

*(Paper No. 4106.)*

**“On Impact Coefficients for Railway Girders.”**

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For some years there has been little discussion of the subject of this Paper by The Institution, probably owing to the lack of recent experimental data which would assist in the formation of more positive conclusions than at present exist as to the proper allowance to be made for the dynamic effect of moving loads.

The subject is of especial interest at the moment to Indian Railway Administrations, in view of the very extensive programmes for strengthening or replacing girder-bridges which the rapid increase in the weight of rolling stock has rendered necessary.

The Author's intention to submit to The Institution a Paper on the subject of impact coincided with the desire of the Indian Railway Board to promote discussion of the matter; and, believing that his Paper would fulfil that purpose, the Board have favoured him with all the documents and experimental data which they have collected, in the expressed hope that they will help to ventilate a vexed question.

Any views which may be expressed in the course of this Paper are not necessarily those of the Railway Board or of any one railway-company.

In order that the discussion may be specially applicable to Indian conditions, it is proposed to commence by recapitulating the Government-of-India Rules as they now stand, and then to sketch briefly the history of their evolution. It is believed that this course will also provide a convenient basis for the consideration of the abstract question.

# SECT. I.—RULES OF THE GOVERNMENT OF INDIA FOR THE DESIGN AND INSPECTION OF GIRDER-BRIDGES.

The first five articles lay down the scope of the rules generally, and define the duties of the Government Inspector and the procedure to be adopted by him and the Railway Administration where a lower standard is considered sufficient to meet the case.

## Maximum Permissible Stress.

6. For any member of a railway bridge of wrought iron or steel the total working load is to be taken as the 'moving load,' increased by an appropriate allowance for 'impact,' and added to the actual 'fixed load.'

7. For the purposes of this rule 'fixed load' is to be taken to mean the weight of the structure itself, with the roadway, flooring, ballast, permanent-way, etc., complete. (*See also 'Wind Pressure.'*)

'Moving load' is to be taken as a train-load on each line of rails as specified in Rule 15; and if there be a road or footway that can be occupied at the same time as the railway track, an additional moving load as specified in Rules 17 and 18.

8. The increment for impact to be allowed in the case of railway load is to be calculated by the formula

$$I = \frac{300}{L + 300} S,$$

where  $S$  is the stress due to the train load specified, considered as at rest in the position which gives the maximum stress in the member under consideration, and  $L$  is the length in feet of that portion of the span which the train has had to traverse to reach that position from the point where it first began to produce stress in that member and  $I$  is the amount to be added to  $S$  to allow for 'impact.' Values of the ratio  $\frac{I}{S}$  for various values of  $L$  are appended as Table I, Appendix I.

*Note (i)* For bending moments caused by an assumed equivalent moving load,  $L$  may be taken as equal to the span of the girder. For shears it will usually be the distance of the point under consideration from the further support. For cross-girder concentrations it will usually be twice the interval between the cross girders.

9. The increment for impact to be allowed in the case of loads moving on the roadway or footways of a combined road and railway bridge shall be half the railway increment specified in Rule 8.

10. The intensity per square inch of different kinds of stress due to the 'total working load' thus calculated is not to exceed the following:—

	Wrought Iron.	Steel.	—
	Tons.	Tons.	
Tension and compression . . . . .	6	8	Per square inch of effective area
Shearing . . . . .	4	5	"
Bearing . . . . .	9	11	"

11. But for occasional loads, such as those due to wind pressure, or exceptional loadings of a separate roadway simultaneously with the absolute maximum train-load, stresses 25 per cent. in excess of the above may be permitted.

[N.B.—This implies that the effect of wind may be neglected when the stresses caused by it are not more than 25 per cent. of those caused by the 'total working load.']

12. For members in tension the working stress per square inch given above is to be taken on the net available area at the weakest part of the member of the structure to which it is applied, after deducting all holes for rivets, pins, bolts, etc.

13. For members in direct compression well-filled rivet holes need not be deducted, but the working stress given above is subject to such reduction as may be necessary according to a suitable column formula. In calculations submitted to Government, the formula given in Table II, Appendix I, should preferably be used. No main compression member should have a greater free length than 100 times its least radius of gyration, but in subsidiary members, such as lateral struts, the ratio may be 120. For plate girders it will suffice if the compression flange has the same sectional area as the tension flange.

14. Members and connections subject to alternating stresses are to be proportioned for tension and compression separately, and half the smaller area is to be added to the larger area to give the total section.

#### *Standard of Permissible Train-load.*

15. The maximum train-load referred to in Rules 7 and 8 is the heaviest train of two engines, followed by as many fully loaded wagons as can get on the bridge, which will not cause in the girders greater bending moments, shears and cross-girder concentrations than those laid down in Tables III, IV and V, Appendix I.

#### *Road or Footway Loads.*

16. For those members of a combined road and railway bridge which carry the roadway only, the moving load on the road or footway shall be taken as 90 lbs. per square foot (4 tons per 100 square feet) of effective surface, or the heaviest elephant, cannon, traction-engine, vehicle or train of vehicles which is likely to be allowed on the same, whichever will give the greatest stress in the members.

17. But for those members which carry both loads, only 20 lbs. per square foot of effective surface need be taken, except in special cases where this moderate load would seem to be inadequate, such as a bridge at a large city, the roadway of which is likely to be constantly crowded. Such cases should be treated on their merits. (*See also Rule 9.*)

#### *Wind Pressure.*

19. Every structure must be capable of bearing a wind pressure of 2·5 tons per 100 square feet (56 lbs. per square foot) when unloaded, or 1·5 ton per 100 square feet (33·6 lbs. per square foot) when loaded with

the maximum moving load, without exceeding the stresses laid down for occasional loads in paragraph 11 above.

22. No allowance for 'impact' is required for wind load, and the excess vertical load on the leeward rail due to a horizontal wind pressure tending to cant the train need not be taken into account. (*See also Note to Rule 11.*)

The remaining rules deal with the applicability of the foregoing rules to existing structures as against new or proposed girders.

The foregoing are the bridge rules which have been in force since 1903. Of the Tables referred to (see Appendix I), Tables II, III, IV and V incorporate the revised standards of 1908. Until 1893 the rules in force were those of the Board of Trade.

*1893 Rules.*—In 1892 the Government of India framed rules designed to supply the deficiencies of the Board-of-Trade rules by ensuring uniformity of design, and circulated the draft rules amongst a large number of representative railway engineers in India for criticism. The only items of these rules which it is necessary to touch upon now are:—

- (a) The coefficient of impact.
- (b) The standard equivalent distributed uniform load to be allowed for moving load.
- (c) The working-stress.

For item (a) the rules were:—

“For any member of a railway bridge of wrought iron or steel the total working load is to be taken as the greatest ‘moving load’ multiplied by a coefficient and added to the actual ‘fixed load.’ The coefficient to be used for this purpose is 2·0 in all cases, except for the upper and lower booms of triangulated girders, for which a coefficient of 1·5 may be used.”

Item (b) was deduced from the maximum bending-moments obtained at any point of a span from two coupled tank engines of imaginary type placed in such position in the span, and in such position with regard to the train, as would produce the greatest effect.

Item (c) allowed a working-stress (for total working load) of 7 tons per square inch for wrought iron and 9 tons per square inch for steel on the net area, the stress in compression being, of course, subject to modification by column formula, the particular formula being unspecified.

Fifty-five of the engineers to whom the draft rules were submitted either approved of them as they stood, or proposed amendments which dealt principally with the coefficient of impact

for the moving load. Some advocated wider limits of the coefficient, but the dissent was only in favour of a refinement of its application, and all agreed that the use of a coefficient of 2 to 1.5 to multiply the moving load and add to the fixed load to arrive at an equivalent total working load would give a closer approximation to the actual effects than that attained by the Board-of-Trade rule, and that the use of the same amount of material would result in a stronger bridge, owing to the disposal of the material to better advantage.

The rules as drafted therefore came into force in 1893.

**1903 Rules.**—In 1903 the rules were revised again, for several reasons. One of these was that the standard equivalent uniform load for bending-moments resulted—for spans up to 100 feet—in the design of girders which were very largely in excess of the strength required to carry the heaviest engines in use or recently proposed for use. Accordingly a number of these engines were selected, and their equivalent loads for bending at a point one-sixth of the span, and also for shear loads, were worked out, with the result that a new and intermediate standard was adopted which gave a moderate margin for bending and shears for all spans.

At the same time the working-stresses of 7 tons and 9 tons per square inch for wrought iron and steel respectively were reduced, on the advice of the consulting engineers, to 6 tons and 8 tons, and the Pencoyd formula for impact replaced the original coefficients applied to the moving load.

The 1903 draft rules were submitted to the railway managers, bridge engineers, consulting engineers and others, of whom about sixty furnished comments. Of these the greater number accepted the standard loads and the Pencoyd formula for impact. Some dissented from the latter as being more suited to American practice, in which the girders are comparatively light, and advocated a less severe formula for Indian use, such as  $\frac{200}{200 + L}$ , or  $\frac{150}{150 + L}$ ; while a considerable number were of opinion that a formula depending rather upon the ratio of the fixed to the moving load than upon span should be adopted. Eventually, after reference to a committee of four consulting engineers, the Pencoyd formula was adopted.

Between 1903 and 1908 it became evident that the standard of equivalent loads no longer gave a sufficient margin to provide for the increasing weight of the rolling stock, and accordingly the 1908 rules specified an increase of 25 per cent. *en bloc* in the loads for which new or strengthened structures were to be designed.

## SECT. II.—STANDARD MOVING LOADS.

In a rule like that of the Board of Trade, which prescribes no definite allowance for the dynamic effect of a moving load upon the structure, and is intended to provide with safety for the strains in that structure from all causes whatsoever, it is evident that there is infinite latitude for the individual designer in considering what proportion of the working-stress per square inch he shall assign to the effect of the load considered as static, and what proportion he shall assign to its dynamic effect.

The Indian rules, on the other hand, by means of the Table of standard equivalent uniformly-distributed loads, specify exactly the proportion of the load which is to be considered as static.

This question of the standard load, as well as the rule for the dynamic increment, has latterly been the subject of discussion, and the Indian railways were invited by the Railway Board to give their views, with the result—the Author understands—that the majority of the railways have concluded that the loads prescribed are not too far in advance of present and probable future requirements. On the question of impact there is a greater diversity of opinion, not only between the various railway-companies, but among their individual engineers. In the meantime articles in the technical press have dealt with the matter in a manner which somewhat blends the two issues of loading and impact. The Author feels that the latter subject cannot be satisfactorily reviewed from an engineering point of view unless it is first dissociated from the former, and with this object he has prepared diagrams (Figs. 1, 2, 3, Plate 3) comparing with the rules the equivalent distributed load for a few typical engines and trains that are in common use, or are likely to be used in the near future.

*Typical Moving Loads.*—The 2-8-0 goods and the De Glehn engines are as used on the Bengal-Nagpur Railway. The 2-6-2 tank engine is one recently passed by the Standards Committee for use on the Great Indian Peninsula Railway. The Mallet and 2-8-0 type goods-engines are possible developments in the near future. As used on the Bengal-Nagpur Railway the latter has 15·15 tons as a maximum axle-load, but the standard for use on several railways is 15·9 tons, and with 90-lb. rails in the track it would be possible to increase the axle-load to 18 tons. It will be seen that, speaking generally, this 18-ton-axle engine leaves but little margin below the Government-of-India 1908 standard loads, either for bending-moments or for shears. The excess of the Government

loads over those of the engines in actual use is most noticeable for spans between 10 and 20 feet. In this is visible the effect of the imaginary-type engine of the 1893 rules with its 15-ton axles at very short distances apart; and although the excess at this point over actual loads was modified in 1903, the subsequent addition of 25 per cent. has perpetuated a larger excess here than at other points of the curve. With this exception there is no noticeably large margin. With the exception of the curve for standard loads all the curves have been deduced from two engines coupled at the head of the train in the ordinary way, although in certain cases greater moments would have been obtained by coupling them funnel to funnel or placing them in the centre of the train in the manner adopted for the 1893 rules. If one engine only had been used, the results for spans over about 60 feet would, of course, be considerably less.

The excess of the standard equivalent loads for metre gauge over those of the actual engines in use on metre-gauge lines is, it is believed, more obvious; but enough has perhaps been said to show that, in considering the subject of the proper increment to the moving load to be allowed for its dynamic effect, the initial magnitude of that load, as it is laid down in the rules, may be accepted as a reasonable basis.

### SECT. III.—THE SAFE WORKING-STRESS.

It is difficult to discuss the correctness of any rule for the coefficient for impact without first coming to a conclusion with regard to the safe working-stress adopted in conjunction with it. The discussion will here be confined to steel for the sake of simplicity, since wrought iron is now seldom used for bridge work.

For all fixed load or its equivalent the existing rules prescribe:—

For tension and compression . . .	8 tons per square inch.
„ shear . . . . .	5 „ „
„ bearing . . . . .	11 „ „

From these again shear and bearing may perhaps be eliminated with propriety. Most engineers will agree that they are safe, though some would prefer a more liberal allowance for bearing-stress. In any case any alteration would make but little difference in the weight of the structure.

It will have been noticed that up to 1903 the safe working-stress for tension and compression was 9 tons per square inch, a factor of safety of 3 being used on the assumed breaking-stress

of 27 tons per square inch. The working-stress of 8 tons now used gives an approximate factor of safety of 2 on the elastic limit, which is probably 15·6 to 16 tons per square inch.

The elastic limit should never be passed. If this be accepted as axiomatic, what has the margin of 8 tons per square inch to cover? Neglecting for the moment the various forces known as "impact," for which a factor is subsequently to provide, the margin evidently has to cover

- (a) Accidental defects in the material.
- (b) Initial stresses in the material due to rolling.
- (c) Stresses due to variation in temperature.
- (d) Loss of section by corrosion.
- (e) Faulty workmanship.
- (f) Secondary and indeterminate stresses due to. eccentric loading, etc. (These are partly dependent upon moving loads and partly upon the fixed load due to the weight of the girder itself.)

(a) Accidental defects in the material are no doubt well guarded against by the preliminary tests and inspection before the material leaves the maker's yard, and need cause little apprehension, but there is at least a possibility of flaws escaping observation.

(b) Initial stresses of considerable magnitude were proved by the late Sir Benjamin Baker to exist in many cold-straightened plates, and he showed that a nominal average stress of 7 tons per square inch on a plate may really be a stress of 14 tons on one side of it and nothing on the other.<sup>1</sup> Extensometer observations go far to prove that such unequal distribution of the stress may continue under a moving load, for they very often show considerable variations on two edges of the same plate. These variations may be due to causes which come under item (f), but it is quite possible that they are caused by the initial stresses set up in the straightening of the material.

(c) Sir Benjamin Baker pointed out on the same occasion that 2 tons per square inch is not an unreasonable addition to make in respect of the variation in temperature. When it is remembered that the temperature between night and the next afternoon may easily vary in India between 80° and 140° F., and that the average roller bearing is not nearly so efficient as it should be, it is clear that, allowing 1 ton per square inch for every 12° change of tempera-

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<sup>1</sup> Discussion on Mr. W. B. Farr's Paper, "Moving Loads on Railway Under-bridges." Minutes of Proceedings Inst. C.E., vol. cxli, p. 2.



ture, a very large part of the margin of safety may be absorbed by this item alone.

(d) In many parts of India the atmosphere is so dry that the original mill-scale may be seen on girders more than 20 years old; on the other hand, near the sea-coast corrosion is a very important factor in the deterioration of girders. The Author knows of one large bridge which has lost on the average at least 9 per cent. of its section, and in spite of every care the corrosion continues.

(e) It is not uncommon to see spaces between the two or more plates forming the boom of a girder. Under these rust forms and sets up local stresses of some magnitude. Also, in the length of its web-members probably no triangulated girder is mathematically correct, and the transmission of stress between boom and web is, at best, only approximately what it should be.

(f) Many indeterminate stresses are caused by one of the agencies already mentioned, but brackets on the vertical struts to carry cross girders are another, and probably the most fruitful, cause of secondary stresses. It is impossible, without very laborious calculation, to say what the secondary stresses from all causes combined will amount to in a bridge of any given type, under any specified type of load, at any particular speed; nor can it be determined at a glance in which members those secondary stresses will reach the maximum.

It would therefore be impossible, even if it were desirable, to calculate them for a given bridge which has to carry a variety of loads; but the increasing attention given to the subject proves that either these stresses must be calculated or something must be allowed for ignorance of them. Mr. C. R. Grimm arrives<sup>1</sup> at magnitudes which may range from 30 per cent. of the primary stresses in the booms to 75 per cent. in the web-members of a Pratt truss with no particular fault in design, the stresses being due simply to the combined effect of moving and fixed load on the riveted joints in the trusses.

A colleague of the Author—Mr. D. H. Remfry, Assoc. M. Inst. C.E.—has furnished an interesting calculation of the secondary stress thrown upon a vertical member by a cross girder attached in the ordinary way in a triangulated deck span.<sup>2</sup>

Certain assumptions are made, as is inevitable in this class of calculation. Mr. Remfry finds that the stress in the vertical is

<sup>1</sup> "Secondary Stresses in Bridge Trusses." New York, 1908.

<sup>2</sup> Attached by horizontal rivets to the angles of the vertical.

3.41 tons per square inch, and as the permissible stress by the Government-of-India rules is 6.8 tons per square inch, the secondary stress is just 50 per cent. of the primary stress, bringing the total up to 10.21 tons per square inch.

