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Producers' and Consumers' Surplus

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PRODUCERS' AND CONSUMERS' SURPLUS.

§ 1. I do not propose in this paper to discuss the difficulties which are encountered when consumers' or producers' surplus measured in money is used as an index of consumers' or producers' surplus of satisfaction. Those difficulties have, I conceive, already been fully explored. My problem—logically a prior one—is concerned with the difficulties in the way of determining producers' and consumers' surplus *as measured in money*. This problem was opened up in Mr. Henry Cunyngame's celebrated article on *Some Improvements in Simple Geometrical Methods of Treating Exchange Value, Monopoly, and Rent*; ¹ it is discussed briefly in Prof. Edgeworth's review of Mr. Cunyngame's *Geometrical Political Economy*, ² and at greater length in my article, *Some Remarks on Utility*. ³ The present paper aims at carrying this discussion somewhat further. It is concerned exclusively with long-period phenomena.

§ 2. There is much to be urged against the employment of novel terminology. In the present instance, however, I cannot but think that there is more to be gained than lost by this procedure. Consequently, I shall begin by setting out a series of definitions, some of which involve new phrases.

Let the *supply price* of a given quantity x of any product in any market be defined as that price which is able, and any lesser price than which is not able, to evoke in that market the production of exactly $(x + \Delta x)$ units. ⁴

In like manner let the *demand price* of a given quantity x of any product in any market be defined as that price which is able, and any greater price than which is not able, to evoke in that market the consumption of exactly x units.

¹ ECONOMIC JOURNAL, vol. ii, p. 35, *et seq.*

² *Ibid.*, vol. xv, p. 62, *et seq.*

³ *Ibid.*, vol. xiii, p. 58, *et seq.*

⁴ The cumbrous phrasing of this definition is necessary in order to avoid ambiguity in cases where the supply curve is inclined negatively; for in these cases a price too small to evoke exactly x units may nevertheless suffice to evoke more than x units. The cumbrousness in the definition of demand price is necessitated by an analogous cause.

Let the *private marginal supply price* of a given quantity x of any product in any market be defined as the difference made to the expenses of production (in terms of money) of the person who would produce the extra Δx units of product, by an increase in total production from x to $(x + \Delta x)$ units.

In like manner let the *private marginal demand price* of a given quantity x of any product in any market be defined as the difference made to the ophelimity¹ (in terms of money) of the person who would consume the extra Δx units of product, by an increase in total consumption from x to $(x + \Delta x)$ units.

Let the *collective marginal supply price* of a given quantity x of any product in any market be defined as the difference made to the aggregate expenses (in terms of money) of all the producers, by an increase in total production from x to $(x + \Delta x)$ units.

In like manner let the *collective marginal demand price* of a given quantity x of any product in any market be defined as the difference made to the aggregate ophelimity (in terms of money) of all the consumers, by an increase in total consumption from x to $(x + \Delta x)$ units.²

Let the *average full expenses of production* of a given quantity x of any product in any market be defined as the sum of the collective marginal supply prices of all quantities between o and x , divided by x .

In like manner let the *average full yield of ophelimity* of a given quantity x of any product in any market be defined as the sum of the collective marginal demand prices of all quantities between o and x , divided by x .

§ 3. The various sorts of prices, thus distinguished, as adjusted to various quantities of output, can be represented by curves. The curves relevant to production are (1) a supply curve SS_1 , (2) a curve of private marginal supply prices SS_2 , (3) a curve of collective marginal supply prices SS_3 , (4) a curve of average full expenses of production SS_4 . Those relevant to consumption are (1) a demand curve DD_1 , (2) a curve of private marginal demand prices DD_2 , (3) a curve of collective marginal demand prices DD_3 , (4) a curve of average full yields of ophelimity DD_4 . It is clear that all the four curves relevant to production start at one point on the axis of Y , and, similarly, that all the four curves relevant

¹ The term ophelimity is due to Professor Pareto. It is free from certain ambiguities involved in the common English term utility.

