



## XL. On the characteristic curves and surfaces of incandescence lamps

J.A. Fleming M.A. D.Sc.

To cite this article: J.A. Fleming M.A. D.Sc. (1885) XL. On the characteristic curves and surfaces of incandescence lamps , Philosophical Magazine Series 5, 19:120, 368-385, DOI: [10.1080/14786448508627687](https://doi.org/10.1080/14786448508627687)

To link to this article: <http://dx.doi.org/10.1080/14786448508627687>



Published online: 29 Apr 2009.



Submit your article to this journal [↗](#)



Article views: 2



View related articles [↗](#)

across the spectrum, and with a small flame a marked thickening of the dark line is seen just above the spoon. The sodium may be burnt in a Bunsen burner or in the flame of a spirit-lamp.

If the sodium-flame is placed between the slit and the lime-light, I have obtained a good result by using a Bunsen burner, the flame of which is cooled down to a proper temperature by a mixture of air and carbon dioxide. Care must be taken not to pass carbon dioxide into the flame to excess, otherwise too great a lowering of its temperature takes place. I have, for instance, obtained in this manner a bluish flame which did not show the slightest trace of sodium in a room where sufficient sodium had been burnt to make every gas-flame give a strong sodium reaction.

The supply of carbon dioxide to the Bunsen burner may be adjusted in the following manner :—A cork is attached to the movable cover which closes the two holes for admitting air, and two holes made in it opposite the air-holes ; to one a glass tube is attached which is connected with a bottle, into which carbon dioxide is passed, fitted with three openings. By opening clamps the carbon dioxide may be all passed into the Bunsen burner, or passed directly out from the bottle. The sodium is first brought into vivid combustion, and then the flame cooled down by admission of the proper supply of carbon dioxide.

The reversed line may also be shown by burning sodium in a spirit-lamp with four wicks, in the centre of which is a jet for the admission of oxygen. This is placed between the slit and the limelight. On passing oxygen into the flame, the heat may be raised sufficiently high to produce a bright Na band upon the screen, especially if the light from the incandescent lime is somewhat moderated, and turned into a dark band when the oxygen is shut off, proving that the production of a dark or bright sodium-band depends upon the temperature of the absorbent vapour.

*XL. On the Characteristic Curves and Surfaces of Incandescence Lamps. By J. A. FLEMING, M.A., D.Sc. (Lond.), Fellow of St. John's College, Cambridge\*.*

RECENT issues of a scientific journal† have contained some interesting letters and notes on the life of incandescence lamps, and on resulting deductions to be made

\* Communicated by the Physical Society: read March 14, 1885.

† 'The Electrician,' vol. xiv. (1885), pp. 246, 294, 311, 347.

therefrom. The difficulty of the full and complete discussion of this subject is the absence of sufficiently prolonged experiments to give statistics reliable for this purpose. These can only be obtained at great expense and by experiments lasting over a considerable time; but the results to hand make a preliminary investigation interesting on the connection which exists between the life of incandescence lamps and other correlated quantities.

The manufacture of incandescence lamps has now advanced to such a condition that the accidents of manufacture are greatly under control. The conditions necessary to get a good lamp are fairly well understood, and the physical actions going on in the lamp are also to a great extent known. We now know that the expectations of earlier investigators of getting an absolutely unalterable carbon incandescence lamp are not destined to be fulfilled; but we know that the gradual destruction of the filament is an operation dependent upon several causes, which may be greatly delayed by attention to, and success in, certain operations of manufacture.

The gradual destruction of the carbon filament in a vacuum lamp is a kind of erosion taking place at one or more points. Observation seems to show that in a single loop-filament this cutting through takes place most generally near the negative side. The determining cause of breakage is, however, temperature; and carbon filaments which present such inequalities of resistance as to give rise to spots of higher temperature might, other things being equal, be expected to be doomed to a short career. Lamp-filaments are therefore like human lives: some come into the world with a taint of disease upon them, in the shape of irregularity of structure, which predisposes to an early death; but nevertheless, as even in the case of suicide, the great law of averages overrides particular instances, and gives us, in the case of a sufficiently extended series of observations, a law connecting the average behaviour under fixed circumstances.

Experience gained during three years of commercial manufacture and use of incandescence lamps has demonstrated that when a large number of filaments are prepared with identical care, and the lamps made with them sorted out into batches, and worked with varying electromotive force, there is a very constant relation between the average efficiency or candles per horse-power and the average duration or life, and the working potential or volts of the lamp. Now, in a general way we do not know what the form of the function is that connects these three quantities, or any two of them, but there have been a large number of observations on the relation of

certain variables which indicate as a most probable form an exponential function.