From extensometer readings taken on the inside and outside of the pairs of bars forming the web-tension members of the Nerbudda Bridge, the stresses on the inside bars were found to be 5 per cent. to 19 per cent. greater than those on the corresponding outside bars. In sending this example Mr. Sales remarks that in all such cases of variation the observations were repeated by changing the extensometer, so as to preclude the possibility of the variation being due to the instrument.

The stresses due to the sources enumerated under (a) to (f) are either not amenable to calculation or not susceptible of exact numerical computation for the purposes of ordinary design. They are, perhaps, not likely to exist simultaneously in any one part of the structure, but there is no certainty that they do not do so in any particular case, and their individual magnitude is considerable. Consequently, after allowing for the fixed or dead load due to the weight of the structure, and for the moving or live load, the nominal working-stress calculated to be 8 tons per square inch may in reality be within measurable distance of the elastic limit of the material, or may here and there be beyond it.

*The Column Formula.*—Table II, Appendix A, shows that the maximum stress permissible in the compression members of a triangulated girder is 6.8 tons per square inch for steel, diminishing for higher ratios of the length of the member to the radius of gyration. In 1903 Mr. John Graham, M. Inst. C.E., put into easy form<sup>1</sup> for reference the results of a large number of column-tests which had been carried out by Messrs. James Christie, Charles A. Marshall and others, and tabulated by Mr. J. Mitchell Moncrieff, M. Inst. C.E.<sup>2</sup> For the failing stresses there given, the factor of safety in the Government-of-India 1908 rules works out at 2.5 to 3.7. The tests referred to were carried out with loads applied centrally, and as it cannot be said that the loads on bridge-struts are always central, the 1908 rules seem to be fairly liberal. A very slight eccentricity in the load causes a very large increase in the buckling tendency.

<sup>1</sup> "Impact and Fatigue in Railway Bridges," *The Engineer*, vol. xciv (1902), p. 465, and vol. xcvi (1903), p. 3.

<sup>2</sup> Trans. Am. Soc. C.E., vol. xlv (1901), p. 334.

## SECT. IV.—IMPACT FORMULAS.

All impact formulas have their origin in the tests of wrought-iron and other bars carried out by Wöhler. The experiments are now so well known, and have been so fully treated by Professor T. Claxton Fidler, M. Inst. C.E., and other writers, that only the outstanding facts which they established need be mentioned. Briefly these were:—

- (i) That for a suddenly applied load varying between zero and a fixed quantity the greatest load which the bar would bear under an indefinite number of repetitions was about half the static breaking-load.
- (ii) That for equal loads applied so as to cause alternating strains of compression and tension in the material the greatest load which the bar would bear indefinitely without breaking was about one-third of the static breaking-load.

These experiments were amplified by Sir Benjamin Baker, Sir William Fairbairn, Professor J. Bauschinger and Professor Osborne Reynolds and Mr. J. H. Smith. At this stage it will be convenient to note that:—

All except Fairbairn's experiments were carried out on practically weightless beams.

The loads were applied more or less rapidly—the frequency ranging from four times per minute in the case of Wöhler's experiments to 2,000 times per minute in those of Messrs. Reynolds and Smith.

The tests which have the greatest application to modern steel bridges are those of Sir Benjamin Baker and Professor Bauschinger. One or two of these results are now given.

SIR BENJAMIN BAKER'S TESTS ON ROTATING BARS 1 INCH IN DIAMETER.  
(Bars Rotated 50 to 60 Times per Minute.)

Material.	Max. S.	Min. S.	Range of Stress.	Number of Repetitions to Cause Fracture.
Fine rivet steel . .	+16·1	−16·1	32·2	From 40,500 to 60,200
Tensile strength 26·8 to 28·6 tons per square inch .	+15·2 +11·6	−15·2 −11·6	30·4 23·2	From 68,400 to 155,295 14,876,432
Elongation 28 per cent. in 8 inches .	..	..	..	..

## SIR BENJAMIN BAKER'S TESTS ON FLAT BARS SUBJECTED TO BENDING.

Material.	Max. S.	Min. S.	Range of Stress.	Number of Repetitions to Cause Fracture.
Soft steel . . .	+19·7	-19·7	39·4	12,240
Tensile strength =	+15·2	-15·2	30·4	262,680
31·3 tons . . .	+12·3	-12·3	24·6	1,183,200
Elongation 20 per cent. in 8 inches .	15·4	0	15·4	{ 3,145,120 unbroken, but deeply flawed.

## PROFESSOR BAUSCHINGER'S TESTS: STRESSES IN TENSION VARYING FROM 0 TO AN UPPER LIMIT.

Material.	Original Elastic Limit.	Acquired Elastic Limit during Tests.	Load Applied.	Number of Repetitions before Fracture.	Tensile Strength in Tons per Square Inch Original.
	Tons per Sq. In.		Tons per Sq. In.	Millions.	
Mild-steel plate .	15·6	18	16	3·55	28·5
"	"	19	23	{ Average } { 0·54 }	"
"	"	16·4	23	0·44	"
"	"	20	26·2	0·34	"
"	"	12·3	26·2	0·11	"
"	"	11·5	26·2	0·04	"

These last tests are chiefly interesting for the evidence they give of the power of steel to acquire a new elastic limit, and Bauschinger was the first to suggest that if the stresses were kept within the elastic limit the material could be loaded indefinitely without suffering, but that if the range of stress exceeded the range between the natural elastic limits, the destruction of the material under repeated loads would only be a matter of time.

It is intended to refer to the other experiments later. Wöhler's experiments resulted in the formulas first of Launhardt and then of Weyrauch, which were intended to apply the new-found knowledge to the design of iron and steel structures. These formulas provided against the so-called "fatigue" to which the reduced breaking-strength of the bars in Wöhler's experiments was attributed.

Launhardt's formula dealt with a maximum and a minimum stress both of the same kind, and he used Wöhler's experiments to promulgate the formula

$$a = u \left( 1 + \frac{t - u \text{ minimum stress}}{u \text{ maximum stress}} \right)$$

where  $u$  denotes the greatest working load already referred to, as found by Wöhler's experiments,

$t$  „ static breaking-stress,

and „  $\frac{t}{2}$ .

Weyrauch applied Launhardt's formula to include the case of alternating stresses by assuming  $u$  in all cases to have the value  $\frac{2t}{3}$ , whether the stresses were of one kind or not. His formula thus became

$$a = u \left( 1 + \frac{1}{2} \frac{\text{minimum stress}}{\text{maximum stress}} \right).$$

Professor Fidler demonstrates<sup>1</sup> clearly that the results of Wöhler's experiments can be shown to be the logical effect of purely dynamic action, and mentions the well-known fact that static tests of presumably "fatigued" material had, when his book was written, so far failed to show a permanently diminished ultimate strength. In this last view Professor Unwin<sup>2</sup> concurs, but he attributes the alteration in breaking-strength in Wöhler's or Bauschinger's tests to the great alterations in the elastic limits. The formula which Professor Fidler has expounded to express his dynamic theory is

$$\Omega = \text{maximum stress} + \omega,$$

where  $\Omega$  represents the "momentary" internal stress caused by a suddenly applied load, and  $\omega$  is the dynamic increment or the quantity by which the maximum apparent stress will be increased momentarily.

For cross girders, vertical suspenders and web diagonals

$$\omega = \text{maximum stress} - \text{minimum stress}.$$

This implies that for these members the actual effect of the

<sup>1</sup> "A Practical Treatise on Bridge-Construction." 4th ed., p. 260. London, 1909.

<sup>2</sup> "The Testing of Materials of Construction." 3rd ed. London, 1910.

moving loads is double that of the same load applied statically, and to obtain the effective stress a coefficient of 2.0 should be applied to the moving load.

For main girders of 100 feet span and upwards the dynamic increment for the booms is

$$w = \frac{\text{maximum stress} - \text{minimum stress}}{2}$$

The coefficient for booms is therefore 1.5.

These are practically the principles of the Government-of-India impact rules of 1893, which have already been indicated.

*Modified Launhardt Formula.*—Launhardt's formula, with the modified notation given below, is much used in America to provide both for the immediate dynamic effect and for "fatigue" or the "cumulative" effect of the repetition of moving loads.

$$a = u \left( 1 + \frac{\text{minimum stress}}{\text{maximum stress}} \right),$$

where  $u = \frac{t}{2}$  assuming that the ultimate breaking-stress will be half the static breaking-stress.

Assuming 27 tons per square inch as the breaking-stress of mild steel and denoting  $\frac{\text{minimum stress}}{\text{maximum stress}}$  by  $\phi$ ,

$$\begin{aligned} a &= u (1 + \phi) \\ &= 13.5 (1 + \phi) \end{aligned}$$

and with a factor of safety of 3, the safe working-stress =  $4.5 (1 + \phi)$ .

*Mr. E. H. Stone's Range Formula.*—Mr. Stone<sup>1</sup> starts from the fact that in any bridge there is a compound stress, due partly to fixed load and partly to moving load, causing a range of stress between maximum and minimum, and develops his formula from the hypothesis that "the total stress from which the bar will ultimately break will vary with the ratio or percentage of the range of stress to the maximum stress." First he takes the results of deflection or extensometer tests as an expression of the immediate dynamic effect of the moving load, and plots these in relation to the ratios of fixed to moving load. Next he uses Gerber's parabola or Launhardt's formula, or the mean of both, and, by applying them

<sup>1</sup> "The Determination of the Safe Working Stress for Railway Bridges of Wrought Iron and Steel," Trans. Am. Soc. C.E., vol. xli (1899), p. 467.

to the altered ratios obtained in the first operation, deduces the ultimate effect upon the bridge for all combinations of fixed and moving load. In his own words: "The total extra effect produced by the moving load as compared with that due to the same weight as fixed load may be taken as that which would be produced by an indefinite number of repetitions of the immediate effect. In other words, the total effect which the bridge should be designed to bear with safety is the ultimate cumulative effect of the immediate effect."

It is interesting to note here the essential difference between Mr. Stone's formula and that adopted by the Government of India in 1893 (see p. 181). The latter assumes that the effect of a ton of moving load on triangulated girders is always twice (for the webs) or one-and-a-half times (for the booms) the corresponding effect of a ton of fixed load. The range formula provides a sliding scale of coefficient depending directly upon the ratio of the moving to the total load.

In actual effect the formulas do not differ so greatly as would appear at first sight, as the use of a fixed coefficient for varying proportions of load automatically gives varying working-stresses of reasonable magnitude, provided that coefficient is chosen sufficiently high.

The expression for Mr. Stone's range formula as applied to steel is safe working-stress =  $9 - (5 \times R^2)$ .

$$\text{In this expression } R = \frac{\text{Moving load}}{\text{Moving load} + \text{fixed load}} = \frac{\text{Range}}{\text{Total}}$$

and the static breaking-stress for steel is assumed to be 27 tons per square inch.

*The Pencoyd Formula.*—The Pencoyd formula differs from the foregoing in that it depends more directly upon span than upon range of stress. If Rule 8 and Table I of Appendix I be examined, it will be seen that this formula provides a complete sliding scale of coefficients for the moving load both for booms and web-members. The formula is  $I = \frac{300}{L + 300} S$ , the terms of which have already been explained in the body of the rules.

In plotting the results of his extensometer readings (Figs. 4, Plate 4), Mr. Sales has given the ratios of total moving load to total fixed load in the case of chords only. The reasons he gives will illustrate the advantages of the Pencoyd formula over formulas sometimes advocated which prescribe for both chords and web-members an allowance for impact based upon the ratio of the total

moving load to the total fixed load of the span. For the chords this principle applies in a perfectly logical manner, as the ratio of live-load stress to dead-load stress follows very nearly the ratio of total live load to total dead load. But this is not the case with the web-members, for if

A denotes the total equivalent moving load due to shears up to the panel point under consideration ;

B „ „ total fixed load due to weight of bridge, track, etc. ;

$S_1$  „ „ stress due to moving load ;

$S_2$  „ „ stress due to fixed load ;

the ratio  $\frac{A}{B}$  for web-members decreases while  $\frac{S_1}{S_2}$  increases. Consequently a formula applied in the manner indicated above would give a smaller coefficient for impact for web-members than for the chord-members of the same span, whereas it is generally conceded that the reverse procedure is the correct one. (See Note i, Table I, Appendix I.)

In Figs. 16, Plate 6, the Author has illustrated the stresses per square inch induced in the chords of typical girders by the use of several different formulas. To obtain the ratio of fixed to moving load the latter has been taken from the 1908 standard loads, and the fixed load is the approximate weight of girders built to carry those loads plus their own weight and that of the track. The abscissæ are then plotted in terms of the span. Two different effective working-stresses are used, 9 tons per square inch for the Launhardt and Stone formula, and 8 tons per square inch for the Pencoyd formula. If 9 tons had been taken for the latter, the results would have coincided very closely with those of the modified Launhardt rule.

The modified Pencoyd formula, in which the coefficient has been taken as  $1 + \frac{50}{50 + L}$ , has no direct scientific basis. A bridge engineer in India is often called upon to estimate the probable effect on a weak girder of the more or less temporary use of greater axle-loads, and the Author has used this formula as a compromise in the belief that it would give an approximation to the immediate stresses to be provided against.<sup>1</sup>

<sup>1</sup> The method of application is the same as in the Pencoyd formula, which is exemplified in the diagram of a 150 feet effective span on p. 209. The increment for impact is about 66 per cent. of the moving load in the case of the chords and increases from 66 per cent. to nearly 80 per cent. at the centre panel in the case of the web-members. (Rule 8 and Table I, note i.)



# SECT. V.—THE EVIDENCE OF THE IMMEDIATE EFFECT OF ROLLING LOADS.

## (A) BY CALCULATION.

One of the principal causes of increased stress, especially on small spans, is the periodical shifting of the weight from one wheel of the engine to another, due to the lurching or rolling of the engine about its vertical plane. Variation of pressure is also caused by the centrifugal force set up by unbalanced revolving parts or the excess vertical action of the balance-weights introduced into the coupled wheels.

Since a very large number of Indian bridges carrying a single track take the wheel-loads immediately over the boom, it is evidently necessary to know the maximum effect upon one main girder for different phases of the engine.

The problem of balancing an engine is somewhat complex, and will evidently present most difficulty where the driving-wheel is small and the space for introducing masses to balance the revolving parts about the crank-pin is limited.

With a view to the investigation of the effects on girder-bridges of a typical engine, the Author had recourse to the makers for their estimate of the various forces causing variation of pressure on the rail at a speed of 40 miles per hour. The engine chosen for illustration is the 2-8-0 goods-engine as used on the Bengal-Nagpur Railway, the calculated bending-moments for which are given in Fig. 1, Plate 3.

Referring now to Fig. 13, Plate 5, the most important causes of variation of pressure on the wheels are :—

- (a) The vertical component of the force  $P$ , due to the piston load acting at the driving crank-pin.
- (b) The excess vertical action of the balance-weights introduced in the leading, intermediate, and trailing wheels to partially balance the reciprocating parts. These forces are called  $B_1$ ,  $B_2$ ,  $B_3$ . The vertical force  $R$ , due to the unbalanced revolving mass about the driving crank-pin.
- (c) The vertical component of the force  $P$ , due to the piston load acting at the crosshead.

The force (c) is equal and opposite to (a) so far as its point of application is concerned, but its effect on the various axle-loads is not readily calculable, as owing to the action of the springs it is distributed over the various wheels in proportions depending not

only on the relative positions of these wheels, but also on the strength of the springs. Further, by its tendency to lift the engine on the side of its application, its effect is felt to some extent on the opposite side of the engine. The manner in which this force acts is shown in Figs. 10 and 11, Plate 5, where it will be seen that the assumption made for purposes of calculation is that the springs deflect equally per unit of load and that the force  $P_1$  raises the rigid frame in a straight line varying from zero at the trailing wheel to 0.436 of the whole force at the radial wheel.<sup>1</sup>

In engines such as that under discussion, having inclined cylinders, there is a further disturbing force, which is the vertical component of the steam-pressure acting on the cylinder-covers. This force,  $S$ , is indicated in Fig. 12, Plate 5, and acts independently of the balancing. In the present instance, with an inclination of 1 in 22 and a maximum pressure on the end of the cylinder of 27.9 tons, it amounts to 1.27 ton, and its effect will be distributed in the same way as the force  $P_1$ . It will act at its maximum up to the point of steam cut-off, which at starting may be as late as 75 per cent. of the stroke, under which circumstances the forces  $P$  and  $P_1$  are also at their maximum. On the other hand, when running at 40 miles per hour with an early cut-off the force  $S$  will be at a maximum only towards the beginning of the stroke, when the forces  $P$  and  $P_1$  are at their minima.

