Crazy Sequential Representations: Simplifications (01)

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Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	(1+2)^(3+4)*5–67+89	(9+8*7*65+4)*3–2*1
10958		(9+8*7*65+4)*3–2+1
10959	12+3+456*(7+8+9)	9+(8*76*(5+4)+3)*2*1
10960	12+(3^4+5+6)*7*(8+9)	9+(8*76*(5+4)+3)*2+1
10961	(1+2+34)*(5*6+7)*8+9	(9+8*7*65+4)*3+2*1
10962	12*3^4*5+678*9	9876+543*2*1

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

Historic Overview

Inder Taneja published five papers on arXiv (for 1 up to 11111):

ARXIV Version	Evaluated Range	Allowed Operations	Missing Increasing	Missing Decreasing	Valid Representations
1 (06-02-2013)[1]	44 to 1000	+ * ^	2	10	1902 (of 1914)
2 (19-03-2013)[2]	44 to 4444	+ * ^	50	53	8699 (of 8802)
3 (05-06-2013)[3]	44 to 11111	+ * ^ ()	590	605	20941 (of 22136)
4 (05-08-2013)[4]	0 to 11111	+ * ^ () -	449	315	21460 (of 22224)
5 (08-01-2014)[5]	0 to 11111	+ * ^ () - /	9	10	22205 (of 22224)

Authors published three papers on Figshare/Zenodo (for -2147483647 up to 2147483647):

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search [6]
14-06-2018	Crazy Sequential Representations: Negative Integers [7]
18-06-2018	Crazy Sequential Representations: Without Subtraction and/or Division [8]

Inder Taneja published three papers on RGMIA (for 11112 up to 30000):

Date	Allowed Operations
12-09-2018	Crazy Representations of Natural Numbers From 11112 to 20000 [9]
10-11-2018	Crazy Representations of Natural Numbers From 20001 to 25000 [10]
10-11-2018	Crazy Representations of Natural Numbers From 25001 to 30000 [11]

Authors published one paper on Figshare/Zenodo (comparing results for 11112 up to 30000):

Date	Allowed Operations
06-12-2018	Crazy Sequential Representations: 11112 up to 30000 [12]

Aim

Simplify the CSR for 11112 up to 2147483647 and the NCSR for -11112 down to -2147483647.

Existing Definitions

Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	(	)

Potential CSR/NCSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

@^ [-+*/^()]* **1** [-+*/^()]* **2** [-+*/^()]* **3** [-+*/^()]* **4** [-+*/^()]* **5** [-+*/^()]* **6** [-+*/^()]* **7** [-+*/^()]* **8** [-+*/^()]* **9** [-+*/^()]* **$**
@^ [-+*/^()]* **9** [-+*/^()]* **8** [-+*/^()]* **7** [-+*/^()]* **6** [-+*/^()]* **5** [-+*/^()]* **4** [-+*/^()]* **3** [-+*/^()]* **2** [-+*/^()]* **1** [-+*/^()]* **$**

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order.
Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9).
Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789).
Negative counterparts of numbers may be used as well (also used by Inder Taneja).

Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Genuine NCSR

Negative integer (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo CSR

Potential non-genuine CSR evaluating to natural number (or zero).
For example, expressions with implicit multiplication by minus one.

Pseudo NCSR

Potential non-genuine CSR evaluating to negative integer (or zero).
For example, expressions with implicit multiplication by minus one.

New Definition

Successful Simplification

> Reduction in length (number of characters in default notation),
> while still being a genuine/pseudo CSR or genuine/pseudo NCSR,
> which results in the same integer as the original CSR/NCSR.

Simplification

Previously authors focused on symbolic simplification, for example:

CSR before symbolic simplification	9+8+7-(-((-6*(-5*-4+3))^2)-1)	19069
Symbolic representation	A+B+C-(-((-D*(-E*-F+G))^H)-I)	-
Symbolic simplification	A+B+C+(-D*(E*F+G))^H+I	-
CSR after symbolic simplification	9+8+7+(-6*(5*4+3))^2+1	19069

However ‘further simplification’ can be achieved, for example:

CSR before ‘further simplification’	9+8+7+(-6*(5*4+3))^2+1	19069
CSR after ‘further simplification’	9+8+7+(6*(5*4+3))^2+1	19069

In other words, while `(-D*(E*F+G))^H` and `(D*(E*F+G))^H` are obviously different,
the final expressions `(-6*(5*4+3))^2` and `(6*(5*4+3))^2` both evaluate to 19044.

Methods

CSR and NCSR were extracted from the following supplements:

Date	Title	
12-06-2018	Crazy Sequential Representations: Exhaustive Search [6]	
	- Supplement 3 : Increasing CSR for 11112 up to 2147483647	n= 828692
	- Supplement 4 : Decreasing CSR for 11112 up to 2147483647	n= 1153402
14-06-2018	Crazy Sequential Representations: Negative Integers [7]	
	- Supplement 3 : Increasing NCSR for -11112 down to -2147483647	n = 845058
	- Supplement 4 : Decreasing NCSR for -11112 down to -2147483647	n = 1201485

Simplification was attempted. Successfully simplified CSR and NCSR were tabulated.
Authors do not guaranty tabulated CSR and NCSR are in their simplest form.