There are four variables between which a relation is required—electromotive force, resistance, candle-power, and life. The third of these is at present somewhat vague and indeterminate. What we really are concerned with is the total eye-affecting radiation; and our present methods only allow of a certain more or less imperfect comparison of this as a whole with that of a standard candle, or a calculation of the integral deduced from observation of the relative intensities of certain rays. Accordingly, of these four quantities two alone can be measured with any great accuracy. One is merely an average, and the other is necessarily a somewhat ill-defined quantity.

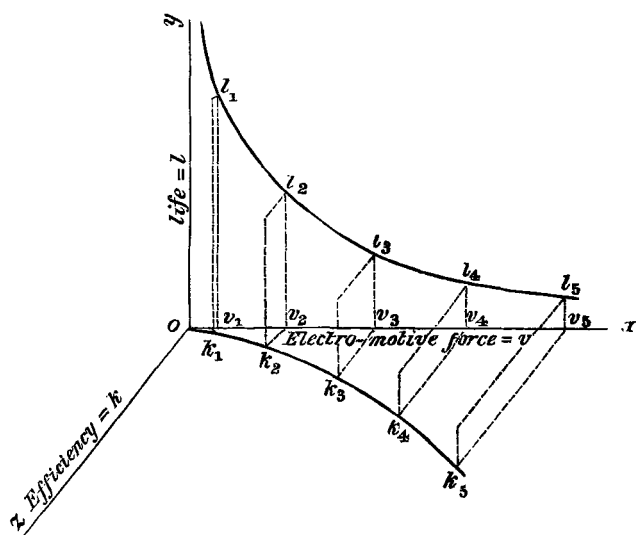
Between any two of these variables we can seek a relation, and having a number of observations we plot down these in what are best called the characteristic curves of the lamp. Now two of the most important of these curves are those connecting the electromotive force and efficiency, or candles per horse-power, and the electromotive force and life; and they may therefore be called the principal characteristic curves of the lamp. Three other useful curves may be obtained by plotting down the curves connecting candle-power and current, candle-power and electromotive force, and electromotive force and resistance. These may be called subsidiary characteristic curves.

Since the life and efficiency of a lamp vary together with the electromotive force, we can only properly represent the relation between the three by a *surface*, which may be called the characteristic surface of the lamp.

Take three rectangular axes,  $x, y, z$  (fig. 1), and let distances measured outwards represent life, candles per horse-power, and electromotive force. Let a curve be drawn on the  $y x$  plane, representing the relation of electromotive force and life, and one on the  $x z$  plane representing electromotive force and efficiency; let the ordinates be drawn at various points to both these curves, starting from abscissæ, representing certain pressures  $v_1, v_2$ , &c. Complete the rectangles on the ordinates  $k, l$ , &c.; and we see that these rectangles form the orthogonal sections of a solid bounded respectively by the two planes  $x y$  and  $x z$ , and two curved surfaces, of which the characteristic curves  $l$  and  $k$  are the traces on these planes. The surface of this volume may be called the characteristic surface of the lamp. We see that the area of the orthogonal section parallel to  $y z$  gradually increases to a maximum, and then decreases. This is obviously because, for zero electromotive force, life is infinite and candles per horse-power zero;

whilst for very high electromotive force, life is zero and candles per horse-power or efficiency a maximum. Now the area of cross section which is a maximum for a certain value  $v_3$  of electromotive force represents the product of life and candles per horse-power, or the maximum candle-hours per horse-power it is possible to get; and the value of  $kl$  is therefore a very important quantity. I shall call this maximum value of  $kl$  the principal modulus of the lamp, because the value of the lamp for commercial purposes is obtained by dividing the numeric representing this principal modulus by the price of the lamp, taking either the cost of manufacture or the selling-price, according as the question is considered from a manufacturer's or purchaser's point of view.

Fig. 1. Diagram of the principal Characteristic Curves of a Lamp.



The product  $kl$  is itself a function of the electromotive force, and the value of the electromotive force which makes this quantity a maximum is an important one to determine. It is the pressure at which the lamp should be worked in order to realize the greatest quantity of light for a given expenditure of energy in the lamp.

Let us next consider the form of the function which expresses these characteristic curves. Take, for instance, the curve of life and electromotive force. We do not know whether this curve is a continuous curve, whether it is asymptotic to axis of  $y$ , or, in fact, how it behaves beyond

the limits of the values of electromotive force  $v$ , for which lamps are incandescent. We have, however, certain values for  $l$  corresponding to values of  $v$  not lying far on either side of the ordinary so-called "marked volts" of the lamp. When the writer was in America he was shown a large number of observations, made with the view of determining the relation between average life and electromotive force, not far on either side of 100 volts, taken for the Edison 108-volt A lamp. For this lamp the life-pressure curve at or near the working-pressure is approximately a logarithmic curve whose equation is

$$l = Av^{-\alpha},$$

where  $A$  and  $\alpha$  are constants; and for the Edison 16-candle 105-volt lamp  $\alpha$  is nearly 25; and, accordingly, life varies inversely as the 25th power, roughly, of the electromotive force. In the figures shown to the writer the life-pressure curve had been drawn for pressures corresponding to lives far in excess of what could actually have been observed during the time filament-lamps have been made, reaching up to 11,793 hours. This is obviously improper: we do not know that  $\alpha$  is not itself a function of  $v$ , and for pressures departing far from the ordinary working-pressure the variation of average life and pressure is not probably expressed by so simple a relation; and we cannot go fairly beyond the limits of actually observed lives. M. Foussat has recently communicated to a scientific journal the results of observations on the life of French Edison lamps.