Fig. 13, Plate 5, shows in diagram form the forces acting when the engine is running at a speed of 40 miles per hour and the crank is at its top and bottom positions respectively. The forces causing downward or positive pressure are in full lines, and those causing upward or negative pressure in dotted lines. The resultant pressure on the rail is their algebraic sum, and is computed in the following manner :—

*Case I. Engine Running Forward at 40 Miles per Hour ;  
Crank Pin at Lowest Position.*

Radial ↓	L ↓	D ↓	I ↓	T ↓
$-\frac{1}{x_0} P_1$	$-\left(\frac{1}{x_1} P_1 + B_1\right)$	$P + R - \frac{1}{x_2} P_1$	$-\left(\frac{1}{x_3} P_1 + B_2\right)$	$-\left(\frac{1}{x_4} P_1 + B_3\right)$
Variation of load on —		$P = 1 \text{ ton.}$		

<sup>1</sup> The proportions of  $P_1$  acting at the different wheels are denoted as  $\frac{1}{x_0} P_1, \frac{1}{x_1} P_1, \dots$

		Tons.
Radial wheel . . .	$= \frac{1}{x_0} P_1$	$= -0.436$
Leading coupled wheel	$= -\left(\frac{1}{x_1} P_1 + B_1\right) = -(0.288 + 1.3)$	$= -1.588$
Driving-wheel . . .	$= P + R - \frac{1}{x_2} P_1 = 1.0 + 2.8 - 0.186$	$= +3.614$
Intermediate wheel .	$= -\left(\frac{1}{x_3} P_1 + B_2\right) = -(0.09 + 0.4)$	$= -0.49$
Trailing wheel . . .	$= -\left(\frac{1}{x_4} P_1 + B_3\right) = -(0.0 + 1.3)$	$= -1.3$

*Case II. Engine Running Forward at 40 Miles per Hour;  
Crank-pin at Top Position.*

P and R, as in Case I, are again at their maxima, but P is positive, whilst R, which in Case I was positive, is now negative. As R is the larger force, the sum of the forces acting on the driving-wheel is negative or acting upwards.

B<sub>1</sub>, B<sub>2</sub> and B<sub>3</sub> have their maximum positive effect at this phase, slightly reduced by the force P<sub>1</sub>, which acts upwards.

Radial ↓	I <sub>1</sub> ↓	D ↓	I ↓	T ↓
$-\frac{1}{x_0} P_1$	$B_1 - \frac{1}{x_1} P_1$	$P - \left(R + \frac{1}{x_2} P_1\right)$	$B_2 - \frac{1}{x_3} P_1$	$B_3 - \frac{1}{x_4} P_1$

Variation of load on —

		Tons.
Radial wheel . . . .	$= -\frac{1}{x_0} P_1$	$= -0.436$
Leading wheel . . . .	$B_1 - \frac{1}{x_1} P_1 = 1.3 - 0.288$	$= +1.012$
Driving-wheel . . . .	$P - \left(R + \frac{1}{x_2} P_1\right) = 1.0 - (2.8 + 0.186)$	$= -1.986$
Intermediate wheel . .	$B_2 - \frac{1}{x_3} P_1 = 0.4 - 0.09$	$= +0.31$
Trailing wheel . . . .	$B_3 - \frac{1}{x_4} P_1 = 1.3 - 0.0$	$= +1.3$

Now the forces P and P<sub>1</sub>, which are opposed to each other and have been defined in clauses (a) and (c), tend to decrease as the speed increases, and, conversely, to increase as the speed decreases, because they depend upon the steam-pressure in the cylinder. This pressure is greatest when the engine starts, and falls when a high speed has been attained. In the engine under consideration P = P<sub>1</sub> = approximately 5 tons when starting.

*Case III. Engine Running Forward at the Moment of Starting.*

In this case there will be no variation of pressure due to  $R$ ,  $B_1$ ,  $B_2$  or  $B_3$ , which forces are all dependent upon speed.  $P = 5$  tons,  $P_1$ , an equal and opposite force distributed in the unit proportions given in Fig. 10, Plate 5, and equal to 2·18 tons at the radial wheel, diminishing to zero at the trailing wheel.

$S$  is at a maximum also, and is 0·554 ton at the radial wheel, diminishing to zero at the trailing wheel. Assuming the steam to be at the back of the cylinder, "S" will be positive, and the variation of pressure due to all causes will be

	Radial ↓	L ↓	D ↓	I ↓	T ↓
	$+\frac{1}{x_0}S_1 - \frac{1}{x_0}P_1$	$+\frac{1}{x_1}S_1 - \frac{1}{x_1}P_1$	$P + \frac{1}{x_2}S_1 - \frac{1}{x_2}P_1$	$\frac{1}{x_3}S_1 - \frac{1}{x_3}P_1$	$\frac{1}{x_4}S_1 - \frac{1}{x_4}P_1$
Variation of Pressure.					
	Tons.				
Radial wheel . . .	$\frac{1}{x_0}S_1 - \frac{1}{x_0}P_1$	=	0·554 - 2·18	=	- 1·626
Leading wheel. . .	$\frac{1}{x_1}S_1 - \frac{1}{x_1}P_1$	=	0·366 - 1·44	=	- 1·074
Driving-wheel . . .	$P + \frac{1}{x_2}S_1 - \frac{1}{x_2}P_1$	=	5 + 0·236 - 0·93	=	+ 4·306
Intermediate wheel .	$\frac{1}{x_3}S_1 - \frac{1}{x_3}P_1$	=	0·114 - 0·45	=	- 0·336
Trailing wheel. . .	$\frac{1}{x_4}S_1 - \frac{1}{x_4}P_1$	=	0·0 - 0·0	=	0·0

There is thus a very considerable increase of pressure on the driving-wheel due to the force  $P$  acting in conjunction with the late cut-off of the steam in the cylinder.

Comparing the effect of these varying pressures with the bending-moments, shears, and cross-girder reactions given in Figs. 1 to 3, Plate 3, the following results are obtained :—

*Bending-Moments.*

Increase per Cent. upon Static Bending-Moments.											
Span in feet . .	5	6·83	12·83	21·11	41·33	61·66	82	105	155	209	308
Conditions of Load.	<sup>1</sup> (a)										
Case I. 40 miles per hour . .	39·6	40	28	17·6	9	6	3·5	2	1·5	1	0·8
Case III. Starting	48·4	58·5	33·8	20·5	14·8	9·7	7	4	3	2	1·6

<sup>1</sup> (a) This apparently anomalous result is accounted for by obtaining the greatest "static" bending-moment from a truck wheel-load, and the greatest "dynamic" moment from an engine wheel-load.

*Shears.*

Increase per Cent. upon Static Shear Loads.													
Span in feet. . .	5	6	83	12	83	21	11	41	33	61	66	82	105 155 209 308
Conditions of Load.													
Case I. 40 miles per hour . . .	39.6	37.25	23	nil	5	1.1	nil	nil	nil	nil	nil	nil	nil
Case III. Starting	48.5	43.6	29	8	8.1	3.6	„	„	„	„	„	„	„

*Cross-Girder Reactions.*

Increase per Cent. on Static Cross-Girder Reactions.													
Spacing of cross girders {	6	7	8	9	10	11	12	13	14	15	16	17	
Conditions of Load.													
Case I. 40 miles per Hour . . .	31	25	21.3	19	16.6	18	16	14.3	13.4	12.75	11.7	10.75	
Case III. Starting	37.6	29.8	25.4	22.5	20.6	21	19	18	17	16	14.7	14	

The salient lessons of this investigation are that with favourable conditions

- (a) Higher stresses may occur with the engine running at low speeds than at high speeds.
- (b) There may be a very great difference between the loads on the two girders of the same span.
- (c) In short spans and cross girders a considerable allowance should be made for variation in wheel-pressure due to the mechanism of the engine alone.

It is interesting to notice here that in a goods-engine of this class, with wheels of comparatively small diameter, it is possible to balance the reciprocating parts to the desired extent (generally about two-thirds) on the leading, intermediate, and trailing wheels, but not always possible to fully balance the revolving mass about the crank-pin in the driving-wheel and at the same time keep within the specified limit of static load upon this wheel. In view of the large variation of pressure on the driving-wheel shown in Case I, due mainly to the unbalanced revolving mass causing the centrifugal force  $R$ , it would appear that better results would be obtained if specifications for engines restricted the makers not to a

hard and fast static load upon the axles, but to the ultimate rail pressure which might be expected at the maximum speed of the engine. For example, in this case the Author understands that if the static load of 15·15 tons had been increased to 15·40 tons by a special arrangement of counterbalance, the nominal load would be increased by 0·25 ton per axle, or 0·125 ton per wheel; but on the other hand the force  $R$ , 2·8 tons, would disappear entirely, and at the maximum speed the bending-moment on short spans would have been reduced about 30 per cent.

In 1893 Professor J. Melan made a mathematical computation<sup>1</sup> of this lurching effect, but, as has been pointed out by Mr. F. Wolley-Dod,<sup>2</sup> the results for short spans were too low and for long spans too high, for the following reasons :—

- (a) The average effect upon the two girders was taken, instead of the maximum effect upon one girder.
- (b) The whole of the weight of the counterbalances was taken as giving rise to centrifugal force resulting in excess vertical pressure on the rail.
- (c) The modifying effect of the pressure on the slide-bars was not taken into account.
- (d) The maximum speed of goods-engines was taken at 27 miles per hour, whereas it is probably 35 to 40 miles per hour.

In his investigations for the 1849 Commission Professor Willis established conclusively that for comparatively weightless beams the curved trajectory of a load crossing a beam at speed had a very great effect upon the magnitude of the stress in the beam. His conclusions were that: "For a given load the increment of deflection due to velocity varies nearly as the square of the velocity directly, and the square of the length inversely." Unfortunately, he was unable for want of funds to determine what effect the mass of such a beam as would be used in actual practice would have upon these conclusions, although he realized that the effect would be very large.

Professor Melan, in the article quoted above, has given an approximate mathematical solution (Appendix II). After determining the equation for the curve of the trajectory of the centre of gravity of

<sup>1</sup> "Ueber die dynamische Wirkung bewegter Lasten auf Brücken." Zeitschrift des oesterr. Ingenieur- und Architekten-Vereines, vol. xlv (1893), p. 293.

<sup>2</sup> Translation of Professor Melan's Paper, published as Technical Paper No. 45, Government of India Technical Section. Note by Mr. Wolley-Dod (p. 11).

the load, his solution depends upon the maximum unit stress in the girder due to the static load and a given value for the modulus of elasticity,  $E$ .

In attempting a solution for the spans and loads which have been considered on pp. 197 and 198, the equation has been accepted, but substitutions have been made for the values of the maximum stress, the modulus  $E$ , and the velocity,  $V$ .

Then if the maximum stress equals 8 tons per square inch,

$$E = 13,000.$$

$$V = 58 \text{ feet per second.}$$

*For Bending-Moments and Shears.*

Increase per Cent. upon Static Bending-Moments and Shears.												
Span in feet . .	5	6·83	12·83	21·11	41·33	61·66	82	105	155	209	308	
Conditions of Load.												
Case I. 40 miles per hour . . . }	79·8	58·4	30·2	17·8	10·85	6·7	4·8	3·3	2	1·3	0·8	

*For Cross Girders.*

Increase per Cent. upon Static Cross-Girder Reactions.												
Spacing of cross girders . . . }	6	7	8	9	10	11	12	13	14	15	16	17
Conditions of Load.												
Case I. 40 miles per hour . . . }	7	5·8	4·9	4·25	3·6	3·2	2·9	2·6	2·3	2·1	1·9	1·8

Adding the maximum possible results for lurching and curved trajectory of the load, the following results are obtained :—

*For Bending-Moments.*

Increase per Cent. upon Static Bending-Moments.												
Span in feet . .	5	6·83	12·83	21·11	41·33	61·66	82	105	155	209	308	
Conditions of Load.												
Case I. Lurching	39·6	40	28	17·6	9	6	3·5	2	1·5	1	0·8	
Curved trajectory	79·8	58·4	30·2	17·8	10·8	6·7	4·8	3·3	2	1·3	0·8	
Total . . .	119·4	98·4	58·2	35·4	19·8	12·7	8·3	5·3	3·5	2·3	1·6	

*Shears.*

Increase per Cent. upon Static Shears.											
Span in feet. .	5	6·83	12·83	21·11	41·33	61·66	82	105	155	209	308
Conditions of Load.											
Case I. Lurching	39·6	37·25	23	nil	5	1·4	nil	nil	nil	nil	nil
Curved trajectory	79·8	58·4	30·2	17·8	10·8	6·7	4·8	3·3	2	1·3	0·8
Total . . .	119·4	95·6	53·2	17·8	15·8	8·1	4·8	3·3	2	1·3	0·8

*Cross Girders.*

Increase per Cent. upon Static Reactions.												
Spacing of cross girders . . }	6	7	8	9	10	11	12	13	14	15	16	17
Conditions of Load.												
Case I. Lurching	31	25	21·3	19	16·6	18	16	14·3	13·4	12·7	11·7	10·75
Curved trajectory	7	5·8	4·9	4·25	3·6	3·2	2·9	2·6	2·3	2·1	1·9	1·8
Total . . .	38	30·8	26·2	23·25	20·2	21·2	18·9	16·9	15·7	14·8	13·6	12·5

There are other causes of increased stress, of which three may be mentioned :—

(i) The effect of vibrations which are the direct result of the velocity with which the load is applied.

(ii) The effect of shocks caused by flat wheels, imperfect track, etc.

(iii) The side pressure of the wheel-flanges against the rails.

Professor Melan has given solutions of (i) and (ii), but it is perhaps a subject of controversy whether velocity alone causes a sensible increase in stress when separated from the other factors already given, except under conditions similar to those of Professor Willis's experiments; and for the effect of shocks the solution is admittedly only very rough.

The side pressure of the wheel-flanges is not calculable. It will, like the lurching effect, vary greatly with every engine, depending largely on the extent to which the reciprocating parts cause a swaying about the horizontal centre. Short, light engines will sway more than heavy engines with the weight well distributed over a long base. The pressure upon one girder must also be considerable when the span is on a curve, unless the centrifugal



force is counteracted by the exact cant of the sleepers which happens to suit the engine and the speed.

Wind-pressure on a train passing over the bridge will, if strong enough, have the effect of throwing a good deal of the total weight on to one girder. It will be noticed that wind-pressure is not taken into account in the Indian Government rules unless it amounts by calculation to more than 25 per cent. of the stresses caused by the total working load.

Finally, something might be allowed for the violent deflection caused by the occasional coincidence of the length of a girder-panel with the wheel-spacing of the load. The Author has noticed marked examples of this upon comparatively light triangulated girders.

#### SECT. V (*continued*).—THE EVIDENCE OF THE IMMEDIATE EFFECT OF MOVING LOADS.

##### (B) EXPERIMENTAL EVIDENCE.

###### *La Touche's Extensometer Results.*

Mr. J. N. D. La Touche, formerly Consulting Engineer and Senior Government Inspector of Railways, made while in India a number of observations of the stresses in small spans, with a strain-gauge of his own invention. This gauge, which the Author has also used, is remarkable for the simplicity of the principle on which it is constructed and the ease with which it can be applied to a girder. By means of a vernier the stress per square inch is read off immediately, on the assumption that the value of the modulus  $E$  for the material is 13,000 tons per square inch.

The results are plotted in Fig. 14, Plate 5. In most cases a reading was taken on both girders, the two girders often exhibiting wide variations of stress. For the reasons given in Section V (A) it is the stress in one of these girders which is required, and the curve of mean stress has therefore been drawn through the mean of the observations, considering the effect upon the highest-stressed girder only. The mean thus arrived at gives about 53 per cent. increment of stress at 5 feet, diminishing to 34 per cent. at 40 feet. Neglecting one or two high results, the maximum for the same spans ranges from 67 per cent. to 46 per cent.

In Section V (A) it has been indicated that marked variations in stress may be expected in short-span girders, either as between two similar spans or between right and left girders of the same span, and that apparently anomalous results of extensometer

readings are explainable by a shifting of the pressure from one wheel to another by the action of the unbalanced parts of an engine, etc. Risk of error in the temporary adjustment of the instrument is small if a consistent method of using it is adopted.

*Government Inspectors' Deflectometer Results* (Fig. 15, Plate 5).

The curves plotted represent the dynamic increment of the load as ascertained by card readings of deflections. These records were made by the senior Government Inspectors for every bridge tested by them, and were kept in the offices of the Railway Board. The method of testing has been described by Sir Guilford Molesworth,<sup>1</sup> Past-President Inst. C.E., and will be familiar to many. As might have been expected, the results vary largely in the small spans, in common with Mr. La Touche's extensometer results. The mean curve is not unlike that obtained in those experiments, ranging from 66 per cent. for 6-foot spans to 31 per cent. at 40 feet. It diminishes to 6 per cent. at 200 feet. The maximum curve has been drawn, neglecting a few abnormal values, and for the corresponding spans it gives 80 per cent., 50 per cent., and 11 per cent.

*Professor F. E. Turneure's Deflectometer Results*<sup>2</sup> (Fig. 15, Plate 5).

These results, which are already familiar to many, have been plotted for comparison. They are slightly smaller in the mean curve, but agree closely with the Indian records in the maxima for spans of 60 feet to 100 feet, and still more closely with Mr. La Touche's maximum extensometer readings from 5 feet to 40 feet.

*Professor Turneure's Extensometer Results* (Fig. 15, Plate 5).

Both chord- and web-member stresses are recorded. From 25 feet to 150 feet the curves are within 7 per cent. of his deflectometer results. The mean curve is only an approximate average; the true average is sometimes above and sometimes below it. The maximum curve has been drawn to omit five values which appear erratic.

*Mr. H. S. Sales's Extensometer Results* (Plate 4, and Fig. 15, Plate 5).

These experiments were made by the Bridge Engineer of the North Western Railway at various times during the last 6 years, and the greatest care was taken to ensure accuracy. A special

<sup>1</sup> Minutes of Proceedings Inst. C.E., vol. cxli, p. 61.

