framework is [H1]; [H-$\mathcal{E}$1] and [C] are not closed by the finite-rank route, their disposition being the zero-form one of Q5a version 3.2. Q5a-O2 [87] establishes the pure-Fourier-mode structure of the pre-saturation pipeline, the truncation status of its three-coordinate reduction (no rank is measurable from it), and an $O(q^{-2})$ upper bound on the admissibility form that rules out the q^{-1} coercivity scale; [H-$\mathcal{E}$1] and [C] are therefore not closed by a finite-rank route, and their disposition is the one fixed by Q5a version 3.2: the canonical filtration is a growing Fourier window of dimension $\min(2n+1, q)$ on which the published form has zero-form Mosco limit. H2 [100] proves [H2] (strong convergence of the rescaled Weil generators $\hat{X}_q \to x$ and $\hat{P}_q \to -i\partial_x$), via a quantitative Poisson aliasing lemma and a discrete Sobolev identity (**proved**). The framework itself is withdrawn in Q5a version 3.2, and the chain's residual conditionality includes the spatial limit hypothesis [H-L] together with the open former hypotheses [H1], [H-$\mathcal{E}$1], and [C] in any non-zero-form reading. **Q11** [103]: Claims temporal rigidity under frozen [H-F], but its absolute value $A_\tau = 2$ additionally imports Q8's scalar normalisation. Schur alone does not fix that scalar; the claimed closure of Q5b-O3 is pending revision, including its algebraic coefficient value, not only its metric interpretation. With the additional, independent and unestablished spatial-limit hypothesis [H-L], and with the identification of the measured admissible space with that module, which Q7 version 2.0 records as supplied by no source, those coefficients give the effective Lorentzian co-metric reading $g^{\mu\nu} = \operatorname{diag}(-2, 2, 2, 2) \propto \eta^{\mu\nu}$ (Minkowski). This entry is pending revision in the present cascade. The derivation chain Q5a → Q5b → Q7 → Q8 → Q9 → Q10 → U1 → W1 → H2 → Q11 → Q5b-O3 was previously recorded as complete *conditionally on [H-F] and [H-L]* (Q5a version 3.2; Q5b version 2.1). That reading is withdrawn at the Q7 step: Q7 version 2.0 no longer supplies the symmetric-square identification the chain consumes, and it withdraws the coefficient values it had imported back from Q8, Q10, U1 and Q11; U1 is now revised, while the Q8, Q10 and Q11 entries await their own stages in this cascade. Q7 further records that the support was circular rather than merely absent: Q8 fixes its scalar by matching A_H, and the absolute bound that delivers A_H inserts the $A_z = 2$ that Q8 itself supplies. Until those revisions are made, the chain is to be read as a sequence of conditional steps with a missing identification at the Q7 step and a circular junction beyond it, not as a closure.

Q12 [116]: Version 1.4 derives Yang–Mills dynamics from the vertical variation of the projective spectral entropy at an arbitrary supplied fixed base metric (**proved**, conditional on a supplied compact gauge group and admissible connection; O31 supplies no SU(3) identification). This fixed-base theorem does not depend on Q11; only its interpretation on the Cosmochrony emergent base is conditional on [H-L]. The Gravity paper [18] derived the Einstein tensor as the infrared-dominant horizontal response of $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ (the a_2 coefficient). Q12 carries out the complementary vertical variation: the operator A_g is extended to $A_{g,\mathcal{A}} = -(\nabla^{\mathcal{A}})^2 + E$ on the associated vector bundle of the admissible principal bundle $P_{G_\Pi}(M, G_\Pi)$, and the Seeley–DeWitt coefficient a_4 is computed:

$$a_4(A_{g,\mathcal{A}})\big|_{\text{gauge}} = \int \frac{1}{12}\,\operatorname{tr}_\rho(F_{\mu\nu}F^{\mu\nu})\sqrt{g}\,\mathrm{d}^4x\,,$$

the gauge component $\Omega^{\text{gauge}}_{\mu\nu} = F_{\mu\nu}$ being isolated from the total bundle curvature and the overall $(4\pi)^{-2}$ carried once by the heat-kernel prefactor. The extended spectral entropy $S_\Pi[g, \mathcal{A}]$ therefore contains a local logarithmic Yang–Mills term, and the vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed base metric yields the Yang–Mills equations $D_\mu F^{a\mu\nu} = 0$ in the current-free sector (**proved** for the a_4 derivation given G_Π). The coefficient of the induced logarithmic running of

g_{YM}^{-2} is $I_\rho/(3\cdot 16\pi^2)$, depending on the representation through the Dynkin index I_ρ and hence not universal; the physical g_{YM} and G_N are renormalized matching data, of which only the logarithmic running is computed. What is structural, within the proper-time scheme, is the difference in UV divergence degree between the two sectors — quadratic at a_2 for gravity, logarithmic at a_4 for gauge; the contrast is not scheme-independent, the zeta-regularized determinant carrying no power divergences. No predictive G_N formula and no tuning-free hierarchy is claimed (aligned with Gravity 3.3). At fixed base metric the vertical variation isolates the Yang–Mills sector, but full horizontal–vertical decoupling does not hold: the metric variation of F^2 is the Yang–Mills stress tensor $T^{\mathrm{YM}}_{\mu\nu}$, an a_4-order back-reaction treated in Q13. Beyond the technical derivation, Q12 offers an interpretive outlook (not a theorem): gravity and gauge dynamics are separated by geometric direction (horizontal vs. vertical variation on the admissible bundle) *and* by spectral order in the Seeley–DeWitt hierarchy, so the difficulty of reconciling gravity with the gauge-theoretic Standard Model may reflect a difference of Seeley–DeWitt order rather than the absence of a common symmetry group G_{unif}. The gauge–gravity synthesis is carried out in Q13.

Q13 [88]: Version 1.4 gives the gauge–gravity spectral synthesis on supplied geometric and gauge data (**conditional** on the matched coefficients, and on Q12, the Gravity paper, matching coefficients and a supplied compact gauge group; O31 supplies no SU(3) identification). Its interpretation on the Cosmochrony emergent base is separately conditional on [H-L]. Q13 joins the metric and connection variations of a single renormalized local action and obtains a *conditional* Einstein–Yang–Mills system. The metric variation of the matched gauge kinetic term supplies the Yang–Mills energy–momentum tensor through $(1/\sqrt{g})\,\delta\int\sqrt{g}\,\mathrm{tr}_\rho(F^2)/\delta g^{\mu\nu} = 2\tau_{\mu\nu}$ (**proved**), while the fixed-metric connection variation supplies $D_\mu F^{a\mu\nu} = 0$; jointly, $G_{\mu\nu} + \Lambda_{\mathrm{eff}} g_{\mu\nu} = 8\pi G_N T^{\mathrm{YM}}_{\mu\nu}$. The normalisation is matching, not prediction: since $\mathrm{tr}_\rho(F^2) = I_\rho F^a F^a$, canonical matching fixes $c_{\mathrm{EH}} = 1/(16\pi G_N)$ and $c_F I_\rho = 1/(4g^2_{\mathrm{YM}})$, so that $8\pi G_N = 1/(2c_{\mathrm{EH}})$ — and *not* $c_{\mathrm{YM}}/c_{\mathrm{EH}}$, which would mix the gauge-action prefactor with the definition of the stress tensor. G_N, g_{YM} and the cosmological coefficient Λ_{ren} (the a_0 term does contribute to the metric equation) are independent renormalized matching data, so the ratio $G_N g^2_{\mathrm{YM}}$ is not an output of the construction. The a_6 sector is given as a representative spanning list of the invariants that can occur, modulo integrations by parts and Bianchi identities: in four dimensions a_6 produces no ultraviolet divergence, its renormalized dimension-six coefficients are themselves matching data, the pure-gauge invariant at that order is $\mathrm{tr}_\rho(F^3)$, and no unique cross-term is determined (**structural**). No non-linear completion of the joint functional is asserted. The derivation is Euclidean; identification with the Lorentzian Einstein–Yang–Mills equations requires a continuation that is not performed. Within the proper-time scheme, the difference in ultraviolet divergence degree between the two sectors is structural ($a_2 \to$ gravity at quadratic UV, $a_4 \to$ gauge at logarithmic UV); this contrast is scheme-dependent and does not hold for the zeta-regularized determinant. The central conceptual contribution is the identification of the spectral stratification principle: gravity and gauge dynamics are not unified by enlarging the symmetry group, but are *stratified by Seeley–DeWitt order* within the single operator $\mathcal{A}_{g,\mathcal{A}}$. The fermionic extension is carried out in Q14.

The Gravity paper [18], Thermodynamics [21], Lorentz/CausalPropagation [37], and BornInfeld [12] form the *spectral gravity sub-programme*, whose synthesis paper is the presentation note **SG-Note** [110] (Presentation Note 4). SG-Note states the conditions under which the effective metric $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision, supports an Einstein sector: from the projective spectral entropy $S_\Pi[g] = \frac{1}{2}\log\det' A_g$ it derives the Einstein tensor as the infrared-dominant horizontal response (the a_2 coefficient, whose finite normalisation remains a matching datum), recasts it as a conditional variational stationarity, analyses the posited four-derivative Lorentzian truncation (whose massless branch is exactly luminal), and presents the determinantal Born–Infeld completion as an admissible member of an explicitly parametrised family resting on the capacity axiom [A-cap]. The a_2 Einstein sector is the gravitational input consumed by the gauge–gravity stratification sub-programme below (Note 8).

Q12 and Q13 form the *gauge–gravity stratification sub-programme*, whose synthesis paper

is the presentation note **GGS-Note** [64] (Presentation Note 8). GGS-Note organises the two constituents into the spectral stratification chain $a_2 \to G_{\mu\nu}$ (horizontal δ_g), $a_4 \to D_\mu F^{a\mu\nu} = 0$ (vertical δ_A), $a_6 \to$ dimension-six invariants (spanning list only), all from the single functional $S_\Pi[g, A] = \frac{1}{2} \log \det' \mathcal{A}_{g,\mathcal{A}}$. Q12 is shared with the gauge-structure sub-programme (Note 3), whose version 2.0 records that the compact gauge group is supplied rather than derived; GGS-Note draws on Q12's a_4 vertical variation, on the fixed-metric isolation of the Yang–Mills sector (Lemma 1; full horizontal–vertical decoupling does *not* hold, the metric variation of F^2 being the a_4 back-reaction $T^{\mathrm{YM}}_{\mu\nu}$), and on the difference in UV divergence degree between the sectors, within the proper-time scheme (a_2 quadratic vs. a_4 logarithmic). Q13 supplies the conditional coupled Einstein–Yang–Mills system and the a_6 invariant inventory. It asserts no numerical hierarchy and no non-linear completion: g_{YM}, G_N and Λ_{ren} are independent renormalized matching data. The note isolates the upstream inputs consumed (Note 4 for $S_\Pi[g]$ and the a_2 sector; Note 3 for G_Π and the principal bundle; Note 2 for the effective Lorentzian base manifold) and records the open deliverables: the quantitative a_6 coefficients, whether admissibility constrains any combination of the matching data, a possible non-linear completion of the joint functional, the Lorentzian continuation of the gauge sector, and the coupled equations with fermionic matter currents from Q14 (Note 6). GGS-Note 2.0 is aligned with Q12 and Q13: on supplied geometric and gauge data it records the fixed-metric isolation of the gauge sector rather than a full horizontal–vertical decoupling, presents a_6 only as a representative spanning list of dimension-six invariants modulo integrations by parts and Bianchi identities, and asserts neither a non-linear completion nor a numerical coupling hierarchy; its proper-time scheme dependence and Euclidean-signature limitation are explicit. Only the emergent-base reading is conditional on [H-L]. O31 version 2.0 withdraws the claimed pointwise result and every derivation of $G_\Pi = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The a_4 calculation of Q12 remains a conditional Yang–Mills result for a supplied compact gauge group; it does not identify the Standard Model group.

Q14 [84]: Version 2.0 treats fermionic matter and chirality (**structural**, conditional on Q13, the Gravity and BI papers, a supplied finite-Heisenberg carrier and central character, the unestablished Lorentzian-geometry hypothesis [H-L] whose coefficient values are pending revision, and supplied gauge data; O31 leaves the colour sector conditional). Q14 extends the gauge–gravity synthesis to the fermionic sector relative to those inputs. On the supplied carrier and conditional geometry, fermions are represented by the spinorial face of the Weil module V_ρ after Lorentzian complexification of its metaplectic lift; neither the carrier nor the spatial limit is selected by the admissibility axioms. Three structural results are proved. (i) The admissible Weil module induces, through its metaplectic lift and the complexification $\mathfrak{mp}(2, \mathbb{R})_{\mathbb{C}} \simeq \mathfrak{sl}_2(\mathbb{C})$, an admissible spinor bundle S_Π whose symmetric and determinant tensor sectors reproduce the $\mathrm{SU}(2)_L \times U(1)_Y$ gauge-sector bundle structure (**structural**); FM-Note and NSR read these as Lorentz-typed adjoint and determinant data, distinct from the independent fermionic weak doublet, which is not produced at this stratum [62, 92]. (ii) The projected Dirac operator $\mathcal{D}_{\Pi,g,A}$ contains a canonical zero-order endomorphism E_Π (the internal spectral residue of non-injective projection) which, via the spinorial lift of BI parity (proved in Q14 as the antilinear $J_\Pi = HK$), is left-admissible and enforces the $V-A$ chiral structure; the γ_5-weighted a_4 Seeley–DeWitt coefficient then imposes anomaly-cancellation trace constraints on the hypercharge weights, making hypercharge a spectral coherence datum rather than a free parameter (**structural**). (iii) The supplied saturation rule $\sigma_c(n_3) = 3$ (a selection rule, not a derived constant) admits a spinorial multiplicity reading, distinct from its geometric reading as three spatial directions, yielding a gauge-singlet three-generation factor $\mathbb{C}^3_{\mathrm{gen}} \subset \ker(\mathrm{ad}_{\mathrm{SU}(2)} \oplus Y)$ (**structural**). The colour-coupled quark sector is conditional on a supplied colour representation. O31 version 2.0 withdraws the claimed pointwise co-admissibility and does not derive that representation [115]. Beyond these three theorems, Q14 §6 establishes a qualitative mechanism for the inter-generation splitting (a diagnostic model, not a fourth theorem). Q14 §6 identifies the rank-three admissible sector with $\mathrm{Sym}^2(V)$ as a structural identification consistent with O29 [74] and O30 [78]; the registry records that neither O29 nor O30's conditional model derives it, so the mechanism is read as conditional

on it: a static J_Π-real, weight-preserving restriction of E_Π^2 has a degenerate outer pair (static obstruction), while the oriented cascade generator ∂_τ of Q11 [103] carries a non-zero J_3 projection that lifts it. This remains a conditional diagnostic mechanism. Its amplitude is deferred to the cascade normalisation, and generation mixing requires further complex metaplectic input.

Q14 is the principal constituent of the *fermionic matter sub-programme*, whose synthesis paper is the presentation note **FM-Note** [62] (Presentation Note 6, working-paper version 2.1). FM-Note organises Theorems A–C into the conditional chain supplied Heisenberg carrier + [H−L] ⇒ $F_n \simeq V_\rho \Rightarrow \mathrm{Mp}(2,\mathbb{R}) \Rightarrow \mathrm{mp}(2,\mathbb{R})_{\mathbb{C}} \simeq \mathfrak{sl}_2(\mathbb{C}) \rightsquigarrow \mathrm{Spin}(3,1) \simeq \mathrm{SL}(2,\mathbb{C}) \Rightarrow \mathcal{S}_\Pi$, isolates the four upstream layers consumed (Notes 1, 5 for the Weil fibre; Note 2 for the Lorentzian closure; Note 3 for the gauge bundle; Q13 [88] for the gauge–gravity synthesis). Hypercharge rigidity additionally consumes a supplied three-dimensional colour module, and generation multiplicity a supplied rank-three carrier. The independent fermionic weak doublet is not produced at this stratum. The open deliverables include explicit construction of E_Π as a spectral operator and the full Lorentzian-signature matter content $S_\Pi^{\mathrm{matter}} = (\mathcal{S}_\Pi \oplus (\mathcal{S}_\Pi \otimes V_{\mathrm{color}})) \otimes \mathbb{C}^3_{\mathrm{gen}}$ with explicit Yukawa structure; the mass spectrum, Yukawa couplings, and CKM/PMNS mixing matrices remain open. The qualitative mechanism of the generation splitting is identified in Q14 §6, conditionally on the supplied $\mathrm{Sym}^2(V)$ identification; its value is dictionary-bound. With PRS [109] the Schur transversality on which the A4 matching was conditional is closed in the present Lorentzian spin stratum: the exact metaplectic opening gives $\alpha(t,s) = ts\,r/\sinh r \not\equiv 0$ with $\mu(t,s) \equiv 0$, and the timelike Clifford symbol excludes the zero-mode locus, so the Schur-transverse branch is selected and the electric genus gives $\mu_\chi^2 < 0$, $u \neq 0$ [105]. The A4 companion extends the radicand to quartic order; on the corpus-derived real cascade the transverse cubic response vanishes, and the boundary-versus-interior selection is closed by admissibility: the saturation contact $\Delta_\chi(s_*) = 0$ is the unique chart-independent A4 lock. The eliminated-block companion [107] constrains the jet of $1-P(s)$ and determines one block of the J_Π-odd tangent, and the contact factorisation of $u(s_*)$ holds only under a localisation hypothesis that its explicit family does not satisfy. The value $|u|$ is dictionary-bound through $\mathcal{N}_A$, so the first-principles front of the sub-programme is upstream and structural (the ADE case selection, the cascade exponent β, and the transfer constant N_{casc}), and beyond it lies the Yukawa sector.

PRS [109]: The projective residue as a Schur complement (**structural**). Companion note of the fermionic matter sub-programme refining the Q14 §6 open deliverable on the inter-generation splitting. Exhibits $E_\Pi = -\Pi_S\, D\,(1-P)\,D\,\Pi_S^* = -M^\dagger M$ as the Schur complement of the spinorial directions eliminated by the non-injective projection (negative semi-definite, zero-order, vanishing in the injective limit), localises the split coefficient u in the $\mathcal{D}^\pm$-transported, generation-projected component of the antiunitary chiral equivariance defect $\Delta_\chi(P) = \pi_{LL} - \tau\,\overline{\pi_{RR}}\,\tau^{-1}$, and stratifies the result on the Cosmochrony axioms: A1–A3 force $u = 0$ (chirally symmetric minimal non-injectivity); a non-zero u requires chiral symmetry-breaking carried by the projection-locking axiom A4. Establishes the finite/Lorentzian separation theorem ($u_{\mathrm{fin}} = 0$ for q an odd prime), locks the Seeley–DeWitt conventions so that the operator-level and spectral-level u coincide on the flat effective metric, and provides a no-go result: chirality is Lorentzian, not a finite-fibre label. Closes the A4 Schur transversality in the present Lorentzian spin stratum: the projected commutator $[\partial_s P|_0, J_\Pi]\big|_{\mathrm{span}(e_+,e_-)}$ has the exact metaplectic opening $\alpha(t,s) = ts\,r/\sinh r \not\equiv 0$, $\mu(t,s) \equiv 0$, and its timelike Clifford symbol is not in the zero-mode locus, so the Schur-transverse branch is selected and, with the electric genus of [105], $u \neq 0$. The remaining open data are the explicit Lorentzian block $1 - P(s)$, the amplitude $|u|$, and the projected Yukawa sector.

AAR [98]: Reduction of the angular amplitude observable (**structural**, working note). Fixes, before any Weil–BFS computation, the precise observable whose measurement tests the absolute normalisation of the Q14 §6 angular generation split. A short Baker–Campbell–Hausdorff reduction identifies the per-step J_Π-odd J_3 component of the metaplectic cascade generator with the oriented symplectic area swept per step — the central Carnot-degree-2 increment of Heis_3 — not a free shear parameter. The quantity to be measured is the accumulated oriented J_3-odd area along the Weil–BFS cascade, normalised by the projected capacity $\widehat{I}(n)$; at n_3^{obs} this equals

$\varepsilon_{\mathrm{Weil}}(n_3^{\mathrm{obs}})$, to be compared with the dictionary value $\varepsilon = 1/10$ without fitting. In the O12 fingerprint the two sectors are the two terms of the exponent: frequency B_c carries the radial capacity σ_c, central phase A_c carries the angular split. A direct evaluation on symmetric BFS shells returns $\Theta_{\mathrm{raw}} = 0$ identically; the orientation prescription is an irreducible input prior to $\mathcal{N}_A$. A subsequent capacity audit on the real Weil–BFS pipeline closes the angular onset coefficient: the directed outgoing-frontier mean is a q-independent exact rational sequence (identical across $q \in \{61, 101, 151, 211, 307\}$, no fit) with saturation onset exactly $\frac{1}{3}$, hence the maximal-locking angular bound $\theta_{\max} = 2\pi/(3q)$ (exact for the standard $\{\pm X, \pm Y\}$ campaign, generator-dependent; only ε and $R = \frac{3}{2}$ are generator-independent). This closes the coefficient, not the dictionary value: $\theta_{\max} \to 0$ with q, so $\varepsilon = 1/10$ still requires the geometric Sym^2 normalisation $\mathcal{N}_A^{\mathrm{geom}} = 3q/(20\pi)$, gated by the same ADE datum [102].

Q11OF [108]: The oriented frontier observable (**structural**, diagnostic). Frontier-transfer diagnostic note on the recursive existence of the J_3-odd cascade signal underlying the Q14 §6 splitting mechanism. The symmetric-capacity obstruction makes the raw shell observable vanish identically ($\Theta_{\mathrm{raw}} = 0$): the involution $g \mapsto g^{-1}$ pairs opposite central phases on symmetric BFS shells, erasing the oriented J_Π-odd component before the radial recursion can act. The honest replacement is a frontier transfer observable on the directed outgoing edges $g \to gs$, whose central increment is the Heisenberg cocycle $a(g)\, s_b$, discrete counterpart of the BCH term $\frac{1}{2}[X, Y]$ and hence of the dynamical J_3. The orientation is the recursive arrow $\partial_\tau = \log g$ from the projection origin. The first test is the existence of an oriented signal $\langle \Delta A_c \rangle_{\partial^+ S_m} \neq 0$; the discriminating anti-bias control is the symmetrised-frontier cancellation. No normalisation, no dictionary value, no ε enters: the note tests only whether the recursion produces a non-zero, bias-free oriented signal. Its resolution section proves that the residual frequency reflection is the central Weil lift $R_b = W(-I) = \mathrm{FT}^2$ up to the metaplectic phase, hence $[\gamma_5, R_b] = 0$.