TABLE I.

Volts.	Life.	$\log_{10}$ (volts).	$\log_{10}$ (life).
95	3595	1.97772	3.55570
96	2751	1.98227	3.43949
97	2135	1.98677	3.32940
98	1645	1.99123	3.21617
99	1277	1.99564	3.10619
100	1000	2.00000	3.00000
101	785	2.00432	2.89487
102	601	2.00860	2.77887
103	477	2.01284	2.67852
104	375	2.01703	2.57403
105	284	2.02119	2.45332

Now the figures given by M. G. Foussat on p. 246 of 'The Electrician' for 1885, stated to be the result of a large number of observations on the relation of average life and pressure at or near 100 volts, conform approximately to the above law. Taking M. Foussat's numbers for life and electromotive force,

and taking logarithms of both, we have the figures given in Table I.

Writing down the differences between successive values of  $\log v$  and  $\log l$ , we get the numbers—

455 .....	11621	432 .....	10513
450 .....	11009	428 .....	11600
446 .....	11323	424 .....	10035
441 .....	10998	419 .....	10449
436 .....	10619	416 .....	12071

Now, if  $l = Av^{-\alpha}$ , then

$$\log l - \log l_1 = -\alpha(\log Av - \log Av_1),$$

where  $l$  and  $l_1$ ,  $v$  and  $v_1$  are adjacent values of  $l$  and  $v$ ; accordingly,

$$\alpha = - \frac{\log l - \log l_1}{\log Av - \log Av_1}.$$

The mean of the differences of  $\log l$  divided by the mean of the differences of  $\log v$  gives 25.4 nearly; and, accordingly, M. Foussat's numbers agree with those found in America in assigning to  $\alpha$  a value not far from 25 for the Edison lamp. In the neighbourhood of 100 volts

$$l = Av^{-25};$$

or, average life varies inversely as the 25th power of electromotive force.

This, however, is easily seen to be a very rough approximation. The successive quotients of life-difference by volt-difference are not constant; and the above simple exponential formula cannot be admitted as anything more than a very imperfect connection. In examining the Carlisle Tables of Mortality, which give the expectation of life at every age drawn from a very large number of observations, it was apparent that an empirical formula connecting the two quantities could be obtained of a form

$$\log e = a + bx + cx^2 + \&c.*,$$

where  $e$  = expectation of life at any age  $x$ .

I was led therefore to try if such an empirical formula better suited the present case; and a process of trial and

\* Let  $x$  be the age, and  $e$  the expectation of life at that age. Then by the Carlisle Tables of Mortality at the several ages 10, 20, 30, 40, 50, 60, 70, 80, 90, the corresponding expectations of life are:—48.82, 41.46, 34.34, 27.61, 21.11, 14.34, 9.18, 5.51, 3.28; and it can easily be found that

$$10 \log e = 17.2 - \frac{x}{100} - \frac{14}{10,000} x^2, \text{ very nearly;}$$

the calculated values from which are:—49.6, 44, 36.6, 28.6, 20.9, 14.3, 9.64, 5.54, 3.14.

failure showed that

$$10 \log l = 135 - v - \frac{v^2}{2000}$$

is a formula which gives very nearly correct results in calculating the life of a lamp  $l$ , given the working-pressure  $v$  in volts. This may be expressed otherwise:—

$$\log l = 13.5 - \frac{v}{10} - \frac{v^2}{20,000},$$

or

$$l = 10^{13.5 - \frac{v}{10} - \frac{v^2}{20,000}}.$$

Calculating by this formula, we get the following values for  $\log l$  and  $l$ ,  $l$  being the average life in hours of the 100-volt Edison lamp as made in France:—

TABLE II.