<sup>2</sup> "Some Experiments on Bridges under Moving Train-Loads." Trans. Am. Soc. C.E., vol. xli (1899), p. 410.

test train, equal in effect to the total equivalent load which would produce the maximum stresses to be anticipated in actual working in the members under consideration, was made up, and the axle-loads were checked by passing the train over a weigh-table. In order to ascertain the speed at which the maximum effect is obtained, the tests were taken at a range of speeds commencing from dead slow and increasing up to the maximum speed possible, having regard to local circumstances.

The extensometer (Fraenkel's) was attached to each member, and was allowed to remain there without further adjustment until the set of tests at the various speeds was completed. The extensometer was thus given every opportunity of getting out of adjustment and itself indicating on the paper if the attachment was not properly made. If by any chance the pencil on the indicating paper did not return to the datum-line on the conclusion of the tests, it was evident that the working parts of the gauge had interfered with the accuracy of the observation, and the test was repeated. These explanations are necessary in view of the doubts sometimes expressed in respect to previous observations of this kind, as to whether the inertia of the instrument had been overcome. In Mr. Sales's experiments all records of which there was the least indication of inaccuracy were discarded.

The moving load was, in all cases except one, arranged to be equivalent to the broad-gauge standard of 1903, and the spans tested were somewhat below the calculated strength required for the loads they have to carry, so that the results are probably slightly more severe than they would be in the case of new spans.

An examination of Figs. 4, 6, 7 and 9, Plate 4, reveals that there is a well-marked point beyond which the value of the coefficient for impact decreases as the speed (represented by the abscissæ) increases. Generally speaking this is 15 to 20 miles per hour. In some cases the coefficient increases up to the maximum speed of 30 miles per hour.

In Fig. 15, Plate 5, the maximum values of the coefficients recorded on the previous drawing for each member have been taken, with the calculated loaded lengths for each case as abscissæ. The resulting maximum values for each loaded length are then joined by a curve B and the mean values by curve A.

On the same drawing the Pencoyd formula (common to the Government-of-India rules of 1903 and 1908) is shown as a curve, for ready comparison with the various extensometer and deflectometer results. That it is largely in excess of these is apparent at a glance. But in Section III the Author has endeavoured to show that the

safe working-stress of 8 tons per square inch nominally provides only for (a) the known effect of the fixed load and (b) the partly known and partly assumed effects of the moving load, and therefore if the allowance for the effect of impact (in the pure sense of the word) provided under (b) is not excessive, it follows that the margin between the calculated stress of 8 tons per square inch and the elastic limit of about 16 tons per square inch is encroached upon by other sources of stress not commonly taken into the calculations. On the other hand, in Section IV it has been shown that the Pencoyd rule (Fig. 16, Plate 6) is very similar to the Launhardt rule if due allowance be made for the difference in the unit stress taken, and, in common with that rule and Stone's range formula, it is really intended to provide not only for the immediate effect but also for the cumulative effect or so-called fatigue induced by an indefinite number of repetitions of a moving load, considered as having already attained a certain increased or dynamic value.

It is on belief or disbelief in the necessity for making such provision that confidence in the justice of the Pencoyd formula will rest.

For comparison with the Pencoyd rule, and in the belief that the provision is necessary, Mr. Sales has taken the curves A as a fair representation of the immediate effect, and, by means of Launhardt's formula, has calculated the extra effect of the indefinite repetition of the forces which they indicate. The resulting coefficients are those shown by the curves C. This is precisely the method recommended by Mr. Stone, as mentioned in Section IV.

#### SECT. VI.—THE EVIDENCE OF THE CUMULATIVE EFFECT OF MOVING LOADS.

The North Western Railway gives the following particulars of 350 wrought-iron spans of 40 feet.

Under the heaviest loads in use on that railway the girders during 33 years of use exhibited signs of weakness.

The maximum stresses are given below. They represent the results given by the use of the Pencoyd rule.

	Tons per Square Inch.
Flange-stress . . . . .	9.39
Permissible by Government rule . . . . .	6.00
Shearing-stress in rivets } . . . . .	4.32
Connecting flange and web } . . . . .	
Permissible ditto . . . . .	4.00
Bearing-stress in rivets . . . . .	19.10
Permissible ditto . . . . .	9.00

The effects upon the girders were:—

- (a) A large number of rivets had become loose.
- (b) The girders had in many cases lost their original camber.
- (c) The ultimate strength of the metal was subsequently found to be very low.
- (d) The metal was found to be brittle.
- (e) A pair of girders when tested to destruction broke at a low stress.

For (a) and (b) no tabulated records are available.

For (c) twenty-seven tensile tests were made from material taken from the destroyed girders. The ultimate strength ranged from 17·9 to 22·5 tons per square inch, and the elastic strength from 12·8 to 16·6 tons.

When the 350 girders were subsequently doubled for heavier loads the flange-stress was 4·86 tons per square inch (using the Pencoyd formula for impact), and 17 per cent. of the rivets have again become loose.

*Girder tested to destruction.*—It was decided to test to destruction a girder of the same type as the 350 mentioned. Certain preliminary rough mechanical tests of the material were made. Pieces of plate were cut from the boom of a similar girder at the centre and from the end of the web. In a bending test the angle of fracture was hardly anything at all, and when bent as columns the test-pieces broke off short, showing that the metal was very brittle. Tensile tests were then made, with the following results:—

No. of Test.	Section of Test-Piece.	Taken from.	How Cut.	Ultimate Strength per Square Inch.
	Inches.			
<i>a</i>	$1\frac{9}{32} \times \frac{3}{4}$	Boom	Lengthwise	18·75
<i>b</i>	"	"	"	17·6
<i>c</i>	"	"	Crosswise	20·4
<i>d</i>	"	"	"	19·7
<i>e</i>	$2\frac{11}{16} \times \frac{3}{8}$	Web	Lengthwise	6·0
<i>f</i>	"	"	"	21·8
<i>g</i>	"	"	Crosswise	19·0
<i>h</i>	"	"	"	19·0

{ Flawed and  
crystalline.

The span tested to destruction was first built up complete and placed upon timber bed-blocks resting upon a foundation of brick in cement. The load consisted of pig iron stacked upon sleepers so arranged as to give a uniformly distributed load on all parts of the span.

Deflections were measured at the centre of the span by an arrangement of levers fitted with a pencil, and so adjusted as to avoid any slackness at the pivot. At the ends the deflections were measured by means of a stretched wire. The deflections at the ends were deducted from those at the centre, and all observations were taken each day at the same time, morning and evening.

Permanent extension was noticeable when the load reached 200 tons and the girder continued to stretch for 4 or 5 days at the rate of  $\frac{1}{32}$  inch to  $\frac{1}{16}$  inch per day.

The load was then increased to 325 tons, when one girder broke at the point marked A (Figs. 17 and 18, Plate 6). The nature of the fracture through bottom cover-plate, web, and angle is indicated in Figs. 19 and 20, Plate 6.

At the moment of permanent extension the stresses were:—

	Tons per Square Inch.
Flange-stress . . . . .	15·2
Rivets connecting flange to web. Shear-stress . . . . .	9·09
" " " " Bearing-stress . . . . .	33·0
Shearing-stress on web . . . . .	5·625

As might be expected, none of the rivets failed through shear, but with such a high bearing-stress it was expected that some sign of crushing would be visible in the rivet-holes. Such, however, was not the case, and the strips of paper which had been glued to angles and web were not torn across, showing that no relative movement had taken place. This is all the more remarkable because the rivet-holes were not drilled but punched, and the rivets did not fill the holes exactly.

The joint at A was seen to stretch when the load was 200 tons, so that from this point in the test the failure of the girder was partly due to undue stress being thrown upon the angles, which were not jointed at A but at B.

No value of the modulus of elasticity was deduced from the experiment, because from this point the deflection of the girders was partly due to the opening of the joint and partly to fibre-extension. The Author thinks that something of the condition of the metal may be deduced if the deflection curve be examined (Fig. 21, Plate 6) up to the point at which the joint began to give.

No stretch could have occurred at least up to the time when the total distributed load was 143 tons, for the flange-stress then was only 7.71 tons. Accordingly, the actual deflection has been equated with Fraenkel's formula for the deflection of a girder of varying moment of inertia, in order to ascertain the value of  $E$  at various stages of the test. The values of the actual deflection are taken from the drawing, which gives the only data available, and slight inaccuracies may thus occur in the measurements, but the values of  $E$  obtained are fairly consistent and, as will be noticed, curiously low.

Load in Tons Distributed.	Value of Deflection $\Delta$ in Inches.	Values of $E$ from $\Delta = 2 \int_0^l \frac{Mmdx}{EI}$	Flange-stress at Centre in Tons per Square Inch.
40	0.35	9,653	2.157
60	0.55	9,214	3.235
73	0.575	10,724	3.937
100	0.80	10,558	5.393
143	1.125	10,737	7.71
200	1.5	11,262	10.786

The average value of  $E$  for the six observations is 10,358 and the minimum for wrought iron is quoted in the text-books as 10,714, the average being about 11,600 to 12,000.

The doubt naturally arises whether the condition of these girders is the result of "fatigue" or simply indicates the use of very poor quality material in their manufacture. If of common quality originally, the average ultimate strength of the material would probably never have exceeded 20 tons per square inch and the elongation 5 per cent. to 10 per cent. in a length of 8 inches. Many of the tensile tests subsequently made on test-pieces from the destroyed girder gave lower values than this. One or two reached a respectable figure in both respects. The reduction of area was generally very poor.

One of the preliminary tests shown on p. 206 gives an ultimate strength of 6 tons per square inch, and the test-piece was found to be flawed. Here again this may have been an original flaw; but it is not inconsistent with the theory of fatigued metal, since there is little doubt that if the elastic limit has been passed there may be a

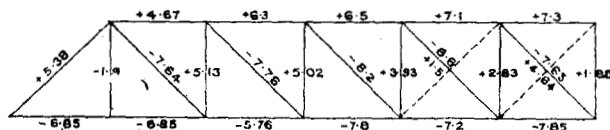
small segregation of the molecules of the metal, which, with repeated loads of a similar magnitude, will gradually become a definite flaw. The fact of the rivets having become loose in the 350 spans of similar type already mentioned appears to point to over-stress. This is perhaps the only real evidence which can be considered to furnish proof of that theory.

*Condition of the Steelwork of the Cantilever Portion of the Lansdowne Bridge over the Rohri Channel, North Western Railway.*—This bridge has been in service 24 years. During that time the loads have gradually increased until they are now practically the equivalent of the Standard B loads of 1903 (Figs. 1, 2 and 3, Plate 3). It was found necessary to strengthen the rail-bearers and cross girders, but the main frames remain as they were built. The induced stresses in these are for the most part the same as permitted by the 1903 rules. The riveting of the bridge was found about 9 months ago to be in a serious condition, roughly 25 per cent. of all the rivets being loose. Since then about 25,000 rivets have been renewed, and a large number still remain loose.

The original rivets were put in by hand, but there are strong grounds for belief that the work was first-class of its kind, and the original rivets when put in were perfectly tight. The conclusion to be derived from the results, therefore, is that the rivets have become loose in service.

Although the calculated stresses in one of the main struts are only 4.3 tons per square inch, including full allowance for impact by the Pencoyd rule, the number of rivets loose in the plates is very large.

The Author has had similar cases to deal with, of which the girder illustrated by the following diagram is one. The spans—three of



150 feet—had been in use about 20 years, of which during the last 2 or 3 they had carried goods-trains equivalent to 1.758 ton per foot for bending-moments and from 1.86 ton for end shears to 2.34 ton for central shears per foot. These loads are approximately equivalent to the 1903 standard. At the end of this time it was found that the girders were losing their camber, and a large



number of loose rivets were appearing in the attachments of the tension bars and vertical struts to the chords. The counterbraces were also badly buckled, some of them being  $3\frac{1}{2}$  inches to 4 inches out of their original line.

The proportions of the loads were  $\frac{\text{Fixed } 32.7}{\text{Moving } 67.3}$  per cent.

The diagram gives the stresses per square inch due to the total load.

In these calculations the Pencoyd impact formula was used when estimating the effect of the moving load.

Precisely similar girders on a part of the line on which these heavy goods-trains were not in use showed no signs of distress except perhaps a slight 'buckle' in the counterbraces. The maximum moving loads used were probably 20 to 25 per cent. less than in the other case. Eliminating the impact factor from the stresses in the above diagram, the greatest stress due to the whole load considered as static would be 5.44 tons per square inch in the booms and 5.35 tons in the tension diagonals. Applying the Stone range formula, the permissible stress in the booms would be 6.74 tons per square inch and 4 tons per square inch in the diagonals, which were therefore, according to this formula, considerably over-stressed by reason of the counterbraces being ineffective.

It has been found on most Indian railways that counterbraces are quite useless for their purpose of taking up the alternation of stress, but their condition is not necessarily due to fatigue, since it is quite possible that in many cases they were fixed in position before the girder was swung on its bearings and had taken up its load. Under these circumstances they would receive an initial "buckle" which never completely straightens out. Consequently, when strengthening a girder, it is now usual to proportion the central ties for both tension and compression.

#### GENERAL CONCLUSIONS.

In Section IV some results of tests carried out by Sir Benjamin Baker have been quoted. Of these the most applicable appear to be the bending tests of flat bars having a tensile strength of 31.3 tons per square inch. If the very liberal assumption is made that a bridge is subject four times per hour to a stress ranging between + 15.2 and - 15.2 tons per square inch, it will, the other conditions being equal, fail in  $7\frac{1}{2}$  years. But the magnitude of this variation may perhaps be ruled out of court as inapplicable to any

moderately well-designed structure. With a range of stress reduced by only 6 tons the life of the bridge will be  $33\frac{1}{4}$  years. With a stress ranging from zero to 15·4 tons fracture will occur in 89 years. It is evident that when the stress varies between smaller limits, both of the same sign, as in the booms of girder-bridges, fracture will only occur when the matter will have no further interest. One of Reynold's and Smith's tests was on a bar which was stressed alternately from + 6·72 to - 5·96, and after 2,025,000 reversals it was not broken.

There is always the doubt, in regard to all these tests, whether the reduced breaking-strength may not have been due to superimposed vibrations set up by a rapidity of application which could not be attained in actual practice. This rate of application is altogether out of proportion to the rate of application of the load for which large main girders are designed. The discrepancy is, however, less marked in the rail-bearers and cross girders, and also in short independent spans. An Indian goods-train may have as many as thirty wagons with axle-loads of 16 tons each. At a speed of, say, 58 feet per second a short rail-bearer will suffer, say, five times per second, a load approximating to that for which it was designed, and the vibrations during the short time in which the train is crossing will be more or less superimposed.

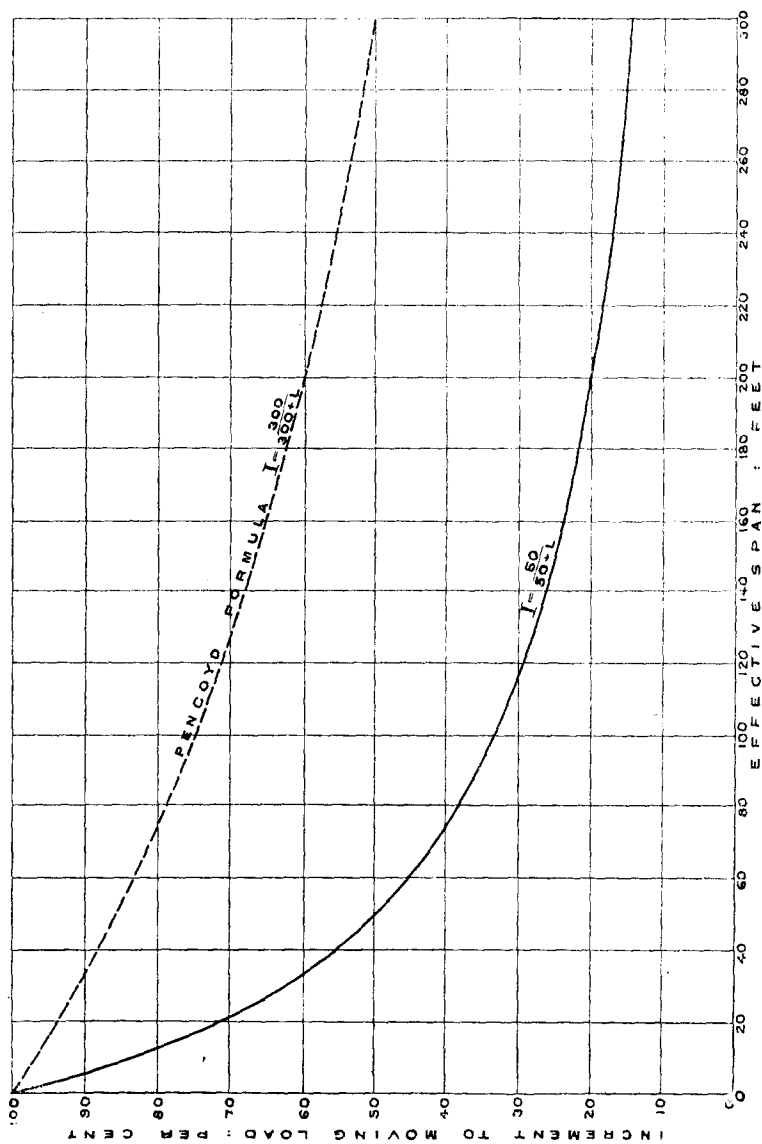
There are further sources of strain in rail-bearers and cross girders which have not yet been mentioned :—

- (a) Distortion due to the ordinary methods of fixing.
- (b) Distortion due to the part which the deck-system takes in resisting the compression or tension of the chords under a moving load. That the deck-system does this to quite a considerable extent may be easily proved by observation, and in some cases can be calculated from the results of a deflection test.

