



# XXIX. Aepinus atomized

Lord Kelvin

To cite this article: Lord Kelvin (1902) XXIX. Aepinus atomized , Philosophical Magazine Series 6, 3:15, 257-283, DOI: [10.1080/14786440209462764](https://doi.org/10.1080/14786440209462764)

To link to this article: <http://dx.doi.org/10.1080/14786440209462764>



Published online: 08 Jun 2010.



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THE  
LONDON, EDINBURGH, AND DUBLIN  
PHILOSOPHICAL MAGAZINE  
AND  
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

MARCH 1902.

XXIX. *Aepinus Atomized.* By Lord KELVIN\*.

§ 1. **A**CCORDING to the well-known doctrine of Aepinus, commonly referred to as the one-fluid theory of electricity, positive and negative electrifications consist in excess above, and deficiency below, a natural quantum of a fluid, called the electric fluid, permeating among the atoms of ponderable matter. Portions of matter void of the electric fluid repel one another; portions of the electric fluid repel one another; portions of the electric fluid and of void matter attract one another.

§ 2. My suggestion is that the Aepinus' fluid consists of exceedingly minute equal and similar atoms, which I call electrions †, much smaller than the atoms of ponderable matter; and that they permeate freely through the spaces occupied by these greater atoms and also freely through space not occupied

\* Communicated by the Author. From the Jubilee Volume presented to Prof. Bosscha in November 1901.

† I ventured to suggest this name in a short article published in 'Nature,' May 27, 1897, in which, after a slight reference to an old idea of a "one-fluid theory of electricity" with *resinous electricity as the electric fluid*, the following expression of my views at that time occurs:—"I prefer to consider an atomic theory of electricity foreseen as worthy of thought by Faraday and Clerk Maxwell, very definitely proposed by Helmholtz in his last lecture to the Royal Institution, and largely accepted by present-day theoretical workers and teachers. Indeed, Faraday's law of electrochemical equivalence seems to necessitate something atomic in electricity, and to justify Johnstone Stoney's word *electron*. The older, and at present even more popular, name *ion* given sixty years ago by Faraday, suggests a convenient modification of it, *electrion*, to denote an atom of resinous electricity. And now, adopting the

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*Phil. Mag.* S. 6. Vol. 3. No. 15. March 1902.

by them. As in Aepinus' theory we must have repulsions between the electrions; and repulsions between the atoms independently of the electrions: and attractions between electrions and atoms without electrions. For brevity, in future by atom I shall mean an atom of ponderable matter, whether it has any electrions within it or not.

§ 3. In virtue of the discovery and experimental proof by Cavendish and Coulomb of the law of inverse square of distance for both electric attractions and repulsions, we may now suppose that the atoms, which I assume to be all of them spherical, repel other atoms outside them with forces inversely as the squares of distances between centres; and that the same is true of electrions, which no doubt occupy finite spaces, although at present we are dealing with them as if they were mere mathematical points, endowed with the property of electric attraction and repulsion. We must now also assume that every atom attracts every electrion outside it with a force inversely as the square of the distance between centres.

§ 4. My assumption that the electrions freely permeate the space occupied by the atoms requires a knowledge of the law of the force experienced by an electrion within an atom. As a tentative hypothesis, I assume for simplicity that the attraction experienced by an electrion approaching an atom varies exactly according to the inverse square of the distance from the centre, as long as the electrion is outside; has no abrupt change when the electrion enters the atom; and decreases to zero simply as the distance from the centre when the electrion, approaching the centre, is within the spherical boundary of the atom. This is just as it would be if the electric virtue of the atom were due to uniform distribution through the atom of an ideal electric substance of which each infinitely small part repels infinitely small portions of

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“essentials of Aepinus' theory, and dealing with it according to the doctrine of Father Boscovich, each atom of ponderable matter is an electron of vitreous electricity; which, with a neutralizing electrion of resinous electricity close to it, produces a resulting force on every distant electron and electrion which varies inversely as the cube of the distance, and is in the direction determined according to the well-known requisite application of the parallelogram of forces.” It will be seen that I had not then thought of the hypothesis suggested in the present communication, that while electrions permeate freely through all space, whether occupied only by ether or occupied also by the volumes of finite spheres constituting the atoms of ponderable matter, each electrion in the interior of an atom of ponderable matter experiences electric force towards the centre of the atom, just as if the atom contained within it, fixed relatively to itself, a uniform distribution of ideal electric matter.

