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found from time to time in the old caverns or weeldons of the limestone. These acknowledged evils undoubtedly affected the economy and prosperity of the trade, until some more fortunate or more ingenious worker applied the bellows to the art of iron-making, which gave rise to the blast bloomery, and occasioned a great revolution and improvement in the fabrication of that valuable and highly useful metal.

Coleford, Gloucestershire.

[To be continued.]

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XXVII. *Suggestions for simplifying Mr. IVORY'S Solution of the Double Altitude Problem.* By Mr. EDWARD RIDDLE, Royal Naval Asylum, Greenwich.

*To the Editors of the Philosophical Magazine and Journal.*

GENTLEMEN.—IN The Philosophical Magazine for August 1821, Mr. Ivory has given a new and direct solution to the well known nautical problem, which requires the latitude to be determined from two altitudes of the sun, and the time elapsed between the observations; and as the rule which he has deduced possesses several advantages over any other that has hitherto been given for the purpose, it may be hoped that teachers of navigation will generally feel it their duty to endeavour as much as possible to introduce it into practice.

I conceive however that, simple and convenient as the rule is, its form yet admits of an advantageous alteration. In the numerical calculation from the formulas which Mr. Ivory has investigated, it is necessary to refer both to a table of *natural sines*, and to one of the *logarithms of numbers*. But in this solution of the problem, the use of both these tables may be very easily dispensed with; as by a simple transformation of two of the expressions the whole of the calculations may be performed with even greater neatness and facility, with the aid of no other table than one of *log. sines*, &c.

In a rule for the guidance of ordinary computers, it is desirable to avoid, when it can conveniently be done, a reference to a variety of tables; as the mere act of turning from one table to another has in itself a tendency to perplex. But by the slight alteration which I have to propose in Mr. Ivory's solution, this objection to it (if an objection it may be called) will be completely obviated; and, besides, the form of calculation admits of an exceedingly convenient arrangement; a circumstance which may recommend the rule to the attention of persons who, otherwise, might not at once be sensible of its value.

In

In this class of persons I must observe that I feel myself included; for it was not till I had given Mr. Ivory's paper a very attentive consideration, and had been at considerable pains in arranging the calculations of an example, that I fully perceived the great practical importance of this result of his able and ingenious view of the problem.

The expressions to which I have alluded as admitting of a convenient transformation are these; viz.

" $A = \frac{\sin h + \sin h'}{2}$ ," and " $B = \frac{\sin h - \sin h'}{2}$ ," where  $h$  and  $h'$  are the altitudes, and the *logarithms* of  $A$  and  $B$  are the quantities required.

$$\text{Now, } \frac{\sin h + \sin h'}{2} = \sin \frac{h + h'}{2} \cdot \cos \frac{h - h'}{2};$$

and  $\frac{\sin h - \sin h'}{2} = \sin \frac{h - h'}{2} \cdot \cos \frac{h + h'}{2}$ ; expressions from which the logarithms of  $A$  and  $B$  may be obtained at once.

With these alterations in the forms of the expressions for  $A$  and  $B$ , Mr. Ivory's formulas for computing the latitude may be arranged as under;  $t$  being the half interval of time between the observations,  $h$  and  $h'$  the altitudes,  $D$  the declination, and  $L$  the latitude.

$$1. \sin a = \cos D \cdot \sin t.$$

$$2. \cos b = \sin D \cdot \sec a.$$

$$3. \sin c = \cos \frac{h + h'}{2} \cdot \sin \frac{h - h'}{2} \cdot \operatorname{cosec} a.$$

$$4. \cos d = \sin \frac{h + h'}{2} \cdot \cos \frac{h - h'}{2} \cdot \sec c \cdot \sec a.$$

$$5. \sin L = \cos c \cdot \cos (b \mp d).$$

It may be observed on these formulas, that  $a$ ,  $\sec a$ , (which is wanted twice,) and  $\operatorname{cosec} a$ , are obtained at the same opening of the table; and so are  $\cos D$  and  $\sin D$ ;  $\sin$  and  $\cos$  of  $\frac{h + h'}{2}$ ;  $\sin$  and  $\cos$  of  $\frac{h - h'}{2}$ ; and  $c$ ,  $\cos c$ , and  $\sec c$ ; so that the number of openings of the book of tables is comparatively small.

