

MATHEMATICS CLUBS IN SECONDARY SCHOOLS.

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No doubt every teacher of secondary mathematics feels that one of his important problems is that of arousing and holding the interest of his pupils in mathematical work. In the elective work of the third and fourth years, one may expect to find some degree of interest on the part of the pupil, but in freshman algebra or second-year geometry, one frequently encounters the pupil who has "no head for mathematics," who announces cheerfully that he has always found the subject difficult, and seems reconciled to the fact that he is invulnerable to any kind of mathematical instruction. Mathematical recreations and diversions are a legitimate and profitable expenditure of time if, without diluting the instruction, they create such an interest that the hard work which is connected with any course that is worth while does not seem like drudgery, for while valuable discipline may be developed by doing unpleasant tasks, the pupil who attacks his problems in a cheerful and interested frame of mind has a great advantage in accomplishing work and overcoming obstacles.

In addition to the problem of arousing the dull or indifferent pupil from his lethargy, there is the difficulty of keeping the brighter and more original pupils working at concert pitch, so that while we are attempting to create interest we may not kill that which already existed. While more intensive work on the subject in hand may be assigned for extra credit to these more ambitious pupils and other devices may be used to retain their interest, still it is a lamentable fact that the amount of uniformity necessary in classroom work often makes it difficult to bring out the capacity of the individual pupil. Yet we owe it to the excellent pupil to hold his interest, to stimulate his enthusiasm, and by opening up to him new fields of thought, to inspire him to the development of mathematical power of which he may be unconscious. While some of these objects may be accomplished in class, every enthusiastic teacher of mathematics must feel that his opportunities for inspiring the really capable student are much too limited.

The Mathematics Club is at least a partial solution of this difficulty. In such a club it is possible to supplement the work of the classroom by offering opportunities that the class does

not afford, to add to the pupils' fund of mathematical knowledge, to inspire pupils to original work, and to lead them to realize that the subject is rich in interest. Boys and girls who become so interested in the classroom work that they voluntarily remain after school for an hour or more to discuss subjects which have been suggested, and who are constantly reaching out for more information, will be glad of an opportunity to meet regularly for more or less informal discussions, and the work of such a club reacts favorably on the attitude toward mathematics throughout the school.

Probably the first Mathematics Club organized in a secondary school was the one in the Shattuck School at Faribault, Minnesota, organized by Mr. Charles Newhall in 1903. It has been a very successful club with a membership of fifteen boys from the senior class who meet in the evening every two weeks and report upon topics in mathematics assigned to them. Interesting articles have appeared in *SCHOOL SCIENCE AND MATHEMATICS*, describing the work of this club and indicating the subjects taken up in the course of the year. These include many topics in the history of arithmetic, algebra and geometry, as well as famous problems, puzzles, tricks and diversions. In a recent article on "Recreations in Secondary Mathematics" in the April number of *SCHOOL SCIENCE AND MATHEMATICS*, Mr. Newhall has given a very comprehensive list of books, magazine articles and general references for such recreations. This article is full of suggestions for teachers who may be interested in organizing clubs in secondary schools, or who may wish to introduce such material for classroom use.

The only clubs with which I am familiar in the Chicago high schools are of recent development. One was organized at Hyde Park High School a little more than two years ago, one at Wendell Phillips soon after, and within the last year one at the Bowen High School. Since these clubs are similar in their organization and general plan, I will describe briefly our experiment at Hyde Park.

Our club came about from a desire on the part of some of the mathematics teachers to appoint a regular time for informal discussions among those groups of interested pupils who often lingered after school to investigate problems which had been suggested to them. It offered an opportunity to bring enthusiastic pupils from different classes together and to direct into profitable channels the interest already created. When the pos-

sibility of organizing a small club was proposed, these pupils were very enthusiastic. From the high school pupils' point of view it was of course imperative that the club be equipped at the outset with constitution and by-laws, a name and a pin. I doubt if any successful club could be organized in high school without these important adjuncts. Officers were elected, a constitution was drawn up, stating that the object of the club should be "to create an interest in and to further the study of mathematics," the organization was called the Pythagorean Club, and it adopted as the official pin the star pentagon, chosen from various mathematical designs submitted by members of the club.

The first members, from twenty-five to thirty in number, were chosen by mathematics teachers and were pupils who had displayed unusual scholarship, interest and originality in regular classroom work. At the beginning of each semester as some members are graduated, they are replaced by new members, but the recommendation by mathematics teachers has continued to be a requirement for election to membership. The club has always been proud of the fact that it is the only one in Hyde Park in which the membership is based upon scholarship and the scholarship requirement has never, to our knowledge, worked an injustice in excluding deserving pupils from membership, since pupils who would be interested in doing the extra work in such a club are almost invariably the ones whose enthusiasm in classroom work would insure their recommendation. On the other hand, it has happened frequently that the possibility of election to membership has spurred a mediocre pupil on to a much higher standard of work, for the club is a popular one and membership in it is regarded as a great honor.