CHO [81]: Chiral orientation of the directed Heisenberg frontier (**conditional** on the inherited central-phase lift). Companion note resolving the chiral weight σ_L left deferred by Q11OF. A parity argument closes the route $\sigma_L = f(T(e))$: edge inversion $U(e^{-1}) = U(e)^{-1}$ and the residual reflection both send the J_Π-odd cascade generator $T(e) \mapsto -T(e)$, so every function of T has equal arrow- and φ-parity, whereas the admissible weight must be arrow-even and φ-odd (**proved**). The resolution is that σ_L is the Weyl representation type $\mathbf{2}$ versus $\bar{\mathbf{2}}$: group inversion preserves the representation and complex conjugation exchanges it, the unique label distinguished by the conjugation that separates φ from arrow inversion (**structural**). In the inherited Heisenberg central-phase lift the edge phase is the linear cyclotomic character $\zeta_q^{\Delta A_c}$, with no Gauss, Maslov, or $\sqrt{5}$ component, and the natural branch gives $\sigma_L = \mathrm{sign}\, \Delta A_c$ on the cocycle support, hence $\rho_\chi = 1$ and $\mathcal{N}_A = \theta_{\max} \neq 0$, q-invariant on $q \in \{61, 101, 151\}$ (**proved** within that convention). Thus [H-orient] is closed in the inherited central-phase lift; the residual point — a possible additional spin-Galois phase of the full Lorentzian chiral lift, in particular a $\sqrt{5}$ or ADE-spin factor orthogonal to ζ_q — is discharged by the chiral-lift corollary of AOG-Note [102]: the same arithmetic orthogonality shows the Lorentzian lift induces no action on $\mathbb{Q}(\sqrt{5})$, so [H-orient] holds unconditionally within the present spin stratum (**proved**). It fixes the frontier amplitude datum $\mathcal{N}_A \neq 0$ but does not derive $u \neq 0$, which still requires the Schur transport [109].

A4-Note [105]: The Born–Infeld saturation margin of the chiral modulus (**structural**). Companion note to PRS fixing the sign of the generation split: definition of the Lorentzian saturation functional $\mathcal{B}_{\mathrm{sat}}(s)$ along the J_Π-odd modulus controlling u. An antisymmetry lemma shows that the generation modulus is J_Π-odd while the symmetric square $M = F_\chi F_\chi$ is J_Π-even from birth, so the primary modulus variable must be an antisymmetric chiral two-form $F_{\chi,\mu\nu}(s)$. $\mathcal{B}_{\mathrm{sat}}(s)$ is then defined as the Born–Infeld determinantal saturation margin of this two-form, with sign locked by the admissibility role of $\mathcal{B}_{\mathrm{sat}}$ (axiom A4 selects saturated minima), not by Maxwell weak-field matching — this neutralises the convention sign-trap on the split sign. The second variation reduces to $\mu_\chi := \partial_s^2 \mathcal{B}_{\mathrm{sat}}(0) = \frac{1}{2} P_{\chi,\mu\nu} P_\chi^{\mu\nu}$, so the existence and stability of the split are governed by the Lorentzian genus of the chiral polarisation P_χ in $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision. The identification of the Schur projector tilt with the Born–Infeld

saturation modulus on the A4 locus rests on the Schur transversality closed in PRS [109] (exact metaplectic opening, timelike Clifford symbol off the zero-mode locus), which selects the Schur-transverse branch; the Lorentzian genus is then determined electric ($\mu_\chi^2 < 0$, $u \neq 0$). Extending the radicand to quartic order, the Born–Infeld pseudoscalar cancels on the electric locus; on the corpus-derived real cascade the transverse cubic response vanishes, and the boundary-versus-interior selection is closed by admissibility: the saturation contact $\Delta_\chi(s_*) = 0$ is the unique chart-independent A4 lock, with $u(s_*)$ invariant under orientation-preserving charts. The contact split factorises as $u_\partial = \sqrt{2}\,\sigma\, s_*\, T_{J_3}(\partial_s \Delta_\chi(P)|_0)/\langle J_3, J_3\rangle$ only under a localisation hypothesis that the explicit family of [107] does not satisfy, so the factorisation is conditional and $|u|$ remains dictionary-bound through $\mathcal{N}_A$. In the affine electric gauge the boundary-branch saturation contact is fixed in closed form, $s_* = \beta/|E_P|$, a radial-form result with no numerical prediction: the P_χ-parallel cubic response is pure reparametrisation gauge, and only the P_χ-transverse curvature, doubly conditional on the non-derived complex phase and the cascade ordering, shifts it.

EBJ [107]: The constrained jet of the Lorentzian eliminated block and the chiral defect-rate identity (**proved** / **open**). For every smooth family and every smooth unitary transport the projector jet $1-P(s) = Q_0+sQ_1+s^2Q_2+O(s^3)$ has $Q_1 = [A, Q_0]$ a purely off-diagonal Grassmann tangent and Q_2 diagonal blocks pinned to Q_1^2 with opposite signs; the parity grading holds under J_Π-equivariance, and with a covariant moving spinorial embedding and a J_Π-even Dirac operator the first-order J_Π-odd part of E_Π^2 is $\{E_0, E_1\}$. Antiunitary parity makes every odd-order generation-mixing coefficient vanish in every covariant family, with or without complex phases, even orders not being excluded; an explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with $u'(0) = 11\cdot 2^{1/4}/9 \neq 0$. In the chiral frame the left-handed block of the odd tangent is half the transported defect rate, one block of the tangent rather than a unique generator; the selection of the family, the localisation of the odd tangent, even-order mixing and the normalisation of $\partial_s\Delta_\chi(P)|_0$ remain open, $|u|$ dictionary-bound.

PYL [93]: Projected Yukawa line and generation-level assignment (**structural**). Companion note opening the mass-sector frontier and closing only its first stage. The determinant line $L_Y = \wedge^2(\mathcal{S}_\Pi)$ is the unique functorial line of the rank-two admissible spinor carrier, invisible to the $\mathrm{SU}(2)_L$ adjoint sector $\mathrm{Sym}^2(\mathcal{S}_\Pi)$ (its induced action is $\det = 1$) and the carrier of the abelian $\mathrm{U}(1)_Y$ weight, so the projected Yukawa sector is the determinant-line sector of the admissible spinor functorial closure, not an external bundle datum. On the gauge-singlet triplet the squared residue $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}} = \mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ orders the three generation levels by exit deficit, with e_0 lightest and e_- heaviest for $u > 0$. The method lock is explicit: L_Y fixes the coupling line, E_Π^2 fixes the levels, and the mass comes after. The open downstream object is the projected Yukawa operator $Y_\Pi : \mathcal{S}_{L,\Pi} \otimes L_Y \to \mathcal{S}_{R,\Pi}$, whose norm, the sign of u, and the complex mixing phase carry the absolute masses and the mixing; no mass value is claimed.

PYO [94]: Projected Yukawa operator and chiral polar factor (**structural**). Companion note taking the second step of the mass-sector frontier, with a sharp partial no-go at its entry. The Schur-residue sector determines the positive hermitian square $H_\Pi := Y_\Pi^\dagger Y_\Pi$, with $\mathrm{Spec}(H_\Pi)|_{\mathbb{C}^3_{\mathrm{gen}}} = \lambda_Y^2\,\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}}$ fixes the squared Yukawa levels, not the morphism. By polar decomposition $Y_\Pi = U_\Pi H_\Pi^{1/2}$ with U_Π a unitary chiral map on the support of H_Π, so H_Π determines Y_Π only up to U_Π: the chiral orientation and the complex mixing phase live in U_Π, while the positive norm λ_Y is a separate scale of H_Π, neither visible to E_Π^2. The CP-real branch $U_\Pi = \mathbf{1}$ is the diagonal no-mixing choice ($v = 0$); a non-trivial U_Π needs the complex metaplectic phase. Front 3b is thereby closed as a squared-Yukawa-levels result, not a mixing result: E_Π^2 can close $H_\Pi = Y_\Pi^\dagger Y_\Pi$ but cannot close Y_Π unless U_Π is fixed. Front 3c then characterises U_Π as a rephasing class, the double quotient $[U_\Pi] \in U(1)_R^3 \backslash U(3)/U(1)_L^3$, with invariants the moduli $|(U_\Pi)_{ij}|$ and a Jarlskog phase (three mixing angles, one CP phase); its non-triviality $[U_\Pi] \neq [I]$ is controlled by the transverse (off-diagonal) component of the metaplectic generator. On the derived real cascade that transverse component is absent ($v = 0$), so $[U_\Pi] = [I]$ and no physical mixing is produced at this stratum; a non-trivial class needs a genuine complex

metaplectic phase, not derived here and observable only relative to a second fermionic sector.

KUD [106]: Charged-lepton Koide relation as a unit-dispersion constraint (**proved**). Source note fixing the phenomenological target of the mass sector, downstream of PYL/PYO, as a single scale-free invariant. The Koide quotient is exactly $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$ in the square-root mass vector r_ℓ, so $Q_\ell = 2/3 \iff \mathrm{CV}(r_\ell) = 1$ (standard deviation equal to mean). This separates the dispersion constraint (Koide) from the concrete hierarchy (the phase placing one generation near a node of the square-root profile), and shows the committed cascade ladder $(1, \frac{1}{2}+u, \frac{1}{2}-u)$ under-disperses, $\mathrm{CV} \le 1/\sqrt{2}$ ($Q \le 1/2$), so it cannot itself reach the Koide value. A maximum-entropy reading (CV = 1 is the unit relative dispersion of the exponential) is recorded as an open test: it requires the projection to fix the first moment $\langle\sqrt{m}\rangle$ alone and a forced discrete-to-continuous bridge. Writing $r_\ell = r_S + r_D$, $s = \|r_S\|$, and $d = \|r_D\|$, KUD proves that no real-linear self-adjoint amplitude map in a positive metric can square to $(s^2 - d^2)P$ on a non-zero projected block; this closes only the linear self-adjoint determinant realisation of the singlet–doublet route, while the route itself and its nonlinear, Krein, and enlarged-tower extensions remain open. It is a target invariant, not a mass formula.

TPC [104]: Ternary phase carrier — colour, generations, and Koide as one carrier (**open**, speculative bridge). A single complex three-phase carrier $z_g = S + A\, e^{i(\delta + 2\pi g/3)}$ read two ways: its pure-phase part sums to zero ($1 + \omega + \omega^2 = 0$), a colour-neutral class; its real part is the square-root mass profile of KUD. The singlet amplitude S is the master dial — $S = 0$ colour (confined), $S \neq 0$ masses, $A = \sqrt{2}\,S$ equipartition (Koide, CV = 1), δ near a node the hierarchy — with a candidate capacity-balance filter $\|r_{\mathrm{dbl}}\| \le \|r_{\mathrm{sing}}\|$ saturated at CV = 1. Explicitly not a derivation: it reorganises the open questions and states the construction charge sheet (exact colour singlet, three generations after neutralisation, electroweak quantum numbers, anomaly cancellation, quark/lepton selection); the colour-to-generation breaking it needs along the neutralisation route is the one already closed by the exact colour degeneracy.

NPI [90]: Neutrino mixing and projective particle identity (**structural**). Structural companion note on the ontology of fermionic identity: the misalignment between the weak flavour basis and the mass propagation basis is the generic situation (neutral meson systems exhibit the same structure; the neutrino is the elementary case), while alignment requires an identity-stabilising projection channel. The large PMNS mixing is then the unfixed default, consistent with the anarchy hypothesis, and the near-diagonality of CKM becomes the explanandum, conditional on the charged channels pinning the up-type and down-type mass bases to a common projective frame. The Dirac–Majorana alternative is reread as projective conjugation resolution. No mixing angles or masses are derived; the quantitative lock remains the trivial chiral polar class $[U_\Pi] = [I]$ of PYO.

NSR-Note [92]: Operator doublets, erased history fibres, and the no-sequential-re-entry theorem (**proved** for the finite obstructions; corpus audit for the O33/EBJ levers). No-go note closing the two deposited candidates for the independent fermionic weak multiplicity $M_L \simeq \mathbb{C}^2$. The commutant of the finite Weil module is spanned by the two parity projectors, so the canonical Hecke $M_2(\mathbb{C})$ doublets of $\mathrm{End}(V_c)$ (O33) admit no fermionic lift to their own carrier; the erased history fibres of the Trajectory Branching canonical channel are structurally abelian (C_4 frame stabiliser, canonical $(\mathbb{Z}_2)^r$ torsor per b-shadow, near-regular induced multiplicities); the flip-antisymmetric fibre line transports exactly into the parity-odd operator sector, but the two odd Hermitian axes $i(A_X \pm A_Y)$ are exchanged by a semilinear automorphism of the cocycle-enriched Heisenberg–Weil datum (RK), with only a coordinate-pinned conditional selector; and the central theorem ($\pi v = 0 \Rightarrow Q\pi v = 0$ for the canonical record quotient, extended to the conditional expectations of arbitrary non-degenerate measures) excludes every sequential re-entry. Corollary: realisation through the history-fibre route requires a parallel substrate channel with an oriented central/Weyl datum — an extension architecture, not a derivation; an independent new multiplicity carrier remains a distinct, unconstrained extension. Three archived scripts certify all computational claims.

AOG-Note [102]: Spin-stratum type-rigidity — orthogonality of the ADE gate and closure

of the chiral lift (**proved** for the gate and the chiral lift; the full-tower generalisation is established for admissible fibre operations). Transversal note isolating the logical status of the generation-split value $\varepsilon = 1/10$. It is shown to be a theorem modulo the ADE case-selection gate: the algebra $\frac{1/2+\varepsilon}{1/2-\varepsilon} = \frac{3}{2} \Rightarrow \varepsilon = 1/10$ is trivial and case-independent (**proved**), so the first-principles status of ε is exactly that of the gate. The obstruction to the gate is characterised as an arithmetic orthogonality: in the compositum $\mathbb{Q}(\zeta_q, \sqrt{5})$ the Born–Infeld parity $c \mapsto q - c$ is complex conjugation ($\zeta_q \mapsto \zeta_q^{-1}$, $\sqrt{5} \mapsto \sqrt{5}$), lying in a different direct factor of the Galois group from the Galois-spin action ($\zeta_q \mapsto \zeta_q$, $\sqrt{5} \mapsto -\sqrt{5}$) that selection of the $\sqrt{5}$-locus would require (**proved**). The same fact places the order-five versus order-ten alternative below projective parity resolution. The negative theorem — that A1–A4, ENI Corollary 6 [38] and the A4 locking do not supply the Galois-spin action — is **proved** as a no-go for the gate, unconditional in the present corpus construction by a character-table audit of $2I$. The full-tower generalisation is a scope hypothesis, not an open dependency lock: a realisation audit shows the Galois exchange $\sqrt{5} \mapsto -\sqrt{5}$ does not preserve the binary-icosahedral carrier — it maps $2I$ onto its opposite-chirality copy, exchanging the two two-dimensional types — whereas the Born–Infeld parity (the entrywise complex conjugation) preserves the carrier and fixes $\sqrt{5}$. Hence every carrier-preserving admissible fibre operation is vertical, and a finite recursive tower preserves the binary-icosahedral arithmetic type (**proved** for admissible fibre operations, in the sense of A1–A4 and Born–Infeld parity; this is a scope hypothesis, not an absolute impossibility of any external equivalence of character tables). The same spin-stratum type-rigidity has a second consequence of opposite sign (the chiral-lift corollary): the Lorentzian chiral lift of the ordered cascade step induces no action on $\mathbb{Q}(\sqrt{5})$ — the chiral weight σ_L is the Weyl-conjugation type, living in the parity factor and not the $\sqrt{5}$ factor — so it discharges the residual obstruction of the chiral orientation note and closes [H-orient] with $\mathcal{N}_A \neq 0$ unconditionally within the present spin stratum [81] (**proved**). For the gate the rigidity is fatal, the required $\sqrt{5}$ action being absent; for the chiral lift it is the closure, the feared $\sqrt{5}$ action being absent. A status sharpening records the concrete generator-set form of the gate. Once the order-five $\sqrt{5}$-locus is selected, $R = 3/2$ is not a fit but a forced Cayley-spectral invariant of $2I$, the outer deficits being $\{|\lambda_1 - 1|, |\lambda_3 - 1|\} = \{\frac{1}{6}, \frac{1}{4}\}$ with central level $\lambda_2 = 1$; the neutral order-four half-turn class — the programme's default admissibility convention — instead gives $\{\frac{1}{5}, \frac{1}{3}\}$, hence $R = \frac{5}{3}$ and $\varepsilon = \frac{1}{8}$, not $\frac{1}{10}$. The residual lock is therefore the order-five-versus-order-four selection (**proved** for the forced ratio, **open** for the selection); the tempting fermionic shortcut does not supply it, the spinorial grading read by the chiral lift being common to both two-dimensional irreducibles and inducing no action on $\mathbb{Q}(\sqrt{5})$.

SRN [101]: Spin as a transformation class of rotating wave modes (**proved** / **open**). Companion audit of the rotating-wave reading of spin: spin is fixed by how components transform under rotations of physical space, not by rotation alone; the central element acts as $(-1)^n$ on $V_\rho^{\otimes n}$; a Veronese-null versus compact-real-form lemma excludes the O28 outer-product real-form selector; the derived data fix the complex carrier and its univalence but neither the spatial realisation nor the rotation action; and, independently, $\overline{\psi_R}\psi_L$ carries exact Higgs-typed electroweak quantum numbers in every tree-level Yukawa sector, permitting but not deriving a composite Higgs condensate, with an ambient enlargement beyond a single copy of V_ρ, a canonical inter-sector identification and the condensation dynamics left open.

5.2 Gravity and Spacetime Track

Spectral2 [51]: Derives operational metric distances from graph Laplacians on a relational substrate without any background manifold (**structural**).

Gravity [18] (*Conditions for an Infrared Einstein Sector from Spectral Geometry*): Determines the conditions under which the metric variation of $S_\Pi[g] = \frac{1}{2} \log \det' A_g$ supports a local infrared Einstein sector (**conditional** on the supplied renormalised coefficients). It separates two regularisation statements that must not be conflated: a physical proper-time cutoff $\Lambda = \ell_{\mathrm{sp}}^{-1}$ carries the power-sensitive terms $a_0\Lambda^4$ and $a_2\Lambda^2$, while zeta regularisation carries the finite de-

terminant and the logarithmic scale dependence governed in four dimensions by $a_4 = \zeta_A(0)$. One minimal scalar contributes $\Delta c_{\rm EH}^{\Lambda} = -\Lambda^2/[12(4\pi)^2]$, which is *negative*: the finite renormalised coefficient $c_{\rm EH}^{\rm ren} = (16\pi G_N)^{-1}$, its sign, and hence a positive Newton constant are *matching data* fixed by a renormalisation condition and the complete operator content — they are not predicted by the minimal determinant (**open**). Once the coefficients are supplied, the variation decomposes into Einstein, cosmological, higher-derivative and non-local sectors, the Einstein term dominating locally when $RL_4^2 \ll 1$ with $L_4^2 = |\beta^{\rm ren}/c_{\rm EH}^{\rm ren}|$ (**proved** given the supplied coefficients). Under the separate coherence-extensivity hypothesis, the determinantal Born–Infeld density $\sqrt{-\det(g_{\mu\nu} + \ell_{\rm sp}^2 R_{\mu\nu})} - \sqrt{-g}$ is an admissible tensorial completion of a supplied Einstein infrared term, but not a uniquely selected one: the admissible completers form an explicitly parametrised family, and the input a uniqueness theorem would require is stated (**structural**).

Thermodynamics [21] (*Einstein–Matter Coupling from Spectral Stationarity: A Conditional Variational Result and the Open Thermodynamic Bridge*): Establishes a conditional variational core, and no thermodynamics (**conditional** on the supplied renormalised coefficients). Assuming a renormalised infrared metric response (supplied by Gravity, where the finite Einstein coefficient is a matching datum) and an effective matter functional $W_\Pi[g, \psi]$, stationarity of $\Gamma_\Pi = S_\Pi^{\rm ren} - W_\Pi$ yields $G_{\mu\nu} + \Lambda_{\rm ren} g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(\Pi)}$ at leading derivative order (**proved** given the supplied coefficients); the factor $8\pi G_N$ is the ratio between the matter variation factor $\frac{1}{2}$ and the geometric coefficient $(16\pi G_N)^{-1}$, not an independent prediction. A dimensional audit of the diagonal heat-kernel quantity $u(x;t) = -\partial_t \log K(x,x;t)$ shows it carries dimension L^{-2} — an eigenvalue scale — with $u = 2/t - R/6 + O(t\mathcal{R}^2, t\nabla^2 R)$ and normalised form $b(x;t) = t\,u(x;t)/2 = 1 - tR/12 + \cdots$. Neither is a temperature, an inverse temperature, or an energy density absent a calibration map; consequently *no local first law follows from the heat kernel alone.* A thermodynamic completion requires an independently defined heat one-form on configuration space, a physical energy notion, and a Carathéodory–Jauch–Roberts integrability theorem establishing $\delta Q = T\,\mathrm{d}S$ (**open**). Reading the stationarity as *equilibrium* is a programme-level interpretation, not a result.

Lorentz [37] (*No Quartic Graviton Dispersion from the Local Covariant Four-Derivative Truncation*): A negative result about a posited kernel (**proved** as a negative result; **open** for the Lorentzian construction). A Lorentzian counterpart of S_Π is not available — an in-in construction on a closed time path is required and is **open** — so the local Lorentzian transverse-traceless kernel $\mathcal{O}_{\rm TT} = \Box(c_{\rm EH} + \beta\Box)$ is posited, not derived. For $c_{\rm EH} \neq 0$ its kernel is the *direct sum* $\ker\Box \oplus \ker(c_{\rm EH} + \beta\Box)$: the massless summand obeys $\omega^2 = |\vec{k}|^2$ exactly, with no correction at any order (**proved** given the posit). No quartic deformation of the graviton dispersion follows, hence $A_4 = 0$ on the massless branch, no mapping $A_4 \leftrightarrow \ell_\chi$, and no bound on ℓ_χ resting on that identification. For $\beta \neq 0$ a helicity-2 pole appears with opposite residue — a ghost independently of the sign of $c_{\rm EH}/\beta$ — at signed mass squared $m_2^2 = -c_{\rm EH}^{\rm ren}/\beta^{\rm ren}$, a matching datum not tied to ℓ_χ. The resemblance to quadratic gravity is a comparison: the five-polarization massive spin-2 multiplet is not established, and the R^2 sector leaves a massive scalar unexcluded (**open**). The vector sector is not derived: the linearized vector equation keeps a $\Box\bar{h}_{0i}$ term that harmonic gauge does not remove, and only a stationarity assumption would reduce it to an elliptic constraint, so a derivation from the full linear system with spin projectors, and with it any frame-dragging reading, remains **open**; so does the first-principles construction of a projected momentum density $T_{0i}^{(\Pi)}$, awaiting the fermionic and coupled-matter sector.