² In a complete discussion it would be necessary to distinguish further between *actual* and *adjudged* private marginal supply and demand prices, because people's estimates of the satisfaction that they will obtain from a purchase or the expenses that they will incur in making a product are sometimes incorrect. In the following pages what is adjudged is assumed to be also actual.

to consumption start at one point on this axis. It is not, however, necessary in regard to either set that coincidence shall be maintained throughout the length of the curves. It is, therefore, desirable to study their relations. All the curves as employed here are, it will be remembered, descriptive of long-period tendencies only.

§ 4. First : consider the relation subsisting between the curve of private marginal supply prices and the curve of collective marginal supply prices.

The private marginal supply price and the collective marginal supply price of any quantity will be equivalent, provided that the difference made by an increase in total production from x to $(x + \Delta x)$ units to the expenses of production of the person who would produce the extra Δx units of product is equal to the difference made to the aggregate expenses of production of all producers. The condition necessary to this result is that an alteration in the output of one supplier does not indirectly alter the expenses associated with a given production of any other supplier. When the whole output is in the hands of a single supplier this condition is always realised. When the output is in the hands of a number of suppliers it is sometimes realised. It would seem, for example, to be realised approximately in some species of agricultural industry. In this case the curves of private marginal supply prices and of collective marginal supply prices will coincide.

The private marginal supply price of any quantity of output will be less than the collective marginal supply price, if an addition to the output of one supplier increases the expenses associated with a given production of any other supplier. This condition is realised, *ceteris paribus*, in an industry worked by more than one supplier, employing a raw material the price of which is raised when its output is increased. In this case the curve of private marginal supply prices, though starting at the same point on the axis of Y as the curve of collective marginal supply prices, lies below that curve through the remainder of its course.

The private marginal supply price of any quantity of output will be greater than the collective marginal supply price if an addition to the output of one supplier diminishes the expenses associated with a given production of any other supplier. This condition is realised, *ceteris paribus*, in an industry worked by more than one supplier, where increased output leads to the *external economies* of improved general organisation—whether production in that industry does, or does not, obey the law of increasing returns as ordinarily understood. In this case the

curve of private marginal supply prices, though starting at the same point on the axis of Y as the curve of collective marginal supply prices, lies above that curve through the remainder of its course.

The relation between the curve of private marginal demand prices and the curve of collective marginal demand prices is exactly analogous to the one I have been describing. The two curves are identical, provided that an alteration in the consumption of one consumer does not indirectly alter the ophelimity associated with a given consumption by any other consumer;—a condition necessarily fulfilled when only a single consumer exists. The curve of private marginal demand prices lies above the curve of collective marginal demand prices if an addition to the consumption of one consumer diminishes the ophelimity associated with a given consumption by other consumers—in cases, for instance, of commodities that are desired partly because they are rare. The curve of private marginal demand prices lies below the curve of collective marginal demand prices if an addition to the consumption of one consumer increases the ophelimity associated with a given consumption by other consumers—in cases, for instance, of commodities that are desired partly because they are common.¹

§ 5. Secondly: consider the relation subsisting between the curve of collective marginal supply prices and the curve of average full expenses of production.

This relation is perfectly precise. Let Q be any point on SS_3 and P the point on SS_4 at which a vertical line through Q cuts SS_4 .

Draw $P R K$ through P , cutting SS_3 in R and the axis of Y in K .

Then, when SS_3 is given, P is determined at such a point that the area $P R Q$ is equal to the area $R K S$.

It is easily seen that, for all values of x in respect of which SS_4 is inclined positively, SS_4 lies below SS_3 : whereas, for all values of x in respect of which SS_4 is inclined negatively, SS_4 lies above SS_3 ; and that, so long as SS_3 remains inclined in one direction, the distance between P and Q increases the further they move towards the right. These propositions are valid alike if the inclination of SS_4 is of the same kind throughout, and if it is first positive and then negative, or *vice versa*.

The relation between the curve of collective marginal demand

¹ The divergences summarily mentioned in the above paragraph are discussed in Mr. Cunyngname's article and in my paper *Some Remarks on Utility*, already cited.

prices and the curve of average full yields of ophelimity is identical with the above. Diagrams drawn on exactly the same plan as that adopted in regard to supply would show the position of DD_4 to be determined by that of DD_3 , lying below it for all values of x in respect of which DD_4 is inclined positively and above it for all values in respect of which DD_4 is inclined negatively.¹

§ 6. Thirdly : consider the relation subsisting between the supply curve and the two curves of private marginal supply prices and average full expenses of production.