For these reasons, and from general consideration of the bad treatment which deck-members have to withstand—treatment realized only by those constantly engaged on the maintenance of bridges—the Author feels that cross girders and rail-bearers should be calculated either for much smaller unit stresses than the 8 tons per square inch allowed, or that even the Pencoyd formula for impact is not quite sufficient to ensure that the limit of elasticity is not passed.

Examination of the values of the deflection and extensometer readings (Fig. 15, Plate 5) reveals a rough similarity. If these be

Fig. 22.



taken as evidence of the immediate effect or amount to be allowed for impact of moving loads, a formula such as that mentioned on p. 193, where the Pencoyd formula  $I = \frac{300}{300 + L} S$  is modified to

$I = \frac{50}{50 + L} S$ , will give increments to the live load which will combine moderately well the features of the maximum and minimum values of those curves. It gives an increment of 91 per cent. at 5 feet to 14.3 per cent. at 300 feet. This curve (*Fig. 22*) is perhaps sufficient for calculation of the immediate impact in the case of girders already in the line, which it is not desired to remove, and could be used in conjunction with periodical inspection where incipient deterioration is suspected. As such, the Author offers it as a basis of discussion. It will be seen that it will not cover all the maximum results of the experiments, and it is to be remembered that the deflection curves only show the average stresses in the flanges. The Author has known cases where a deflection test failed to indicate any dynamic increment at the centre, but a marked increment at the one-fifth or one-sixth point of a span of 150 feet. The distortion thus, so to speak, ignored by the test at the centre had probably a worse effect than if it had extended to the centre and the radius of the curve been constant.

It is evident that it is not desirable to apply a reduced impact formula to the design of new girders intended to last indefinitely, unless it is absolutely certain that if used in conjunction with the unit working-stress that formula will ensure that the elastic limit will never be exceeded.

Remembering the sources of stress dealt with in Section III, the Author confesses he feels no certainty that the formula

$I = \left( \frac{50}{50 + L} S \right)$  is sufficiently liberal for new girders.<sup>1</sup>

The ratio of the moving load to the fixed has probably a very large influence upon its cumulative effect, and the remarks in this Paper have been confined for the sake of simplicity to single-track bridges in which the fixed load is at its smallest. Also, the examples are taken mainly from triangulated girders of simple construction.

It is believed that certain English bridges, such as the Britannia Bridge, are, neglecting an allowance for dynamic increment, stressed to about 7 tons per square inch, and that they show no signs of deterioration. This fact may well be due to the very high propor-

<sup>1</sup> If the Author's conclusions as to cross girders and rail-bearers are correct, the formula is obviously inadequate for those members.

tion of fixed load. The longest span of the Britannia Bridge is only 460 feet in length, but it weighs 1,587 tons per tube, a figure to which the average moving load must bear only a small ratio. On such members as cross girders and rail-bearers it appears from the discussion on Mr. Farr's Paper, already mentioned, that bridge engineers in England reduce the stresses allowed by the Board of Trade by 57 per cent. or less, according to their position in the structure.<sup>1</sup> This means that a calculated static stress of 2.79 tons per square inch is sometimes taken as a maximum.

*Weight of Girders.*—To illustrate the comparative weights of main girders for three different rules for calculation the Author has drawn the curves shown in Fig. 23, Plate 6.

The lower or theoretical curves represent the theoretical weight of the main girders necessary to carry the Government-of-India standard moving loads, using the impact-factor applicable to the case, and taking the stresses due to the weight of the girders, the deck-system and the track into account in their effect upon that weight. The result of this calculation, which is made for spans of several different lengths, but of similar type, is a parabolic curve which represents the weight of the main girders if the exigencies of manufacture be neglected. Allowing for gusset-plates, loss of area in rivet-holes, weight of rivet-heads, cover-plates, and unavoidable excess thickness in plates towards the end of the girders, the final weight will be anything from 50 per cent. to 90 per cent. greater than indicated by the theoretical curve. The upper curves have been drawn on the supposition that the proper addition is 90 per cent., because girders built to a new design will have the maximum of "waste" or excess material. When old girders are strengthened at site, the material is added only where required, and the resulting girder more nearly approaches uniform strength. The weights of some actual girders, both new and strengthened, have been plotted for comparison.

This method of estimating the weights of parallel triangulated girders is an adaptation of the principles advocated by Professor Fidler<sup>2</sup> and is here applied to spans ranging from 100 feet to 300 feet (Appendix III). Plate girders, owing to the excess of material in the web-plates, do not in practice so easily follow a parabolic law. To obtain the weight of the whole span, of course, an arbitrary addition must be made for the deck-system estimated from actual practice.

<sup>1</sup> Minutes of Proceedings Inst. C.E., vol. cxli, p. 32.

<sup>2</sup> "Practical Treatise on Bridge Construction." Chaps. VII and XVII.

Curiously enough, the theoretical weights of main girders calculated upon the Board-of-Trade rule and the rule just suggested, namely,  $I = \frac{50}{50 + L} S$ , are practically identical if the calculations are made for the limiting unit stresses in each case, which are 6·5 tons per square inch in compression and tension for the Board-of-Trade rule, and 6·8 tons in compression and 8 tons in tension for the other rule. It is perhaps necessary to point out here that the similarity of the two rules ends with the weight, for the one insists upon no allowance for impact, while the other provides, in common with the Pencoyd rule, an allowance for impact dependent upon the loaded length which produces the greatest stress in the member under consideration.

Although the weight of the two girders of the diagrams Fig. 23, Plate 6, can be shown to be similar (Appendix III), the material is perhaps disposed to better advantage in that of the Indian rule, as the tensile strength of the steel is more fully taken into account. Thus, in the central panel of the bottom boom the section necessary would be for the Indian rule 59·8 square inches as against 63·8 square inches in the other.

The exact stresses would, of course, be altered in the final girder; for instance, the alternating stress in the third diagonal would disappear when the added dead weight came into operation, and the stresses in the struts would require modification by the application of the column formula.

The Author feels that, in the endeavour to arrange his subject in a suitable form for discussion, he has repeated much which is familiar to engineers who have studied the problem before, but he trusts he has erred on the right side.

In conclusion, the Author desires to express his thanks to the Indian Railway Board, to Mr. J. N. D. La Touche, M. Inst. C.E., and to Messrs. Robert Stephenson & Co. for the information kindly furnished by them, and also to acknowledge the assistance rendered by the members of Sir John Wolfe Barry's staff in the preparation of drawings. Especial acknowledgments are due to Mr. Sales of the North Western Railway, who has very kindly furnished the results of his experiments, and has presented these and the data supplied by the Railway Board in a form which has saved the Author much labour.

The Paper is accompanied by twenty-one drawings and tracings, from which Plates 3-6 and the Figures in the text have been prepared.

[APPENDIXES.

## APPENDIXES.

## APPENDIX I.

TABLE I (*ABRIDGED*).—COEFFICIENTS OF IMPACT FOR TRAIN LOAD.

L = length in feet loaded to produce Maximum Stress in member under consideration.

L.	$\frac{300}{L+300}$	L.	$\frac{300}{L+300}$	L.	$\frac{300}{L+300}$	L.	$\frac{300}{L+300}$	L.	$\frac{300}{L+300}$
5	0.984	60	0.833	115	0.725	170	0.638	250	0.546
10	0.968	65	0.822	120	0.714	175	0.632	260	0.536
15	0.952	70	0.811	125	0.706	180	0.625	270	0.526
20	0.937	75	0.800	130	0.698	185	0.619	280	0.517
25	0.923	80	0.789	135	0.690	190	0.612	290	0.508
30	0.909	85	0.779	140	0.682	195	0.606	300	0.500
35	0.896	90	0.769	145	0.674	200	0.600	400	0.429
40	0.882	95	0.759	150	0.667	210	0.588	500	0.375
45	0.870	100	0.750	155	0.659	220	0.577	600	0.333
50	0.857	105	0.741	160	0.652	230	0.566		
55	0.845	110	0.732	165	0.645	240	0.556		

Note (i).—For bending-moments caused by an assumed equivalent moving load L will be equal to the span of the girder. For shears it will usually be the distance of the point under consideration from the further support. For cross-girder concentrations it will usually be twice the interval between the cross girders.

Note (ii).—In combined railway and road or footway bridges the coefficients of impact for the roadway or footway loads will be half those given in above Table.

TABLE II.—PERMISSIBLE COMPRESSIVE STRESSES.

Note (i).—The Table printed below is based upon the following formulas:—

$$P = 8 \left( 0.95 - 0.003 \frac{L}{r} \right), \text{ with a maximum of } P = (8 \times 0.85) \text{ for riveted ends.}$$

$$P = 8 \left( 0.95 - 0.0045 \frac{L}{r} \right), \text{ with a maximum of } P = (8 \times 0.85) \text{ for pin ends.}$$

Note (ii).—The stresses are for steel of the quality assumed in Rule 10, i.e., for which 8 tons per square inch is permissible for direct tensile stress.

Note (iii).—For wrought iron 75 per cent. of the stresses may be used.

P = stress allowed, in tons per square inch.

L = length in inches, centre to centre of connections.

r = least radius of gyration in inches.

L r	Riveted Ends. P.	Pin Ends. P.	L r	Riveted Ends. P.	Pin Ends. P.	L r	Riveted Ends. P.	Pin Ends. P.
10	6.80	6.80	48	6.45	5.87	86	5.54	4.50
12	6.80	6.80	50	6.40	5.80	88	5.49	4.43
14	6.80	6.80	52	6.35	5.73	90	5.44	4.36
16	6.80	6.80	54	6.30	5.66	92	5.39	4.29
18	6.80	6.80	56	6.26	5.58	94	5.34	4.22
20	6.80	6.80	58	6.21	5.51	96	5.30	4.14
22	6.80	6.80	60	6.16	5.44	98	5.25	4.07
24	6.80	6.74	62	6.11	5.37	100	5.20	4.00
26	6.80	6.66	64	6.06	5.30	102	5.15	3.93
28	6.80	6.59	66	6.02	5.22	104	5.10	3.86
30	6.80	6.52	68	5.97	5.15	106	5.06	3.78
32	6.80	6.45	70	5.92	5.08	108	5.01	3.71
34	6.78	6.38	72	5.87	5.01	110	4.96	3.64
36	6.74	6.30	74	5.82	4.94	112	4.91	3.57
38	6.69	6.23	76	5.78	4.86	114	4.86	3.50
40	6.64	6.16	78	5.73	4.79	116	4.82	3.42
42	6.59	6.09	80	5.68	4.72	118	4.77	3.35
44	6.54	6.02	82	5.63	4.65	120	4.72	3.28
46	6.50	5.94	84	5.58	4.58			

<sup>1</sup> The original 1903 rule was based upon the following formula:—

$$P = \left( 8 - 0.025 \frac{L}{r} \right) \text{ for riveted ends.}$$

$$P = \left( 8 - 0.04 \frac{L}{r} \right) \text{ for pin ends.}$$



TABLE III (ABRIDGED).—STANDARD<sup>1</sup> EQUIVALENT UNIFORM LOAD IN TONS PER FOOT OF EACH TRACK, TO BE USED FOR CALCULATING BENDING-MOMENTS.

Effective Span in Feet.	5-foot 6-inch Gauge.		Metre Gauge.		Effective Span in Feet.	5-foot 6-inch Gauge.		Metre Gauge.	
	Total Load.	Load per Foot.	Total Load.	Load per Foot.		Total Load.	Load per Foot.	Total Load.	Load per Foot.
5	45.0	9.000	25.0	5.000	170	357	2.101	234	1.375
10	45.0	4.500	33.1	3.313	180	373	2.073	244	1.354
15	62.8	4.184	41.3	2.750	190	389	2.046	254	1.335
20	75.5	3.775	49.3	2.463	200	404	2.021	264	1.319
25	86.4	3.455	57.1	2.285	210	420	1.998	274	1.304
30	96.8	3.225	65.0	2.166	220	434	1.974	284	1.290
35	106.8	3.050	72.5	2.071	230	449	1.953	294	1.278
40	116.8	2.919	80.0	2.000	240	464	1.933	304	1.265
45	127.0	2.818	87.5	1.945	250	478	1.913	314	1.255
50	137.0	2.738	95.0	1.900	260	493	1.896	324	1.245
60	157.0	2.616	109.4	1.823	270	508	1.880	334	1.236
70	177.0	2.529	123.1	1.759	280	522	1.865	344	1.228
80	197.0	2.463	137.0	1.711	290	537	1.853	352	1.215
90	217.0	2.408	149.0	1.660	300	552	1.841	361	1.204
100	236.0	2.360	161.0	1.613	325	589	1.811	385	1.186
105	245.0	2.335	167.0	1.588	350	625	1.786	410	1.171
110	254.0	2.313	172.0	1.564	375	662	1.764	435	1.159
120	272.0	2.270	183.0	1.523	400	698	1.744	459	1.148
130	291.0	2.235	193.0	1.485	425	734	1.726	484	1.139
140	308.0	2.200	204.0	1.454	450	770	1.711	508	1.128
150	325.0	2.166	214.0	1.425	475	817	1.698	532	1.120
160	341.0	2.133	224.0	1.399	500	843	1.685	557	1.113

Note (i).—For purposes of comparison the load equivalent to any train may be calculated either by the method given in the Bulletin of the International Railway Congress, August, 1901 (also in the Technical Section Paper No. 121), for moments at the sixth point of span, or by the actual wheel loads.

Note (ii).—For spans below 35 feet the maximum moment, wherever it occurs, is to be taken as the centre ordinate to the parabola of uniform load, as the girders will usually be of uniform section.

Note (iii).—In this Table L is the effective, not the clear, span, i.e., the span from centre to centre of bearings.

<sup>1</sup> 25 per cent. higher than the standard of 1903.

TABLE IV (*ABRIDGED*).—STANDARD<sup>1</sup> EQUIVALENT UNIFORM LOAD IN TONS PER FOOT OF EACH TRACK TO BE USED FOR CALCULATING SHEARS.

L = Loaded length in feet, which produces the maximum shear in the member under consideration.

L in Feet.	Load per Foot.		L in Feet.	Load per Foot.	
	5-foot 6-inch Gauge.	Metre Gauge.		5-foot 6-inch Gauge.	Metre Gauge.
5	9.0	5.4	170	2.199	1.426
10	6.1	4.05	180	2.166	1.405
15	5.018	3.334	190	2.139	1.385
20	4.425	2.879	200	2.111	1.369
25	4.03	2.608	210	2.088	1.354
30	3.733	2.425	220	2.065	1.340
35	3.523	2.296	230	2.044	1.328
40	3.358	2.199	240	2.024	1.315
45	3.223	2.123	250	2.006	1.304
50	3.11	2.061	260	1.990	1.294
60	2.933	1.971	270	1.974	1.285
70	2.8	1.896	280	1.959	1.276
80	2.695	1.816	290	1.945	1.268
90	2.605	1.739	300	1.931	1.260
100	2.53	1.675	325	1.903	1.244
105	2.498	1.648	350	1.878	1.229
110	2.466	1.621	375	1.855	1.216
120	2.410	1.576	400	1.835	1.205
130	2.36	1.538	425	1.818	1.195
140	2.313	1.504	450	1.801	1.186
150	2.271	1.475	475	1.786	1.179
160	2.233	1.449	500	1.774	1.171

<sup>1</sup> 25 per cent. higher than the standard of 1903.

TABLE V (ABRIDGED).—STANDARD<sup>1</sup> CROSS-GIRDER REACTIONS IN TONS.

Note (i). The Table printed below is derived from the load specified in Table III according to the formula—

$$R = \frac{2M}{S}, \text{ where } S = \text{spacing of the cross girders in inches.}$$

M = maximum bending-moment on the stringer, considered as of span 2S.

R = reaction at middle point of said stringer.

Note (ii). Half of the reaction given in this Table is to be considered as applied at each point where a rail-bearer rests on the cross girder.

Distance apart of Cross Girders, Centre to Centre, in Feet.	Reaction in Tons.		Distance apart of Cross Girders, Centre to Centre, in Feet.	Reaction in Tons.	
	5-foot 6-inch Gauge.	Metre Gauge.		5-foot 6-inch Gauge.	Metre Gauge.
8	32.8	21.5	22	62.4	43.0
10	37.8	24.7	24	66.5	46.0
12	42.1	27.8	26	70.5	49.0
14	46.4	31.0	28	74.5	51.9
16	50.4	34.0	30	78.5	54.7
18	54.4	37.0	32	82.5	57.5
20	58.4	40.0	34	86.5	60.3

## APPENDIX II.

### INCREASE OR DECREASE DUE TO THE FACT THAT THE LOAD DESCRIBES A CURVED TRAJECTORY.

Professor J. MELAN.

A load passing in a curved trajectory due to the deflection of the bridge gives rise to centrifugal action, added to the force of gravity if the curve be concave upwards, and deducted from it if it be convex. Let  $x, y$ , be the co-ordinates of the deflection curve, G the load moving with velocity V.

Then the centrifugal force is  $\frac{GV^2}{y} \frac{d^2y}{dx^2}$ , and the equation of the curve  $y = G \left( 1 - \frac{V^2}{y} \frac{d^2y}{dx^2} \right) f(x)$ .