5.3 Lorentz Capacity Sub-Programme

The Lorentz-capacity (LC) sub-programme isolates a single structural question: where does the Lorentz factor γ originate, given that the Lorentzian metric is itself an effective object reconstructed from admissible non-injective projection? The answer is that γ is not a kinematic postulate but the temporal residual of a bounded Born–Infeld capacity budget, lifted to special-relativistic observer transformations through the Q5b–Q11 effective co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose asserted coefficient values are pending revision in the present cascade. A guiding distinction runs

throughout: the Born–Infeld capacity sphere $(F^\tau)^2 + \beta^2 = 1$ supplies the load β and the clock factor $1/\gamma$, while the Lorentz group acts as the isometry group of the effective metric, not of the capacity sphere. The chain assembled by the sub-programme is

$$\text{BI budget} \to F^\tau = 1/\gamma \to g^{\mu\nu} = 2\eta^{\mu\nu} \to \Lambda(\beta) \to R(x)^2 = 1 - \epsilon(x) \to \text{finite-}q \text{ domain},$$

with the co-metric step carrying the coefficient values now pending revision. The synthesis paper **LC-Synthesis** [66] (**synthesis**) gathers the six constituent papers below into this single account; their individual contributions follow.

LorentzCapacity [57]: Derives the Lorentz factor and relativistic mobility $\beta\gamma$ from the O7 saturation parameter η_n^{O7} under the Born–Infeld admissibility budget (**conditional** on that budget). The analytic factorization (central coordinate c contributes only a global phase) shows the total O7 residual satisfies a linear filling law and does not by itself yield the Lorentz factor. The conditional mechanism via the temporally projected residual $\Sigma_n^\tau := \mathcal{P}_\tau^{\mathrm{Q5b}}\,\Sigma_n^{\mathrm{tot}}$ and a quadratic normalization gives $\eta_n^\tau \to \beta\gamma$ and $\Phi(\eta_n^\tau) \to 1/\gamma$ (**proved** for the algebraic BI target and the factorization; $\mathcal{P}_\tau^{\mathrm{Q5b}}$ construction: closed by TempProj [60]).

TempProj [60]: Closes the open construction of $\mathcal{P}_\tau^{\mathrm{Q5b}}$ from LorentzCapacity (**conditional** on [U] and the Born–Infeld lift). Constructs the temporal residual map as the unique positive function compatible with the Lorentzian splitting of the Q5b co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose asserted coefficient values are pending revision in the present cascade, the Born–Infeld unit capacity-flow sphere, and the exact Weil filling law $\Sigma_n^{\mathrm{tot}} = 1 - B_n$. The explicit formula $\mathcal{P}_\tau^{\mathrm{Q5b}}\Sigma_n^{\mathrm{tot}} = \sqrt{1 - B_n^2}$ follows from a fibre-by-fibre uniqueness argument; Lemma 1 and Theorem 1 of LorentzCapacity become unconditional (**proved** conditional on [U]; three-input necessity: **proved**).

LCII [27]: Extends TempProj to the non-homogeneous fibrewise regime (**conditional** on the imported occupancy–lapse identification). A localised spectral occupancy $\epsilon(x)$ reduces the local BI capacity radius to $R(x)^2 = 1 - \epsilon(x)$; the fibrewise map factorises as $\mathcal{P}_{\tau,x}^{\mathrm{Q5b}}\Sigma^{\mathrm{tot}}(x,n) = R(x)\sqrt{f(x,n)(2 - f(x,n))}$. Gravitational time dilation is identified as $d\tau/dn|_x = R(x)/\gamma$; LCIIo1 [25] refutes the free-fraction identification of the capacity radius with the metric radius, which agree only at $\epsilon(x) = 0$, while leaving scalar closure of the bridge open. The paper establishes a unified capacity arbitration picture: special-relativistic time dilation $(1/\gamma)$ and gravitational time dilation $(R(x))$ are two independent manifestations of the same BI capacity trade-off, separated in the product $R(x)/\gamma$ (**structural**; open problem LC-O2-O1 remains open, see [25]).

LCIIo1 [25] (version 2.0): Establishes a *no-go* for the scalar free-fraction reduction of the capacity–metric bridge of LCII. Version 1.0 claimed to close that bridge via $A_\tau(x) = A_\tau^\infty(1-\epsilon(x))$ together with $g^{\tau\tau}(x) = -A_\tau(x)/4$; that claim is withdrawn. With the Q11 identification $g^{\tau\tau} = -A_\tau$ (flat-space value -2, pending revision) and a diagonal temporal block, the free-fraction law gives $g_{\tau\tau}(x)/g_{\tau\tau}^\infty = 1/(1-\epsilon(x))$, which agrees with $R(x)^2 = 1 - \epsilon(x)$ only at $\epsilon(x) = 0$. The paper also shows that the partial-trace form of the local coefficient is a fresh postulate rather than a consequence of [114], and that it is mistyped (a norm-valued occupancy is not a trace fraction). What survives is the co-metric statement $g^{\tau\tau}/g_\infty^{\tau\tau} = 1 - \epsilon$, which does not govern proper-time dilation. The result is narrow: a reciprocal scalar law would close the bridge algebraically. **LC-O2-O1 is reopened** (**proved**, as a negative result).

LCo1 [33]: Closes LC-O1, the reconstruction of effective Lorentz transformations between inertial observers from projective temporal ordering data. Given a projective temporal capacity load β from LorCap/TempProj, the unique proper orthochronous transformation between the corresponding inertial splittings of the effective co-metric $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision, is the standard Lorentz boost; length contraction $\ell = \ell_0/\gamma$ follows as a corollary (**proved**).

LCo3 [29]: Closes LC-O3, the finite-q correction problem near the saturation boundary. Setting $R := (q-1)/2$ and $n = R + m$, the parabolic Carnot formula deviates from the exact torus capacity by $\Delta_q(n) = 4m^2/q^2$; the quadratic identity $B_n^2 + (F_n^\tau)^2 = 1$ receives no correction, while the Carnot identification holds for $n \le R$ and asymptotically as $m/q \to 0$ (**proved**).

With LC-O1 and LC-O3 closed, the Lorentz-capacity sub-programme is complete for *inertial* kinematics, up to the co-metric coefficient values it takes from Q5b–Q11, which are pending revision. Its non-homogeneous extension is not: LC-O2-O1 is reopened, and the identification of the local capacity radius with the metric lapse remains the declared postulate of Remark 3.2 of [27].

5.4 Standard Model Companion Papers

The papers Foundation, TopologicalInvariants, SMchargemass, O23, O27–O32, Q6a, Q7, and Q12 form (in their gauge-group identification aspect) the *gauge structure sub-programme*, whose synthesis paper is the working paper **The Gauge Structure Sub-Programme** (GS-Note, Presentation Note 3, version 2.0) [65]. Its former identification of $G_\Pi = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ used the withdrawn O31 results. The colour factor and complete group are therefore open; O32 measurements do not repair the missing group-identification bridge. The note covers gauge group identification only; the derivation of Yang–Mills dynamics belongs to the gauge–gravity spectral stratification sub-programme.

BornInfeld [12]: capacity axiom [A-cap] bounding the per-node relaxation rate, with the BI action as the saturation candidate compatible with it; the constitutive, propagation, and gauge-sector bridges to continuum physics are stated as open problems (**structural**).

TopologicalInvariants [72]: Charge quantisation ($\pi_1(S^1) \cong \mathbb{Z}$) and absence of magnetic monopoles from the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$ of the projection fibre (**proved**; $\pi_2(\mathcal{C}_{\mathrm{eff}}) = 0$ beyond the canonical model: **open**, Remark 4.2).

SMchargemass [17]: Mass and electric charge as BI-saturated responses (**structural**; baryon-to-photon ratio η: **open**).

Bell [14]: The claimed derivation of Bell-inequality violations from non-injectivity alone is refuted by a finite Bell-local countermodel; the required cross-context global-coupling obstruction is not derived (**open**).

BellNoGo [91]: Proves that non-injectivity alone has no Bell-discriminating power (redundant-extension and universal pushforward theorems), exhibits the explicit Bell-local countermodel, disproves output entropy as a non-injectivity measure, and identifies the correct obstruction as the absence of a setting-independent global coupling across measurement contexts (Fine's theorem) (**proved**).

The Bell paper opens the *entanglement sub-programme*, whose presentation paper is **ENT-Note** [52] (v1.2). The note locates entanglement *inside* the spectral admissibility cascade: for a closed conjugate Weil pair $\{c, q-c\}$, the reduced entropy on the residual fibre-level support of the Gram–Schmidt span is exactly $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$, the logarithm of the residual Gram–Schmidt rank, with no surviving spectral weighting. Structural monotonicity of S_{ent} along the cascade follows from the Gram–Schmidt construction alone. The note integrates the analytical results of ENT-E1 (Weil fingerprint orthogonality theorem) and ENT-E2 (separation of the admissible pair entropy $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ from the full residual-rank entropy), upgrading several previously numerical hypotheses to theorems and recording the residual epistemic stratification. The note thus reduces entanglement to a pure O-series observable and connects the pair-capacity exponent δ_{pair} — whose numerical comparison with the mass hierarchy has no native capacity-to-rate derivation — to a downstream trace of the stabilization profile.

ENT-E1 [76]: closes the open analytic strengthening of ENT-Note by explicit computation (**proved** / **open**). The k=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ for any block (c_1, c_2, c_3) factors as a global phase times a single discrete Fourier mode at frequency $A = c_1 b_1 + c_2 b_2 + c_3 b_3 \bmod q$. Since distinct Fourier modes are orthogonal, every newly admitted fingerprint is already orthogonal to the current span, so the admission residual $w_j = 1$ is an *exact* algebraic identity — not a numerical accident. For matched single-character blocks the conjugate pair samples sign-reflected frequencies and the closure hypothesis of ENT-Note becomes a theorem; the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ is then derived analytically (**proved**; literal Schmidt

pairing for independently sampled canonical O25 blocks: **open**).

ENT-E2 [73]: separates two genuinely distinct entanglement entropies that ENT-Note and ENT-E1 establish on two different projections of the conjugate Weil pair. The *admissible pair entropy* lives on a hypothesised carrier: the explicit hypothesis (H-carrier), stated by E2 itself and derived nowhere, fixes the admissible sector of each factor as the atomic Fourier triple $\mathrm{ran}(\Pi_q^{(c)}) = \mathrm{span}\{e_0, e_{\xi_c}, e_{q-\xi_c}\}$, whose parity-pair component $\{e_{\xi_c}, e_{q-\xi_c}\}$ is identified with the spin-$\frac{1}{2}$ representation. The hypothesis is motivated by the pure-mode structure of the pre-saturation pipeline (*Q5a-O2* [87]); its rank-three input is the supplied selection rule of *O23* [70], within the open fibre-identification problem of *O18* [22]; no mean-zero theorem removes the zero mode (the passage to the pair sector is part of the sector definition); the singlet identification of *Q3* [83] then gives $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ for general canonical blocks, conditional on both (H-carrier) and Q3's explicit, undemonstrated diagonal-invariance hypothesis [H-inv] (**conditional** on (H-carrier) and [H-inv]). Q1 does not establish a singlet correlator or a Tsirelson bound. The *full residual-rank entropy* $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ lives on the full Gram–Schmidt basis (rank q at saturation, not rank 3) and is a pipeline-level contraction observable, unconditional for matched blocks (ENT-E1) and otherwise conditional on the residual admissibility indiscernibility hypothesis [$\mathrm{H_{res}}$] and on extending *O17*'s equal-residual-dimension result beyond its own toy model to the real pipeline; the metaplectic dilation of *O31* [115] is excluded as a bridge for [$\mathrm{H_{res}}$] (**proved**; [$\mathrm{H_{res}}$] for general canonical blocks: **open**).

SchwingerLamb [63]: Fine-structure constant α as a projective invariant; Lamb shift and Schwinger correction rederived in the pre-geometric framework (**structural**; quantitative radiative corrections: **heuristic**).

5.5 Cosmological and Condensed-Matter Applications

Cosmology [3]: Flat galaxy rotation curves and the Hubble tension from a common effective mechanism; no dark matter or modified gravity (**heuristic**; quantitative fit: **heuristic**). Version 2.0 adds a structural expansion sector: under the rank–time dictionary stated as an explicit hypothesis ($dt = H^{-1}dn$, hence $n = \log a$, bridging the temporal-ordering sector and the operational closure of its Appendix A), the accelerated component is carried by projective history branching [97] rather than by a dark-energy fluid, and the naive state-volume reading is quantitatively excluded ($N_{\mathrm{dist}} \sim a^{0.881} \neq a^3$); the QUMOND relation is repositioned precisely (interpolation structure as phenomenological input; falsifiable content in the $a_\star \sim \kappa c H(z)$ closure, the environmental H_0 bias, and void signatures); the dimensional chain of the saturation scale is corrected ($a_\star \sim \kappa\, c_{\mathrm{BI}}^2/\ell$); H_∞, $w(n)$ and the low-ℓ D4 re-audit are left open, with the phenomenological core independent of them (H-dict: explicit hypothesis; expansion reading: **structural**; open items: **open**). Version 2.2 upgrades the asymptotic de Sitter limit from a free late-time fit to a calibrated spectral steady state driven by the history-entropy production rate (structural carrier: **structural**, conditional on the [H-ent-id] identification; numerical value of H_∞: explicitly not derived, with the two extraction routes closed conditionally), and gates the $w(n)$ candidate by the conditional no-go of [97]. Cosmology 2.2 also records the interpretive consequence of the low-rank Pell count: the earliest projectable epoch is naturally read as finite-resolution rather than infinitely hot, while the standard hot classical regime is left unchanged.

TrajectoryBranching [97]: Projective trajectory branching and the de Sitter limit (endpoint no-gos, b-only reduction and generic-probe Pell rate: **proved**; full-history compression rate $\log 2$: **proved**; exact class count: **proved**; cosmological reading: **structural**). The LPS $\to$ Heisenberg transfer relocates exponential growth from endpoint volumes (polynomial by Bass–Guivarc'h, degree 4) to projectively distinguishable histories (D4): any endpoint-based distinguishability collapses, while the canonical O12-compatible residual channel is b-only (the central charge is unconditionally erased from mono-path norms) and counts histories by the Pell recurrence $N(n) = 2N(n-1) + N(n-2)$, rate $h_b = \log(1+\sqrt{2})$, proved for a generic probe and witnessed exactly at $q = 211$. Supplies the structural bridge from the admissibility cascade to a de Sitter-like

component independent of matter content. Since v1.1.0 the full-history Gabor channel is itself a theorem: the Gabor span exhausts the abelian ℓ^1 disk, so nonzero symbols are confined to ℓ^1-outward geodesics (geodesic death), and for a generic probe (Lawrence–Pfander–Walnut full spark plus a Chebotarëv-based separation lemma) the compression rate is exactly pinned at $\log 2 < \log(1+\sqrt{2})$. Version 1.2.0 upgrades the class count itself to a theorem, $N_{\mathrm{dist}}^{\mathrm{V3}}(n) = 2^{n+2} - 4n - 1$ ($n \geq 2$, window $q > (4n-1)(2n+3)$): the last separation step (a-mirror pairs at equal $|b|$) is closed by the generic non-vanishing of a mixed two-anchor coefficient — a Hermitian jet at the orthogonal point of an anchor moment domain — combined with a Klein rigidity property of outward geodesics, so the b-mirror is the only generic degeneracy of the projective history record. Version 1.3.0 gates the equation-of-state candidate by an explicit conditional no-go: the finite-n rate correction decays like $n\,2^{-n}$, so under the rank–time dictionary hypothesis any transient built on the full-history profile is confined to the early low-rank window, and under the minimal count-to-density mapping class the exact counts do not supply an observable evolving equation of state (conditional no-go: **proved** given the stated hypotheses; remaining evolving-w routes: rank-dependent dictionary corrections or the spectral-stationarity route [21]); it also records the arithmetic identity of the canonical count (Pell equations, $\sqrt{2}$ convergents) and its effective memory depth one (transfer matrix).

LowLCapacity [89]: Low-multipole suppression from distinguishable-history capacity (**numerical** for the re-audited 8/3 fibre-to-base stiffness; capacity ceiling: conditional theorem chain in ProjCE [95]; saturation: **open**; no-fit grid: numerical benchmark; TE/EE transfer: **open**). Replaces the white-paper phenomenological low-ℓ envelope (two free parameters, illustrative comparison) by a small-N mechanism: the largest angular modes probe the first admissibility ranks, which contain only 3, 7, 17 distinguishable histories (canonical Pell count [97]). Under a separate saturation hypothesis, a quadrupole-anchored, parameter-free 2×2 grid of envelopes and angular dictionaries defines a saturation benchmark, not a derived prediction. Version 1.5.0 applies the Planck PR3 map calibration and the real-$m = 0$ full-sky multiplet correction to the TT Bessel cells. The leading TT diagnostic changes from the historical $27.39 \to 17.29$ reported in version 1.4.0 to the active $27.26 \to 17.70$. TE is neutral and EE mildly disfavoured under the uncorrected minimal-transfer convention; the envelope is a staircase in ℓ (integer ranks), a discrete signature. The S^3 8/3 stiffness witness is re-audited (geometric, not LPS-bound) and carries the separate modulation observable $\sqrt{8/3} \simeq 1.633$.

Superconductivity [4]: Projective phase locking as the structural origin of superconducting coherence across BCS, cuprate, and nickelate systems (**structural**).

Quantum Fisher Information [50]: The scale-free growth of the quantum Fisher information density in the strange metal $Ce_3Pd_{20}Si_6$ reduces to projective spectral data; it is a property of the full nilpotent $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ substrate (edge spectral dimension $d_s \approx 4$, $f_Q(T) \sim T^{-1}$) and only marginal for the single $w = 2$ sector, with a thermal closure $\Delta_\Pi(T) = \frac{d_s}{2} T$ derived from the heat-kernel response (regime **structural**; observed exponent **heuristic**; asymptotic value **open**). Condensed-matter companion to the superconductivity note, and a new spectral entanglement witness complementary to the entanglement sub-programme.

6 Dependency Structure

Figure 1 presents the logical dependency DAG. Node status and edge status are deliberately independent. The interactive graph types audited claim-bearing edges as *proved*, *conditional*, *postulated*, *interpretive*, or *withdrawn*, and renders synthesis links separately. Every substantive edge not yet audited defaults to *dependency*: it records use or citation and carries no derivational status. Thus a proved source node and a proved target node never make their connecting edge proved by default. Three entry paths emerge for external readers.

Foundational path.

$$\text{ENI} \to \text{Found} \to \text{HeisStr} \to \text{noscale}.$$

Requires least prior knowledge; separates the axiomatic core from the open carrier-selection edge.

Spectral path. BornInfeld $\to$ SpAdm $\to$ SpRel $\to$ O1 $\to \cdots \to$ O24 $\to$ O25 $\to$ O28 $\to$ O29. Develops the spectral machinery, the fibre-level chain, and the SGN no-go for its proposed capacity-to-rate continuation.

Physical path. Foundation $\to$ Q1 $\to$ Q2 $\to$ Q3, with O18 + O22 + O23 supplying a proposed auxiliary fibre chain whose O18 bridge is open; Q2 $\to$ Q5a $\to$ Q5b $\to$ Q6a + Q6b; O23 + O28 + Q5b $\dashrightarrow$ Q7 (requirements for an equivariant bridge; the identification is missing, verdict 3, with O29 auditing the measured rank rather than supplying it); parallel: Spectral2 $\to$ Gravity $\to$ Thermodynamics $\to$ Lorentz; Q5b + Q11 + PTO + BI $\to$ LorCap $\to$ TempProj $\to$ LCII $\to$ LCIIo1; LCo1, LCo3.

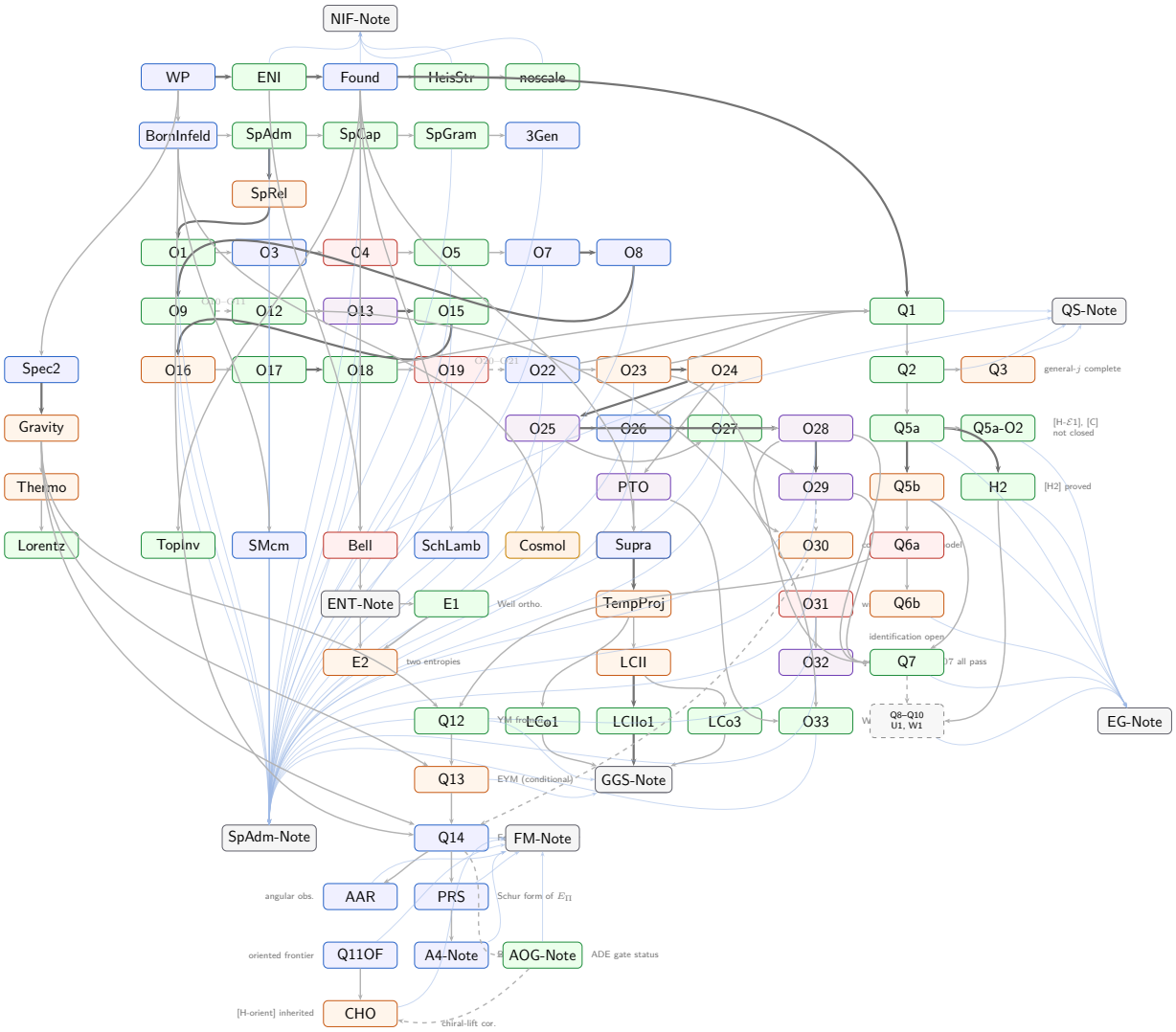


Figure 1: Logical dependency DAG of the Cosmochrony programme. Colour coding: ■ **proved**, ■ **structural**, ■ **numerical**, ■ **conditional**, ■ **heuristic**, ■ **open**, ■ **synthesis**; each single-paper node carries the primary label of its inventory row, while the dashed composite block gathers a sub-chain and carries no paper label. In both figures the grey side annotations and the dashed result boxes carry no status colour: where their text states a status it is that of the component named, not the primary label of the node. The interactive graph's conditional-reading and postulated edges (EG-Note to Q12, Q13 and Q14) are not drawn in this figure. Thick arrows indicate the main logical spine; thin arrows indicate supporting dependencies. Dashed arrows with labels abbreviate linear sub-chains. The faint converging edges gather the constituent papers of each thematic sub-programme into its synthesis node: the Branch I foundational papers (ENI, Foundation, HeisStr, noscale) into **NIF-Note**, the spectral admissibility papers into **SpAdm-Note**, the Lorentz-capacity papers into **LC-Synth**, the geometric emergence papers into **EG-Note**, Q1/Q2/Q3 and the Bell paper into the quantum-structure **QS-Note**, Q12 and Q13 into the gauge–gravity stratification **GGS-Note**, and Q14 with the companion notes PRS, AAR, Q11OF, CHO, A4-Note and AOG-Note into the fermionic-matter **FM-Note**. The dashed uncoloured arrow from Q7 leads to the geometric emergence block Q8–Q10, U1 and W1, detailed in Figure 2; it carries no status colour because what Q7 version 2.0 supplies forward is conditional on an identification its own inventory row records as open. The interactive version with clickable nodes is provided as a companion HTML file.