Results

In total 2806762 out of 4028637 expressions were successfully simplified:

	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
Previously Published	828692	1153402	845058	1201485
Successfully Simplified	610647	806459	600985	788671

Resulting in significant reduction of length:

Length Reduction	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
1	12017	12496	10692	9723
2	80114	109358	77515	102561
3	30409	25812	30926	25829
4	26310	42702	24518	41593
5	9372	10779	10158	12218
6	4453	8887	4353	9173
7	2815	3716	2841	4639
8	540	1232	609	1341
9	393	678	538	839
10	91	89	65	142
11	15	45	45	62
12	4	6	14	12
13	2	3	2	2
14	1	1	0	2

For example, for two CSR and two NCSR, length was reduced by 14 characters:

	Previously Published	Successfully Simplified
-732421853	-9-(8+(-7-(6-(5^-(-4*3))))*(2+1))	98-76-5^(4*3)*(2+1)
-3644901	-9*-(-(-8+(-7-(-((-6*-5)^4)-3)))/2)+1)	9*(8+(7-(6*5)^4-3)/2+1)
77824815	-(-1/(-2*-3)+(-4-5)/(-6*-(7^-8)))*9	1/2*3+(-4-5)/6*-7^8*9
2382138	-9*-((-8-(-(7^6)+5))/(-4*-(3^-2))+1)	9*((8-7^6+5)/4*-3^2+1)

Successfully simplified CSR and NCSR were tabulated in the following supplements:

- Supplement 1 : Simplified increasing CSR within 11112 up to 2147483647 range
- Supplement 2 : Simplified decreasing CSR within 11112 up to 2147483647 range
- Supplement 3 : Simplified increasing NCSR within -11112 down to -2147483647 range
- Supplement 4 : Simplified decreasing NCSR within -11112 down to -2147483647 range

Discussion

Authors prefer genuine CSR/NCSR (without negation) over pseudo CSR/NCSR (with negation). Surprisingly, 752752 pseudo CSR/NCSR became genuine CSR/NCSR after simplification:

Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
166536	215804	162276	208136

Typically by getting rid of things like `/-(`, `^-(`, `*-(`, and `(-(`.

Examples where `/-(` was removed:

		Previously Published (Pseudo)	Simplified (Genuine)
Increasing CSR	**63439**	12^3-4/-(5^-6)-789	12^3-4/-5^-6-789
Decreasing CSR	**49044**	-9*(8-76*-5*43)/-(2+1)	9*(8-76*-5*43)/(2+1)
Increasing NCSR	**-115173**	(-12^3+4+5)*67/-(8-9)	(12^3-4-5)*67/(8-9)
Decreasing NCSR	**-41186**	9+8-7*-654/-(3^-2)-1	9+8-7*654/3^-2-1

Examples where `^-(` was removed:

		Previously Published (Pseudo)	Simplified (Genuine)
Increasing NCSR	**40457**	1-2*(-3^-(-4-5)-67*8-9)	1+2*(3^(4+5)+67*8+9)
Decreasing NSR	**50824**	-9*(-87*65+4^-(-3/2))+1	-9*(-87*65+4^(3/2))+1
Increasing CSR	**-73232**	(-1-2^-(-3-4))*567-89	(-1-2^(3+4))*567-89
Decreasing CSR	**-28828**	-9*8-7*(6+5+4^-(-3*2)+1)	-9*8-7*(6+5+4^(3*2)+1)

Examples where `*-(` was removed:

		Previously Published (Pseudo)	Simplified (Genuine)
Increasing NCSR	**13810**	-123*-(-4*-5*6-7)-89	123*(4*5*6-7)-89
Decreasing NSR	**20291**	-9+8+76*-(54-321)	-9+8-76*(54-321)
Increasing CSR	**-13965**	1+2+3*-(4567+89)	1+2-3*(4567+89)
Decreasing CSR	**-22215**	-9*-(-876+54)*3-21	9*(-876+54)*3-21

Examples where `(-(` was removed:

		Previously Published (Pseudo)	Simplified (Genuine)
Increasing NCSR	**19300**	-12+34*-(-(567-8)-9)	-12+34*((567-8)+9)
Decreasing NSR	**27596**	9-8-7*-6*-(-(5^4)-32)+1	9-8-7*6*(-5^4-32)+1
Increasing CSR	**-29144**	1-(-(-23-4*5)*678-9)	1-((23+4*5)*678-9)
Decreasing CSR	**-41102**	-9*(-(8-7*654)-3)+2-1	9*((8-7*654)+3)+2-1

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