the ideal substance in other atoms, and attracts electrions, according to the inverse square of the distance. But we cannot make the corresponding supposition for the mutual force between two *overlapping* atoms; because we must keep ourselves free to add a repulsion or attraction according to any law of force, that we may find convenient for the explanation of electric, elastic, and chemical properties of matter.

§ 5. The neutralizing quantum of electrions for any atom or group of atoms has exactly the same quantity of electricity of one kind as the atom or group of atoms has of electricity of the opposite kind. The quantum for any single atom may be one or two or three or any integral number, and need not be the same for all atoms. The designations monelectronic, dielectronic, trielectronic, tetraelectronic, polyelectronic, &c., will accordingly be convenient. It is possible that the differences of quality of the atoms of different substances may be partially due to the quantum-numbers of their electrions being different; but it is possible that the differences of quality are to be wholly explained in merely Boscovichian fashion by differences in the laws of force between the atoms, and may not imply any differences in the numbers of electrions constituting their quanta.

§ 6. Another possibility to be kept in view is that the neutralizing quantum for an atom may not be any integral number of electrions. Thus for example the molecule of a diatomic gas, oxygen, or nitrogen, or hydrogen, or chlorine, might conceivably have three electrions or some odd number of electrions for its quantum so that the single atoms, O, N, H, Cl, if they could exist separately, must be either vitreously or resinously electrified and cannot be neutral.

§ 7. The present usage of the designations, positive and negative, for the two modes of electrification originated no doubt with the use of glass globes or cylinders in ordinary electric machines giving vitreous electricity to the insulated prime conductor, and resinous electricity to the not always insulated rubber. Thus Aepinus and his followers regarded the prime conductors of their machines as giving the true electric fluid, and leaving a deficiency of it in the rubbers to be supplied from the earth. It is curious, in Beccaria's account of his observations made about 1760 at Garzegna in Piedmont on atmospheric electricity, to read of "The mild excessive electricity of the air in fair weather." This in modern usage would be called mild positive electricity. The meaning of either expression, stated in non-hypothetical language, is, the mild vitreous electricity of the air in fair weather.

§ 8. In the mathematical theory of electricity in equilibrium, it is a matter of perfect indifference which of the opposite electric manifestations we call positive and which negative. But the great differences in the disruptive and luminous effects, when the forces are too strong for electric equilibrium, presented by the two modes of electrification, which have been known from the earliest times of electric science, show physical properties not touched by the mathematical theory. And Varley's comparatively recent discovery\* of the molecular torrent of resinously electrified particles from the "kathode" or resinous electrode in apparatus for the transmission of electricity through vacuum or highly rarefied air, gives strong reason for believing that the mobile electricity of Aepinus' theory is resinous, and not vitreous as he accidentally made it. I shall therefore assume that our electrions act as extremely minute particles of *resinously* electrified matter; that a void atom acts simply as a little globe of atomic substance, possessing as an essential quality vitreous electricity uniformly distributed through it or through a smaller concentric globe; and that ordinary ponderable matter, not electrified, consists of a vast assemblage of atoms, not void, but having within the portions of space which they occupy just enough of electrions to annul electric force for all places of which the distance from the nearest atom is large in comparison with the diameter of an atom, or molecular cluster of atoms.