Table 1 lists all 116 papers of the programme, the present registry excepted. The **Key** column gives the citation key used in `cosmochrony-bibliography`. The final block gathers the sub-programme presentation and synthesis notes, marked **synthesis**: they organise results established elsewhere and are not themselves sources of new results.

Table 1: Complete inventory of the 116 programme papers, excluding the present registry.

Label	Short title	Key	Main contribution	Status
WP	White Paper	[19]	Programme overview; substrate χ; iterated projection; BI constraint; relaxation cascade	**structural**
ENI	Emergence & non-injectivity	[38]	No-go: genuine emergence $\Leftrightarrow$ Π non-injective; projection entropy S_Π	**proved**
Found	Foundation	[6]	Axioms A1–A4; the published Heisenberg/Weil selection theorem is not established; version 2.2: continuum-limit passages conditioned on the unestablished spatial-limit hypothesis [H-L] (Q5b version 2.0); co-metric completion conditional on [H-L]; relation to HJ, symplectic, WKB, FRG formalisms	**structural** / **conditional** on [H-L] / **open**
HeisStr	Heisenberg carrier obstruction	[2]	Contains the explicit S_3 counter-model to the carrier-selection implication and corrects the central-subgroup/Schur step (version 2.1); the conditional Stone–von Neumann endpoint remains valid when the Heisenberg carrier and central character are supplied	**proved** / **conditional** on a supplied carrier and central character / **open**
noscale	No external scale	[36]	No independent dimensional parameter beyond c_{BI}	**proved**
EffComp	Effective composition prerequisites (K4)	[86]	Independence blocks = bicliques of the projected support; full simplex lifts all products; cylindrical-conditioning closure iff $\operatorname{rank} W = 1$; support rectangularity does not imply preparation independence; contract proposed, not derived from A1–A4	**proved** / **open**

continued...

Label	Short title	Key	Main contribution	Status
STEFT	Simplicial states & local tomography (K4 cont'd)	[85]	Primitive family never convex ($m \geq 2$, unconditional); classical randomisation $P1$ closes it rank-independently; affine effects = cube (representation, not availability); coarse-graining gives the Boolean skeleton, AX-RND the full cube; $P1$/AX-RND independent, unified only under an explicit control-independence clause; pointwise product forced given composed-effect existence; local tomography iff local families span their ambient spaces	**proved**
HMPL	Power-lift refinement obstruction (STEFT cont'd)	[99]	μ^α Kolmogorov-consistent iff $\alpha = 1$ (general, self-contained); escort renormalisation restores it trivially but departs from the pointwise lift; exact $\mathbb{Q}(\sqrt{2})$ instantiation under its own postulate H2 on the Trajectory-Branching automaton, distinct from the Q5a [H2]; endpoint/shadow quotients incomparable; $X \leftrightarrow X^{-1}$ transports the endpoint by an exact automorphism, forcing real co-b_n phase sums; bounded exact search finds no cancellation among 62 nontrivial-phase classes of 64	**proved** / **conditional** on its postulate H2 for the instantiation
Born-Infeld	Bounded relational capacity	[12]	Capacity axiom [A-cap]; BI action as saturation candidate; bridges open	**structural** / **open**
SpAdm	Spectral admissibility (Q_8)	[55]	$A_n^{\max} = c_{\mathrm{BI}}/\sqrt{\lambda_n}$; first tractable instance	**proved**
SpCap	Spectral capacity	[56]	Capacity functional $\mathcal{C}(G, S)$; binary-polyhedral maximality proved via adjoint identity $\hat{A}_{\mathrm{adj}} = 2M - I$ and icosahedral isotropy	**proved**
SpGram	Gram rigidity	[41]	Neutral-generator geometry from $G_{st} = -\frac{1}{2}\chi_\rho(st)$	**proved**
3Gen	Three generations	[71]	Three-generation structure from ADE stratigraphic levels	**structural**
SpRel	Spectral relaxation	[9]	$\Lambda_{\mathrm{proj}} \asymp \lambda_2$ for LPS expander families	**conditional** on the capacity–isoperimetry closure and spectral-isoperimetric saturation hypotheses

continued...

Label	Short title	Key	Main contribution	Status
O1	Variable-valence graphs	[47]	LPS saturation is a fixed-valence artefact; growing valence restores window	**proved**
O3	Mass hierarchy	[45]	First SM connection: $\beta^* \in (0.09, 0.13)$ from lepton mass ratios	**structural**
O4	No cascade-exponent bound from bounded flux	[13]	Bounded relational flux and Ramanujan expansion do not yet yield a cascade-exponent bound: the former derivation of $\beta \leq 1$ does not follow from the stated argument, and any structural upper bound on β by this route is open	**open**
O5	Rep-theoretic obstruction	[28]	Fixed-repr. approaches cannot close the β gap	**proved**
O6	Matrix redundancy	[34]	Matrix-level dynamic redundancy: $R_n \to 0$ for the Steinberg fingerprint (proved); the confinement corollary $\beta_{\text{eff}} \in \beta^*$ is conditional on a power-law Ansatz that the paper's own fingerprint does not satisfy, so a genuine power-law regime remains open	**proved** / **conditional** on the power-law Ansatz / **open**
O7	Continuum target	[43]	Target: $\delta \in [7.4, 10.6]$ on 3-step fingerprints	**structural**
O8	LPS geometric obstruction	[1]	Exponential shell growth $\Rightarrow$ usable window $= O(\log q)$ BFS steps, on a shell-growth premise the paper records as an empirical observation for its family	**structural** / **numerical**
O9	Heisenberg graphs	[44]	Structural transition: $\text{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ with $\lvert B_n\rvert \sim n^4$ replaces LPS	**proved**
O10	Large-q computation	[42]	Ball growth $\hat{D} = 3.05$ at $q = 101$, continuing the convergence toward $D_{\text{hom}} = 4$; TensorSketch obstruction identified	**numerical**
O11	Fourier-character proxy	[75]	Bidimensional proxy overcomes TensorSketch obstruction	**numerical**
O12	Exact Weil blocks	[24]	$\delta_{\text{exact}} = 3$ exactly (unfolded, $q \to \infty$); finite-q fitted slopes are crossover statistics, independent of the central coordinate γ	**proved**
O13	Extended prime range	[10]	Exponent sequence at $q \in \{29, 61, 101, 151, 211\}$ does not drift toward the phenomenological target; consistent with $\delta_{\text{exact}} = 3$	**numerical**

continued...

Label	Short title	Key	Main contribution	Status
O14	Observable-class mismatch	[77]	Fixed-q normalisation $q^{-\eta}$ changes intercepts, not intra-q slopes, and defines no endpoint correction from a fitted slope alone; the central-phase correction vanishes identically; no inter-q estimator or β^* diagnostic follows; SGN places the shell factor additively	**structural**
O15	Block-level no-go	[39]	$\beta^* = 1/(\delta + \frac{1}{2})$ does not transfer to the exact-block setting; aggregation bound $\alpha_{\mathrm{dyn}} \leq \hat{\delta}_{\mathrm{exact}}$ holds conditionally, verified at $q = 151, 211$	**proved** / **conditional** on the stated aggregation hypothesis
O16	Pair observable	[26]	Conjugate pairs $\{c, q-c\}$ are fibres of Π under two independent open hypotheses; exponent doubling $\delta_{\mathrm{pair}} = 2\delta_c$ holds conditionally, trivially on the real pipeline	**conditional** on the parity and realisation hypotheses
O17	Block-independence, toy model	[23]	Raw Gram-Schmidt redundancy is exactly independent of block parameters for any two blocks, not only conjugate ones; scoped to a scalar toy model, not shown for the real pipeline	**proved**
O18	BI action parity	[22]	$S_{\mathrm{BI}}[-\chi] = S_{\mathrm{BI}}[\chi]$; neither $\Pi(\chi) = \Pi(-\chi)$ nor the character pairing $c \leftrightarrow q - c$ follows	**proved** / **open**
O19	Proposed canonical normalisation	[16]	Generic Gram–Schmidt identities survive; the residual factor, completed tests, and pipeline-independent observable are not established for the real pipeline, the conjugate-block identity of O16 [26] forcing $r(c, q) \equiv 1$ exactly on the O12/O13 exact block observable	**open** / **proved**
O20	Conditional persistence template	[46]	Threshold-crossing algebra under explicit hypotheses; no intrinsic threshold or selected exponent window	**conditional** on the monotone-amplitude and capacity-bound hypotheses

continued...

Label	Short title	Key	Main contribution	Status
O21	Local crossing diagnostic	[54]	Two-point crossing formula under a fixed normalisation; it scales under amplitude rescaling, while shell alignment and the capacity-to-rate carrier remain open	**conditional** on the shell-alignment conjecture and the cross-substrate identification / **open**
O22	Projection locking	[53]	BI projection locking forces the O21-defined n_3^{obs} onto a BFS shell within the stated model, on a sketch-level continuous decay law and the observable-space shell decomposition; this does not make that rank amplitude-invariant	**structural** / **open**
O23	Conditional adjoint dimension	[70]	For a supplied irreducible $\mathbb{C}^2$ carrier, the traceless neutral sector is $\mathfrak{su}(2) \cong \mathrm{Im}\,\mathbb{H}$, dimension three; Q_8's two-axis neutral generating set rules out generator-axis counting; carrier selection and the Σ_c-identification are open, so $\Sigma_c(n_3) = 3$ is a supplied selection rule	**conditional** on the supplied carrier and the Σ_c selection rule / **open**
O24	Observable rank rigidity	[40]	Verticality Lemma develops $c_{\mathrm{BI}} \to \delta_{\mathrm{pair}}$ under the stated fibre hypotheses; no first-principles fibre identification and no native derivation of $\delta_{\mathrm{pair}} \to \beta^*$	**conditional** on the supplied fibre and rank-three hypotheses / **open**
O25	Numerical campaign (extended to $q = 601$)	[59]	Exhaustive pair sampling through $q = 211$; representative sampling of 24 pairs at $q = 307$ and 12 at $q = 401, 601$; raw $\delta_{\mathrm{global}} \to 7.61$ at $q = 601$ under the frozen $\log(n+1)$ estimator; fixed-q pair concentration; O14 correction overcorrects at large q; δ_∞ not pinned; reciprocal image 0.108–0.123 only as a phenomenological comparison	**numerical**

continued...

Label	Short title	Key	Main contribution	Status
O26	Quadratic completion	[49]	Dictionary: Weil blocks $\leftrightarrow$ rank-1 matrices in $M_2(\mathbb{C})$; connects Heis_3 to $2I \subset \mathrm{SU}(2)$; pair-sector fibre structure taken as a motivated hypothesis; Level I growth-exponent identification of σ_{pair} with a Hilbert–Schmidt trajectory norm proved; Level II a conjecture (n-independence of the proportionality open); Level III conditional on the admissible embedding $\Phi_{q,\rho}$, version 2.1 proves the symmetric-square rank bound under Hypothesis 4.4(a) (at most three for $d_\rho = 2$, excluding the rank-four target under that contract); measured H_{eff} ranks do not test the embedded covariance; no rate equation follows	**structural** / **conditional** on the admissible embedding / **open**
O27	Quaternionic rigidity	[5]	Quotient factorisation does not imply equivariance; under explicit saturation, carrier, parity and equivariance inputs, the Schur-level counts are proved; canonical equivariant vector lift and any β^* interpretation remain open	**proved** / **open**
O28	Window calibration & r_{eff}	[8]	Calibrated linear extrapolation of n_1 fails out of sample (to $q = 601$, up to $+88\%$); the declared adjustment δ_{corr} lies in $[7.4, 10.6]$ over $q \leq 211$ and, by O14, does not correct the intra-q fitted exponent (a diagnostic, not a corrected exponent); O28 reports the 1% threshold rank $r_{\mathrm{eff}}^{1\%} = 3$ and the resolved spectrum $[1 : \frac{1}{2} : \frac{1}{2}]$ in its stated perimeter; O29's extended recomputation finds rank three modal rather than invariant (32/50 pairs at $q = 101$, 103/105 at $q = 211$)	**numerical**
CC-Note	Inverse-square coverage & constant depth	[32]	Fingerprints are Fourier characters, so $r_c(n) = \lvert T_c' + C\, I_{n+1} \rvert$ with shellwise ceiling 18; deterministically $n_1(q) \leq 22$ and $x_1(q) \asymp q^{-2}$ for $q \geq 311$; under generic sampling $n_1(q) \to 22$ and $q^2 x_1(q) \to 99{,}689$; O28's depths are the transitional regime	**proved**

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Label	Short title	Key	Main contribution	Status
SGN	Native span growth and transfer no-go	[35]	$\Delta r = \sigma_c \lvert S_n \rvert$; real-weight three-branch theorem and rigorous integer-rank finite-window law; corpus-native valence–exploration, frontier, pair-carrier and exponent-coordinate obstructions refute a Heisenberg derivation of $1/(\delta_{\mathrm{pair}} + \frac{1}{2})$	**proved**
O29	Numerical Veronese and carrier diagnostics	[74]	Finite-checkpoint numerical $6 \to 3$ collapse; fitted commutators and commutant are compatible with a supplied adjoint carrier but do not identify it; W_{adm}, the auxiliary PCA truncation $U_c^{(2)}$, and V_ρ remain distinct; neither $d_\rho = 2$ nor a Veronese root is measured	**numerical**
O30	Conditional 3×3 model	[78]	Under (H0)–(H3), $\beta_j = \varepsilon_j \bar{\alpha}_j$; two weighted cancellations give $\mathcal{C}_c = \langle r^4 \rangle \, \mathrm{diag}(1/4, 1/2, 1/4)$; this object is distinct from O28's measured 9×9 covariance and does not derive its spectrum; factor 2 comes from symmetric-tensor normalisation	**conditional** on (H0)–(H3)
O31	Withdrawal notice for colour-triplet and gauge-group claims	[115]	Withdraws the proposed individual-profile co-admissibility, arithmetic criterion, metaplectic step, $\Delta(27)$ proposition, rank-eight test, SU(3) uniqueness, irreducibility of the Born–Infeld-compatible fibre-preserving group, and every gauge-group derivation; gives explicit SO(3), T^2 and $\{1, 2, q-3\}$ counterexamples; remaining material under re-audit	**open** / **proved**
O32	[H-color] numerical test	[82]	[H-color] passes for all $q \in \{61, 151, 211, 307\}$ ($R \leq 3$, ctrl_ref $= 1.657 \times 10^{-3}$; $R = 2.1, 0.4, 1.0, 0.7$ respectively); C_{color} rank 1 at $q \in \{61, 151, 211\}$; $\delta_{\mathrm{tri}} \approx 3\delta_c$ at the percent level (0.7% at $q = 151$, 1.3 to 2.2% elsewhere); the rank and pointwise levels of [H-color] rested on the superseded O31 and are open, the averaged ($O(q^{-1})$, O32 Proposition 1) and effective levels stand; every gauge-group reading is withdrawn with O31	**numerical** / **open**

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Label	Short title	Key	Main contribution	Status
O33	Arithmetic Weil doublets and mixing	[79]	Closed sumset formula and spacing theorem proved; stabiliser reduced exactly to A_K, with an orbit-divisibility sieve, uniform exact C_4, C_6 families, and an additive rigidity theorem $A_K = \{\pm 1\}$ outside a degenerate boundary regime (completeness open only there); internal Mackey–Hecke $M_2(\mathbb{C})$ doublets and Harper split derived; boundary mixing reduced to the character sum $C_\theta(K)$ by Galois separation, with its zeros completely classified on at most four hole pairs; degenerate regime: infinite Gaussian-box C_4 family, all 75 exceptional sets through $q = 151$ in three mechanisms, no order eight at $q = 193, 241, 257$; porous depths converge to an explicit point process with mean 28/15; every proper Gaussian box, direct or wrapped, and every free four-point complement orbit proved to have exact stabiliser C_4 by a fourth-moment argument, leaving completeness and the Eisenstein top level open; odd sector protected by parity closure	**proved** / **open**
PTO	Temporal ordering	[48]	$I(n) = \sigma_{\text{pair}}(0) - \sigma_{\text{pair}}(n)$ is a non-decreasing temporal proxy; confirmed for $q \in \{29, 61, 101, 151, 211\}$	**numerical**
Fibre-Erase	Fibre erasure	[96]	Monotone $I(c; \sigma(\ell))$ on BFS coarse-graining axis ℓ via DPI (Thm 4.1, conditional on [H-suff]); [H-suff] proved at the rank level (Prop. 3.6); capacity-level numerical evidence for $q \in \{61, 151, 211\}$; explicitly not a c-theorem	**conditional** on [H-suff] and [H-sg]
Q1	Fourier-support rigidity	[30]	Exact, unconditional conjugate-sector subspace identity and its rank/support no-go corollary; no phase coherence, singlet correlator, Tsirelson bound, or Born rule	**proved**
Q2	$2I$ eigenvalue degeneracy	[7]	Exact finite-group fact: spin-$\frac{1}{2}$/spin-$\frac{3}{2}$ share $\lambda = 18$ on $2I$; no phase coherence, Born rule, Tsirelson bound, or fixed point	**proved**

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Label	Short title	Key	Main contribution	Status
Q3	Conditional universal singlet	[83]	Given an explicit, undemonstrated diagonal-$2I$-invariance hypothesis [H-inv], Schur's lemma identifies the proto-state as the singlet $\|\Omega_j\rangle$ for all $j \in \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$, with an explicit Casimir correlator; [H-inv] itself, phase coherence, and the Born rule are not derived	**conditional** on the diagonal-invariance hypothesis [H-inv] and a supplied composition
Q5a	Canonical Fourier filtration	[15]	On a supplied finite-Heisenberg carrier and non-trivial central character (version 3.2): canonical filtration identified exactly (toric Fourier window span$\{e^{2\pi i b x/q} : \|b\| \le n\}$); published form converges to the zero form (rate q^{-2}); under [H-depth] and measured positive finite weight limits, normalisation no-go (no common scalar normalisation yields a non-trivial toric differential operator); conditional dual-window Dirichlet limit in the rescaled frequency variable. No Mosco derivation of $L_\Pi = -A\partial_x^2$; the spatial limit is left as the hypothesis that Q5b names [H-L]; Q5 open	**proved** (negative, under [H-depth] and [H-w']); **open** (continuum)
Q5a-O2	Pipeline Fourier structure & truncation status	[87]	Pre-saturation fingerprints and selected GS vectors are pure Fourier modes; displacements are $o(q)$, frequencies $o(q)$ for fixed blocks but macroscopic per-q under the implemented sampling (no q-independent limit); the three-coordinate reduction is a truncation from which no rank is measurable (HEFF_DIM = 3 fixed in advance); the form is $O(q^{-2})$ on the whole unit sphere (normalisation fact), ruling out the q^{-1} coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route (zero-form limit on the canonical filtration)	**proved** / **open**

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Label	Short title	Key	Main contribution	Status
Q5b	4D Lorentzian geometry	[11]	BFS stratification of $\mathrm{Heis}_3(\mathbb{R}) \to$ 4D Lorentzian spacetime; $g^{\mu\nu} = 2\,\eta^{\mu\nu}$ asserted under stated inputs (Q9, Q10, Q11), and, since the Q7 version 2.0 audit, under the identification [ID] that no source supplies; that reading is pending revision. Spatial limit input formalised as the unestablished hypothesis [H-L], on a supplied Heisenberg carrier (version 2.1); Q9 gives a conditional kinetic limit under [K] and [R], without discharging [H-lift]; metric results remain conditional on [H-L] and [H-lift]; Carnot/$D_{\mathrm{hom}} = 4$ geometry independent	**conditional** on [H-L] and [H-lift] (metric chain); **structural** (geometry)
Q6a	Gauge structure	[31]	U(1) and SU(2) from fibre fixed points; former full-group derivation relied on withdrawn O31 claims; colour factor and complete gauge group open	**open**
Q6b	Effective spacetime	[20]	$g^{\mu\nu} = \mathrm{diag}(-2,2,2,2)$ from Π_q; Schwarzschild from flux conservation; Einstein equations as consistency conditions; closes chain $\Pi_q \to \mathcal{L}_\Pi \to g^{\mu\nu} \to G_{\mu\nu}$ conditionally; Q9 supplies only a kinetic limit under [K] and [R]; metric claims conditional on the independent [H-L] and [H-lift] and on the identification Q7 version 2.0 records as supplied by no source. Q6b's own remarks call the co-metric "fully explicit" and "fully determined" on the strength of Q8, Q10 and Q11, so those conclusions are pending revision	**conditional** on the Q5a Mosco hypotheses ([H1], [H-w], [H-$\mathcal{E}$1], Conjecture C), [H-L], [H-lift] and that identification