Primâ facie, it seems evident that the supply curve SS_1 and the curve of private marginal supply prices SS_2 must be identical. Equivalence between these two curves is not, however, possible if the private marginal supply price of any quantity x multiplied by x is less than the average full expenses of production

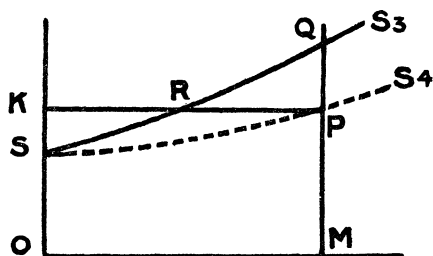


FIG. I.

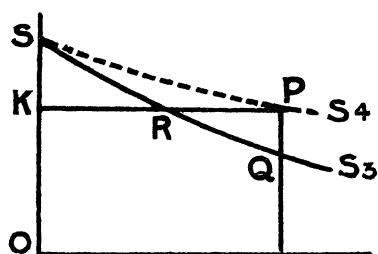


FIG. II.

multiplied by x . For if, in these conditions, a quantity x were sold at the private marginal supply price, the industry as a whole would be yielding less than normal returns, capital and labour would be driven out of it, and the quantity x would not continue to be called out by the private marginal supply price of the quantity x . Therefore, in the conditions stated, the supply price of a quantity x is greater than the private marginal supply price of this quantity. It is evidently equal to the average full expenses of production of a quantity x . Hence it follows that the supply curve SS_1 is coincident with the curve of private marginal supply prices SS_2 for values of x in respect of which SS_2 lies above SS_4 : and is coincident with the curve of average full expenses of production SS_4 for values of x in respect of which SS_2 lies below SS_4 . This proposition holds good whether SS_4 is inclined positively or negatively. If in the annexed figure $S R K S_2$ represent the curve of private marginal supply prices and $S Q K S_4$

¹ Cf. my paper "Monopoly and Consumers' Surplus," *ECONOMIC JOURNAL*, xiv., p. 388.

the curve of average full expenses of production, the supply curve will be represented by $S R K S_4$.

The general result thus reached has a corollary in the special case where the curve of private marginal supply prices and the curve of collective marginal supply prices are identical. For in that case our reasoning shows that the supply curve is coincident with the curve of collective marginal supply prices SS_3 for values of x in respect of which SS_3 lies above SS_4 , and is coincident with the curve of average full costs of production in respect of all other values. But in § 5 it was proved that SS_3 necessarily lies above SS_4 for all values in respect of which SS_4 is inclined positively and below SS_4 for all values in respect of which that curve is inclined

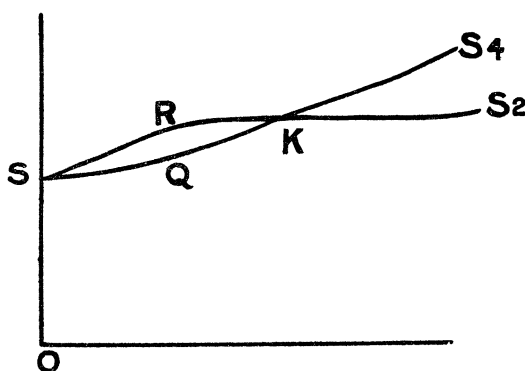


FIG. III.

negatively. Hence, in the special case when the curves of private marginal supply prices and collective marginal supply prices are identical, the supply curve, for all values in respect of which it is inclined positively, coincides with the curve of collective marginal supply prices, and for all values in respect of which it is inclined negatively, with the curve of average full expenses of production.