Calculated by formula.		Observed.	
$l$ .	$\log l$ .	$\log l$ .	$l$ .
3539	3.5488	3.5557	3595
2749	3.4392	3.4395	2751
2136	3.3296	3.3294	2135
1658	3.2196	3.2167	1645
1289	3.1100	3.1062	1277
1000	3.0000	3.0000	1000
776	2.8900	2.8948	785
602	2.7798	2.7788	601
467	2.6698	2.6785	477
362	2.5592	2.5740	375
281	2.4488	2.4533	284

It is evident, then, that the simple exponential function does not give nearly so good results as a formula of this latter description; and there is no doubt but that by a suitable selection of constants a formula can be obtained expressing the life of the lamp as a function of electromotive force throughout an observed range, which shall be closely in accordance with observed facts\*. Until, however, a much

\* Mr. F. M. Wright has given, in 'The Electrician,' p. 311 (1885), a formula—

$$100 - V = 9.098213 \log \frac{L}{1000}.$$

This is equivalent to

$$9.09 \log L = 127 - V;$$

and Professors Ayrton and Perry have given

$$L = 10^{14 - 11v}$$

or

$$10 \log L = 140 - 1.1 v,$$

both of which are nearly equivalent to the formula given in the text.



larger collection of statistics is obtained we shall not be in any position to determine if these constants are definite for each type of lamp or kind of carbon, and whether the average life of a lamp can be predicted from a knowledge of these constants.

An attempt was made, in the next place, to endeavour to obtain an approximate empirical formula connecting the efficiency of a lamp, or the candles per horse-power, and the electromotive force. On April 13, 1882, Prof. A. Jamieson read a paper before the Society of Telegraph Engineers and of Electricians (*Journ. Soc. Tel. Eng.* vol. xi. p. 164), "On Tests of Incandescent Lamps," and he has there given a number of tables and curves for different lamps, giving the efficiencies and resistances for various electromotive forces.

These observations afford a convenient means of putting to the test empirical formulæ, because the observations seem to have been carried out with very great care, and being done with secondary and primary batteries as current generators, the observations are more likely to be accurate than when a current from a dynamo machine is used; also because the observations for candle-power were entrusted to Dr. Wallace, gas-analyst for Glasgow, and were therefore in the hands of an observer whose eye was probably more trained to detect minute differences of illumination than one not so familiar with such work.

Professor Jamieson gives one complete table of the constants of an Edison 8-candle lamp over a great range of candle-power. Selecting that portion of the table in which the electromotive force was high enough to illuminate the lamp, we have as follows :—

TABLE III.

Tests of an Edison 8-candle Lamp, made by Prof. Jamieson, March 8, 1882.

Resistance, R.	E.M.F. in volts, V.	Current, in amperes, A.	Candle-power, K.
63·7	45·9	0·722	5·2
63·3	46·7	0·737	6·2
62·7	48·3	0·77	8·2
61	51·9	0·85	12·9
60·6	53	0·874	14·3
59·3	56·2	0·948	21·3
58·4	58	0·995	25·3
57·8	61·1	1·06	35·8
57	63·1	1·21	43·8

If we take the logarithms of these numbers we have the following table :—

## Edison 8-candle lamp.

log R.	log V.	log A.	log K.
1·80414	1·66181	1·85854	0·71600
1·80140	1·66932	1·86747	0·79239
1·79729	1·68395	1·88649	0·91381
1·78533	1·71517	1·92942	1·11059
1·78247	1·72428	1·94151	1·15534
1·77305	1·74974	1·97681	1·32838
1·76641	1·76343	1·99782	1·40312
1·76193	1·78604	0·02531	1·55388
1·75587	1·80003	0·08279	1·64147

Let E be the watts of the lamp and  $k$  the efficiency or candles per horse-power. Then

$$k = \frac{K 746}{AV};$$

and if we calculate the logarithms of the efficiency corresponding to each electromotive-force value, and compare these with four times the value of the corresponding electromotive force, we have the following table:—

log $k$ .	4 log $v$ .	4 log $v$ - log $k$ .
2·06839	6·64724	4·57885
2·13834	6·67728	4·53894
2·21611	6·73580	4·51969
2·33879	6·86068	4·52189
2·36229	6·89712	4·53483
2·47457	6·99896	4·52439
2·51461	7·05372	4·53911
2·61527	7·14416	4·52889
2·63137	7·20012	4·56885

Excepting the first and last values, which are the result of observations on the candle-power at extreme values, the intermediate figures are not very far from constant, and indicate, as a first rough approximation, that efficiency varies as the fourth power of the electromotive force.

At both high and low candle-powers the comparison of the light with a standard candle is difficult. In one case an excess of red, and in the other an excess of violet rays makes the comparison of the naked lights much more difficult at extremes, and, as is well known, the efficiency for very high or low candle-power must be stated for a definite radiation. The only other comparison between these two variables

attempted has been in the case of the experiments made by the Committee appointed to report on the incandescence lamps in the Paris Electrical Exhibition, an abstract of which appears in the same volume of the 'Journal of the Society of Telegraph Engineers.' The Committee condense these results on four varieties of lamps into the following numbers for the efficiencies, or candles per horse-power and electromotive force working them, taken at two very different values.