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Label	Short title	Key	Main contribution	Status
Q7	Requirements for an equivariant bridge	[112]	Proves $[\mathfrak{su}(2)] \not\cong \mathfrak{heis}_3$, the uniqueness of the spin-1 module and $[F_c, L_{\mathrm{Weil}}] = 0$, and gives the closed form of the compressions on the O25 data; version 2.0 withdraws the Casimir reading, the isotropy fit and the transfer to the continuum symbol. The identification $H_{\mathrm{eff}} \cong \mathrm{Sym}^2(V_\rho)$ is supplied by no source. The exact Q5b principal-symbol target is excluded by rank; a new compatible positive target remains open	**proved** (requirements on the supplied carrier); **open** (the identification, and the new target under [ACT], [INV] and [ADAPT])
Q8	Casimir rigidity	[58]	Within the frozen, unestablished Q5a hypothesis [H-F], Casimir rigidity gives $A_Z = C_{\mathfrak{su}(2)} = 2$ and fixes λ by horizontal-sector consistency; any cometric reading additionally requires [H-L] and the identification [ID] that Q7 version 2.0 records as supplied by no source. Q7 records the support for these values as circular: the scalar was fixed by matching a value itself obtained by inserting A_Z. Pending revision	**structural / open**
Q9	Kinetic limit under [K] and [R]	[111]	A nonnegative modulation term leaves a supplied kinetic Mosco limit unchanged when compatible recovery sequences suppress it. The bound is $\mathcal{V}_q(f_q) \le 8\pi^2 r_q a_q(X)\|kf_q\|^2/q^2$. The free form selects no nonzero central character; a relative moment bound gives only an elementwise inequality on a cone. Q5b-O1, [H-lift] and bridge non-obstruction remain open (version 2.0)	**conditional** on [K] and [R]
Q10	Asymptotic isotropy	[80]	$A_H(q) \to 2$ with $\lvert A_H - 2\rvert \le C\,\varepsilon(q)$; universality [U] $\Rightarrow$ $\mathfrak{su}(2)$-isotropy $\Rightarrow$ Casimir; the isotropic case of Q7's bridge conjecture was read as resolved under [U]; Q7 version 2.0 records that conjecture as neither confirmed nor refuted, and this row is pending revision (as are the Q5b, Q6b, Q8, W1 and Q11 rows; U1 is revised below)	**conditional** on [U]

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Label	Short title	Key	Main contribution	Status
U1	Obstruction to the generator route to [U]	[113]	Version 2.0 proves the exact equidistance of finite Heisenberg multiplication generators at distinct characters. A separate example excludes the claimed modulus for their sum. It withdraws the former proof of [U], its $O(q^{-1/2})$ rate, the small-θ conclusion and the closure of Q10 and Q7. The O25 observable is a sampled-block rank increment, not an identified compressed operator norm. [U] remains open	**proved** / **open**
W1	[H-w] under [H1]	[114]	$a_q(s) \to A > 0$ uniformly in s: fibre-invariance of admissible observables ($\mathcal{O} = \operatorname{Im}\Pi$) is the proposed structural origin; the U1 rate imported for quantitative control is withdrawn, so W1's result awaits audit. Its passage to the effective Lorentzian geometry consumes $A_H \to 2$ from Q10, so that step is pending revision in the present cascade; after W1 and H2 [100], which closes [H2], the open hypothesis of the former Q5a framework is [H1], with [H-$\mathcal{E}$1] and [C] not closed by Q5a-O2's finite-rank route [87]	**conditional** on [H1]
H2	[H2] proved	[100]	Strong convergence $\hat{X}_q \to x$, $\hat{P}_q \to -i\partial_x$ on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: quantitative Poisson aliasing lemma (rate $O(q^{-1})$), discrete Sobolev identity, quasi-isometry of sinc embedding; proves hypothesis [H2] of the former Q5a framework, whose [H-$\mathcal{E}$1] and [C] are not closed by the finite-rank route	**proved**
Q11	Temporal Casimir rigidity	[103]	Claims $A_\tau = C_{\mathfrak{su}(2)} = 2$ under frozen [H-F], but fixes the scalar by importing Q8's normalisation; the absolute temporal value therefore also requires revision; the co-metric reading $g^{\mu\nu} = \mathrm{diag}(-2,2,2,2) \propto \eta^{\mu\nu}$ additionally requires the independent, unestablished [H-L] and the identification Q7 version 2.0 records as supplied by no source. Pending revision	**structural** / **open**

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Label	Short title	Key	Main contribution	Status
Q12	Yang–Mills from a_4 (version 1.4)	[116]	Seeley–DeWitt $a_4 \supset \frac{1}{12}\mathrm{tr}_\rho(F^2)$ (gauge component); vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed metric yields $D_\mu F^{a\mu\nu} = 0$; induced log running $\propto I_\rho$ (non-universal), g_{YM} and G_N renormalized matching data; sectors differ in UV divergence degree (proper-time scheme), no tuning-free hierarchy claimed (**proved** for the a_4 derivation at a supplied fixed metric and supplied compact gauge data; only the emergent-base reading consumes [H-L])	**proved** / **conditional** on [H-L] for the emergent-base reading
Q13	Gauge–gravity spectral synthesis (version 1.4)	[88]	On supplied geometric and gauge data, joint $\delta_{g,\mathcal{A}} S^{\mathrm{ren}} = 0$ yields a conditional Einstein–Yang–Mills system; metric variation of the gauge kinetic term $= 2\tau_{\mu\nu}$; a_6 given as a spanning list modulo IBP and Bianchi (no unique cross-term, no UV divergence in $d = 4$); no nonlinear completion asserted; $8\pi G_N = 1/(2c_{\mathrm{EH}})$ by canonical matching with $c_F I_\rho = 1/(4g^2_{\mathrm{YM}})$; couplings are matching data, no hierarchy claimed; Euclidean; only the emergent-base reading consumes [H-L]	**conditional** on the matched coefficients and, for the emergent-base reading, on [H-L]
Q14	Fermionic matter & chirality (version 2.0)	[84]	On a supplied Heisenberg carrier and a conditional [H-L] geometry whose coefficient values are pending revision, the spinor bundle S_Π reproduces the $\mathrm{SU}(2)_L \times U(1)_Y$ gauge-sector data (no independent fermionic weak doublet: NSR-Note); projected Dirac endomorphism E_Π enforces $V-A$ chirality via the spinorial BI parity lift (proved); γ_5-weighted a_4 constrains hypercharge, with rigidity conditional on a supplied colour module; a supplied rank-three carrier yields $\mathbb{C}^3_{\mathrm{gen}}$; colour, masses, Yukawa couplings and CKM/PMNS mixing remain open	**structural** / **conditional** on the supplied colour module / **open**

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Label	Short title	Key	Main contribution	Status
PRS	Schur form of E_Π + localisation of u	[109]	$E_\Pi = -\Pi_S\, D\,(1-P)\, D\,\Pi_S^* = -M^\dagger M$ Schur complement of eliminated sector; generation split u controlled by $\mathcal{D}^\pm$-transported antiunitary chiral equivariance defect $\Delta_\chi(P) = \pi_{LL} - \tau\,\overline{\pi_{RR}}\,\tau^{-1}$; A1–A3 force $u = 0$, nonzero u requires A4 chiral symmetry-breaking; finite/Lorentzian separation: $u_{\text{fin}} = 0$ for q odd prime; operator u = spectral u on flat effective metric; closes A4 Schur transversality in the present Lorentzian spin stratum (exact metaplectic opening $\alpha = ts\,r/\sinh r \not\equiv 0$, $\mu \equiv 0$, timelike Clifford symbol off the zero-mode locus ⇒ Schur-transverse, electric genus $\mu_\chi^2 < 0$, $u \neq 0$); left-admissibility branch $\pi_{LL} = 0$ taken as the orientation-compatible branch, its derivation from A4 open (version 2.1); remaining open: explicit $1 - P(s)$ (structurally constrained by EBJ 2.0), amplitude $\lvert u\rvert$, Yukawa	**structural** / **open**
AAR	Angular amplitude observable	[98]	BCH reduction identifies per-step J_Π-odd J_3 component of metaplectic cascade generator with oriented symplectic area swept per step (Carnot-degree-2 increment of Heis_3); observable is accumulated oriented J_3-odd area along Weil–BFS cascade normalised by $\widehat{I}(n)$, equals $\varepsilon_{\text{Weil}}(n_3^{\text{obs}})$ to be compared with dictionary $\varepsilon = 1/10$ without fitting; O12 fingerprint: B_c carries radial σ_c, A_c carries angular split; symmetric BFS shells give $\Theta_{\text{raw}} = 0$ identically; capacity audit closes the onset exactly $\frac{1}{3}$, $\theta_{\max} = 2\pi/(3q)$ (generator-dependent), still gated for $\varepsilon = 1/10$	**structural**

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Label	Short title	Key	Main contribution	Status
Q11OF	Oriented frontier observable	[108]	Symmetric-capacity obstruction makes raw shell observable vanish identically ($g \mapsto g^{-1}$ pairs opposite central phases); frontier transfer observable on directed outgoing edges $g \to gs$ with central increment Heisenberg cocycle $a(g)\, s_b$ (discrete BCH $\frac{1}{2}[X, Y]$); orientation is recursive arrow $\partial_\tau = \log g$; first test is existence of oriented signal $\langle \Delta A_c \rangle_{\partial^+ S_m} \neq 0$; anti-bias control via symmetrised-frontier cancellation; resolves the operator condition of the chiral lift: the residual frequency reflection is the central Weil lift $R_b = W(-I) = \mathrm{FT}^2$ up to the metaplectic phase, hence $[\gamma_5, R_b] = 0$	**structural**
CHO	Chiral orientation of the directed frontier	[81]	Resolves the chiral weight σ_L deferred by Q11OF: the route $\sigma_L = f(T(e))$ is closed by a parity argument (edge inversion and residual reflection both send $T(e) \mapsto -T(e)$, whereas the admissible weight is arrow-even and φ-odd, proved); σ_L is instead the Weyl representation type $\mathbf{2}$ vs $\bar{\mathbf{2}}$, the unique label separated by conjugation; edge phase is the linear cyclotomic character $\zeta_q^{\Delta A_c}$, with no Gauss or Maslov factor	**conditional** on the inherited central-phase lift / **proved**

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Label	Short title	Key	Main contribution	Status
A4-Note	BI saturation margin of the chiral modulus	[105]	Fixes the sign of the generation split: defines Lorentzian $\mathcal{B}_{\mathrm{sat}}(s)$ along J_Π-odd modulus; antisymmetry lemma forces antisymmetric chiral two-form $F_{\chi,\mu\nu}(s)$; sign locked by admissibility role of $\mathcal{B}_{\mathrm{sat}}$ (A4 selects saturated minima), not by Maxwell weak-field matching; genus reduction $\mu_\chi = \partial_s^2\mathcal{B}_{\mathrm{sat}}(0) = \frac{1}{2}P_{\chi,\mu\nu}P_\chi^{\mu\nu}$; split existence/stability reduced to Lorentzian genus of P_χ in $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision; genus determined electric in the Schur-transverse branch ($\mu_\chi^2 < 0$, $u \neq 0$); version 2.1: the saturation contact $\Delta_\chi(s_*) = 0$ is the unique chart-independent A4 lock, $s_* = \beta/\lvert\mathbf{E}_\mathcal{P}\rvert$ in the affine electric gauge, $u(s_*)$ invariant under orientation-preserving charts; the Front D factorisation of $u(s_*)$ is conditional on a localisation hypothesis not satisfied by the EBJ witness; amplitude $\lvert u\rvert$ dictionary-bound through $\mathcal{N}_A$	**structural** / **conditional** on the localisation hypothesis

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Label	Short title	Key	Main contribution	Status
EBJ	Constrained jet of $1 - P(s) +$ chiral defect-rate identity	[107]	Constrains PRS open deliverable 1, the explicit Lorentzian $1 - P(s)$, structurally: for every smooth family and every smooth unitary transport the self-adjoint projector jet $1 - P(s) = Q_0 + sQ_1 + s^2Q_2 + O(s^3)$ is constrained — $Q_1 = [A, Q_0]$ is a purely off-diagonal Grassmann tangent ($Q_0Q_1Q_0 = 0 = (1 - Q_0)Q_1(1 - Q_0)$), Q_2 diagonal blocks pinned to Q_1^2 with opposite signs; the parity grading holds under J_Π-equivariance, and with a covariant moving spinorial embedding and a J_Π-even Dirac operator the residue jet is graded E_0 even, E_1 odd, the first-order J_Π-odd part of E_Π^2 is $\{E_0, E_1\}$, and $u'(0) = \langle J_3, \{E_0, E_1\}\rangle / \langle J_3, J_3\rangle$; antiunitary parity makes every odd-order generation-mixing coefficient vanish in every covariant family, with or without complex phases (even orders not excluded); an explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with $u'(0) = 11 \cdot 2^{1/4}/9 \neq 0$; for the parity written in the chiral frame as $\Sigma_\tau \circ \mathrm{conj}$ with $\Sigma_\tau = \left(\begin{smallmatrix} 0 & \tau \\ \tau^{-1} & 0 \end{smallmatrix}\right)$, $[\Pi_{J_\Pi\text{-odd}}(1 - P)]_{LL} = \frac{1}{2}\Delta_\chi(P)$ and $[\Pi_{J_\Pi\text{-odd}}\dot{Q}(0)]_{LL} = \frac{1}{2}\partial_s\Delta_\chi(P)|_0$ determine one block of the odd tangent, not a unique generator; selection of the family, localisation of the odd tangent, normalisation of $\partial_s\Delta_\chi(P)|_0$, and generation mixing at even orders open, $\lvert u\rvert$ dictionary-bound	**proved** / **open**

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Label	Short title	Key	Main contribution	Status
PYL	Projected Yukawa line + generation-level assignment	[93]	Opens the mass-sector frontier, closing its first stage: the determinant line $L_Y = \wedge^2(\mathcal{S}_\Pi)$ is the unique functorial line of the rank-two carrier, invisible to the $\mathrm{SU}(2)_L$ adjoint $\mathrm{Sym}^2(\mathcal{S}_\Pi)$ (action det = 1) and carrier of the abelian $\mathrm{U}(1)_Y$ weight, so the projected Yukawa sector is the determinant-line sector of the spinor functorial closure, not an external bundle; generation-level assignment from $E_\Pi^2|_{\mathbb{C}^3_{\mathrm{gen}}} = \mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ via exit deficits (e_0 lightest, e_- heaviest for $u > 0$); method lock: L_Y fixes the coupling line, E_Π^2 fixes the levels, the mass comes after; open downstream: the Yukawa operator Y_Π, its norm, $\mathrm{sign}(u)$, and the mixing phase	**structural** / **open**
PYO	Projected Yukawa operator + chiral polar factor	[94]	Second mass-sector step, partial no-go: the Schur-residue sector fixes the positive hermitian square $H_\Pi := Y_\Pi^\dagger Y_\Pi$ with $\mathrm{Spec}(H_\Pi)|_{\mathbb{C}^3_{\mathrm{gen}}} = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so E_Π^2 fixes the squared Yukawa levels, not the morphism; polar decomposition $Y_\Pi = U_\Pi H_\Pi^{1/2}$ leaves the unitary chiral factor U_Π (chiral orientation + complex mixing phase) free, with the norm λ_Y a separate scale of H_Π, neither seen by E_Π^2; CP-real branch $U_\Pi = \mathbf{1}$ is diagonal no-mixing ($v = 0$); Front 3b closed as squared-Yukawa levels, not mixing; Front 3c: U_Π is a rephasing class $[U_\Pi] \in U(1)_R^3 \backslash U(3) / U(1)_L^3$ (moduli + Jarlskog phase, 3 angles + 1 phase), non-trivial iff the metaplectic generator is transverse; $[U_\Pi] = [I]$ on the derived real cascade ($v = 0$, no mixing)	**structural** / **open**

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Label	Short title	Key	Main contribution	Status
KUD	Charged-lepton Koide relation as unit dispersion	[106]	Mass-sector target note: $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$, so Koide $\Longleftrightarrow$ $\mathrm{CV}(r_\ell) = 1$ ($\sigma_r = \bar{r}$); separates the dispersion constraint from the hierarchy phase (node placement); the cascade ladder $(1, \frac{1}{2} + u, \frac{1}{2} - u)$ under-disperses ($\mathrm{CV} \leq 1/\sqrt{2}$, $Q \leq 1/2$), so it cannot reach $Q = 2/3$; maximum-entropy reading ($\mathrm{CV} = 1$) recorded as an open test (first-moment constraint + forced discrete-to-continuous bridge); target invariant, not a mass formula; scoped no-go excludes the linear self-adjoint determinant realisation of $s^2 - d^2$ (equipartition route otherwise open)	**proved** / **open**
TPC	Ternary phase carrier: colour, generations, Koide as one carrier	[104]	Speculative bridge: one complex three-phase carrier $z_g = S + A\, e^{i(\delta + 2\pi g/3)}$ — pure-phase part sum-zero ($1 + \omega + \omega^2 = 0$, colour-neutral), real part the square-root mass profile of KUD; singlet amplitude S the master dial ($S = 0$ colour, $S \neq 0$ masses, $A = \sqrt{2}S$ equipartition/Koide, δ-node the hierarchy); candidate capacity filter $\|r_{\mathrm{dbl}}\| \leq \|r_{\mathrm{sing}}\|$ saturated at $\mathrm{CV} = 1$; not a derivation, states the construction charge sheet	**open**
NPI	Neutrino mixing and projective particle identity	[90]	Structural note: interaction–propagation basis misalignment is generic, alignment requires an identity-stabilising projection channel; PMNS large mixing the unfixed default (anarchy-consistent), CKM near-diagonality the explanandum (conditional: common projective frame); Dirac–Majorana as conjugation resolution; falsifiable expectation: fewer stabilising channels $\Rightarrow$ larger mixing; no angles or masses derived	**structural** / **conditional** on a common projective frame

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Label	Short title	Key	Main contribution	Status
NSR-Note	No-go: operator doublets and erased history fibres	[92]	Weil carrier commutant = $\mathbb{C}P_+ \oplus \mathbb{C}P_-$ (no fermionic lift of the Hecke doublets); C_4 frame stabiliser; erased fibres canonical $(\mathbb{Z}_2)^r$ torsors; semi-linear RK exchange of the odd axes; no-sequential-re-entry theorem $\pi v = 0 \Rightarrow Q\pi v = 0$ (all measures); history-fibre route requires a parallel oriented substrate channel (extension, not derivation)	**proved**
AOG-Note	Spin-stratum type-rigidity: ADE gate orthogonality + chiral-lift closure	[102]	$\varepsilon = 1/10$ is a theorem modulo the ADE case-selection gate: $\frac{1/2+\varepsilon}{1/2-\varepsilon} = \frac{3}{2} \Rightarrow \varepsilon = 1/10$ trivial and case-independent (proved); obstruction characterised as arithmetic orthogonality in $\mathbb{Q}(\zeta_q, \sqrt{5})$: BI parity is complex conjugation ($\zeta_q \mapsto \zeta_q^{-1}, \sqrt{5} \mapsto \sqrt{5}$), in a distinct Galois factor from the Galois-spin action ($\zeta_q \mapsto \zeta_q, \sqrt{5} \mapsto -\sqrt{5}$) (proved); order-five vs order-ten below projective parity resolution; negative theorem a no-go for the gate, unconditional in the present construction by a character-table audit of $2I$; the full-tower generalisation established for admissible (carrier-preserving) fibre operations — the Galois exchange $\sqrt{5} \mapsto -\sqrt{5}$ does not preserve the binary-icosahedral carrier, so a finite recursive tower preserves the arithmetic type (a scope hypothesis, not an absolute impossibility); chiral-lift corollary: same rigidity discharges the residual obstruction of CHO, the Lorentzian lift inducing no $\sqrt{5}$ action, closing [H-orient] with $\mathcal{N}_A \neq 0$ unconditionally in the present spin stratum (proved); concrete form: $R = 3/2$ a forced Cayley invariant on the order-five locus ($\{\frac{1}{6}, \frac{1}{4}\}$) vs $R = \frac{5}{3}$, $\varepsilon = \frac{1}{8}$ on neutral order-four, residue = the order-five selection (proved/open)	**proved** / **open**

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Label	Short title	Key	Main contribution	Status
SRN	Spin as a transformation class of rotating wave modes	[101]	Conceptual and structural audit of the rotating-wave reading of spin, companion to the fermionic-matter sub-programme: spin is fixed by how components transform under rotations of physical space, not by rotation alone (scalar counterexample); central grading on tensor powers of the admissible module, with the central element acting as $(-1)^n$ on $V_\rho^{\otimes n}$ (proved); Veronese-null vs compact-real-form lemma excludes the O28 outer-product real-form selector (proved); spinorial-soldering indeterminacy: the derived data fix the complex carrier and its univalence but neither the spatial realisation, the rotation action ρ_{sp}, nor its normalisation; corrects Q7 version 1.3 (e_0 is the weight $m = 0$ of the $j = 1$ module, not a $j = 0$ state; version 2.0 carries the corrected reading); Version 1.4 adds an independent downstream reconnaissance: $\overline{\psi_R}\psi_L$ carries exact Higgs-typed electroweak quantum numbers (H or $\widetilde{H}$) in every tree-level Yukawa sector under $\mathrm{Spin}(3,1) \times SU(3)_c \times SU(2)_L \times U(1)_Y$ (proved, exact representation audit), permitting but not deriving a composite Higgs condensate; independent of the soldering audit above; three data left explicitly open (an ambient enlargement beyond a single copy of V_ρ, a canonical inter-sector identification, and the condensation dynamics itself)	**proved** / **open**
Spec2	Relational spacetime	[51]	Operational metric from graph Laplacians; spacetime from spectral correlations	**structural**

continued...