When  $y_1 = f(x)$  then  $G = 1$  for a static load, and for all practical purposes  $y = Gf(x)$ .

<sup>1</sup> 25 per cent. higher than the standard of 1903.

For a perfectly straight beam of uniform section lying horizontally and traversed by a single load  $G$ ,

$$y = \frac{G}{3 EI} \frac{x^2 (L - x^2)}{L} \quad \therefore \frac{d^2 y}{dx^2} = \frac{2 G}{3 EIL} (L^2 - 6 Lx + 6 x^2).$$

The curve<sup>1</sup> for a single rolling load is therefore convex upwards near the ends (for  $x < 0.212 L$ ), and concave upwards at the centre, and the maximum curvature is at the centre where  $\frac{d^2 y}{dx^2} = \frac{GL}{3 EI}$ ; hence the total pressure at the centre is

$$G_1 = G \left( 1 + \frac{V^2}{g} \frac{GL}{3 EI} \right).$$

If  $\sigma$  be the greatest stress due to the static load, and  $h$  the depth of the beam, then

$$\sigma = \frac{GLh}{8 I} \text{ and } G_1 = G \left( 1 + \frac{8 \sigma V^2}{3 Egh} \right) \quad \dots \dots (1)$$

In the case of several concentrated loads we take a uniform equivalent load  $w$  per foot,

then

$$y = \frac{w}{6 EI} \left( 4 L - \frac{39}{4} x^2 + 6 \frac{x^3}{L} \right) x^3$$

$$\therefore \frac{d^2 y}{dx^2} = \frac{w}{6 EI} \left( 24 L - 117 x + 120 \frac{x^2}{L} \right)$$

when  $x$  is the abscissa of the centre of gravity of the load. This curve is convex upwards for  $0.293 L$  at each end, and concave upwards in the middle.

At the centre  $\frac{d^2 y}{dx^2} = -\frac{3 wL^2}{8 EI}$  and the increase of load is

$$w_1 = w \left( 1 + \frac{3 V^2 wL^2}{8 g EI} \right).$$

If  $\sigma$  be the greatest stress due to the static load,

then

$$\sigma = \frac{wL^2 h}{16 I} \quad \therefore \frac{wL^2}{I} = \frac{16 \sigma}{h}$$

and

$$w_1 = w \left( 1 + 6 \frac{\sigma V^2}{Egh} \right) \quad \dots \dots (2)$$

In the above equation the girders are assumed of uniform cross section; but in practice girders approach more closely the condition of uniform resistance, and in this case the greatest deflection is 1.2 times for parallel, and 1.66 times for parabolic girders, as much as that of girders of uniform section: and in framed girders the deflection is increased by alteration in the length of the bracing. The coefficient 6 in equation (2) should therefore be increased by the above proportion. It is further assumed that the line of the unloaded bridge is straight vertically, and that sufficient camber has been introduced to counteract the deflection of the dead load. Now the camber, as a rule, is greater than this; but this cannot always be taken into consideration, e.g., in bridges which have been in use some time, or in cases of difference in temperature

<sup>1</sup> The curve is not the deflection curve, but the curve described by the centre of gravity of the moving load.

between upper and lower booms, also because the deflection of the cross girders is not taken into consideration.

We may therefore take a coefficient of 7.5 instead of 6 in equation (2) as an average value.

Then if  $d$  be the dead weight per foot run and  $\sigma$  maximum = 5 tons per square inch stress in the girders, then

$$\sigma = \sigma \text{ maximum } \frac{w}{w+d} \text{ and } \frac{\sigma}{E} = \frac{5}{12,500} \frac{w}{w+d} = 0.0004 \frac{w}{w+d}.$$

For girders of ordinary proportions  $gxh = 3L$  nearly; and for  $V = 66$  feet per second equation (2) becomes

$$w_1 = \left(1 + 4.35 \frac{w}{L(w+d)}\right) w \quad \dots \quad (3)$$

### APPENDIX III.

WEIGHT OF PARALLEL FRAMED GIRDERS CALCULATED FROM THE DIRECT STRESSES. (Fig. 23, Plate 6, and pp. 213 and 214.)

THEORETIC WEIGHTS OF MAIN GIRDERS DESIGNED (a) TO MODIFIED PENCROY

RULE I =  $\frac{50}{50+L} S$ , AND (b) TO BOARD-OF-TRADE RULES.

The weight of a bar of steel, 1 foot in length, of 1 square inch sectional area, is taken at 0.00153 ton.

Then the equation for the theoretic weight of the girders is :

$$W_1 = L\gamma_c \Sigma S \text{ for compression members,}$$

$$\text{and } W_2 = L\gamma_t \Sigma S \text{ for tension members,}$$

where

$$W_1 + W_2 = W = \text{weight of all the members,}$$

$$L = \text{length in feet of each member,}$$

$$S = \text{direct stress in each member,}$$

$$c = \text{unit stress per square inch for compression,}$$

$$t = \text{,, ,, ,, tension,}$$

$$\gamma_c = \text{weight per foot lineal of a bar in compression,}$$

$$\gamma_t = \text{,, ,, ,, tension,}$$

$$\left(\gamma_c = \frac{0.00153}{c} \quad \gamma_t = \frac{0.00153}{t}\right).$$

*Calculation A.*

The lineal dimensions of the girder being given, the stresses under the moving load are first calculated, and the theoretic weight of the girder necessary to carry these stresses is computed by the above formula.

*Calculation B.*

The weight of the main girders as ascertained by Calculation A is added to an arbitrary allowance for the weight of the deck system, bracing and track, and the fixed load stresses are computed.

*Calculation C.*

The moving load stresses found under Calculation A are combined with the fixed load stresses of Calculation B and give the total stresses in the theoretic girder.

The stress diagrams on Fig. 23, Plate 4, represent the result of Calculation C for two girders under a common moving load but differing in the unit stresses and in the allowance for impact.

The weight of the theoretic girder may be calculated as follows :

- (a) 209 feet effective span to rule for impact  $I = \frac{50}{50 + L}$  S. Unit stress for compression 6·8 tons. Tension 8 tons.

*Compression Members.*

$$\text{Top boom } \frac{14 \cdot 92 \times 0 \cdot 00153 \times 2511 \cdot 01 \times 2}{6 \cdot 8} = 16 \cdot 858 \text{ tons.}$$

$$\text{Struts } \frac{17 \cdot 083 \times 0 \cdot 00153 \times 716 \cdot 785^* \times 2}{6 \cdot 8} = 5 \cdot 510 \text{ ,,}$$

*Tension Members.*

$$\text{Bottom boom } \frac{14 \cdot 92 \times 0 \cdot 00153 \times 2022 \cdot 71 \times 2}{8} = 11 \cdot 543 \text{ tons.}$$

$$\text{Diagonal ties } \frac{22 \cdot 68 \times 0 \cdot 00153 \times 1009 \cdot 79^* \times 2}{8} = 8 \cdot 760 \text{ ,,}$$

*Summary.*

Top boom . . . .	16·858 tons
Struts . . . . .	5·510 ,,
Bottom boom . . . .	11·543 ,,
Diagonals . . . . .	8·760 ,,

$$42 \cdot 671 \times 2 \text{ girders} = 85 \cdot 34 \text{ tons.}$$

(b) Similarly the calculated theoretic weight of the girders to Board of Trade Rules is 82·59 tons.

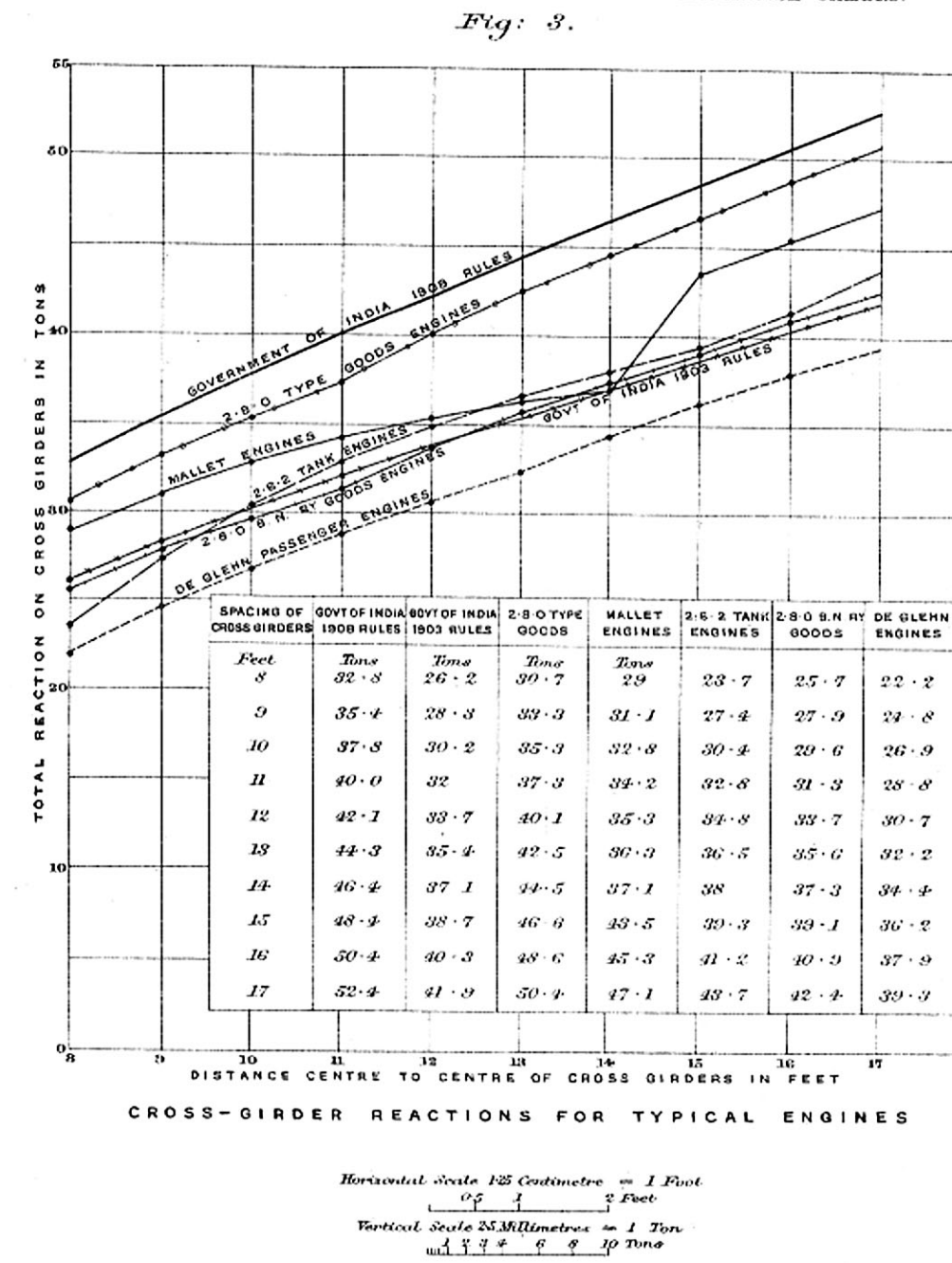
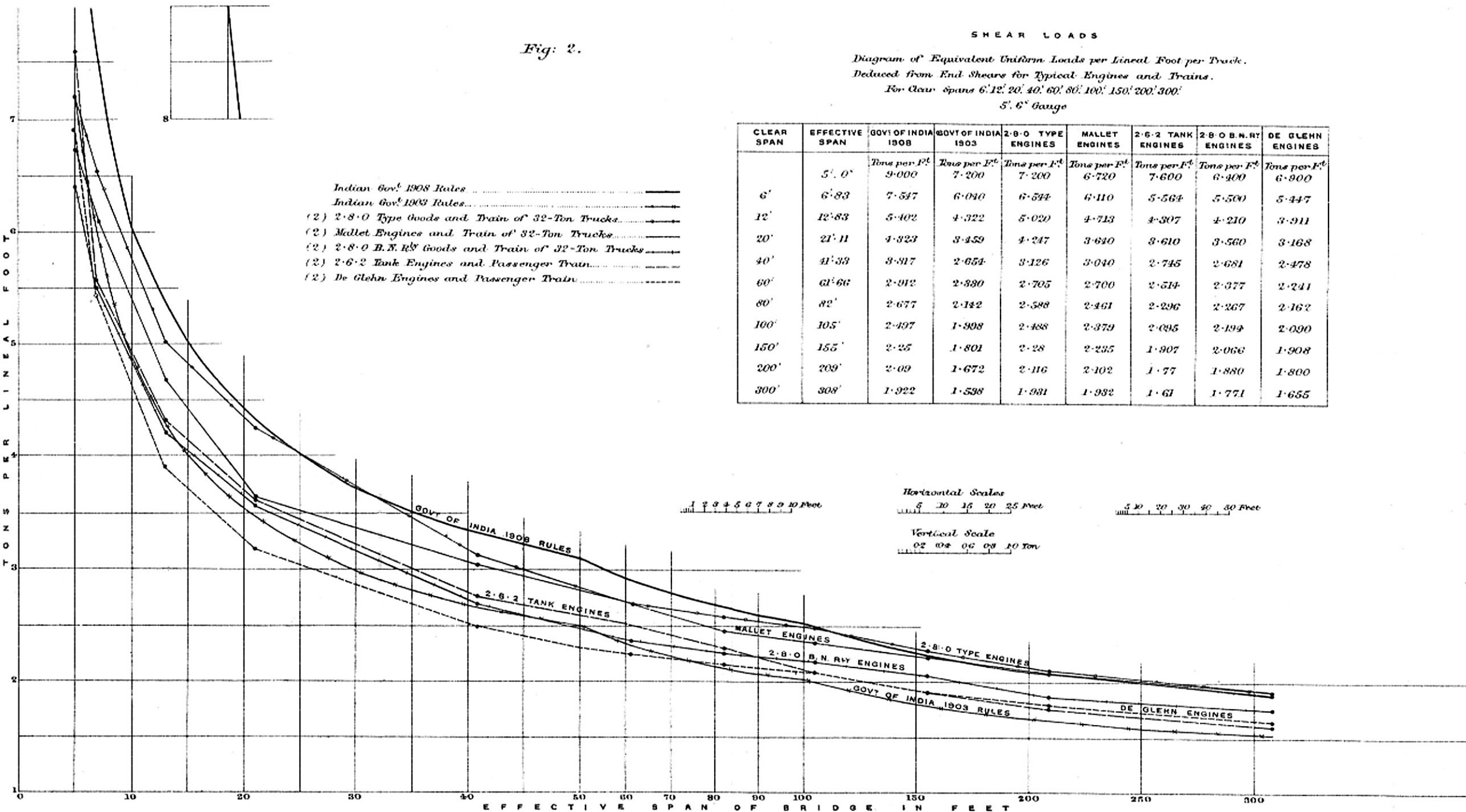
The corresponding equations to the parabolic curves of Fig. 23, Plate 4, are in round figures  $y = \frac{x^2}{520}$  and  $y = \frac{x^2}{530}$ , giving  $y = W = 84$  tons and 82·41 tons respectively.

To allow for the practical "waste" of manufacture these figures must be increased from 50 to 90 per cent. For new girders the increment will be approximately 90 per cent. For old girders strengthened at site it will seldom be more than 75 per cent.

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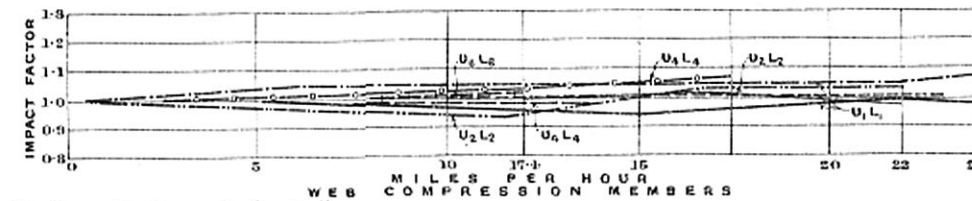
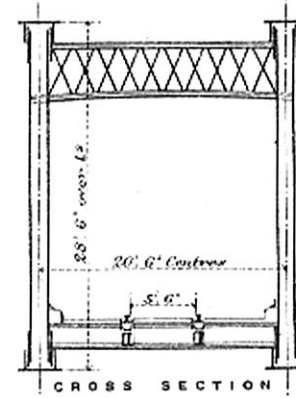
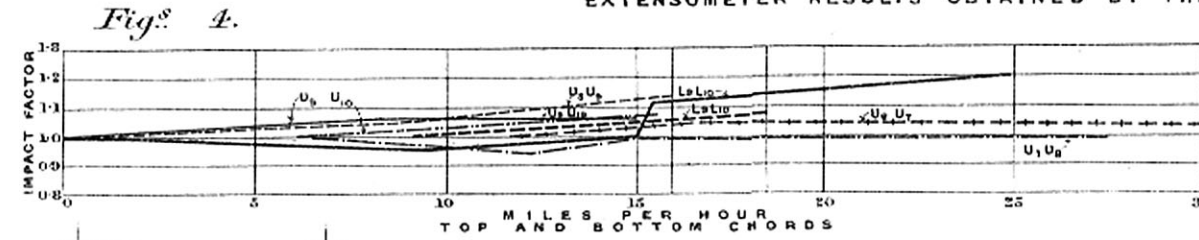
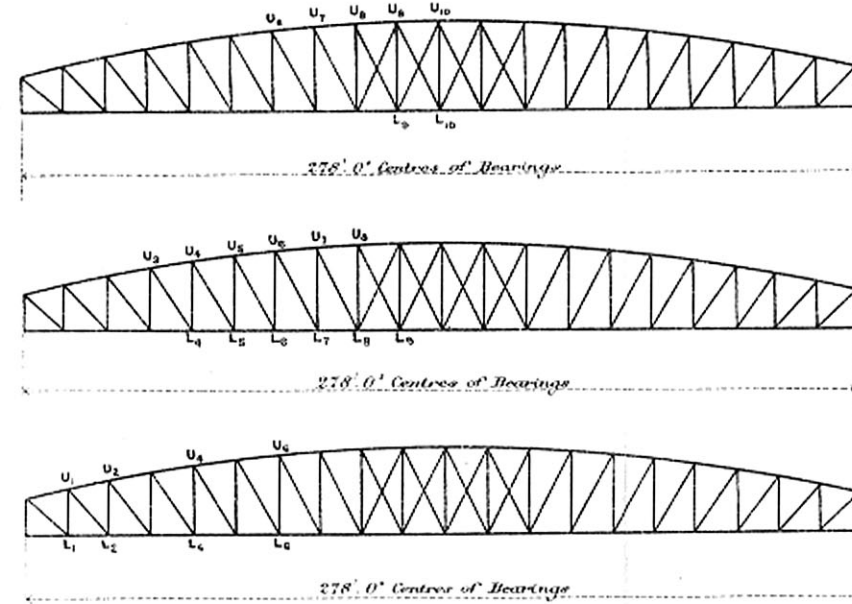
\* Where the stresses are alternating, half the smaller stress is added to the larger. (See Rule 14, Government-of-India Rules.)