	Edison.		Swan.	
E.M.F. ....	89.11	98.39	47.3	54.21
Candles per horse-power ...	196.4	307.2	177.9	262.5
	Lane-Fox.		Maxim.	
E.M.F. ....	43.63	48.22	56.49	62.27
Candles per horse-power ...	173.6	276.9	151.3	239.4

If we take the logarithm of each number, and compare the difference of the logarithm of the efficiencies with the difference of the logarithm of the corresponding electromotive forces for each lamp, we have the following ratios :—

$$\begin{array}{rcl}
 \text{Edison.} & & \text{Swan.} \\
 \frac{19428}{4302} = 4.5, & & \frac{16895}{5922} = 2.9. \\
 \text{Lane-Fox.} & & \text{Maxim.} \\
 \frac{20277}{4344} = 4.6, & & \frac{19928}{4231} = 4.7.
 \end{array}$$

If, therefore,  $k_1$  and  $k_2$  be the efficiencies corresponding to two observed values  $v_1$  and  $v_2$  of electromotive force,

$$\frac{\log k_1 - \log k_2}{\log v_1 - \log v_2} = 4.5;$$

or

$$k = Cv^{4.5}$$

represents an approximate formula for calculating the efficiency. If, now, approximately, in the case of an Edison lamp, the candles per horse-power vary as the 4.5 power of the electromotive force, and the life varies inversely as the 25th power of the electromotive force, it follows that

$$l \propto \frac{1}{k^{5.7}}$$

or, roughly, that the life varies inversely as some power between 5 and 6 of the candles per horse-power.

The writer was given in America an extensive series of observations on the relation between the candles per horse-power and the average life, which were as follows :—

Life.	Candles per H. P.
11793 .....	100
7150 .....	110
4528 .....	120
2974 .....	130
2015 .....	140
1403 .....	150
1000 .....	160
727 .....	170
539 .....	180
406 .....	190
310 .....	200
236 .....	210
102 .....	220

The length of life, 11,793 hours, given as corresponding to an efficiency of 100 candles per horse-power, is nearly equal to four years of average burning in working hours, and could not have been the result of actual observation; but these numbers agree very closely with the law that

$$l \propto \frac{1}{k^{5.25}},$$

where  $l$  represents life in hours and  $k$  candles per horse-power. It is most probable that this law has been deduced from observations on the life lying within an observed range and then extended by calculation to efficiencies below those actually observed. In any case these observations are not in great discord with the above deductions made from the numbers furnished by M. Foussat on the connection between life and electromotive force, in conjunction with other observations on the relation of efficiency to working pressure in the case of Edison lamps.

It is, however, far more probable that the connection between the candles per horse-power and the working pressure is expressible by a formula of this kind,

$$\log k = \alpha + \beta v + \gamma v^2 + \&c.,$$

where  $\alpha, \beta, \gamma$  are known constants; and in this case we should have then both the life-pressure characteristic curve, and the efficiency-pressure characteristic curve expressed by analogous equations:—

$$\log l = a + bv + cv^2 + \&c.,$$

$$\log k = \alpha + \beta v + \gamma v^2 + \&c.$$

We have seen above that such a formula does fit in with observed values for one pair of variables. Further examination of this point is desirable.

Professors Ayrton and Perry have drawn attention \* to a connection between the candle-power of a lamp and the working potential, and they find that in very many cases the cube root of the candle-power is proportional to the working potential minus a constant. Now, this is equivalent to saying that the candle-power of the lamp is proportional to the cube of the potential measured above a certain point.

I have examined some records of measurements of Edison lamps to put this law to further test, and the results are given below for two 16-candle Edison lamps measured by myself, and one 8-candle lamp measured by Prof. Jamieson.

Edison 16-candle Lamp.—No. 1.

Candle-power= $K$ .	Volts, $v$ .	$\sqrt[3]{K}$ .	$v-57.17$ .	$\frac{v-57.17}{\sqrt[3]{K}}$ .
16	105.27	2.5726	48.10	18.8
11.5	99.56	2.2572	42.39	18.8
8.25	94.32	2.0206	37.15	18.4
4.8	90.03	1.6869	32.86	19
2.8	85.74	1.4095	28.57	20
2	81.22	1.2599	24.05	19.08

Edison 16-candle Lamp.—No. 2.

$K$ .	$v$ .	$\sqrt[3]{K}$ .	$v-50$ .	$\frac{v-50}{\sqrt[3]{K}}$ .
16	103.84	2.5198	53.84	21.5
13	100.03	2.3513	50	21.3
9.5	97.65	2.118	47.65	22.4
8.25	94.79	2.019	44.79	22.1
7	92.89	1.913	42.89	22.3
5.5	88.12	1.765	38.12	22.2

Edison 8-candle Lamp used by Professor Jamieson.