Label	Short title	Key	Main contribution	Status
Gravity	Conditions for an IR Einstein sector	[18]	Cutoff ($a_0\Lambda^4$, $a_2\Lambda^2$) and zeta ($a_4 = \zeta_A(0)$) sectors separated; $c_{\rm EH}^{\rm ren}$, its sign and $G_N > 0$ are matching data, not predicted; BI determinantal UV completion under coherence-extensivity	**conditional** on supplied renormalised coefficients; **open** (gravitational coupling)
Thermo	Conditional Einstein–matter coupling	[21]	$G_{\mu\nu} + \Lambda_{\rm ren} g_{\mu\nu} = 8\pi G_N T^{(\Pi)}_{\mu\nu}$ from stationarity of Γ_Π, given supplied coefficients; $u \sim L^{-2}$ is not a temperature; no local first law	**conditional** on the supplied renormalised coefficients
Lorentz	No quartic graviton dispersion	[37]	Posited local four-derivative TT kernel factorizes; massless branch exactly $\omega^2 = \lvert\vec{k}\rvert^2$, no A_4, no ℓ_χ bound; pole mass and in-in construction open; vector sector and any frame-dragging reading open (a $\Box\bar{h}_{0i}$ term survives harmonic gauge)	**proved** (negative, about the posited truncation); **open** (Lorentzian construction, vector sector)
LorCap	Lorentz factor from capacity	[57]	$\eta_n^\tau \to \beta\gamma$; $\Phi(\eta_n^\tau) \to 1/\gamma$; BI factorization and central-phase factorization (**proved**); $\mathcal{P}_\tau^{\rm Q5b}$ construction: closed by TempProj [60]	**conditional** on the Born–Infeld mobility budget
TempProj	Temporal residual map	[60]	$\mathcal{P}_\tau^{\rm Q5b}\Sigma_n^{\rm tot} = \sqrt{1-B_n^2}$; uniqueness (three-input necessity); Lemma 1 of LorCap proved under [U] and the Born–Infeld lift	**conditional** on [U] and the Born–Infeld lift
LCII	Fibrewise capacity lapse	[27]	Fibrewise extension to non-homogeneous regime; $d\tau/dn\vert_x = R(x)/\gamma$; unified capacity arbitration: $1/\gamma$ (kinematic) and $R(x)$ (gravitational) as two facets of the same BI trade-off	**conditional** on the imported occupancy–lapse identification
LCIIo1	No-go for the scalar free-fraction closure of the capacity–metric bridge	[25]	The free-fraction law $A_\tau = A_\tau^\infty(1-\epsilon)$ with $g^{\tau\tau} = -A_\tau$ gives $g_{\tau\tau}/g_{\tau\tau}^\infty = 1/(1-\epsilon)$, matching $1-\epsilon$ only at $\epsilon = 0$; LC-O2-O1 reopened	**proved**

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Label	Short title	Key	Main contribution	Status
LCo1	Lorentz transformations from projective temporal ordering	[33]	Effective Lorentz transformations between inertial observers are uniquely fixed by the projective temporal capacity load β; length contraction $\ell = \ell_0/\gamma$ as a corollary	**proved**
LCo3	Finite-q corrections to the Carnot capacity identification	[29]	Parabolic Carnot formula deviates from the exact torus capacity by $\Delta_q(n) = 4m^2/q^2$ ($n = R + m$, $R = (q-1)/2$); quadratic identity exact, Carnot identification holds for $n \leq R$	**proved**
TopInv	Topological invariants	[72]	Charge quantisation ($\pi_1 \cong \mathbb{Z}$) and no monopoles on the canonical model (proved); projective chirality (Hopf fibration) established at the framework level, qualitatively; the extension to the full effective configuration space is open	**proved** / **open**
SMcm	Charge, mass, inertia	[17]	Mass and charge as BI-saturated responses	**structural**
Bell	Bell factorisation bridge	[14]	Non-injectivity alone refuted as sufficient; global-coupling obstruction not derived	**open**
BellNoGo	Redundant-extension no-go theorem	[91]	Non-injectivity has no Bell-discriminating power; correct obstruction is cross-context global-coupling absence (Fine's theorem)	**proved**
ENT-E1	Weil fingerprint orthogonality	[76]	The k=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ factors as a global phase times a single discrete Fourier mode, so the admission residual $w_j = 1$ is an exact algebraic identity and not a numerical accident; for matched single-character blocks the closure hypothesis of ENT-Note becomes a theorem and the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ is derived analytically under the companion's supplied purity of the joint state (literal Schmidt pairing for independently sampled canonical O25 blocks: open)	**proved** / **open**

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Label	Short title	Key	Main contribution	Status
ENT-E2	Two entanglement entropies of the conjugate pair	[73]	Separates the admissible pair entropy $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ on a hypothesised atomic carrier (the explicit hypothesis (H-carrier), stated by E2 itself and derived nowhere; the singlet identification further conditional on Q3's [H-inv], for general canonical blocks) from the full residual-rank entropy $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ on the full Gram–Schmidt basis (unconditional for matched blocks by ENT-E1, conditional on $[\mathrm{H_{res}}]$ and the O17 toy-model transfer gap for general blocks); excludes the metaplectic dilation as a bridge for $[\mathrm{H_{res}}]$	**conditional** on (H-carrier) and [H-inv]; **proved** for matched blocks
SchLamb	Fine-structure constant	[63]	α as projective invariant; Lamb shift and Schwinger correction	**structural**
Cosmol	Rotation curves / Hubble	[3]	Flat rotation curves and Hubble tension from a common effective mechanism; v2.0 adds the history-branching expansion sector under the explicit rank–time dictionary hypothesis ($n = \log a$), with the state-volume reading quantitatively excluded and QUMOND repositioned as input	**heuristic** / **conditional** on the rank–time dictionary hypothesis
Supra	Superconductivity	[4]	Projective phase locking as origin of superconducting coherence	**structural**
QFI	Quantum Fisher information as spectral witness	[50]	Condensed-matter companion to Supra: the scale-free growth of the quantum Fisher information density in $Ce_3Pd_{20}Si_6$ reduces to projective spectral data of the full nilpotent $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ substrate (edge spectral dimension $d_s \approx 4$, $f_Q(T) \sim T^{-1}$), only marginal for the single $w = 2$ sector, with the thermal closure $\Delta_\Pi(T) = \frac{d_s}{2} T$ from the heat-kernel response (regime structural; observed exponent heuristic; asymptotic value open)	**structural** / **heuristic** / **open**

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Label	Short title	Key	Main contribution	Status
TrajBr	Trajectory branching / de Sitter limit	[97]	Exponential growth relocated from endpoint volumes (polynomial, Bass–Guivarc'h) to projectively distinguishable histories; canonical O12-compatible channel is b-only with exact Pell count, $h_b = \log(1+\sqrt{2})$ (generic probe, exact $q = 211$ witness); full-history channel proved strictly compressive by geodesic death, pinned rate $\log 2$ and exact class count $2^{n+2} - 4n - 1$ (generic a-mirror separation via the mixed two-anchor coefficient); v1.3.0 gates the $w(n)$ candidate by a conditional no-go (early low-rank window only, under dictionary and minimal mappings)	**proved** / **conditional** on the dictionary and minimal mappings
LowLCap	Low-ℓ from history capacity	[89]	Conditional small-N capacity ceiling evaluated under a separate saturation hypothesis: Planck-calibrated, real-$m = 0$-corrected TT benchmark changes χ^2 27.26 $\to$ 17.70 in the leading Bessel+n_{H} cell; TE/EE transfer and saturation remain open; staircase envelope; 8/3 witness re-audited with observable $\sqrt{8/3}$	**numerical** / **conditional** on the saturation hypothesis / **open**
ProjCE	Projection kernel / capacity ceiling	[95]	Amplitude-level projection derived as conditional expectation onto the distinguishable-record algebra (two classical characterisations, graded hypotheses, operator invariant under degraded selection); strict ensemble-power suppression by total variance, and the parameter-free ceiling $(N-1)/N$ — identically the Bessel envelope of the low-ℓ note	**structural**

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Label	Short title	Key	Main contribution	Status
SpAdm-Note	The spectral admissibility sub-programme (Note 1)	[69]	Version 2.0 maps the five precursor papers, O1 and O3–O33 and the SGN/CC companions along bounded flux ⇒ envelope ⇒ Weil-sector capacity ⇒ $\dim_{\text{pair}}$ on a supplied carrier; the reciprocal β^* prescription has no carrier on the measurement substrate (coincidence check); SU(2) carrier supplied conditionally; O31 withdrawal; O32 measurements; O33 exact finite structure; open: [H-color] beyond O32's two levels, the spin-$\frac{1}{2}$ sector, campaign beyond $q = 211$	**synthesis**
EGNote	The emergent geometry sub-programme (Note 2)	[61]	Maps the twelve constituents (Q5a, Q5a-O2, Q5b, Q6b, Q7–Q11, U1, W1, H2). Q8 consumes frozen, unestablished [H-F]; Q9 gives a conditional kinetic-limit criterion under [K] and [R] but does not discharge [H-lift]. Every effective co-metric or Lorentzian reading, including $g^{\mu\nu} = 2\eta^{\mu\nu}$, additionally requires unestablished [H-L], [H-lift] and the identification Q7 version 2.0 records as supplied by no source	**synthesis**
GSNote	The Gauge Structure Sub-Programme (Note 3, version 2.0)	[65]	Organises Foundation, TopInv, SMcm, O23, O27–O32, Q6a, Q7 and Q12. Its former full-group identification used withdrawn O31 claims; the colour factor and complete group are open. Gauge group identification only; Yang–Mills dynamics belongs to Note 8. Consolidation of the constituent papers; no new result	**synthesis**
SGNote	The spectral gravity sub-programme (Note 4)	[110]	Organises Gravity, Thermo, Lorentz and BornInfeld into the conditions under which $g^{\mu\nu} = 2\eta^{\mu\nu}$, whose coefficient values are pending revision, supports an Einstein sector: a_2 infrared response of the projective spectral entropy, its variational restatement, the four-derivative Lorentzian truncation, and the determinantal BI completion. Consolidation of the constituent papers; no new result	**synthesis**

continued. . .

Label	Short title	Key	Main contribution	Status
NIFNote	The non-injective foundations sub-programme (Note 5, version 1.5)	[67]	Synthesises ENI, Foundation, HeisenbergStructure and noscale with separate proved, conditional and supplied ingredients: ENI's one-way implication under its own hypotheses; immediate information loss and projective incompleteness under A2; temporal order conditional on [H-acyc]; irreducibility conditional on a representation dictionary, with non-commutation requiring dimension greater than one and a generating pair. The finite S_3 obstruction is sourced in HeisenbergStructure 2.1. The Heisenberg group, non-trivial central character and associated Weil action are supplied; carrier selection, the Born rule and Q5b's spatial limit [H-L] remain open. Consolidation of the constituent papers; no new result	**synthesis**
FMNote	The fermionic matter sub-programme (Note 6)	[62]	Version 2.1 organises Theorems A–C of Q14, conditional on a supplied Heisenberg carrier and [H-L], into the chain to the spinorial face of the Weil module: $\mathrm{Sym}^2(\mathcal{S}_\Pi) \simeq \mathrm{ad}_{\mathbb{C}}(P_\Pi)$ and $\wedge^2(\mathcal{S}_\Pi) = L_Y$ reproducing $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, $V-A$ chirality and hypercharge constraints; rigidity additionally uses a supplied colour module and the three-generation factor uses a supplied rank-three carrier; the eliminated-block companion constrains one block of the odd tangent, not a unique carrier, and the contact factorisation is conditional. Masses, Yukawa couplings and CKM/PMNS mixing remain open. Consolidation of the constituent papers; no new result	**synthesis**

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Label	Short title	Key	Main contribution	Status
QSNote	The quantum structure sub-programme (Note 7)	[68]	States the current, narrow scope of Q1 (unconditional rigidity theorem), Q2 (exact eigenvalue degeneracy), and Q3 (conditional singlet theorem) and records that no chain from Bell–Infeld indiscernibility to phase coherence, a Tsirelson bound, or the Born rule is currently established. Consolidation of the constituent papers; no new result	**synthesis**
GGSNote	The gauge–gravity stratification sub-programme (Note 8, version 2.0)	[64]	On supplied geometric and gauge data, organises Q12 and Q13 around the spectral stratification principle ($a_2 \to$ gravity, $a_4 \to$ gauge), with fixed-metric isolation of the gauge sector, a_6 as a representative spanning list, and neither a non-linear completion nor a numerical coupling hierarchy claimed; only the emergent-base reading is conditional on [H-L]. Consolidation of the constituent papers; no new result	**synthesis**
ENTNote	The entanglement sub-programme presentation note	[52]	Locates entanglement inside the spectral admissibility cascade: reduced entropy of a closed conjugate Weil pair as $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$, with structural monotonicity along the cascade; integrates the ENT-E1 and ENT-E2 results (version 1.2), the pair-sector $\log 2$ being conditional on E2's atomic-carrier hypothesis (H-carrier) and [H-inv], and records the residual epistemic stratification. Consolidation of the constituent papers; no new result	**synthesis**
LC-Synthesis	The Lorentz-capacity sub-programme synthesis	[66]	Gathers LorCap, TempProj, LCII, LCIIo1, LCo1 and LCo3 into the single chain BI budget $\to F^\tau = 1/\gamma \to g^{\mu\nu} = 2\eta^{\mu\nu} \to \Lambda(\beta) \to R(x)^2 = 1 - \epsilon(x) \to$ finite-q domain, distinguishing the BI capacity sphere from the Lorentz group as isometry group of the effective metric; the co-metric coefficient values are pending revision. Consolidation of the constituent papers; no new result	**synthesis**

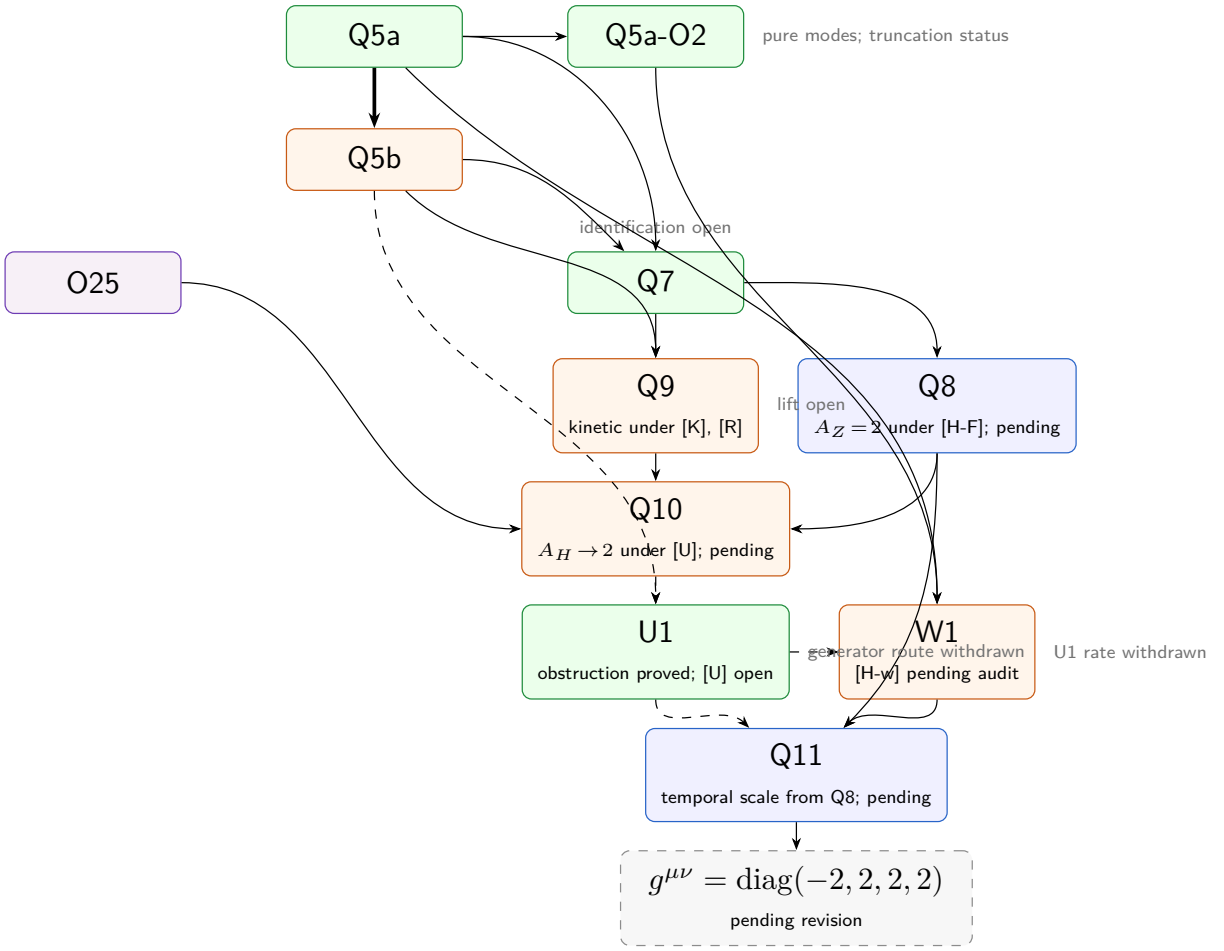


Figure 2: Geometric emergence sub-programme: dependency map from Q5a to the proposed effective Lorentzian co-metric. **Q5a-O2** [87] (**proved** for its stated results): pure Fourier modes of the pre-saturation pipeline; truncation status of the three-coordinate reduction; rules out the q^{-1} coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route (zero-form limit on the canonical filtration, Q5a version 3.2). **Q9** (**conditional** on [K] and [R]): a nonnegative modulation term leaves a supplied kinetic Mosco limit unchanged when each finite-energy vector has one kinetic recovery sequence on which the modulation energy vanishes. The bound retains its energy scale; the free limit does not select a Schrödinger character or close [H-lift]. **Q8** (**structural**, within the frozen [H-F]; pending revision): $A_Z = C_{\mathfrak{su}(2)} = 2$ by Casimir rigidity; normalisation fixed by horizontal sector. **Q10** (**conditional** on [U]; pending revision): universality [U] $\Rightarrow$ $\mathfrak{su}(2)$-isotropy $\Rightarrow$ $A_H \to 2$. **U1** (**proved** for an unconditional obstruction; **open** for [U]): exact equidistance of distinct-character Heisenberg multiplication generators; a separate adjacent-character example obstructs the claimed summed-operator modulus. The former [U] proof and $O(q^{-1/2})$ rate are withdrawn. **W1** (**conditional** on [H1]): [H-w] as a structural consequence of fibre-invariance ($\mathcal{O} = \mathrm{Im}\,\Pi$); quantitative control formerly imported from U1 is withdrawn, leaving W1 pending its own audit. The metric reading of $g^{\mu\nu} = \mathrm{diag}(-A_\tau, 2, 2, 2)$ remains conditional on the independent, unestablished spatial-limit hypothesis [H-L] of Q5a version 3.2/Q5b version 2.1, and on the identification of the measured admissible space with the spin-1 module, which Q7 version 2.0 records as supplied by no source: Q7 is **proved** for the requirements it states on the supplied carrier and **open** for that identification. [H-F] is also unestablished.

7 Open Problems

7.1 Branch II (O-Series)

O29 at larger primes. The effective rank $r_{\text{eff}} = 3$ is observed for all tested pairs at $q \in \{61, 151\}$, modally with seed sensitivity at $q = 101$, and for 103/105 pairs in the single-sample run at $q = 211$. Extension of the carrier diagnostics — Veronese collapse, $\mathfrak{so}(3)$ span and Schur commutant — to $q \in \{307, 401\}$ is pending.

Non-conjugate block protocol. A measurement protocol using non-conjugate block data remains an open construction. To probe $\mathrm{End}(V_\rho)$ it would require a typed bridge to that module; measured H_{eff} ranks alone cannot supply it. O26 2.1 excludes the rank-four target for $d_\rho = 2$ under its conjugate-product Hypothesis 4.4(a), without excluding other constructions.

Notation consolidation: $c_{\text{BI}} \to c_{\text{BI}}$. A coordinated rename of the BI saturation constant across all papers is planned and deferred pending consistency with already-deposited O-series papers.

7.2 Branch III (Q-Series and Applications)

Bridge existence at finite q (Q7–Q10). Q10 still asserts $A_H \to 2 = A_Z$ conditionally on [U], which U1 version 2.0 leaves open rather than proving. Q7 version 2.0 no longer reads that as establishing the isotropic limit of its former bridge claim; the revised Conjecture 4.7 concerns a new positive target, and it withdraws its own use of those results; Q10 remains pending revision in this cascade. The exact Q5b rank-two principal symbol cannot realise the positive rank-three target. The revised Conjecture 4.7 asks instead for a new spatial construction supplying the target, action and compatible adapted transport. Abstract intertwiner existence is already guaranteed for the supplied isomorphic modules.

$\pi_2(\mathcal{C}_{\text{eff}}) = 0$ beyond the canonical model (TopologicalInvariants, Remark 4.2). General proof of this topological property is open.

Full Standard Model gauge group (Q6a, O31–O32). O31 version 2.0 withdraws every derivation of $\mathrm{SU}(3)$ and of $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. O32 reports finite-sample numerical observations, but these do not identify a group or restore the withdrawn individual-profile claim. The colour factor, the complete gauge group, the interpretation of the O32 measurements, a transfer chain to α_s, and the identification of quarks remain open.

Baryon-to-photon ratio (SMchargemass). Quantitative derivation of $\eta \sim 10^{-10}$ is open.

