It certainly seems best that (*b*) and (*c*), like (*a*), should have affixes above the line, and that (*d*), (*e*), and (*f*), should have affixes below the line. The most suggestive notation would be

$$(a)\ \overset{(r)}{x},\quad (b)\ {}^{(r)}n,\quad (c)\ n^{(r)},$$
$$(d)\ \underset{(r)}{x},\quad (e)\ {}_{(r)}n,\quad (f)\ n_{(r)},$$

but this of course is impossible, and (*b*) and (*e*) would be open to the objection mentioned in the second paragraph above.

On the whole, since (*e*) is, after (*a*), the most important, I would suggest alternative symbols for it, with corresponding alternatives for (*f*). The following seem best :

$$(a)\ x^r \equiv x\,.\,x\ldots\{r\},\quad (b)\ n^{(r)} \equiv n(n-1)\ldots\{r\},\quad (c)\ n^{[r]} = n(n+1)\ldots\{r\},$$

$$(d)\ x_r \equiv \frac{x\,.\,x\ldots\{r\}}{1\,.\,2\ldots\{r\}},\quad (e)\ n_{(r)} \equiv \binom{n}{r} \equiv \frac{n(n-1)\ldots\{r\}}{1\,.\,2\ldots\{r\}},\quad (f)\ n_{[r]} \equiv \begin{bmatrix} n \\ r \end{bmatrix} \equiv \frac{n(n+1)\ldots\{r\}}{1\,.\,2\ldots\{r\}}.$$

Thus the formulae (1)-(6) above become

$$(1)\ (x+y)^n = x^n + \binom{n}{1}x^{n-1}y^1 + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots,$$

$$(2)\ f(x+y) = f(x) + y_1\,.\,D_x f(x) + \ldots + y_r\,.\,D_x{}^r f(x) + \ldots,$$

$$(3)\ (x+y)_n = x_n + x_{n-1}y_1 + \ldots + x_{n-r}y_r + \ldots,$$

$$(4)\ (x+y)^{(n)} = x^{(n)} + \binom{n}{1}x^{(n-1)}y^{(1)} + \ldots + \binom{n}{r}x^{(n-r)}y^{(r)} + \ldots,$$

$$(5)\ f(x+y) = f(x) + \binom{y}{1}\Delta_x f(x) + \ldots + \binom{y}{r}\Delta_x{}^r f(x) + \ldots,$$

$$(6)\ (x+y)_{(n)} = x_{(n)} + x_{(n-1)}y_{(1)} + \ldots + x_{(n-r)}y_{(r)} + \ldots,$$

or

$$\binom{x+y}{n} = \binom{x}{n} + \binom{x}{n-1}\binom{y}{1} + \ldots + \binom{x}{n-r}\binom{y}{r} + \ldots,$$

with corresponding formulae in place of (7) and (8). W. F. SHEPPARD.

279. [K. 21.]. *Two approximate geometrical constructions for inscribing a Nonagon in a circle.*

1. Col. Weldon's Construction :

Let $\alpha\beta\gamma$ be the inscribed circle, centre I, of the equilateral triangle ABC cutting IA, IB, IC at α, β, γ respectively. With centre β and radius $\beta\gamma$ describe a circle cutting the circle whose centre is A and radius AB, at X. Then BX is a side of the nonagon.

2. Mr. J. Houghton Spencer's Construction :

Let PQ, QR be two sides of a hexagon escribed to a circle, touching the circle at B and C. Bisect QC at a. Join Pa cutting the circle at X. Then BX is a side of the nonagon.

Error. The angle $BAX = 40^\circ\ 5'\ 57{\cdot}6''$ showing an error of approximately $6'$ equivalent to $\frac{1}{30}$ part of an inch on an arc of 20 inch radius.

The remarkable part of the two constructions given above, and discovered independently, is that they are in reality identical as is seen from the following geometrical proof.

Let QR be one side of an escribed hexagon touching $\odot$ at C.

Bisect QC at a. Join Pa.

It is required to prove that Pa passes through X, the point where the arc centre β and radius $\beta\gamma$ cuts the circle BXC, Y being the other point of intersection of these two circles.

Produce Pa to cut AC at N.

Produce $A\beta$ to cut Pa at M.

XY is the radical axis of the two circles $Y\gamma X$, BXC and $\therefore$ passes through a, since aC, aQ are tangents to these two $\odot^s$ and equal.

If $AB=2d$, then $\beta E=\frac{2d}{\sqrt{3}}$, $AE=d$. Hence $\frac{\beta E}{EA}=\frac{2}{\sqrt{3}}$.

$$\left.\begin{array}{l} CQ \text{ and } AP \text{ are parallel and } Ca=\tfrac{1}{4}AP=\frac{d}{\sqrt{3}} \\ \text{and } CN=\tfrac{1}{4}AN=\tfrac{1}{3}AC=\frac{2d}{3} \end{array}\right\};$$

$$\therefore \frac{aC}{CN}=\frac{\sqrt{3}}{2}=\frac{EA}{E\beta}.$$

Hence angle $\beta AE=$angle $CaN=$angle QaP.

Hence angle $QPM=$angle BAM.

Hence $BPAM$ are concyclic.

$\therefore$ Angle $PMA=$angle $PBA=$a right angle.

$\therefore$ $A\beta$ is perpendicular to Pa.

But $A\beta$ (line of centres) is perpendicular to radical axis YXa,

$\therefore$ YXa and Pa coincide,

i.e., Pa passes through X. S. DE J. LENFESTEY.

280. [R. 6. a. β.] *On the Kinetic Measure of a Force.*

That the force on a particle is proportional to the time-rate of change of its momentum is, of course, an article of faith; but that the same statement may be applied to a body without any reservation as to what is meant by this term appears to be open to discussion.

To take an example: Suppose a railway train, moving uniformly, to enter a tunnel; the part within the tunnel at any instant is clearly a "body" and is acquiring momentum so long as the train is only partly within the tunnel. But we do not say that this fact points to the existence of unbalanced force on this portion of the train, for the change of momentum arises here from a flux of mass into the tunnel, and our "body" does not consist from time to time of the same particles.

Let μ be the mass per unit length of the train, v its velocity. In the time dt the momentum brought into the tunnel is clearly $\mu v dt\times v$. We must then subtract μv^2 from the rate at which momentum is accumulating in the "body" to arrive at the true kinetic measure of the force upon it. Similarly, if we fix attention on the part left within the tunnel as the train is emerging we must add μv^2 to the rate of change of its momentum in order to form a just estimate of the resultant force upon it; which force should be zero in both cases, as the motion is uniform. (Cf. Lamb's *Motion of Fluids*, 1st ed., § 12.)

The problem of snow-sliding down a roof, alluded to by Prof. G. H. Bryan in our May number, is, in my opinion, incorrectly solved in one of our text-books. No friction or adhesion are allowed for at all, and yet, wonderful to say, the acceleration of the portion left on the roof is only $\frac{1}{3}$ that of a freely-sliding particle!