$K$ .	$v$ .	$v-28.7$ .	$\sqrt[3]{K}$ .	$\frac{v-28.7}{\sqrt[3]{K}}$ .
5.2	45.9	17.2	1.7324	9.928
6.2	46.7	18	1.8371	9.798
8.2	48.3	19.6	2.0165	9.720
12.9	51.9	23.2	2.3453	9.892
14.3	53	24.3	2.446	9.937
21.3	56.2	27.5	2.772	9.921
25.3	58	29.3	2.936	9.981
35.8	61.1	32.4	3.296	9.831
43.8	63.1	34.4	3.525	9.759

\* "On the most Economical Potential-Difference to Employ with Incandescent Lamps:," 'The Electrician,' March 7, 1885, p. 348.

In these cases we see that the cube root of the candle-power is very nearly proportional to the excess of the electromotive force above a certain point. Now, on examining a number of such cases, it appears that this constant is the value of that electromotive force at which the lamp just begins to give signs of incandescence. In the case of the 8-candle lamp above, Professor Jamieson marks in his table against the E.M.F. 28·7 "filament bright red." It will be very interesting if further examination should give confirmation to this surmise; but the above figures seem to indicate that the cube root of the candle-power is proportional to the electromotive force reckoned from the neighbourhood of that pressure at which the filament begins to give out light. If we call this excess pressure the "effective volts," then we can state the rule that the cube root of the candle-power is proportional to the effective volts. We have then, according to the observations of Professors Ayrton and Perry, in a certain number of cases an empirical law of this kind,

$$\sqrt[3]{K} = a(v - b),$$

in which  $K$  is the candle-power measured at a pressure  $v$ .

This may be written

$$\frac{1}{3} \log K = \log a + \log (v - b).$$

By ordinary algebraic theory we have

$$\log a = \overline{a-1} - \frac{1}{2}\overline{a-1}^2 + \frac{1}{3}\overline{a-1}^3 - \&c.;$$

$\therefore$  if  $c = b + 1$ , we can write  $\log (v - b)$  as equal to the series

$$\overline{v-c} - \frac{1}{2}\overline{v-c}^2 + \frac{1}{3}\overline{v-c}^3 + \&c.,$$

and

$$\log K = A + B\overline{v-c} + C\overline{v-c}^2 + D\overline{v-c}^3 + \&c.$$

If the supposition above made is correct, that  $b$  is a value not far from that pressure at which the lamp becomes incandescent, then  $k$  is seen to be an exponential function of the effective volts of a kind similar to that which has been found to reconcile very well the values of observed life and corresponding working electromotive force.

Other observers have before now called attention to the fact, that within a certain range the candle-power of a lamp varies approximately as the sixth power of the current passing\*.

In the case of the above-mentioned Edison 16-candle lamp,

\* At the British-Association Meeting at Montreal, Mr. Preece read a note on this subject, confirming previous observations made in 1883, and showing, from observations of his own, Professor Kittler, and Captain Abney, that incandescence varies very nearly as sixth power of current.

No. 1, the following values were obtained, connecting current and candle-power :—

Candle-power, K.	$\sqrt[6]{K}$ .	Current in amperes, $a$ .	$\sqrt[6]{K \div a}$ .
16	1.5874	.7582	2.09
11.5	1.5024	.7065	2.12
8.25	1.4215	.6720	2.11
4.8	1.2988	.6203	2.09
2.8	1.1872	.5859	2.03
2	1.1225	.5514	2.02

The table shows that, with considerable accuracy, over a range from two to sixteen candles, the incandescence varies as the sixth power of the current.

In those characteristics into which candle-power enters in any way, there is a considerable difficulty in getting results with sufficient accuracy to determine the constants of the characteristic equations. There is, however, one pair of variables, namely electromotive force and resistance, both of which can be measured with very high accuracy over a great range; and some very interesting examples of these pressure-resistance curves are given by Professor Jamieson in his memoir above alluded to.

A very short examination of the way in which an incandescence lamp behaves under increasing electromotive force, shows that the resistance decreases with increase of electromotive force, but that it does not decrease without limit; it tends to a minimum value, beyond which it appears to be constant. This is very strikingly shown for some of the Swan lamps tested by Professor Jamieson. In his paper certain formulæ are given connecting various lamp-constants, and as a first approximation to a pressure-resistance equation is given the following :—  $\log P = \log E + ar$ ,

where  $P$  is a constant,  $E = \text{E.M.F.}$ , and  $r = \text{resistance}$ .

It is not possible that such a formula should represent correctly the relation of resistance to pressure at high pressures, because it in no way expresses the fact that resistance tends to a minimum with increasing electromotive force.