§ 9. This condition respecting distance would, because of the inverse square of the distance law for the forces, be unnecessary and the electric force would be rigorously null throughout all space outside the atoms, if every atom had only a single electrion at its centre, provided that the electric quantities of the opposite electricities (reckoned according to the old definition of mathematical electrostatics) are equal in the atom and in the electrion. But even if every neutralized separate atom contains just one electrion in stable equilibrium at its centre, it is obvious that, when two atoms overlap so far that the centre of one of them is within the spherical boundary of the other, the previous equilibrium of the two electrions is upset, and they must find positions of equilibrium elsewhere than at the centres. Thus in fig. 1 each electrion is at the centre of its atom, and is attracted and repelled with equal forces by the neighbouring atom and electrion at *its* centre. In fig. 2, if E and E' were at the centres C, C', of the two atoms, E would be repelled by E' more than it would be

\* Proc. Roy. Soc. vol. xix. pp. 239, 240 (1871).

attracted by the atom  $A'$ . Hence both electrions being supposed free,  $E$  will move to the right; and because of its diminished repulsion on  $E'$ ,  $E'$  will follow it in the same direction. The equations of equilibrium of the two are easily written down, not so easily solved without some slight arithmetical artifice. The solution is correctly shown in fig. 2, for the case in which one radius is three times the

Fig. 1.

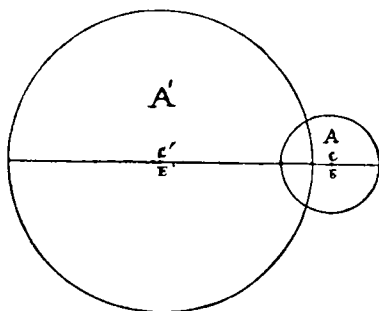
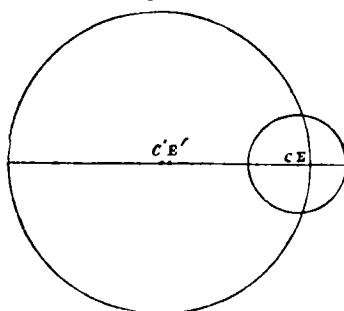


Fig. 2.



Radii 3 and 1.  
 $C'C = 2.7$ .  $C'E' = .1458$ .  $CE = .0462$ .

other, and the distance between the centres is 2.7 times the smaller radius \*. The investigation in the footnote shows that if the atoms are brought a little nearer, the equilibrium becomes unstable; and we may infer that both electrions jump to the right,  $E'$  to settle at a point within the atom  $A$  on the left-hand side of its centre; and  $E$  outside  $A'$ , to settle at a point still within  $A$ . If, lastly, we bring the centres closer and closer together till they coincide,  $E$  comes again within  $A'$ , and the two electrions settle, as shown in fig. 3, at distances on the two sides of the common centre, each equal to

$$\frac{1}{2} \sqrt[3]{\frac{2}{\frac{1}{a^3} + \frac{1}{a'^3}}}$$

\* Calling  $e$  the quantity of electricity, vitreous or resinous, in each atom or electrion;  $\zeta$  the distance between the centres of the atoms;  $a, a'$  the radii of the two atoms;  $x, x'$  the displacements of the electrions from the centres;  $X, X'$  the forces experienced by the electrions; we have

$$X = e^2 \left[ -\frac{x}{a^3} + \frac{1}{(\zeta + x - x')^2} - \frac{\zeta + x}{a'^3} \right];$$

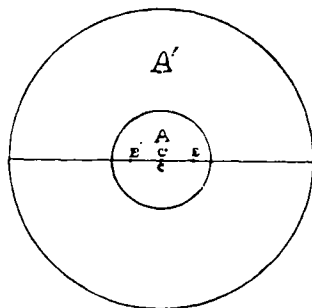
$$X' = e^2 \left[ -\frac{x'}{a'^3} + \frac{1}{(\zeta - x')^2} - \frac{1}{(\zeta + x - x')^2} \right].$$

Each of these being equated to zero for equilibrium gives us two equations

which for the case  $\alpha' = 3\alpha$  is

$$\frac{1}{2}\alpha \sqrt[3]{\frac{2.27}{28}} = .622\alpha.$$

Fig. 3.