PLATE 3.  
IMPACT COEFFICIENTS  
FOR RAILWAY GIRDERS

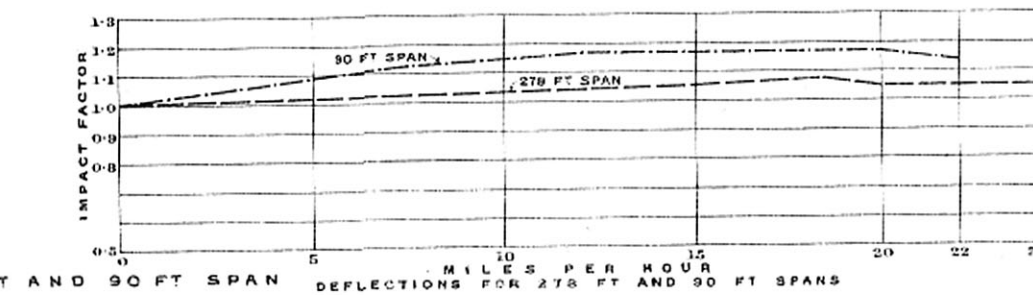
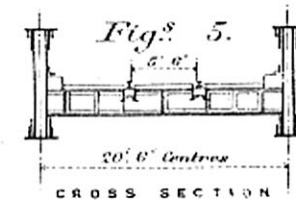
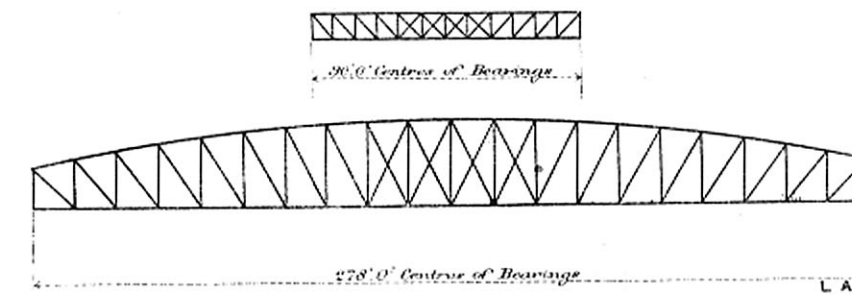




# IMPACT COEFFICIENTS FOR RAILWAY GIRDERS. EXTENSOMETER RESULTS OBTAINED BY THE BRIDGE ENGINEER N.W.RY



LANSDOWNE BRIDGE 278 FT SPAN  
Dead load 520 tons. Live load equivalent on full span without impact 416 tons. Live/Dead 0.8. (Experiments carried out on one girder only of the same span.)



LANSDOWNE BRIDGE 278 FT AND 90 FT SPAN  
DEFLECTIONS FOR 278 FT AND 90 FT SPANS

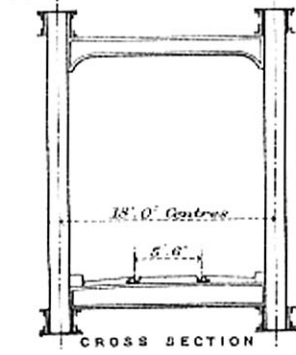
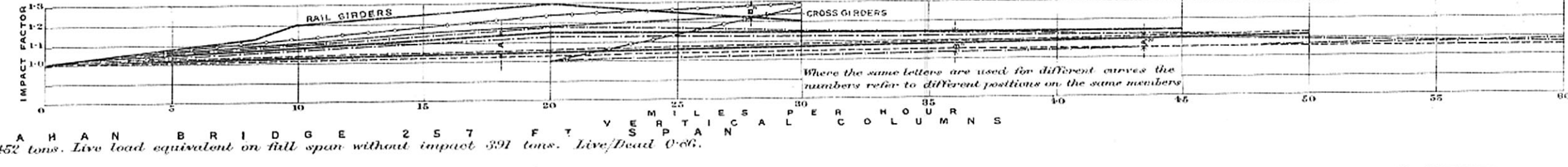


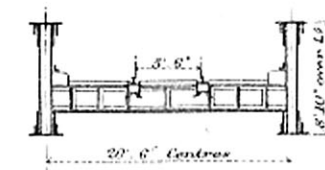
Fig. 9.



ADAMWAHAN BRIDGE 257 FT SPAN  
Dead load 452 tons. Live load equivalent on full span without impact 391 tons. Live/Dead 0.86.

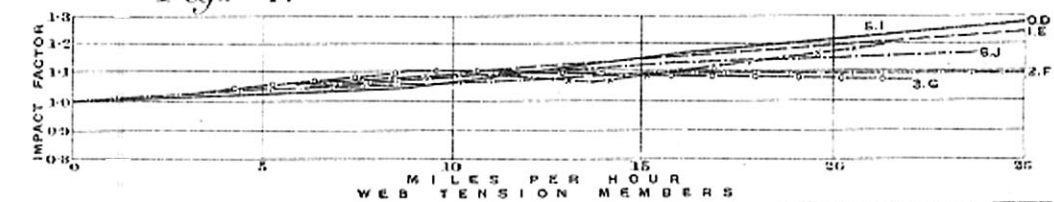
Minutes of Proceedings of The Institution of Civil Engineers. Vol. C.C. Session 1914-15, Part II.

Fig. 6.

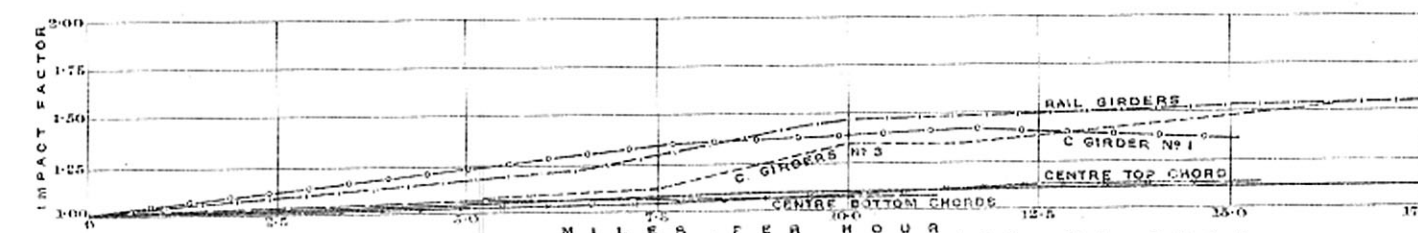
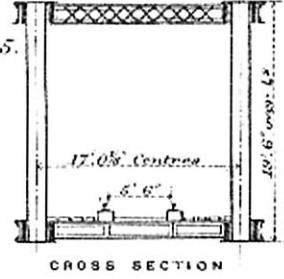
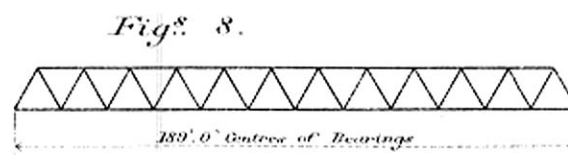
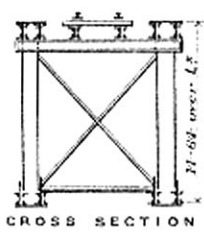


CROSS SECTION  
LANSDOWNE BRIDGE 90 FT SPAN  
Dead load 124 tons. Live load equivalent on full span without impact 181 tons. Live/Dead 1.45.

Fig. 7.



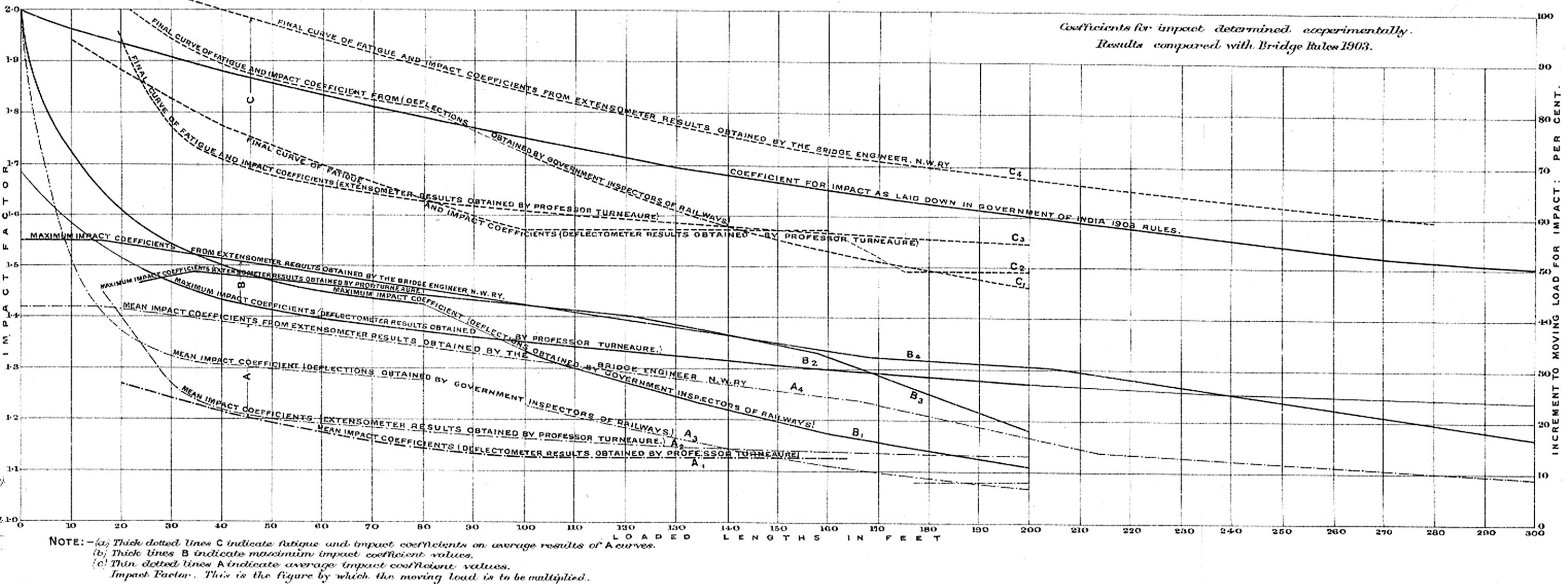
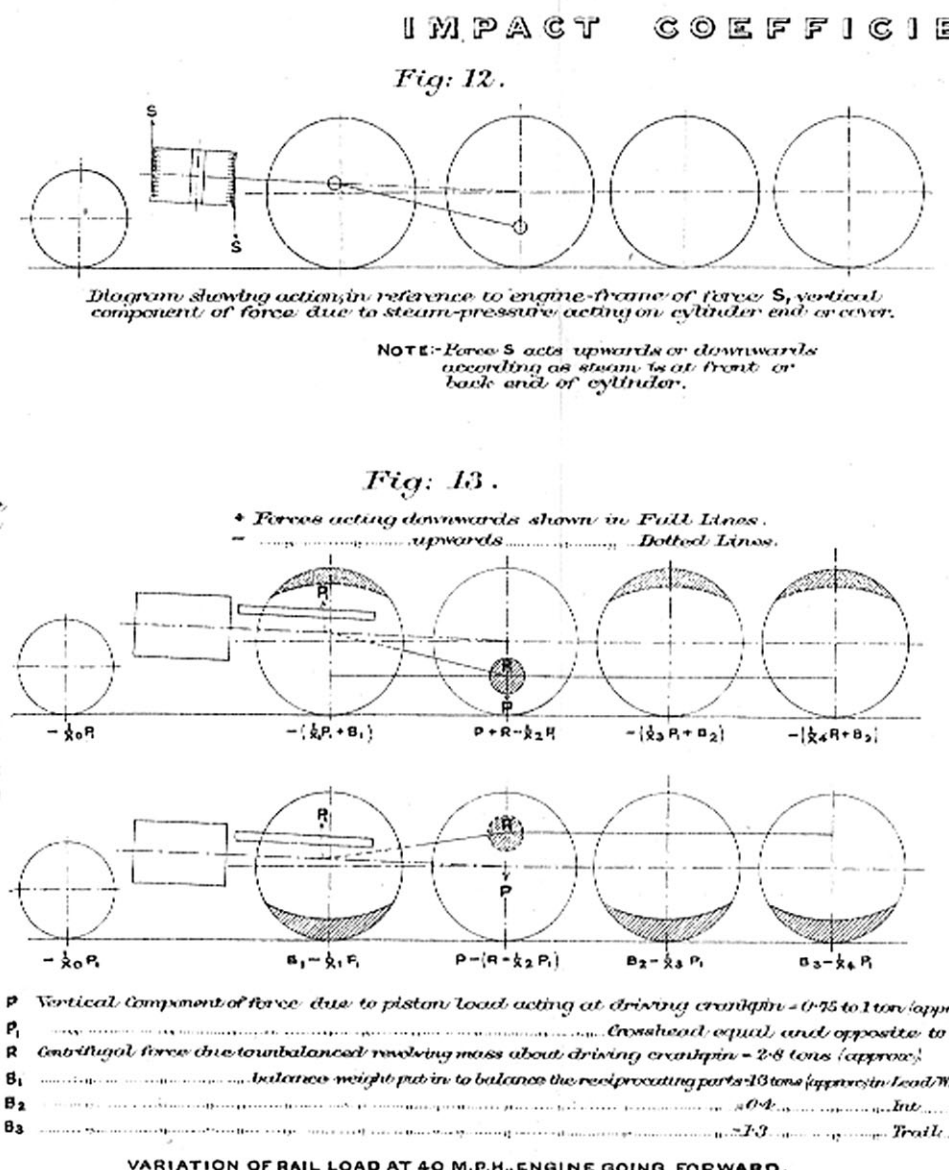
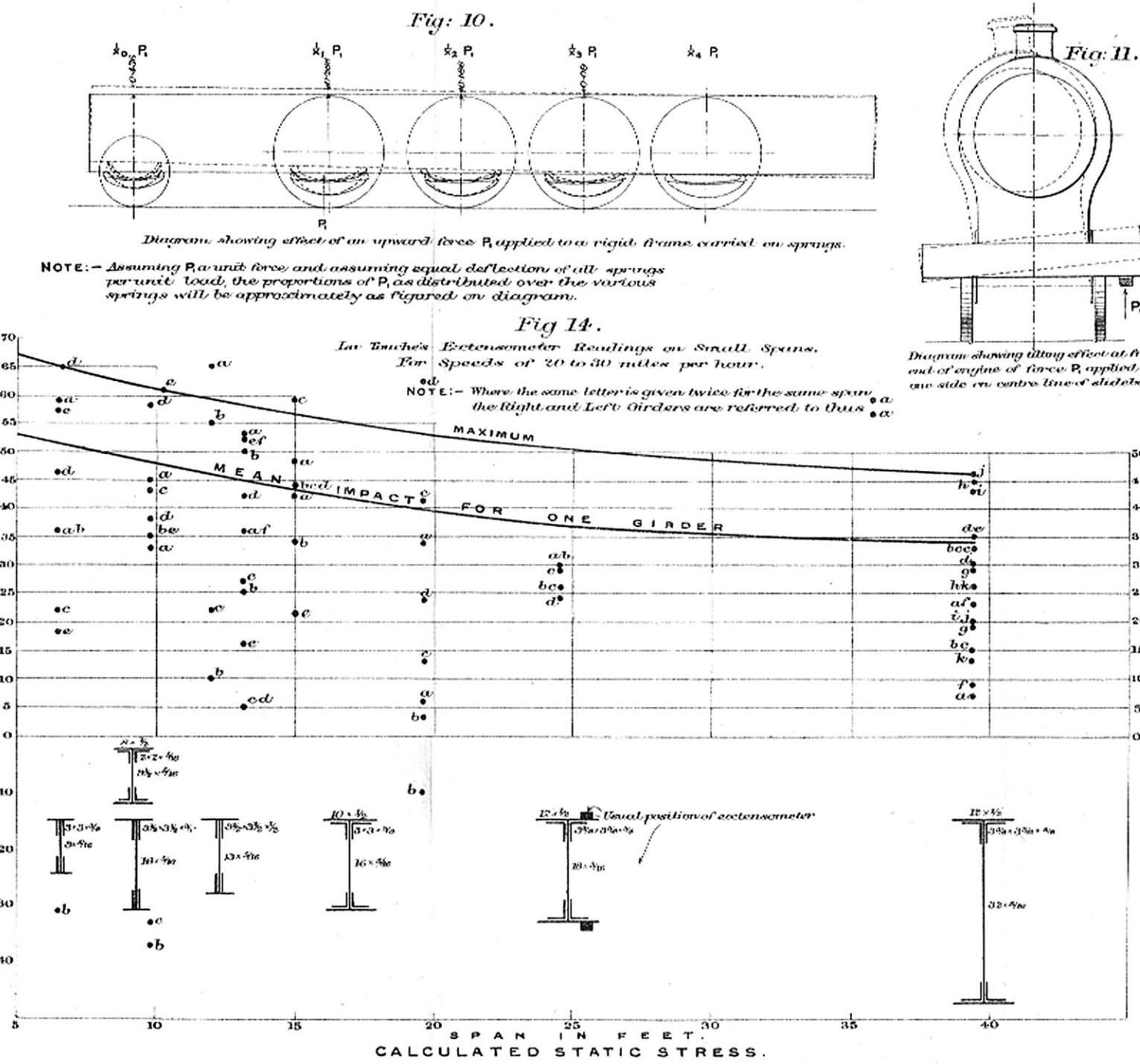
NERBUDDA BRIDGE 182 FT 3 1/2 INCHES SPAN  
Dead load 299 tons. Live load equivalent on full span without impact 301 tons. Live/Dead 1.5.