While pupils completing the second semester of plane geometry are eligible to membership, the members are usually elected from classes in advanced algebra, solid geometry, trigonometry or college algebra. Lack of absolute uniformity in preparation has not seriously hampered the Program Committee in the selection of topics, since many subjects are of general interest to third- and fourth-year pupils. Moreover, pupils whose knowledge of trigonometry is limited to the elementary work in this subject done in connection with similar triangles in geometry, are interested in the occasional trigonometric solution of a problem; those whose knowledge of equations does not extend beyond quadratics are glad to hear an occasional cubic or biquadratic discussed by members of college algebra classes, and these discus-

sions often create an interest in the elective courses in mathematics.

Regular program meetings, usually an hour and a half in length, are held every two weeks at the close of school. The President of the club, who is usually a senior who is taking advanced work in mathematics, presides at the meetings while the programs are arranged by a Program Committee appointed by the President at the beginning of each semester. This committee confers with mathematics teachers in regard to the subject matter for each program and also urges club members to propose problems or topics of special interest which they may wish to hear discussed. At each meeting of the club, programs for the next meeting are distributed so that members may be informed two weeks in advance of the topics which will be up for consideration.

While two or three mathematics teachers are usually present at each meeting and contribute something to the general discussion, it is the general policy to leave the meeting in the hands of the members as much as possible and to encourage among them free and informal discussion of any topics in mathematics which may appeal to them. When problems have been proposed for solution and are presented, the teachers place the responsibility upon the club of deciding upon the validity of the proofs presented and of accepting or rejecting the solutions. This custom has tended to make the members more alert, with the result that the solution of an unusually difficult problem frequently provokes animated discussion and argument which continue long past the usual time for adjournment.

The programs have been of great variety. Several meetings have been given over to the discussion of famous problems in geometry. The circle and its triangles have proved to be of great interest, and the nine-point circle always challenges members to an unusual amount of investigation. When one problem of this character furnishes the basis for discussion, the rivalry among the members and the great variety of solutions presented is one of the chief factors of interest.

Early in the organization of the club, a program was announced on the Pythagorean theorem, inasmuch as that theorem had been of special interest in the second semester of geometry and many pupils had indicated a desire to investigate other proofs than those brought up in class. An enormous amount of work was done by the club and the teachers present at the meeting considered the results surprising. In some cases possible figures

were suggested, but there was a lively competition among the members in working out original proofs and in the majority of cases both the figure and the proof were original. From the proofs submitted to the Program Committee, ten or twelve were presented at the club meeting. In connection with these proofs, one pupil gave a short account of the history of the theorem, and another gave a sketch of the life of Pythagoras and the work of the Pythagorean school. So much interest was shown in the proofs presented, that a committee was appointed to investigate the published proofs in the city libraries to determine which of our proofs were really new discoveries; figures which were devised by members but which seemed to defy solution were submitted to the club for further consideration; and so much enthusiasm for this noted theorem was created, that geometry pupils began to inquire eagerly when they could take up the Pythagorean theorem.

Concurrency of lines and collinearity of points has opened up an interesting field to the enthusiastic student in geometry, and teachers have been glad to suggest material in these subjects to supplement the meager work which they can do in the average class. The interest in working out the theorems of Menelaus, Ceva and Carnot reached its climax in the solution of Pascal's Mystic Hexagram.

Two of our most successful meetings have been on the general subject of mathematical fallacies, a subject of unfailing interest to the majority of high school students, for they seem to feel for some mysterious reason that it is a much more wonderful feat to prove a fact that cannot possibly be true than to work out rigid and logical demonstrations of the more plausible theorems of algebra or geometry. A notice of a meeting on fallacies created a great demand for library books and other material on the subject, and aroused the interest of pupils who were not members of the club to such an extent that there were numerous requests for permission to attend the meetings. Some of the fallacies were quickly detected, such as the fact that all triangles are isosceles, that an obtuse angle is equal to a right angle, that a quadrilateral with two opposite sides equal is a trapezoid, that two perpendiculars may be drawn from a point to a line, and that the locus of points in a plane equidistant from a given point is not a circle. Some of the algebraic fallacies presented more difficulty, especially those involving the interpretation of equations. The one which defied their efforts for the greatest

length of time, and in consequence caused the greatest amount of excitement, was the proof that a part of a line is equal to the whole line. The hope of finding the flaw in the proof incited pupils to wrestle with intricate points which would have seemed commonplace and prosaic in other situations. They carefully examined the proofs for similarity of triangles; they questioned the comparison of areas; they investigated the operations in proportion involved; they demanded proofs for the theorems of the square on the sides opposite the acute and obtuse angles of a triangle; they cross-questioned themselves and one another with a zeal that would gladden the hearts of any pedagogue; and when the fallacy was finally detected, the fact was emphasized that when two equal fractions have equal numerators, the denominators are not necessarily equal.