$V-A$ chiral structure and fermionic matter (Q14). Q14 [84] addresses the formal derivation of the $V-A$ coupling: the projected Dirac endomorphism E_Π, via the spinorial lift of BI parity (proved), is left-admissible and enforces chiral selection. Hypercharge is not a free parameter but is fixed by anomaly-cancellation constraints on the γ_5-weighted a_4 Seeley–DeWitt coefficient. The three-generation multiplicity follows from the supplied saturation rule $\sigma_c(n_3) = 3$. A qualitative mechanism for the inter-generation splitting is identified (Q14 §6, conditionally on the supplied $\mathrm{Sym}^2(V)$ identification: static obstruction, oriented J_Π-odd lift, non-zero cascade J_3 projection). Residual open points: the amplitude of the splitting (the cascade normalisation $\varepsilon = \mathcal{N}_{\text{casc}}|\alpha|$; the $c_{\text{BI}} \to \delta_{\text{pair}} \to \beta^*$ route has no native Heisenberg derivation), the explicit mass generation mechanism, generation mixing via the complex metaplectic phase, and the Lorentzian extension of the projected Dirac operator. In the mass sector the Yukawa line is closed: $L_Y = \wedge^2(\mathcal{S}_\Pi)$ and the generation-level assignment from $E_\Pi^2|_{\mathbb{C}^3_{\text{gen}}} = \mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ are

structural [93]. The Yukawa operator splits further: its hermitian square $H_\Pi = Y_\Pi^\dagger Y_\Pi$ is closed, with $\mathrm{Spec}(H_\Pi)|_{\mathbb{C}^3_{\mathrm{gen}}} = \lambda_Y^2\{1, \frac{1}{2}+u, \frac{1}{2}-u\}$, so E_Π^2 fixes the squared Yukawa levels, not the morphism [94]; what stays open is the chiral polar factor U_Π in $Y_\Pi = U_\Pi H_\Pi^{1/2}$ (chiral orientation and complex mixing phase), the separate norm λ_Y, and hence the absolute masses and mixing. The polar factor is moreover not canonical but a rephasing class $[U_\Pi] \in U(1)^3_R\backslash U(3)/U(1)^3_L$ (moduli and a Jarlskog phase: three mixing angles, one CP phase), its non-triviality controlled by the transverse component of the metaplectic generator; on the derived real cascade that component is absent ($v = 0$), so $[U_\Pi] = [I]$ and no mixing is produced at this stratum [94]. A bridge to the spectral-admissibility programme refines this: the transverse datum splits into an internal block $e_0 \leftrightarrow e_\pm$, sourcable by a genuinely complex non-central $\mathfrak{sl}_2$ metaplectic coefficient, and an external block $e_+ \leftrightarrow e_-$ outside the $\mathrm{Sym}^2(\mathfrak{sl}_2)$ lift (a new-tower-stratum datum, gated by the same rigidity [102] as $\varepsilon = 1/10$). PYO then closes the internal block negatively at the present stratum: the non-central coefficients are forced real (the cascade step is real and the inherited lift carries only the central phase [81]), so $\Im p = \Im q = 0$, $[U_\Pi] = [I]$, and the stratum derives the diagonal split but not the mixing [94]. The oriented area N_A is the real Cartan channel and may fix the diagonal split normalisation without predicting the polar mixing class [94]. The radial factor of the split modulus is fixed in form, $s_* = \beta/|E_P|$ in the affine electric gauge [105], without numerical prediction. The angular normalisation is closed at the coefficient level: a capacity audit on the real Weil–BFS pipeline returns the q-independent exact onset $\frac{1}{3}$, hence $\theta_{\max} = 2\pi/(3q)$ for the standard campaign, while the dictionary value $\varepsilon = 1/10$ is neither a capacity fit nor fixed by $\mathrm{Sym}^2(\mathbb{C}^2)$ alone [98, 102]. The mass-sector frontier thus separates into three layers: the present stratum derives the diagonal split (angular coefficient closed, $u \neq 0$ by Schur transport), it does not derive polar mixing ($[U_\Pi] = [I]$), and the dictionary value reduces — once the order-five $\sqrt{5}$-locus is selected, giving the forced invariant $R = 3/2$ against the neutral order-four alternative $R = \frac{5}{3}$, $\varepsilon = \frac{1}{8}$ — to the single ADE / type-rigidity datum $R = 3/2$, equivalently the prime 5 or the $\sqrt{5}$-gate [102]. The third layer is a status statement: the projected-Yukawa split parameter u and the angular datum ε are the same functional of the level ratio, $\varepsilon(R_\Pi) = u$ with $R_\Pi = (\frac{1}{2}+u)/(\frac{1}{2}-u)$, so the equality $u = \varepsilon = 1/10$ is exactly the dictionary gate $R_\Pi = R_{\mathrm{ADE}} = 3/2$; the representation $\mathrm{Sym}^2(\mathbb{C}^2)$ leaves u free and the carrier-preserving recursion cannot supply the order-five value [94, 102]. The split is moreover not the mass hierarchy: at the gate the squared Yukawa levels are $\lambda_Y^2\{1, \frac{3}{5}, \frac{2}{5}\}$, order-one ratios that cannot be the observed charged-fermion hierarchy by direct assignment, and the single-level cascade factor is itself order one [9]. The hierarchy is therefore carried, if at all, by generation-dependent cascade depths n_g in the projective amplification law $\Lambda_{\mathrm{proj}}(n)$ in the exponential growth regime: the cascade exponent fixes the amplification rate, while the generation stabilisation depths remain the next structural datum, with λ_Y excluded as a hierarchy carrier [94, 9]. The linear-growth route to these depths is closed: every depth extracted from the growth law $N(\lambda; n) \sim F_{\mathrm{KM}}(\lambda)\, n$ [9] — the level-crossing depth, the saturation rank, or any intermediate fraction — is a function of the same Kesten–McKay coefficients $c_g(p)$, hence inherits both the band-edge divergence and the symmetry pin $c_2 = \frac{1}{2}$, and cannot be at once stable in the cascade parameter and correctly ordered, for the inter-block and the stratigraphic inter-level partition alike [94]. The remaining positive route — a decaying capacity-controlled per-level profile $\sigma_{\lambda_g}(n)$ transported onto the cascade — is likewise tested and closed as a static route. Quantising the per-level resolution density rather than the count removes the band-edge denominator obstruction, the depth gap becoming independent of $c_g(p)$; but an exact audit shows the Weil–Schur transport is scale-free, so the required stabilisation window is reached only by re-inserting the Laplacian valency $|S|$, the count normalisation, which returns to the already closed law $n_g = \Delta I_g / c_g(p)$ [94]. The static admissibility spectrum therefore fixes the generation structure and the order-one level split, but not the charged-fermion hierarchy. The remaining dynamical route — reading the depths off the relaxation trajectory rather than the static spectrum — is also tested at first order and reproduces the same obstruction [94]. A parallel reading, with thresholds fixed by the generation carrier, makes the depth

gap independent of the band-edge coefficient but leaves it on the order-one level-crossing scale; the sequential Born–Infeld and renewed-floor readings reach the hierarchy scale only through the cascade amplification itself or an unpinned ceiling-to-floor ratio, both of which reintroduce a free normalisation. The hierarchy is thus carried by no forced static or first-order dynamical quantity of the projective cascade: the cascade fixes generation structure, the order-one splitting, and a band-edge-independent order-one depth gap, but not the charged-fermion hierarchy at this stratum.

Gauge–gravity synthesis (Q12 → Q13). Gravity and Q12 together establish the Einstein (a_2) and Yang–Mills (a_4) sectors of the single functional $S_\Pi[g, \mathcal{A}]$ — the former from the horizontal variation, the latter from the vertical one: at fixed base metric the vertical variation isolates the Yang–Mills equation, but the two sectors are *not* independent under a joint variation, the metric variation of F^2 being the Yang–Mills back-reaction $T^{\mathrm{YM}}_{\mu\nu}$. Q13 [88] carries out the joint variation of a single matched local action and obtains the conditional coupled Einstein–Yang–Mills system, with $8\pi G_N = 1/(2c_{\mathrm{EH}})$ fixed by canonical matching. No numerical hierarchy ratio and no non-linear completion are asserted: g_{YM}, G_N and Λ_{ren} are independent renormalized matching data. Residual open points: whether a well-defined non-linear completion of the joint functional exists at all, the quantitative a_6 coefficients (after reduction of the spanning list to a minimal independent set), whether admissibility constrains any combination of the matching data, the Lorentzian continuation of the gauge sector, and the extension to projected matter currents $J^{a\nu}$.

Gauge-group identification upstream of Yang–Mills dynamics (Q12, O31). Q12 derives the Yang–Mills equations for a supplied compact G_Π from the a_4 heat-kernel coefficient. O31 version 2.0 withdraws the proposed SU(3) identification, so the colour factor and the complete gauge group remain inputs rather than derived results.

Lorentz Capacity Sub-Programme. The Lorentz-capacity sub-programme is closed for inertial kinematics, up to the co-metric coefficient values it takes from Q5b–Q11, which are pending revision; it is open in its non-homogeneous extension. TempProj closes the $\mathcal{P}^{\mathrm{Q5b}}_\tau$ construction of LorCap; LCIIo1 [25] (version 2.0) *reopens* LC-O2-O1, establishing that the scalar free-fraction law $A_\tau(x) = A^\infty_\tau(1-\epsilon(x))$ does not derive $R(x)$ from $\epsilon(x)$: with $g^{\tau\tau} = -A_\tau$ it yields $g_{\tau\tau}/g^\infty_{\tau\tau} = 1/(1-\epsilon)$, agreeing with the capacity definition only in the homogeneous vacuum. LCo1 [33] closes LC-O1: the effective Lorentz transformations between inertial observers are uniquely fixed by the projective temporal capacity load β, yielding length contraction $\ell = \ell_0/\gamma$ as a corollary. LCo3 [29] closes LC-O3: the parabolic Carnot identification holds for $n \le R := (q-1)/2$ with deviation $\Delta_q(n) = 4m^2/q^2$ for $n = R + m$, while the quadratic identity $B_n^2 + (F_n^\tau)^2 = 1$ is exact.

Thermodynamics: the integrability bridge, Lorentzian extension and matter coupling. The v1.1 local spectral first law was withdrawn in v2.0: the heat-kernel quantity $u \sim L^{-2}$ is an eigenvalue scale, not a temperature, so no first law follows from the determinant alone. Establishing one requires an independently defined heat one-form ξ_{sp} on configuration space, a physical energy notion, and a Carathéodory–Jauch–Roberts integrability result showing $\ker \xi_{\mathrm{sp}}$ is flat, hence $\delta Q = T\,\mathrm{d}S$. Analytic continuation and the explicit form of W_Π for matter fields also remain open.

8 Conclusion

The Cosmochrony corpus is organised as a coherent, multi-level programme toward the derivation of physical structure from a single primitive. Branch I (Foundation, ENI, and HeisenbergStructure [2]) is not formally closed. The S_3 countermodel proves that the published inputs do not force centrality or a Heisenberg/Weil carrier. The carrier remains a supplied realisation for the downstream mathematical programme. The binary-polyhedral maximality conjecture (SpectralCapacity [56]) is proved: $2I$ uniquely maximises $\mathcal{C}_\alpha(G,S)$ for all $\alpha > 0$ among finite SU(2) subgroups with neutral generating sets, closing the precursor programme for the O-series. Branch II (O-series) has measured the pair exponent on the supplied conjugate-pair sector through $q = 601$ (O24, O25), the Born–Infeld fibre identification remaining open (O18). SGN proves that the subsequent conversion to β^* is not derivable natively on the Heisenberg measurement substrate: its two corpus-defined closures, square-root frontier, pair-growth carrier, and exponent-coordinate identification fail. The observed numerical agreement with the mass-fitted window therefore remains a phenomenological coincidence check rather than a mass-hierarchy derivation. O29 reports finite-checkpoint numerical rank, Veronese, commutator, and commutant diagnostics that are compatible with a supplied adjoint carrier but do not identify it. Its auxiliary PCA plane is not the spin-$\frac{1}{2}$ module, and neither $d_\rho = 2$ nor a Veronese square root follows from the measurements. O30 studies a distinct conditional 3×3 model on $\mathrm{Sym}^2(V_\rho)$: (H0)–(H3) give the equatorial form, and two weighted cancellations give the ratio $[1 : \frac{1}{2} : \frac{1}{2}]$ within that model. It does not derive O28's measured 9×9 spectrum, which remains empirical. O31 version 2.0 is a withdrawal notice: the proposed colour-triplet co-admissibility, arithmetic and group-identification results do not survive audit. The colour factor and complete Standard Model gauge group are open, and O32 numerical measurements supply no replacement group derivation [115]. Branch III (Q-series and companions) develops conditional mathematical models toward quantum mechanics, spacetime, and Standard Model observables. It contains proved internal results but does not derive those physical structures from the foundational axioms as a closed chain. The SU(2) quantum sector is not established: Q3 proves, conditional on an explicit, undemonstrated diagonal-$2I$-invariance hypothesis, that the proto-state is the singlet for all five admissible sectors of $2I$ via Schur's lemma, with the universal correlator $E(\hat{a},\hat{b}) = -j(j+1)/3 \cdot (\hat{a}\cdot\hat{b})$ (**structural**); the invariance hypothesis itself, phase coherence, and the Born rule remain open (**open**). The geometric emergence sub-programme has advanced substantially: Q9 gives a kinetic Mosco limit under [K] and [R], with no derived [H-lift] or $A_H > 0$ coefficient; Q10 asserts $A_H \to 2$ under spectral universality [U]; U1 version 2.0 withdraws its proof of [U] and its $O(q^{-1/2})$ rate; it proves an unconditional equidistance obstruction for finite Heisenberg generators and leaves [U] open. W1's claim of [H-w] imports the withdrawn U1 rate and awaits its own audit. The effective Lorentzian co-metric $g^{\mu\nu} = \mathrm{diag}(-2,2,2,2) \propto \eta^{\mu\nu}$ is claimed by Q11 [103], whose absolute temporal value $A_\tau = 2$ imports Q8's scalar normalisation and is itself pending revision. Its proposed metric interpretation additionally requires the independent, unestablished spatial-limit hypothesis [H-L] and the identification of the measured admissible space with the spin-1 module, which Q7 version 2.0 records as supplied by no source; [H-F] remains separately open. The Q5b, Q6b, Q8, Q10, W1 and Q11 entries are pending revision in the cascade that Q7 version 2.0 opens; the marker is carried on the surfaces that assert the affected results. Q12 [116] completes the spectral derivation of gauge dynamics: the gauge component of the Seeley–DeWitt a_4 coefficient of the operator $A_{g,\mathcal{A}}$ on the admissible vector bundle contains the Yang–Mills density $\frac{1}{12}\mathrm{tr}_\rho(F^2)$, and the vertical variation $\delta_{\mathcal{A}} S_\Pi = 0$ at fixed metric yields $D_\mu F^{a\mu\nu} = 0$; the induced logarithmic running is non-universal (Dynkin index I_ρ), while g_{YM} and G_N remain renormalized matching data. Q13 [88] carries out the gauge–gravity synthesis, obtaining the conditional coupled Einstein–Yang–Mills system from the joint variation of a single matched local action, with $8\pi G_N = 1/(2c_{\mathrm{EH}})$ fixed by canonical matching. What is structural, within the proper-time scheme, is the difference in UV divergence degree between the sectors (a_2 quadratic vs. a_4 logarithmic); the numerical hierarchy $G_N g_{\mathrm{YM}}^2$ is not a prediction, both couplings being independent matching data. Q14 [84] extends

this to the fermionic sector: the admissible Weil module induces a spinor bundle S_Π reproducing the $\mathrm{SU}(2)_L \times U(1)_Y$ gauge-sector data (the independent fermionic weak doublet is a distinct object, excluded at the operator and history strata [92]); the projected Dirac endomorphism E_Π enforces $V-A$ chirality via the spinorial lift of BI parity (proved); hypercharge is fixed by anomaly-cancellation constraints on the γ_5-weighted a_4 coefficient; and the supplied saturation rule $\sigma_c(n_3) = 3$ yields a gauge-singlet three-generation factor $\mathbb{C}^3_{\mathrm{gen}}$. A qualitative mechanism for the inter-generation splitting is identified in Q14 §6, conditionally on the supplied $\mathrm{Sym}^2(V)$ identification (a static J_Π-protected degeneracy lifted by the oriented cascade generator), its value being dictionary-bound. Together, Q12–Q14 organise a conditional common spectral framework for supplied geometric, gauge, colour, and carrier data. They do not derive the complete Standard Model field content from non-injective projection: the colour factor, several carrier selections, [H-L], masses, Yukawa couplings, and mixing data remain open or supplied. *Interpretive outlook.* Beyond the individually labelled results above, the programme's unifying reading can be stated in one sentence — it is a *spectrally stratified theory of emergent interactions*, in which the type of a fundamental interaction is determined by the Seeley–DeWitt order at which the admissible operator responds. This reading is an interpretive synthesis, not a theorem: its force rests on the proved and structural results catalogued above, and specifically on the qualitative separation of spectral orders, not on any quantitative claim about the couplings, which are renormalized matching data throughout. Q5a-O2 [87] establishes the pure-Fourier-mode structure of the pre-saturation pipeline, the truncation status of its three-coordinate reduction (no rank measurable from it), and the $O(q^{-2})$ upper bound ruling out the q^{-1} coercivity scale; [H-$\mathcal{E}$1] and [C] are not closed by a finite-rank route, their disposition being the zero-form Mosco limit on the canonical Fourier-window filtration of Q5a version 3.2. The H2 paper proves [H2] (strong convergence of the rescaled Weil generators $\hat{X}_q \to x$ and $\hat{P}_q \to -i\partial_x$) via a quantitative Poisson aliasing lemma and a discrete Sobolev identity.

For an external reader, three entry sequences are suggested. For the quantum structure thread: ENI → Foundation → Q1, with the proposed O18 → O22 → O23 fibre chain treated as an open auxiliary route rather than a proved foundation. For the spectral mass-hierarchy question: ENI → Foundation → O16 → O24 → SGN, ending with the native-transfer no-go and leaving the numerical mass comparison phenomenological. For the spacetime thread: Spectral2 → Gravity → (Thermodynamics or Lorentz); for the geometric emergence thread: Q5a, Q5b, Q7–Q11, U1, W1 and H2, with Q9's kinetic result independent of Q7 and Q8 and with the geometric and coefficient steps still open.

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[65] J. Beau. The Gauge Structure Sub-Programme, 2026. Presentation Note 3: audits the gauge-structure chain. The phase-fibre and supplied SU(2) sectors are status-separated, while O31 version 2.0 withdraws the claimed colour-triplet derivation; SU(3) and the complete Standard Model gauge group remain open, doi:10.5281/zenodo.20435029.

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[67] J. Beau. The Non-Injective Foundations Sub-Programme, 2026. Presentation Note 5, version 1.5: synthesises ENI's one-way implication under its hypotheses, Foundation's immediate information loss and A2 projective incompleteness, temporal order conditional on [H-acyc], and irreducibility conditional on a representation dictionary. Non-commutation requires dimension greater than one and a pair generating the acting group. HeisenbergStructure 2.1 supplies the finite carrier-selection obstruction. The Heisenberg group, non-trivial central character and associated Weil action remain supplied inputs; the Born rule and Q5b spatial-limit hypothesis [H-L] remain open, doi:10.5281/zenodo.20548383.

[68] J. Beau. The Quantum Structure Sub-Programme, 2026. Presentation Note 7: status-separated map of Q1, Q2, Q3, the published Bell paper, and BellNoGo. Q1 proves Fourier-support rigidity and a no-go result; Q2 proves one finite Laplacian degeneracy; Q3's singlet and Casimir correlator require its explicit undemonstrated invariance hypothesis; and BellNoGo proves that bare non-injectivity does not explain Bell violations. Phase coherence, the Born rule, a Tsirelson bound, sector selection, and an SU(2) fixed point remain open, doi:10.5281/zenodo.20562949.

[69] J. Beau. The Spectral Admissibility Sub-Programme, 2026. Presentation Note 1, version 2.0: maps the five precursor papers, O1 and O3–O33 and the Span-Growth and Critical Coverage companion notes along bounded flux $\Rightarrow$ admissibility envelope $\Rightarrow$ Weil-sector capacity $\Rightarrow \dim_{\mathrm{pair}}$ on a supplied Heisenberg carrier and its associated Weil action; records that the reciprocal β^* prescription has no carrier on the Heisenberg measurement substrate (coincidence check), the conditionally supplied SU(2) carrier, the O31 withdrawal, O32 measurements and O33 exact finite structure; open: [H-color] beyond O32's two levels, the spin-$\frac{1}{2}$ sector, and a campaign beyond $q = 211$, doi:10.5281/zenodo.20402946.

[70] J. Beau. Three Neutral Directions from a Spinor Carrier: Conditional Status of the Threefold Admissibility Threshold, 2026. Spectral admissibility sub-program "O23" paper, doi:10.5281/zenodo.19375136.

[71] J. Beau. Three Particle Generations and Mass Hierarchy from Spectral Stratigraphy, 2026. Spectral admissibility sub-program paper, doi:10.5281/zenodo.19040490.

[72] J. Beau. Topological Invariants of Admissible Configurations: Charge Quantisation, Absence of Magnetic Monopoles, and Projective Chirality in Cosmochrony, 2026. Cosmochrony companion paper K, doi:10.5281/zenodo.19282962.

[73] J. Beau. Two Entanglement Entropies of the Conjugate Weil Pair: the Admissible Pair Sector and the Full Residual Rank, 2026. Entanglement sub-programme "E2" paper: separates the admissible pair entropy $S_{\mathrm{ent}}^{\mathrm{adm}} = \log 2$ from the full residual-rank entropy $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$. The first requires E2's own explicit hypothesis [H-carrier] and Q3's independent [H-inv]; the rank-three threshold is a supplied O23 selection rule and no mean-zero theorem removes the zero mode. The second is unconditional for matched blocks by E1 and conditional on [$\mathrm{H_{res}}$] for general blocks; the metaplectic dilation is excluded as a bridge, doi:10.5281/zenodo.20499440.

[74] J. Beau. Veronese Collapse in the Measured Trajectory Span: What the O28 Covariance Does and Does Not Identify, 2026. O29 (version 2.0): audits what the O28 rank identifies; rank three modal, not invariant (32/50 at $q = 101$, 103/105 at $q = 211$); measured span, PCA truncation and supplied spin-$\frac{1}{2}$ module separated; the projected-coordinate relation is satisfied only approximately; finite-precision Veronese, commutator and commutant diagnostics compatible with the supplied adjoint reading but deriving no carrier, doi:10.5281/zenodo.19787221.

[75] J. Beau. Weil-Block Projective Capacity on Heisenberg Graphs: Resolving the Representation Obstruction and Extracting the Capacity Exponent, 2026. Spectral admissibility sub-program "O11" paper, doi:10.5281/zenodo.19169987.

[76] J. Beau. Weil Fingerprint Orthogonality and the Flat Schmidt Spectrum of Conjugate Pairs, 2026. Entanglement sub-programme "E1" paper: proves analytically that the k=3 Weil fingerprint of $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$ is a discrete Fourier mode, upgrading the numerical hypothesis $w_j = 1$ of the entanglement-note to a theorem and deriving the flat Schmidt spectrum $S_{\mathrm{ent}}(n) = \log r_{\mathrm{pair}}(n)$ for matched conjugate-pair blocks, doi:10.5281/zenodo.20499309.

[77] J. Beau. Why Exact Heisenberg Capacity Does Not Determine the Phenomenological Cascade Rate: Observable Heterogeneity, Normalisation, and the Transfer Boundary, 2026. O14 (version 1.1): a fixed-q normalisation factor $q^{-\eta}$ changes intercepts, not slopes, and defines no endpoint correction from a fitted slope alone; the central-phase correction vanishes identically; no endpoint or inter-q estimator and no β^* diagnostic follow from the exact pipeline, doi:10.5281/zenodo.19226272.

[78] Jérôme Beau. A Conditional 3×3 Model on $\mathrm{Sym}^2(V_\rho)$: Equatorial Fibres from a Fixed Symmetric Square, and the Exact Decorrelation Conditions for its Spectrum, 2026. O30 (version 2.0): conditional 3×3 model on $\mathrm{Sym}^2(V_\rho)$, distinct from O28's measured 9×9 covariance; under the Veronese-cone, canonical inter-block identification and fixed projected-point assumptions the equatorial form follows, and the spectrum $[1 : \frac{1}{2} : \frac{1}{2}]$ holds exactly under two weighted cancellation conditions not implied by the unweighted second-harmonic condition, doi:10.5281/zenodo.20014707.