$$E'C = CE = .622.$$

§ 10. Mutual action of this kind might probably be presented in such binary combinations as  $O_2$ ,  $N_2$ ,  $H_2$ ,  $Cl_2$ ,  $CO$ ,  $SO$ ,  $NaCl$  (dry common salt) if each single atom,  $O$ ,  $N$ ,  $H$ ,  $Cl$ ,  $C^*$ ,  $S$ ,  $Na^\dagger$ , had just one electron for its neutralizing

which are not easily dealt with by frontal attack for the determination of two unknown quantities  $x, x'$ ; but which may be solved by a method of successive approximations, as follows:—Let  $x_0, x_1, \dots, x_i, x'_0, x'_1, \dots, x'_i$ , be successive approximations to the values of  $x$  and  $x'$ , and take

$$x_{i+1} = \frac{1}{\alpha^3} + \frac{1}{\alpha'^3} \left( \frac{1}{D_i^2} - \frac{\zeta}{\alpha'^3} \right); \quad x'_{i+1} = \alpha'^3 \left\{ \frac{1}{(\zeta - x_i)^2} - \frac{1}{(\zeta + x_{i+1} - x'_i)^2} \right\};$$

where  $D_i^2 = (\zeta + x_i - x'_i)^2$ . As an example, take  $\alpha = 1$ ,  $\alpha' = 3$ . To find solutions for gradual approach between centres, take successively  $\zeta = 2.9, 2.8, 2.7, 2.6$ . Begin with  $x_0 = 0, x'_0 = 0$ , we find  $x_1 = .01243, x'_1 = .0297$ , and the same values for  $x_2$ , and  $x'_2$ . Take next  $\zeta = 2.8, x_0 = .01243, x'_0 = .0297$ ; we find  $x_1 = x_2 = .0269, x'_1 = x'_2 = .0702$ . Thus we have the solution for the second distance between centres. Next take  $\zeta = 2.7, x_0 = .0269, x'_0 = .0702$ ; we find  $x_1 = x_2 = .0462, x'_1 = x'_2 = .1458$ . Working similarly for  $\zeta = 2.6$ , we do not find convergence, and we infer that a position of unstable equilibrium is reached by the electrions for some value of  $\zeta$  between 2.7 and 2.6.

\* The complexity of the hydrocarbons and the Van't Hoff and Le Bel doctrine of the asymmetric results (chirality) produced by the quadrivalence of carbon makes it probable that the carbon atom takes at least four electrons to neutralize it electrically.

† The fact that sodium, solid or liquid, is a metallic conductor of electricity makes it probable that the sodium atom, as all other metallic elements, takes a large number of electrons to neutralize it (see below, § 30).

quantum. If the combination is so close that the centres coincide, the two electrions will rest stably at equal distances on the two sides of the common centre as at the end of § 9. I see at present no reason for considering it excessively improbable that this may be the case for SO, or for any other binary combinations of *two atoms of different quality* for neither of which there is reason to believe that its neutralizing quantum is not exactly one electrion. But for the binary combinations of two atoms of identical quality which the chemists have discovered in diatomic gases (O<sub>2</sub>, N<sub>2</sub>, &c.) there must, over and above the electric repulsion of the two similar electric globes, be a strong atomic repulsion preventing stable equilibrium with coincident centres, however strongly the atoms may be drawn together by the attractions of a pair of mutually repellent electrions within them; because without such a repulsion the two similar atoms would become one, which no possible action in nature could split into two.

§ 11. Returning to fig. 3, let us pull the two atoms gradually asunder from the concentric position to which we had brought them. It is easily seen that the electrions will both remain within the smaller atom A, slightly disturbed from equality of distance on the two sides of its centre by attractions towards the centre of A'; and that when A' is infinitely distant they will settle at distances each equal to  $\frac{1}{2}a\sqrt[3]{2} = \cdot 62996 a$  on the two sides of the centre of A. If, instead of two monelectronic atoms, we deal with two polyelectronic atoms as in § 9, we find after separation the number of electrions in the smaller atom increased and in the larger decreased; and this with much smaller difference of magnitude than the three to one of diameters which we had for our monelectronic atoms of § 9. This is a very remarkable conclusion, pointing to what is probably the true explanation of the first known of the electric properties of matter; attractions and repulsions produced by rubbed amber. Two ideal solids consisting of assemblages of monelectronic atoms of largely different sizes would certainly, when pressed and rubbed together and separated, show the properties of oppositely electrified bodies; and the preponderance of the electronic quality would be in the assemblage of which the atoms are the smaller. Assuming as we do that the electricity of the electrions is of the resinous kind, we say that after pressing and rubbing together and separating the two assemblages, the assemblage of the smaller atoms is resinously electrified and the assemblage of the larger atoms is vitreously electrified. This is probably the