The demonstrations of the formulas may be seen in Mr. Ivory's paper, which also contains the investigation of a very neat method of estimating the effect of the change of the sun's declination, in the interval between the observations, when it is deemed of any consequence to advert to it. The practical rule from the above formulas may be thus given in words, observing that in every case the tens are to be rejected from the indices of the logarithms.

#### *Practical Rule.*

1. Add together the *sine* of half the elapsed time, and the *cosine*

*cosine* of the declination, and the sum will be the *sine* of arc first.

2. Add together the *secant* of arc first, and the *sine* of the declination, and the sum will be the *cosine* of arc second; which will be *acute* when the latitude and declination are of the same denomination, but *obtuse* when they are of different ones.

3. Add together the *cosect.* of arc first, the *cosine* of half the sum of the altitudes, and the *sine* of half their difference; and the sum will be the *sine* of arc third.

4. Add together the *secant* of arc first, the *sine* of half the sum of the altitudes, the *cosine* of half their difference, and the *secant* of arc third, and the sum will be the *cosine* of arc fourth.

5. When the zenith and elevated pole are on the same side of the great circle passing through the places of the sun at the times of observation, the *difference* of arcs second and fourth will be arc fifth, otherwise their *sum* will be arc fifth.

6. Add together the *cosines* of arcs third and fifth, and their sum will be the *sine* of the latitude.

*Note.* When the declination and latitude are nearly equal, and of the same name, it may sometimes be doubtful whether the *sum* or *difference* of arcs second and fourth ought to be taken for arc fifth. But the computation is soon made on both suppositions; for *cosine* of arc fifth is the last logarithm which is taken from the tables, and the other parts of the calculation are therefore not affected by the change. One of the results must certainly be the required latitude, and the latitude by account will generally be sufficient to determine which of them ought to be taken. But when the sum of arcs second and fourth is equal to  $90^\circ$ , or greater, it can only be their difference which is arc fifth.

*Form of Calculation.*

sin H.E.T. ° ' _____					
sin Dec _____	cos				
sin Arc I _____	sect			cosect	
Arc II _____	cos				
		sect Arc I _____			
		sin $\frac{1}{2}$ sum alts. ° ' _____	cos		
		cos $\frac{1}{2}$ diff. alts. _____	sin		
		sect Arc III _____	sin		
Arc IV _____	cos		cos		
Arc V .....			cos		
		Lat. _____	sin		

*Example.*—Given the altitudes of the sun  $19^{\circ} 41'$  and  $17^{\circ} 13'$ , interval one hour, and the sun's declination  $20^{\circ}$  S. to find the latitude, it being by account about  $50^{\circ}$  N.

		$19^{\circ} 41'$			
		$17^{\circ} 13'$			
Sum	36	54,	half sum	$18^{\circ} 27'$	half interval $7^{\circ} 30'$
Diff.	2	28,	half diff.	1	14
9.115698	sin	$7^{\circ} 30'$			
9.972986	cos	20	0	sin	9.534052
9.088684	sin I	7	3	sect	.003296
					cosect .911030
	II	110	10	cos	9.537348
					.003296
				9.500342	sin $18^{\circ} 27'$ cos 9.977003
				9.999899	cos 1 14 sin 8.332924
				.006103	sect III 9 35 sin 9.221037
	IV	71	8	cos	9.509640
					cos 9.993897
	V	39	2	.....	cos 9.890298
				Lat.	49 59 sin 9.884195

I have introduced the rule in the above form among the young navigators in this Institution; and if what I have said may be the means of drawing the attention of practical men to the subject, my object in writing these observations will be attained.

XXVIII. *On the Relation of Acids and Alkalies to vegetable Colours, and their Mutations thereby.* By JOHN MURRAY, F.L.S. M.G.S. M.W.S. &c. &c.

*To the Editors of the Philosophical Magazine and Journal.*

July 9, 1822.

GENTLEMEN,—YOU had the goodness to insert in a former Number of the Philosophical Magazine (volume lviii. p. 273) a few remarks of mine on the change of vegetable colours by metallic salts; and it is more than a twelvemonth since I have in my prælections pointed out that the mere change of colour produced by acids and alkalis afforded no certain index of their nature. I showed the action of salts of iron, &c. on syrup of violets, &c. in my experiments at the Surry Institution, both in my public discussions and to several individuals in the laboratory of that Institution.

At page 274 of the Philosophical Magazine, vol. lviii. my language, one would think, is emphatic enough. "It seems evident