An empirical formula can, however, be obtained which will express this in the following way :—

Let  $R$  be the resistance of a lamp measured with any electromotive force,  $E$ , at the terminals. Let  $E_0$  represent the electromotive force at which the lamp just becomes incandescent, and let  $R_0$  be the corresponding resistance. Let  $r$  be the minimum resistance to which the lamp approximates, as

*Phil. Mag.* S. 5. Vol. 19. No. 120. May 1885. 2 D

$E$  increases, until the filament breaks. Then

$$R = r + 10^{\log(R_0 - r) - A(E - E_0)}$$

is an expression which has the following properties:—

If  $E = E_0$ , then  $R = R_0$ ;

if  $E = \infty$ ,  $R = r$ .

The above formula is otherwise written

$$\log(R - r) = \log(R_0 - r) - A(E - E_0).$$

This formula has been tested for an Edison 8-candle lamp of which the resistance was measured over a considerable range by Professor Jamieson, and the resistance-pressure curve given in his paper. For this lamp luminosity commenced at 28·7 volts, and at this pressure it had a resistance of 73·4 ohms; its resistance gradually decreased with increasing pressure until it became apparently constant and equal to 53·5 ohms. Taking  $A$  equal to  $\frac{1}{50}$ , we have

$$50 \log(R - 53\cdot5) + (E - 28\cdot7) = 50 \log(73\cdot4 - 53\cdot5) = 64\cdot945.$$

Calculating  $R$  by this formula for various values of electromotive force  $E$ , we have the following table of observed and calculated resistances:—

Edison 8-candle Lamp.

E.M.F.	Resistance observed.	Resistance calculated by above formula.	E.M.F.	Resistance observed.	Resistance calculated by above formula.
28·7	73·4	73·4	48·3	62·7	61·57
32·7	71·1	70·05	51·9	61	60·34
36·1	68·1	67·65	53	60·6	60
39·7	66·4	65·49	56·2	59·3	59·13
43·1	64·7	63·78	58	58·4	58·66
45·9	63·7	62·51	61·1	57·8	57·98
46·7	63·3	62·19	63·1	57	57·58

The accordance between the observed and calculated resistances is fairly close. It seems very probable, from an inspection of the values at very low electromotive forces, that the resistance is a function of the electromotive force, reckoned from the pressure  $b$ , at which the lamp-filament begins to be bright red, and that of  $R$  is the resistance corresponding to any electromotive force,  $E$ , and  $r$  is the minimum resistance to which the lamp tends. Then

$$\log(R - r) = A + B(E - b) + C(E - b)^2 + \&c.,$$

where  $A$ ,  $B$ ,  $C$ , &c. are constants. If this should be the case, then it may be possible to express the life, candles per horsepower, candle-power, and resistance of a lamp, all as similar functions of the electromotive force, knowing certain constants



and the two values of electromotive force at which the lamp becomes incandescent, and the resistance to which it finally tends to obtain. When, however, we are dealing with comparatively small variations of electromotive force, it is possible to calculate both life and efficiency from a simpler exponential function of the form—

$$l \propto v^x$$

$$R \propto v^y$$

when  $x$  and  $y$  are numerics. Assuming such an approximate formula, it becomes possible to deal with an interesting question, which has been discussed also by Professors Ayrton and Perry\*, but which is capable of being investigated in a slightly different manner from that adopted by them in their paper. We shall take the warrant we have in the above figures for the assumption that for electromotive forces not far from those at which the lamp is intended to be used in the case of an Edison lamp, say 100 volts,

Average life varies inversely as the twenty-fifth power of the electromotive force ;

Efficiency, or candles per horse-power, varies as the fourth power of the electromotive force ;

Candle's light varies as the sixth power of the current, and therefore as the sixth power of the electromotive force, seeing that the resistance alters very little after the lamp has reached fair incandescence ;

And that therefore life varies inversely as the sixth-and-a-quarter power of the efficiency, and also inversely as the fourth-and-a-quarter power of the candle-power.

Let  $p$  be the price of a lamp in pounds sterling, or fractions of a pound.

Let  $l$  be the average life in hours,  $c$  the actual candle-power, and  $k$  the candles per horse-power, when run at a certain electromotive force  $v$ .

Then  $\frac{p}{cl}$  is the cost of one candle-light per hour as far as the lamp itself is concerned. Let  $P$  be the cost of 1 horse-power hour of electric energy expended in the filament.  $P$  will not be the same for all amounts, 1000  $P$  costs less than ten times 100  $P$ , but for the small variations we are considering we shall consider  $P$  to be a constant. Then  $\frac{P}{k}$  is the cost of the power in making one candle-light for one hour, and the total cost of getting one candle-light for one hour is

$$\frac{p}{cl} + \frac{P}{k} = T.$$

\* "On Potential-difference, &c."

The first term is the expense of the translating device, and it may be called the cost of *lampage*.