NERBUDDA BRIDGE 182 FT 3 1/2 INCHES SPAN  
Dead load 160.8 tons. Live load equivalent on full span without impact 175.7 tons. Live/Dead 1.09.

Where the same letters are used for different curves the numbers refer to different positions on the same members





**NOTE:**—(a) Thick dotted lines C indicate fatigue and impact coefficients on average results of A curves.  
(b) Thick lines B indicate maximum impact coefficient values.  
(c) Thin dotted lines A indicate average impact coefficient values.  
Impact Factor. This is the figure by which the moving load is to be multiplied.

GOODS-ENGINE CAUSING VARIATIONS OF RAIL LOAD.

GOVT. INSPECTORS OF RAILWAYS RESULTS DEFLECTIONS.

METHOD ADOPTED IN CARRYING OUT THE TESTS.

PROFESSOR TURNEAURE'S RESULTS, DEFLECTOMETER AND EXTENSOMETER.

H.S. SALES'S (BRIDGE ENGINEER N.W.R.Y.) RESULTS, EXTENSOMETER.

STANDING. SPEED.  $\frac{b-a}{a}$  Coefficient for impact  
Card diagrams both taken by the same test train.  
 $a$ —height of diagram when standing.  
 $b$ —maximum height of diagram for any speed.

COEFFICIENT for impact  
No special test train is used, one diagram only is taken for one test.

DEAD BLOW.  
10 MILES PER HOUR.  
20 MILES PER HOUR ETC.  
(Greatest of  $b, b_1, b_2$ , etc.)— $a$ —Coefficient for impact  
Special test trains of known weight are used  
all diagrams taken with same test train.

Fig. 16.

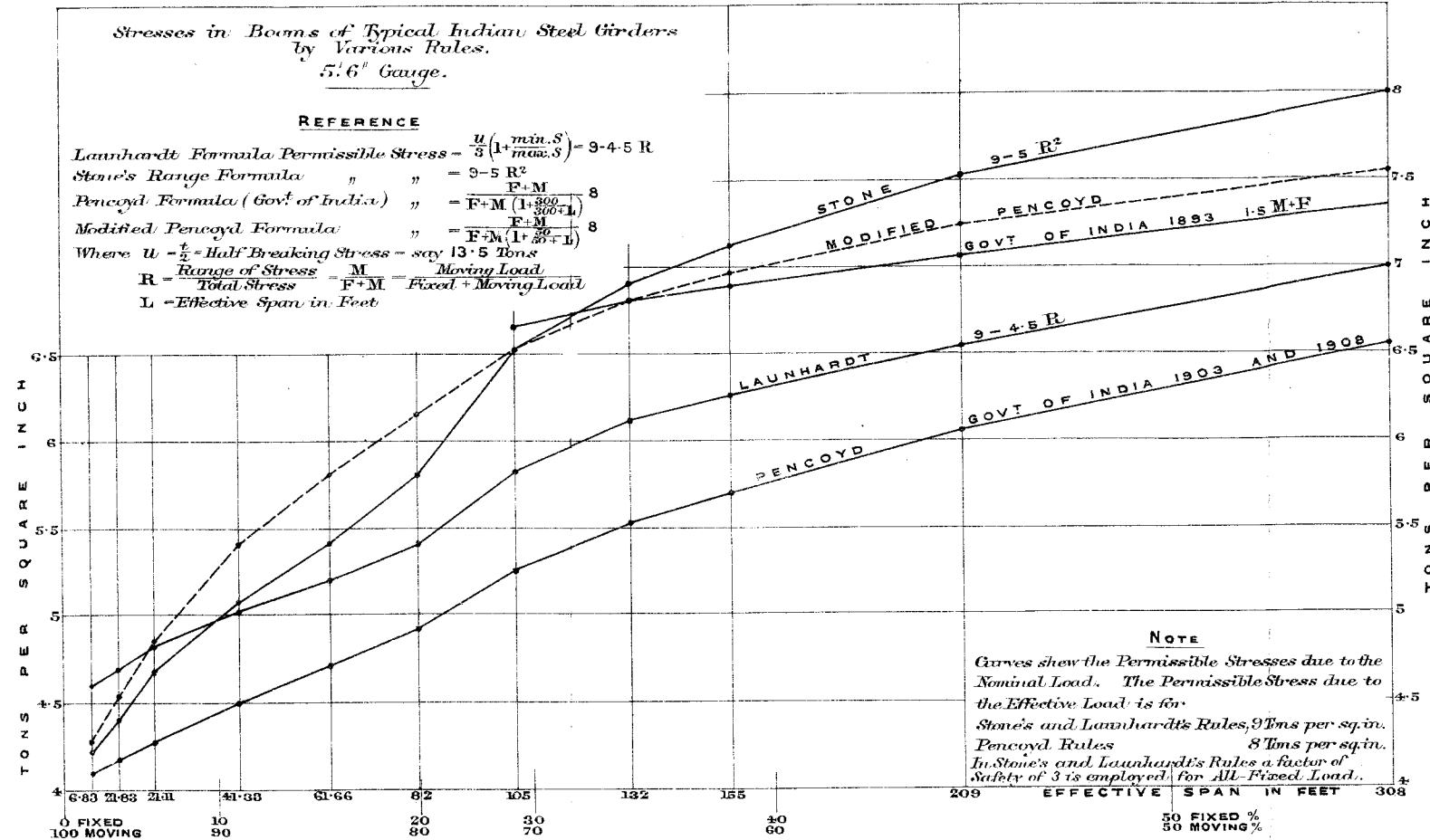
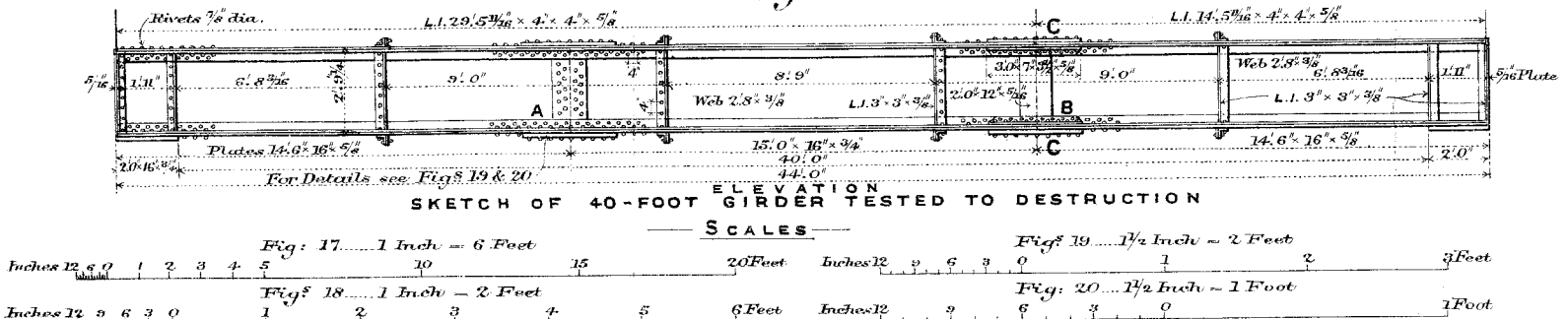


Fig. 17.



# IMPACT COEFFICIENTS FOR RAILWAY GIRDERS.

Fig. 18.

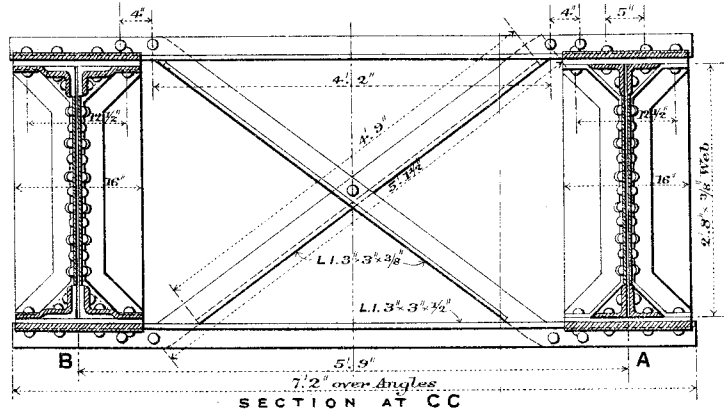
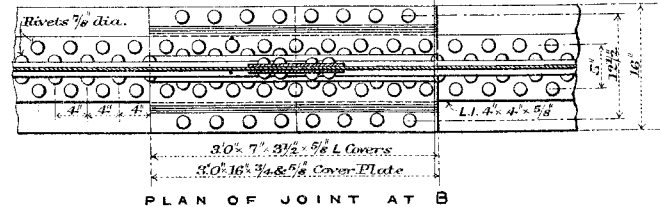
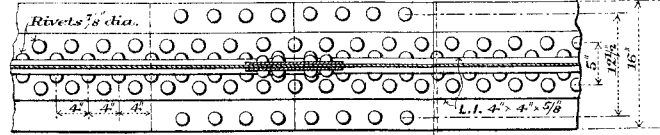
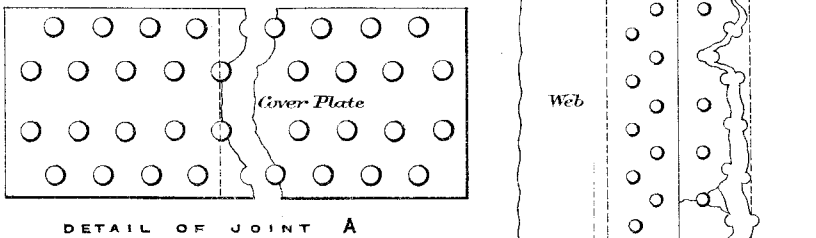


Fig. 19.



SKETCHES ILLUSTRATING FRACTURES

Fig. 21.

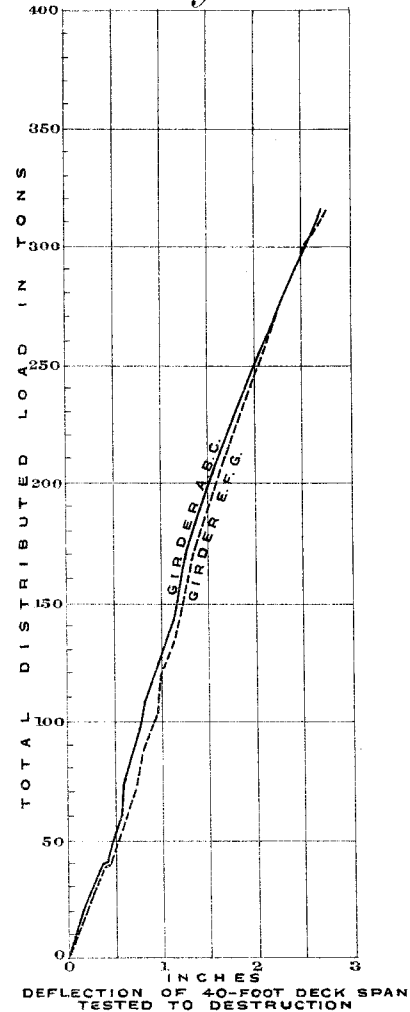
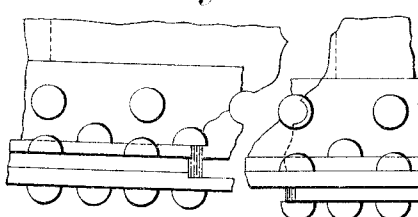


Fig. 20.



EFFECTIVE SPAN IN FEET	THEORETIC WEIGHTS			
	STRESS	IMPACT	STRESS	IMPACT
	B.O.F.T. NIL	G.O.F.I. 50+L	G.O.F.I. 300+L	
	$Y = \frac{30^2}{530}$	$Y = \frac{30^2}{520}$	$Y = \frac{30^2}{372}$	
105 Feet	20.8	21.2	29.63	
155 "	45.33	46.2	64.58	
209 "	82.41	84	117.42	
308 "	178.98	182.43	255	

EFFECTIVE SPAN IN FEET	ACTUAL WEIGHTS			
	STRESS	IMPACT	STRESS	IMPACT
	B.O.F.T. NIL	G.O.F.I. 50+L	G.O.F.I. 300+L	
	$Y = \frac{30^2}{258.57}$	$Y = \frac{30^2}{255.7}$	$Y = \frac{30^2}{185.32}$	
105 Feet	39.52	40.28	56.3	
155 "	86.13	87.78	122.7	
209 "	156.6	159.6	223.1	
308 "	340.06	346.6	484.5	

Fig. 23.

Diagrammatic Comparison of Weights of Parallel Main Girders built to different Rules To carry Govt. of India Standard 1908 Loads.  
5.6" Gauge Single Track.

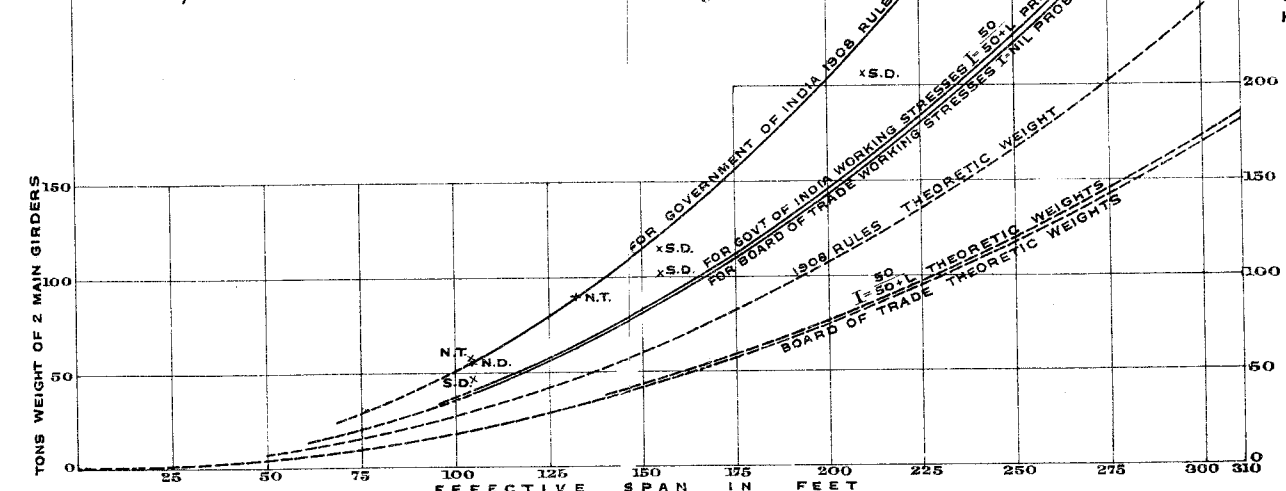
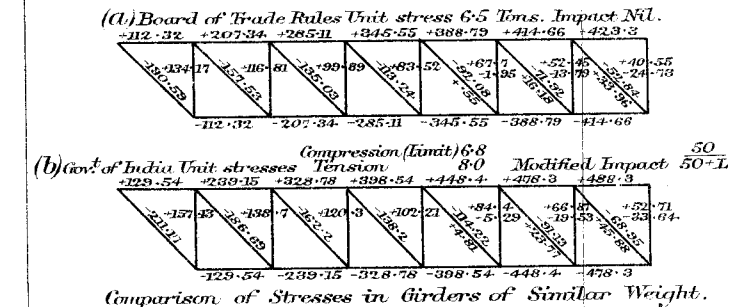


PLATE 6.  
IMPACT COEFFICIENTS  
FOR RAILWAY GIRDERS.

## NOTE

Actual Weights on Bengal-Nagpur R.R. plotted thus:—  
 S.D.=Strengthened Deck Span.  
 N.D.=New Deck Span.  
 N.T.=New Through Span.