The relation subsisting between the demand curve and the two curves of private marginal demand prices and average full yields of opheimlimity is exactly analogous to the above relation. The demand curve DD_1 is coincident with the curve of private marginal demand prices DD_2 for values of x in respect of which DD_2 lies below DD_4 ; and is coincident with the curve of average full yields of opheimlimity DD_4 for values of x in respect of which DD_2 lies above DD_4 . This proposition holds good whether DD_4 is inclined positively or negatively. The general result has a corollary in the special case where the curve of private marginal demand prices and the curve of collective marginal demand prices

$ORPM$ minus the area $OSQM$. If the conditions are such that the curve of collective marginal supply prices coincides with the supply curve, this magnitude is equal to the area SPR . If SS_3 lies below SS_1 , it is equal to the area SPR plus the area SPQ ; if SS_3 lies above SS_1 , it is equal to the area SPR minus the area SPQ .

This completes the argument as regards producers' surplus. To adapt it to the case of consumers' surplus merely verbal changes are required.

§ 8. These results lead to a conclusion of some importance in connection with the doctrine of *maximum satisfaction*. Draw the supply and demand curves SS_1 and DD_1 , and also the curves

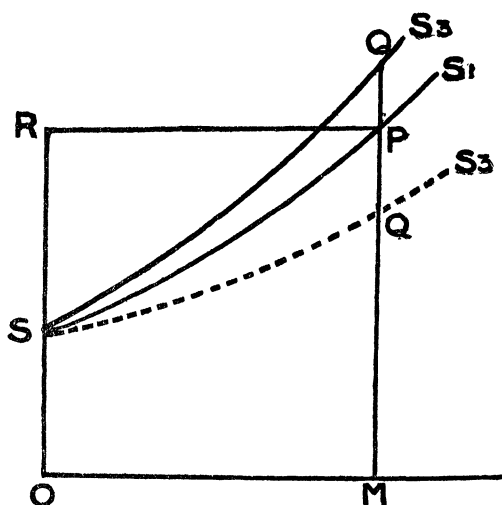


FIG. V.

of collective marginal supply prices and collective marginal demand prices SS_3 and DD_3 . Let SS_1 and DD_1 intersect at P , and let PM be drawn vertically to the axis of X . It is often implied in popular argument that producers' surplus and consumers' surplus in terms of money are necessarily maximised, in these circumstances, by the production and consumption of OM units of the commodity. From our analysis, however, it is clear that the sum of these two surpluses is a maximum when ON units are being produced and consumed, ON being the quantity corresponding to the intersection of the curves SS_3 and DD_3 . This quantity ON is only identical with OM provided that the distance by which SS_3 lies below (or above) SS_1 at the point corresponding to the production of OM units is equal to the

distance by which DD_3 lies below (or above) DD_1 at this point. This condition obviously includes the special case in which SS_1 coincides with SS_3 , and DD_1 with DD_3 . Apart from this condition, ON may be either greater or less than OM , according to the relations that subsist between the curves. Where ON is greater than OM , the grant of a bounty on the production of the commodity, provided that the bounty is not large enough to expand production beyond ON , and provided that no cost is involved in the levy of the funds from which the bounty is drawn, must, other things equal, be socially advantageous. Where ON is less than OM , the imposition of a tax on the production of the commodity, provided that the tax is not large enough to

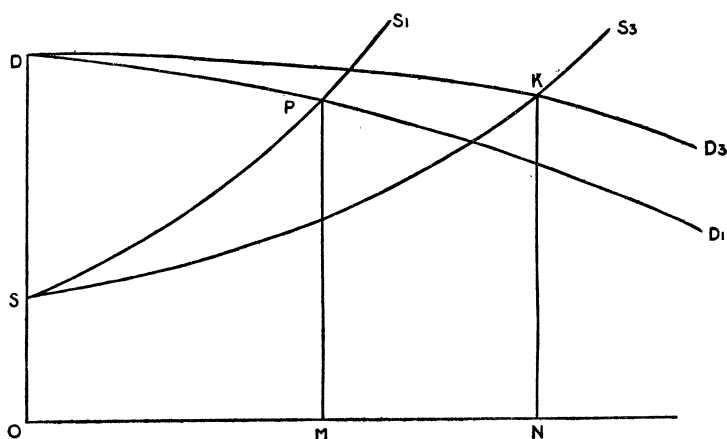


FIG. VI.

contract production below ON , must, other things equal, be socially advantageous.¹