Certain types of construction problems have been very popular in the club, and for nearly a year the famous theorem of Apollonius, to construct a circle which shall be tangent to each of three given circles, has been up for consideration. Several members of the club have solved many of the series of construction problems leading up to the final and difficult construction, and recently two members of the club obtained the final construction by independent and original solutions. One of the boys who was successful in arriving at the final solution declared that he had learned more about mathematics in his attempts to solve the theorem of Apollonius than in any course in mathematics which he had taken, and at the two meetings of the club held for the purpose of hearing the two solutions, members agreed that the solutions obtained represented the best work done in the history of the club.

Occasionally the program has consisted of a paper or a talk followed by a discussion. At the request of the members, one of the teachers gave a talk at one of the meetings on "The Fourth Dimension"; a member of the club who had become interested in the parallel postulate gave a talk on that subject; another member who had done a considerable amount of work in astronomy and who edited an astronomical journal, gave an interesting paper on "The Mathematical Phases of the Discovery of Neptune" which he was afterwards invited to read before the Wendell Phillips Club; a member of one of the beginning classes in geometry who had worked out twelve proofs for the fact that the sum of the interior angles of a triangle is equal to two right angles was asked to present his proofs at one of the club meetings.

A Committee on Proofs asks members who have done especially original work to write out demonstrations in permanent form so that they may be preserved. It is hoped that this collection may in time become an interesting one, and that it may be an incentive to future members of the club.

When a program is unusually serious in its nature, the Program Committee often provides for diversion in the way of mathematical tricks, peculiarities, magic squares, etc. One of the greatest sources of amusement has been the initiation of new members at the beginning of each semester, when one of the mathematics teachers publicly tests the ability of new members to perform the fundamental operations of addition, subtraction, multiplication and division, not, however, in their familiar decimal system, but in duodecimals or a system of fives or eights. The struggles and mistakes of those who are being initiated are thoroughly enjoyed by all of the old members of the club and considerable information in regard to these systems of notation is absorbed.

While it is not the intention of teachers or of members of the club to emphasize the social side of the organization, one social meeting is usually held at the beginning of each semester when new members are taken into the club, for it is felt that in a large school an occasional social meeting results in greater freedom, and spontaneity in the regular program meetings. At these social affairs all of the entertainment is of a mathematical character, consisting of mathematical charades, contests and games. Members of the club have displayed great cleverness and ingenuity in devising mathematical contests for these occasions. There have been question contests in which all the answers were mathematical terms, art exhibits in which the guests were to guess what well-known propositions were illustrated by the sketches, and thrilling tales have been composed from mathematical symbols and presented to the guests for translation, but while the parties of the Mathematics Club have been lively affairs, it has not been necessary to sugar coat the regular proceedings of the club by holding social meetings frequently. The sustained interest of the members in the serious work of the club has been quite remarkable and very gratifying. The club has been and is a popular one, the meetings are well attended, and the organization is just as enthusiastic now as when it was new and had all of the charm of novelty.

While we consider that our two years' experience with a Mathematics Club at Hyde Park has been largely an experiment,

there is a general agreement that the experiment has been a successful one, resulting in arousing interest in mathematical literature, developing imagination along mathematical lines, cultivating and encouraging the development of power of initiative and original work, and in creating greater interest in all lines of mathematical work throughout the school.

THE MAGNETIC FIELD SURROUNDING A VOLTAIC CELL.

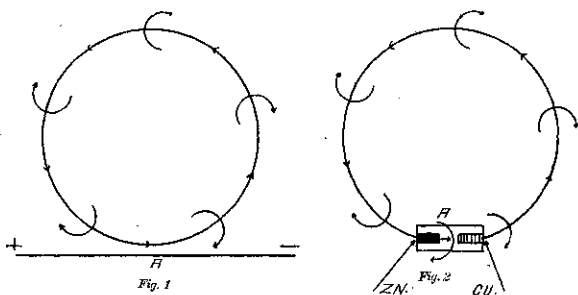
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The following facts are well known and their demonstration by the teacher or in the laboratory is included in every course in electricity. (1) A solid conductor carrying a current of electricity is surrounded by a magnetic field, in which (2) the lines of force are in concentric circles with the conductor at the center. (3) The plane of these circles is at right angles to conductor and (4) if you look along the conductor in the direction in which the current is flowing, the lines of force will seem to take a *clockwise* direction around the conductor.

That a voltaic cell in action is surrounded by a magnetic field in every way similar to the field in the wire is not so well known. Textbook writers have either ignored, or assumed, or been ignorant of the fact. The present writer has not found in any book that he has consulted any reference to it.

That such a field should exist in the cell as well as in the external circuit seems a reasonable supposition.



Suppose as in Figure 1 a wire carrying a current to be bent into a large circle. The direction of the current being indicated by the arrows on the wire, the direction of the lines of force