The second term is the cost of the production of the energy which passes through the translating device or lamp-filament and is converted into eye-affecting radiant energy, and this may be called the *machinage*; hence the total expense of keeping going incandescent light is made up of lampage and machinage. Now, we can procure a given amount of light either by running the lamps very high, in which case lamps will cost a great deal, and power less in proportion, or we can run the lamps very low and save in lampage, whilst expending more in power in a greater number of lamps; and the question arises, apart from capital expenditure, at what point is the greatest economy obtained, or, in other words, what proportion ought lampage to bear to machinage in order that the total cost may be a minimum? To solve this we shall assume, as is very probable, that for the limit of variation of E.M.F. employed the average life of the lamp is an exponential function either of the candles per horse-power  $k$ , or of the candle's light  $c$ , for the particular lamps considered.

Let  $l = \frac{A}{k^\alpha}$  and  $l = \frac{B}{c^\beta}$  be the functions.  $\alpha$  and  $\beta$  have definite values at a given E.M.F.;  $A$  and  $B$  are constants.

Then  $cl = B'l^{1-\frac{1}{\beta}}$ ;  $k = A'l^{-\frac{1}{\alpha}}$ .

Substituting, we have  $B'pl^{\frac{1}{\beta}-1} + A'Pl^{\frac{1}{\alpha}} = T$ ;

in which  $B'$  and  $A'$  are constants.

Now this expresses the total cost of working as a function of the average life at a certain E.M.F.

Let us also take that  $l = \frac{1}{v^\gamma}$ , where  $v$  is the E.M.F. at which the lamps are being run.

Then

$$B'p\left(\frac{1}{v^\gamma}\right)^{\frac{1-\beta}{\beta}} + A'P\left(\frac{1}{v^\gamma}\right)^{\frac{1}{\alpha}} = T,$$

or

$$B'pv^{-\frac{1-\beta}{\beta}\gamma} + A'Pv^{-\frac{\gamma}{\alpha}} = T.$$

Let this be varied by varying  $v$ ; to find at which value it becomes a minimum with respect to  $v$ , differentiate with respect to  $v$ , and equate to zero.

$$\frac{dT}{dv} = 0 = -\frac{B'\frac{1-\beta}{\beta}\gamma pv^{-\frac{1-\beta}{\beta}\gamma}}{v} - \frac{A'\frac{\gamma}{\alpha}Pv^{-\frac{\gamma}{\alpha}}}{v},$$

or

$$-\alpha\frac{1-\beta}{\beta}B'pv^{-\frac{1-\beta}{\beta}\gamma} = A'Pv^{-\frac{\gamma}{\alpha}}.$$

Hence the total cost of working is a minimum when the cost of power  $A'Pv^{\frac{2}{\beta}}$  is to the lampage  $B'pv^{-\frac{1-\beta}{\beta}}$  as  $\alpha \frac{1+\beta}{\beta}$  to unity.

Now the above investigations show that for Edison lamps  $\alpha$  is a quantity in the neighbourhood of  $6\frac{1}{4}$ , and  $\beta$  near  $4\frac{1}{8}$ ; hence

$$-\alpha \frac{1-\beta}{\beta} = \frac{19}{4},$$

and hence ratio of lampage to total cost is  $\frac{4}{23} = 17.4$  per cent.

Hence we arrive at this curious result, that independently of the cost of the lamp, or electrical energy, we must run at such a pressure that lampage is about 18 per cent. of the total cost. Now, if instead of employing these approximate exponential expressions for the values of  $l$ ,  $c$ , and  $k$  in terms of the electromotive force, we had introduced the more accurate forms of equation indicated above, we should have had an equation in terms of  $v$  to solve as the result of equating the differential to zero, which would, by the introduction of the proper constants, give the value of electromotive force at which the total cost becomes a minimum. In their paper, Professors Ayrton and Perry have calculated to a fraction of a volt what this economical potential is. As, however, the characteristic equations are only approximate, it seems hardly necessary to do more than obtain a similar approximate expression for the economical working.

At the Edison lamp-factory in America calculations were made, on the assumption of a particular type of lamp and length of life and cost of power, to ascertain the ratio of lampage to total cost, which made the total cost a minimum, and the result appeared to be to fix it at about 16 per cent. These calculations are therefore in singular accord with the deduction of theory based on determination of the constants of the characteristic curves, arrived at both by graphic and analytical methods.

---

XLI. *Experiments on the Electromagnetic Action of Dielectric Polarization.* By Prof. W. C. RÖNTGEN\*.

THE theory of electrical and magnetic phenomena proposed by Faraday and elaborated by Clerk-Maxwell, is based upon the assumption that in insulators bounded by electrified conductors there exists a dielectric polarization or displacement—a change which, in whatever way produced, exerts electrodynamic effects exactly like an electric current flowing in a conductor.

\* Translated from the *Sitzungsberichte der Berliner Akad. der Wissenschaften* for February 26, 1885.