§ 9. Up to this point the argument has been absolute, and no reference has been made to matters of probability. One step into that debatable ground may, however, be taken with advantage at this point. We may conceive the private marginal supply price of any quantity x as made up of two parts, the magnitude of one of which depends on the aggregate output x and that of the other on the output x_r of a typical individual producer. That is to say, the private marginal supply price of x unit is $\{fx_r + \phi x\}$. Let the output increase from x to $(x + \Delta x)$, and let Δp_1 be the corresponding change in private marginal supply price, and Δp_2

¹ These results are not, of course, contradictory, but merely supplementary, to those established by Dr. Marshall in his *Principles* (p. 467, *et seq.*).

the corresponding change in collective marginal supply price. Then it is plain that

$$\Delta p_1 = \Delta x f' + \Delta x \phi' \quad \text{and} \quad \Delta p_2 = \Delta x f' + \Delta x \phi' + x \phi'.$$

Now in some cases where f' differs in sign from ϕ' , the signs of Δp_1 and Δp_2 will be different. But in all cases where f' and ϕ' have the same sign, and in many cases where they have a different sign, the signs of Δp_1 and Δp_2 will not be different. Therefore, it is *probable* in general that these signs will not be different. That is to say, it is probable that the curve of private marginal supply prices SS_2 and the curve of collective marginal supply prices SS_3 will be inclined in the same direction. But, when SS_3 is inclined negatively, it lies below SS_4 , which, when SS_1 coincides with SS_2 , cannot lie above SS_2 . Therefore, when SS_1 coincides with SS_2 , and is inclined negatively, it probably lies above SS_3 . When SS_1 coincides with SS_4 and is inclined negatively, it necessarily lies above SS_3 . But SS_1 must coincide with either SS_2 or SS_4 . Therefore, it is probable that when SS_1 is inclined negatively, it lies above SS_3 . But, if SS_1 lies above SS_3 , it is probable, since DD_1 is as likely to lie below as above DD_3 , that the point of intersection between SS_1 and DD_1 will lie to the left of that between SS_3 and DD_3 . Hence, when the supply curve SS_1 is inclined negatively, that is to say, in cases of increasing return as ordinarily understood, it is probable that ON will be greater than OM . In all cases, therefore, when increasing return is known to prevail, and when nothing else is known, it is probable that the grant of a bounty at a rate not exceeding some definite but unknown amount, would prove socially advantageous.

An analogous proposition, to which, however, no practical importance attaches, can be established in regard to cases where, instead of the supply curve being inclined negatively, the demand curve is inclined positively.

§ 10. There remains one further question upon which, as it seems to me, the preceding analysis throws light, the question, namely, to what extent output will be contracted if, all other things remaining the same, an element of monopolistic power is introduced among the sellers or the buyers. As the analysis is substantially the same in both cases, I shall deal here with a sellers' monopoly only.

This problem is complicated by the circumstance that monopolisation, apart altogether from the introduction of new economies, which are not relevant to this discussion, may alter the position of the curve of private marginal supply prices. If,

before monopolisation, the curve of private marginal supply prices and the curve of collective marginal supply prices coincide, it will not, of course, have this effect. If, however, before monopolisation, the curve of private marginal supply prices differ from the curve of collective marginal supply prices, it must have this effect, because, when there is only one producer, private and collective marginal supply prices are necessarily identical.

Let us begin with the case in which SS_2 and SS_3 are identical both before and after monopolisation. Then, as explained in § 6, the supply curve SS_1 , for all values in respect of which it is

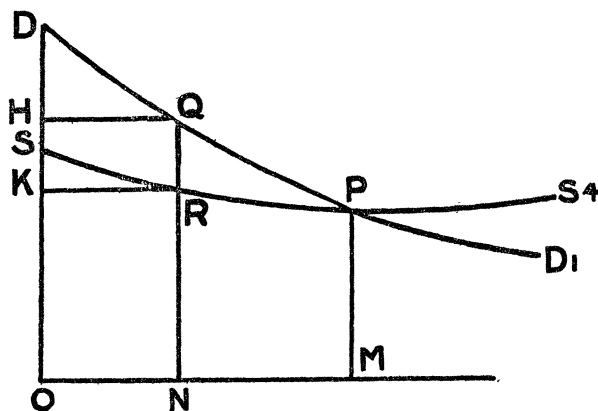


FIG. VII.

inclined negatively, coincides with SS_4 , and, for all values in respect of which it is inclined positively, with SS_3 .