[79] Jérôme Beau. A Finite Spectral Architecture for Mass-Square Splitting and Mixing: Exact Doublets and Protected Sectors from Weil Admissibility, 2026. O33: exact cyclic-sumset closed form of the three-character Weil filtration, spacing criterion for porosity, full stabiliser through A_K, Mackey–Hecke doublets with $M_2(\mathbb{C})$ commutant and Harper split $1 \pm q^{-1/2}$, Galois separation of the boundary sum with its zero locus classified, odd-sector parity protection, exact C_4 and C_6 families with completeness and the Eisenstein top level open, doi:10.5281/zenodo.21360722.

[80] Jérôme Beau. Asymptotic $\mathfrak{su}(2)$-Isotropy of the Effective Quadratic Form and the Identification $A_H = 2$, 2026. "Q10" paper, doi:10.5281/zenodo.19880900.

[81] Jérôme Beau. Chiral Orientation of the Directed Heisenberg Frontier, 2026. Chiral orientation of the directed frontier: the chiral weight σ_L is not a function of the cascade generator but the Weyl representation type $\mathbf{2}$ versus $\bar{\mathbf{2}}$; the edge phase is the linear cyclotomic character with no Gauss, Maslov or $\sqrt{5}$ component, giving $\rho_\chi = 1$ and $\mathcal{N}_A = \theta_{\max}$; $u \neq 0$ not derived, doi:10.5281/zenodo.20735907.

[82] Jérôme Beau. Colour Triplet Co-admissibility on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: Numerical Test of Hypothesis [H-color], 2026. O32: numerical test of [H-color] on $q \in \{61, 151, 211, 307\}$ with controls $q \in \{29, 101\}$; inter-triplet variance ratio at or below the noise floor, $\delta_{\mathrm{tri}} \approx 3\delta_c$ at the percent level, triplet covariances of numerical rank one at three primes; the averaged level of [H-color] is proved to $O(q^{-1})$ (Proposition 1) and the effective level in the $q \to \infty$ limit,

both standing, while the pointwise and rank levels rested on the superseded O31 and are open; no group identification, doi:10.5281/zenodo.20145607.

[83] Jérôme Beau. Diagonal 2I-Invariance Selects the Universal Spin-j Singlet and Its Casimir Correlator, 2026. "Q3" paper, doi:10.5281/zenodo.19979535.

[84] Jérôme Beau. Fermionic Matter and Chirality from Projective Dirac Admissibility, 2026. Fermionic-matter paper Q14 (version 2.0): on a supplied Heisenberg carrier and conditionally on [H-L], the Weil module induces an admissible spinor bundle whose symmetric and determinant sectors reproduce the $\mathrm{SU}(2)_L \times U(1)_Y$ bundle structure, the projected Dirac residue enforces $V-A$ chirality, hypercharge rigidity uses a supplied colour module, and the three-generation factor a supplied rank-three carrier, doi:10.5281/zenodo.20218409.

[85] Jérôme Beau. Finite Criteria for Simplicial States and Local Tomography toward Emergent Operational Composition. Working paper, 2026. Companion to Effective Composition, resolving K4's finite, mono-system, and bipartite-composition states-and-effects content: the raw cylindrical-conditioning family is never itself convex; an explicit classical-randomisation postulate closes it onto the full simplex rank-independently; affine effects correspond to a cube of vectors (representation, not physical availability); the pointwise product of two local effects is forced given physical existence of a composed effect; local tomography holds exactly when local effect families linearly span their ambient spaces.

[86] Jérôme Beau. Finite Support and Preparation Criteria for Independent Effective Subsystems: A Contract for Non-Injective Admissibility. Working paper, 2026. Proposed finite prerequisite contract for effective composition (K4): independence blocks are exactly the bicliques of the projected support relation; on a block the full simplex lifts every product preparation; for strictly positive cylindrical conditionings, product closure holds iff the fibre-weight matrix has rank one; support rectangularity does not imply preparation independence.

[87] Jérôme Beau. Fourier Structure of the Pre-Saturation Admissible Pipeline: What the Three-Coordinate Projection Does and Does Not Measure, 2026. "Q5a-O2" paper, doi:10.5281/zenodo.19915951.

[88] Jérôme Beau. Gauge–Gravity Spectral Synthesis: A Conditional Einstein–Yang–Mills System from Projective Spectral Entropy, 2026. "Q13" paper, doi:10.5281/zenodo.20209859.

[89] Jérôme Beau. Low-Multipole Suppression from Distinguishable-History Capacity, 2026. Replaces the white-paper phenomenological low-ℓ envelope by a parameter-free small-N distinguishable-history mechanism (canonical Pell count). The conditional capacity ceiling is evaluated under a separate saturation hypothesis, so the comparison is a benchmark rather than a derived prediction. Version 1.5.0 applies the Planck PR3 map calibration and the real-$m = 0$ multiplet correction to the TT Bessel cells: the leading diagnostic changes from the historical 27.39 to 17.29 in version 1.4.0 to the active 27.26 to 17.70. TE/EE transfer and saturation remain open; staircase envelope as discrete signature; S^3 8/3 witness re-audited with its own observable $\sqrt{8/3}$. doi:10.5281/zenodo.21204352.

[90] Jérôme Beau. Neutrino Mixing and Projective Particle Identity: A Structural Note, 2026. Fermionic matter sub-programme companion note: basis misalignment as the generic situation, alignment as requiring an identity-stabilising projection channel; PMNS large mixing as the unfixed default and CKM near-diagonality as the explanandum, conditional on a common projective frame; Dirac–Majorana as projective conjugation resolution. Structural throughout, no mixing angles or masses derived, doi:10.5281/zenodo.21207750.

[91] Jérôme Beau. Non-Injective Observable Maps Do Not Explain Bell Violations: A Redundant-Extension No-Go Theorem, 2026. Cosmochrony no-go theorem paper, doi:10.5281/zenodo.21837950.

[92] Jérôme Beau. Operator Doublets, Erased History Fibres, and the No-Sequential-Re-entry Theorem, 2026. Fermionic-matter no-go note. Exact finite obstructions closing the two deposited candidates for the independent fermionic weak multiplicity: abelian carrier commutant of the finite Weil module (no fermionic lift of the Hecke doublets); C_4 frame stabiliser and canonical $(\mathbb{Z}_2)^r$ erased-fibre torsor; semilinear RK exchange of the two odd Hermitian axes; the no-sequential-re-entry theorem ($\pi v = 0 \Rightarrow Q\pi v = 0$, extended to conditional expectations of arbitrary non-degenerate measures) and the O33/EBJ lever audit; corollary: realisation through the history-fibre route requires a parallel substrate channel with an oriented central/Weyl datum, doi:10.5281/zenodo.21381089.

[93] Jérôme Beau. Projected Yukawa Line and Generation-Level Assignment, 2026. Fermionic matter sub-programme companion note: closes the first stage of the Yukawa sector — the functorial determinant line $L_Y = \wedge^2(\mathcal{S}_\Pi)$ and the generation-level assignment from $E^2_\Pi|_{\mathrm{gen}} = \mathrm{diag}(1, \frac{1}{2}+u, \frac{1}{2}-u)$ — without deriving mass values, doi:10.5281/zenodo.20767265.

[94] Jérôme Beau. Projected Yukawa Operator and Chiral Polar Factor, 2026. Fermionic matter sub-programme companion note: the squared Yukawa observable $H_\Pi = Y_\Pi^\dagger Y_\Pi$ and the chiral polar ambiguity $Y_\Pi = U_\Pi H_\Pi^{1/2}$ — E^2_Π fixes the squared levels, not the morphism, doi:10.5281/zenodo.20767498.

[95] Jérôme Beau. Projection as Conditional Expectation and the Capacity Ceiling, 2026. Derives the amplitude-level action of the non-injective projection as the conditional expectation onto the distinguishable-record sigma-algebra, under four graded hypotheses and by two independent classical characterisations (contractive projections; Moy–Rota–Douglas averaging operators); converts it into strict ensemble-power suppression (law of total variance) and the parameter-free capacity ceiling $(N-1)/N$ for a complex Gaussian mode on N distinguishable histories — identically the Bessel envelope of the low-ℓ capacity note. doi:10.5281/zenodo.21227547.

[96] Jérôme Beau. Projective Information Loss and Fibre Erasure in Admissible Non-Injective Projection, 2026. doi:10.5281/zenodo.20466459.

[97] Jérôme Beau. Projective Trajectory Branching and the de Sitter Limit, 2026. Result note: the LPS $\to$ Heisenberg transfer relocates exponential growth from endpoint volumes (polynomial, Bass–Guivarc'h degree 4) to projectively distinguishable histories. Endpoint-based distinguishability (D1–D3) provably collapses; the canonical O12-compatible residual channel is b-only (the central charge is unconditionally erased from mono-path norms) and its distinguishable-history count is the Pell count $N(n) = 2N(n-1) + N(n-2)$, rate $h_b = \log(1+\sqrt{2})$, proved for a generic probe and witnessed exactly at $q = 211$. Structural bridge from the admissibility cascade to a de Sitter-like component independent of matter content; the full-history Gabor channel keeps a projective-compression profile as the open candidate for an evolving effective equation of state. doi:10.5281/zenodo.21197757.

[98] Jérôme Beau. Reduction of the Angular Amplitude Observable: the J_3 Split as Accumulated Oriented Symplectic Area, 2026. Fermionic matter sub-programme working note: identifies, before any Weil–BFS computation, the numerical observable for the absolute normalisation of the angular generation split of Q14 §6. A Baker–Campbell–Hausdorff reduction identifies the per-step J_Π-odd J_3 component of the metaplectic cascade generator with the oriented symplectic area swept per step (Carnot-degree-2 central increment of Heis_3), not a free shear parameter. The quantity to be measured is the accumulated oriented J_3-odd area

along the Weil–BFS cascade, normalised by the projected capacity $\widehat{I}(n)$; at n_3^{obs} this equals $\varepsilon_{\mathrm{Weil}}(n_3^{\mathrm{obs}})$, to be compared with the dictionary value $\varepsilon = 1/10$ without fitting. In the O12 fingerprint the two sectors are the two terms of the exponent: frequency B_c carries the radial capacity σ_c, central phase A_c carries the angular split. Direct evaluation on symmetric BFS shells returns $\Theta_{\mathrm{raw}} = 0$ identically; the orientation prescription is an irreducible input prior to $\mathcal{N}_A$, doi:10.5281/zenodo.20601228.

[99] Jérôme Beau. Refinement Consistency Obstructs Nonlinear Power Lifts of Finite History Measures. Working paper, 2026. Takes up the phase/modulus law left open by STEFT: for any finite branching system, the power-lift family μ^α of its induced valuation is Kolmogorov-consistent, at a branching state reached with positive μ-valuation, iff $\alpha = 1$, by a self-contained comparison of x^α against x on $[0, 1]$; a row-renormalised escort lift restores consistency trivially but generally departs from the pointwise lift. Specialised, conditional on a named, non-derived maximal-entropy/Parry postulate H2, to the trajectory-branching automaton: exact instantiation in $\mathbb{Q}(\sqrt{2})$; endpoint/shadow quotients exactly incomparable; $X \leftrightarrow X^{-1}$ transports the Heisenberg endpoint by an exact automorphism, forcing real co-b_n phase sums; bounded exact search over 13120 words finds no destructive cancellation among 62 nontrivial-phase classes of 64.

[100] Jérôme Beau. Semiclassical Consistency of the Weil Representation on $\mathrm{Heis}_3(\mathbb{Z}/q\mathbb{Z})$: Quantitative Sinc Embedding, Generator Convergence, and Aliasing Control, 2026. "H2" paper, doi:10.5281/zenodo.19962051.

[101] Jérôme Beau. Spin as a Transformation Class of Rotating Wave Modes, 2026. Spin as a transformation class of rotating wave modes: spin is fixed by how components transform under spatial rotations; central grading $(-1)^n$ on tensor powers of the admissible module; Veronese-null versus compact-real-form lemma; soldering indeterminacy; version 1.4 adds the exact Higgs-typed quantum numbers of the Dirac bilinear in every tree-level Yukawa sector, permitting but not deriving a composite condensate, doi:10.5281/zenodo.21380026.

[102] Jérôme Beau. Spin-Stratum Type-Rigidity: Orthogonality of the ADE Gate and Closure of the Chiral Lift, 2026. Arithmetic orthogonality of the ADE gate: $\varepsilon = 1/10$ is a theorem modulo the ADE case-selection gate, whose derivation requires the Galois action $\sqrt{5} \mapsto -\sqrt{5}$, arithmetically orthogonal to the Born–Infeld parity $\zeta_q \mapsto \zeta_q^{-1}$; the gate cannot be discharged from the present axioms, and the chiral lift induces no $\sqrt{5}$ action, doi:10.5281/zenodo.20693084.

[103] Jérôme Beau. Temporal Casimir Rigidity and Closure of the Effective Metric: Identification of $A_\tau = 2$, 2026. "Q11" paper, doi:10.5281/zenodo.20098387.

[104] Jérôme Beau. Ternary Phase Carrier: Colour Confinement, Generation Splitting, and Koide Dispersion, 2026. Fermionic matter sub-programme note (speculative bridge): one complex three-phase carrier $z_g = S + A\, e^{i(\delta + 2\pi g/3)}$ read two ways – pure-phase part sums to zero ($1 + \omega + \omega^2 = 0$, colour-neutral class), real part $S + A\cos(\delta + 2\pi g/3)$ is the square-root mass profile of the unit-dispersion note. The singlet amplitude S is the master dial: $S = 0$ pure-phase (colour, confined); $S \neq 0$ real offset (masses); $A = \sqrt{2}S$ singlet-doublet equipartition (Koide, CV=1); δ near a node gives the hierarchy. Candidate admissibility filter: capacity balance $\|r_{dbl}\| \leq \|r_{sing}\|$ saturated at CV=1. Not a derivation; states the construction charge sheet. doi:10.5281/zenodo.21115977.

[105] Jérôme Beau. The Born–Infeld Saturation Margin of the Chiral Modulus: Antisymmetric Structure, Lorentzian Genus, and Electric Determination of the Generation Split, 2026. Fermionic matter sub-programme companion note to PRS. Fixes the sign of the generation split through the definition of the Lorentzian saturation functional $\mathcal{B}_{\mathrm{sat}}(s)$ along the J_Π-odd

modulus controlling the three-generation split coefficient u. Three results: (i) antisymmetry lemma — the generation modulus is J_Π-odd and the symmetric square $M = F_\chi F_\chi$ is J_Π-even from birth, so the primary modulus variable must be an antisymmetric chiral two-form $F_{\chi,\mu\nu}(s)$; (ii) $\mathcal{B}_{\rm sat}(s)$ defined as the Born–Infeld determinantal saturation margin of this two-form, with sign locked by the admissibility role of $\mathcal{B}_{\rm sat}$ (axiom A4 selects saturated minima), not by Maxwell weak-field matching; (iii) genus reduction — $\mu_\chi := \partial_s^2 \mathcal{B}_{\rm sat}(0) = \frac{1}{2} P_{\chi,\mu\nu} P_\chi^{\mu\nu}$, so the existence and stability of the split are governed entirely by the Lorentzian genus of the chiral polarisation P_χ in the effective metric $g^{\mu\nu} = 2\eta^{\mu\nu}$. The Schur transversality is closed in PRS, so the genus is determined electric ($\mu_\chi^2 < 0$, $u \neq 0$); the radicand is extended to quartic order, the boundary contact $\Delta_\chi(s_*) = 0$ is selected as the unique chart-independent A4 lock ($s_* = \beta/|\mathbf{E}_\mathcal{P}|$ in the affine electric gauge), the contact factorisation is conditional on a localisation hypothesis that the EBJ witness does not satisfy, and $|u|$ remains dictionary-bound through $\mathcal{N}_A$, doi:10.5281/zenodo.20633931.

[106] Jérôme Beau. The Charged-Lepton Koide Relation as a Unit-Dispersion Constraint, 2026. Fermionic matter sub-programme source note: recasts the charged-lepton Koide relation as a unit coefficient-of-variation condition on the square-root mass vector, $Q_\ell = (1 + \mathrm{CV}(r_\ell)^2)/3$, hence $Q_\ell = 2/3 \iff \mathrm{CV}(r_\ell) = 1$ ($\sigma_r = \bar{r}$); separates the dispersion constraint from the hierarchy phase (node placement); records the cascade-ladder dispersion obstruction $\mathrm{CV} \leq 1/\sqrt{2}$, $Q \leq 1/2$; states the maximum-entropy reading as an open S_Π-variational test. Target invariant for the fermionic-mass sector, not a mass formula. doi:10.5281/zenodo.21109812.

[107] Jérôme Beau. The Constrained Jet of the Lorentzian Eliminated Block and the Chiral Defect-Rate Identity: A Moving-Schur Existence Theorem for the Generation Split, 2026. Fermionic matter sub-programme companion note, logically between the Schur reduction (PRS) and the physical identification of the chiral generator. For every smooth self-adjoint projector family $1 - P(s) = Q_0 + sQ_1 + s^2 Q_2 + O(s^3)$ the projector constraints force $Q_1 = [A, Q_0]$ to be a purely off-diagonal Grassmann tangent and pin the diagonal blocks of Q_2 to Q_1^2 with opposite signs; the jet parities alternate under J_Π-equivariance. With a covariant moving spinorial embedding and a J_Π-even Dirac operator the first-order J_Π-odd part of E_Π^2 is $\{E_0, E_1\}$, and antiunitary parity makes every odd-order generation-mixing coefficient vanish, mixing at even orders not being excluded. An explicit covariant moving family realises $E_0^2 = \mathrm{diag}(1, \frac{1}{2}, \frac{1}{2})$ with a non-zero split rate $u'(0) = 11 \cdot 2^{1/4}/9$. In the chiral frame $[\Pi_{J_\Pi\text{-odd}} \dot{Q}(0)]_{LL} = \frac{1}{2} \partial_s \Delta_\chi(P)|_0$ determines one block of the odd tangent, not a unique generator; the selection of the family, the localisation, and the normalisation of $|u|$ remain open, doi:10.5281/zenodo.20763532.

[108] Jérôme Beau. The Oriented Frontier Observable: a Recursive Q11 Diagnostic for the Angular Generation Split, 2026. Fermionic matter sub-programme diagnostic note. The symmetric-capacity obstruction makes the raw shell observable of the angular generation split vanish identically ($\Theta_{\rm raw} = 0$): the involution $g \mapsto g^{-1}$ pairs opposite central phases on symmetric BFS shells, erasing the oriented J_Π-odd component before the radial recursion can act. This note defines the honest replacement: a frontier transfer observable on the directed outgoing edges $g \to gs$, whose central increment is the Heisenberg cocycle $a(g)\, s_b$, discrete counterpart of the BCH term $\frac{1}{2}[X, Y]$ and hence of the dynamical J_3. The orientation is the recursive arrow $\partial_\tau = \log g$ from the projection origin. The first test is the existence of an oriented signal $\langle \Delta A_c \rangle_{\partial^+ S_m} \neq 0$, which may fail under residual frontier re-pairing; the discriminating anti-bias control is the symmetrised-frontier cancellation. No normalisation, no dictionary value, no ε enters; the note tests only whether the recursion produces a non-zero, bias-free oriented signal, doi:10.5281/zenodo.20601245.

[109] Jérôme Beau. The Projective Residue as a Schur Complement: Reduction of the Generation Split to the Chiral Asymmetry of Projection Locking, 2026. Fermionic mat-

ter sub-programme note: exhibits the projective endomorphism as the Schur complement $E_\Pi = -\Pi_S D (1-P) D \Pi_S^* = -M^\dagger M$ of the spinorial directions eliminated by the non-injective projection; reduces the inter-generation split coefficient u to the $\mathcal{D}^\pm$-transported, generation-projected component of the antiunitary chiral equivariance defect $\Delta_\chi(P) = \pi_{LL} - \tau\,\overline{\pi_{RR}}\,\tau^{-1}$; stratifies the result on the Cosmochrony axioms (A1–A3 force $u = 0$; non-zero u requires A4 chiral symmetry-breaking); proves finite/Lorentzian separation (finite locking is J_Π-equivariant, so $u_{\mathrm{fin}} = 0$); locks Seeley–DeWitt conventions so operator and spectral u coincide; no-go result: chirality is Lorentzian, not a finite-fibre label. Version 2.1 closes the A4 Schur transversality in the present Lorentzian spin stratum (exact metaplectic opening $\alpha(t,s) = ts\,r/\sinh r \not\equiv 0$, $\mu \equiv 0$, timelike Clifford symbol off the zero-mode locus), so that with the electric genus of the A4 companion $u \neq 0$; the eliminated-block companion constrains the explicit block $1 - P(s)$; open: $1 - P(s)$ itself, the normalisation $|u|$, and the projected Yukawa sector, doi:10.5281/zenodo.20601040.

[110] Jérôme Beau. The Spectral Gravity Sub-Programme, 2026. Presentation Note 4, doi:10.5281/zenodo.20533351.

[111] Jérôme Beau. Toward a Kinetic Limit on a Supplied Schrödinger Fibre: Conditions for Generator Suppression, 2026. "Q9" paper, version 2.0, doi:10.5281/zenodo.19880574.

[112] Jérôme Beau. Toward Three Spatial Directions from a Supplied Spinor Carrier: Requirements for an Equivariant Bridge, 2026. Q7 (version 2.0): states the requirements an equivariant bridge would have to meet and proves those that follow from representation theory on a supplied carrier; the identification of the measured space with the spin-one module is an explicit hypothesis supplied by no source. The numerical section is re-typed: the stored basis rows are pure Fourier modes, so the flat-mode entry of the compression, historically named A_z, is the Rayleigh quotient $4 - 2 - 0 = 2$, while A_z itself is left an unevaluated parameter, and the gap between the two distinct entries is the identity $4\sin^2(\pi k/q)$. The Casimir reading, the isotropy fit and the downstream imports are withdrawn. Verdict on the bridge: missing identification, neither derived nor ruled out, doi:10.5281/zenodo.19802123.

[113] Jérôme Beau. Uniformity Requirements for Weil Fingerprint Energies: Obstructions to the Lipschitz Route to [U], 2026. "U1" paper, version 2.0, doi:10.5281/zenodo.19881146.

[114] Jérôme Beau. Weight Stabilisation of the Admissible Dirichlet Form: Proof of Hypothesis [H-w] from Spectral Universality, 2026. "W1" paper, doi:10.5281/zenodo.19886319.

[115] Jérôme Beau. Withdrawal Notice for O31: Unsupported Colour-Triplet Co-admissibility and Gauge-Group Claims, 2026. O31 withdrawal notice, doi:10.5281/zenodo.20030491.

[116] Jérôme Beau. Yang–Mills Dynamics from Projective Spectral Entropy: Vertical Heat-Kernel Variation on the Admissible Fibre, 2026. "Q12" paper, doi:10.5281/zenodo.20189137.