First, let the supply curve be inclined negatively, and, therefore, coincide with SS_4 (Fig. VII.). It is then obvious that if P be the point of intersection of DD_1 and SS_4 , and if PM be drawn vertically to the axis of x , the output under free competition will be measured by OM . In like manner the output under monopoly will be measured by ON where ON is such that the rectangle $Q R K H$ is a maximum.

Secondly: let the supply curve be inclined positively and, therefore, coincide with SS_3 . If (Fig. VIII.) SS_3 and DD_1 intersect at P and PM be drawn vertically to the axis of x , it is obvious, as before, that the output under free competition will be measured by OM . The output under monopoly is not, however, in this case determined unambiguously.

If the monopolistic body is interested in producers' surplus, the quantity of output ON produced under monopoly will be such that ON multiplied by the vertical distance cut off between DD_1

and SS_4 by a vertical line drawn through N —i.e., the rectangle $Q K R T$ —is a maximum. This case, together with the case represented in Fig. VII., is covered by the analysis employed by Dr. Marshall in the fourteenth chapter of his fifth book.

If, on the other hand, the monopolistic body is of such a sort that all producers' surplus accruing to its members has to be paid away to others—a state of things approximately realised in the case of a group of tenant farmers combined for selling purposes but competing with one another for the hire of land—considerations of the magnitude of this surplus do not enter into the monopolistic body's calculations. In these circumstances the quantity of output ON produced under monopoly will be such that ON multiplied by the vertical distance cut off between DD_1 and

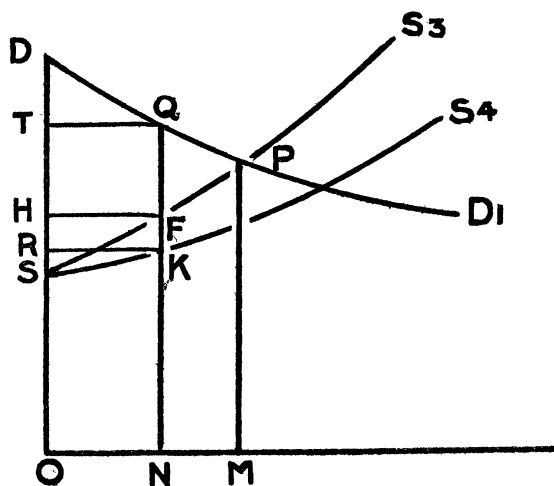


FIG. VIII.

SS_3 by a vertical line drawn through N —i.e., the rectangle $Q F H T$ in Fig. VIII.—is a maximum.

A comparison between the effects of monopolisation in the above two cases, where producers' surplus does and does not enter into the monopolistic body's consideration is easily made. It is evident that, so long as SS_3 remains inclined positively, any point P upon SS_3 will lie further above the corresponding point on SS_4 the further P moves towards the right. Therefore the value of ON which will maximise the rectangle $Q F H T$ will in general be smaller than the value which will maximise the rectangle $Q K R T$. That is to say, the introduction of monopoly will in general check output more when the monopolistic body is indifferent to, than when it is interested in, producers' surplus.

It is interesting to note that, when the curves involved are straight lines, this comparison has an exact result. Let DD_1 be inclined to the horizontal at an angle θ and let SS_3 be inclined at an angle ϕ . Let the output under free competition be α . Then, if the monopolistic body disregards producers' surplus, output under monopoly will be equal to $\frac{1}{2}\alpha$: if it regards producers' surplus, output will be

$$\frac{\tan \theta + \tan \phi}{2 \tan \theta + \tan \phi} \alpha.$$

There remains the case in which monopolisation alters the position of the supply curve, the case, that is to say, in which, under free competition, the curve of private marginal supply prices differs from the curve of collective marginal supply prices. In that case output under free competition is determined by the intersection of DD_1 with SS_2 instead of with SS_4 or with SS_3 . Output under monopoly is determined according to circumstances in one of the ways described in this section. Hence the difference made to output by monopolisation is determined. This completes the solution of the problem.

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