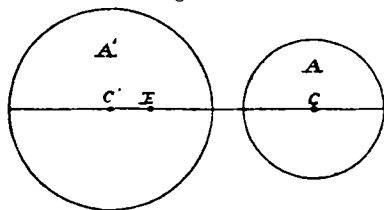
true explanation of the old-known fact that ground glass is resinous relatively to polished glass. The process of polishing might be expected to smooth down the smaller atoms, and to leave the larger atoms more effective in the surface.

§ 12. It probably contains also the principle of the explanation of Erskine Murray's\* experimental discovery that surfaces of metals, well cleaned by rubbing with glass-paper or emery-paper, become more positive or less negative in the Volta contact electricity scale by being burnished with a smooth round hard steel burnisher. Thus a zinc plate brightened by rubbing on glass-paper rose by  $\cdot 23$  volt by repeated burnishing with a hard steel burnisher, and fell again by the same difference when rubbed again with glass-paper. Copper plates showed differences of about the same amount and in the same direction when similarly treated. Between highly burnished zinc and emery-cleaned copper, Murray found a Volta-difference of  $1\cdot 13$  volts, which is, I believe, considerably greater than the greatest previously found Volta-difference between pure metallic surfaces of zinc and copper.

§ 13. To further illustrate the tendency (§ 9) of the smaller atom to take electrions from the larger, consider two atoms :  $A'$ , of radius  $a'$ , the greater, having an electrion in it to begin with ; and  $A$ , radius  $a$ , the smaller, void.

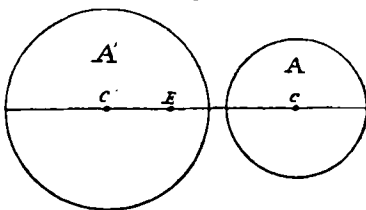
By ideal forces applied to the atoms while the electrion is free let them approach gradually from a very great distance apart. The attraction of  $A$  draws the electrion from the centre of  $A'$ ; at first very slightly, but farther and farther as the distance between the atoms is diminished. What will be

Fig. 4.



$$a' = 1. \quad C'C = 2. \quad C'E = \cdot 38.$$

Fig. 5.



$$C'C = 1\cdot 89. \quad C'E = 0\cdot 63.$$

the position of the electrion when the distance between the centres is, as in fig. 4,  $2a'$ ? Without calculation we see that the electrion would be in equilibrium if placed at the point

\* "On Contact Electricity of Metals," Proc. Roy. Soc. vol. lxiii. p. 113 (1898). See also Lord Kelvin, "Contact Electricity of Metals," Phil. Mag. vol. xlv. pp. 96-98 (1898).

in which the surface of  $A'$  is cut by the line of centres; but the equilibrium would be obviously unstable, and a simple calculation\* shows that the stable position actually taken by the electrion is  $\cdot38a'$  from  $C'$ , when the distance between the centres is  $2a'$  (fig. 4). If the distance between the centres is now diminished from  $2a'$  to  $1\cdot89a'$  ( $a'$  being now supposed to be anything less than  $\cdot89a'$ ) the electrion comes gradually to distance  $\cdot63a'$  from  $C'$  (fig. 5); its equilibrium there becomes unstable; and it jumps out of  $A'$  towards  $A$  (like a cork jumping out of a bottle). It will shoot through  $A$  ( $A'$  and  $A$  being held fixed); and after several oscillations to and fro, perhaps† ten or twenty, if it has only quasi inertia due to condensation or rarefaction‡ produced by it in ether; or perhaps many times more if it has intrinsic inertia of its own; it will settle, with decreasing range of excursions, sensibly to rest within  $A$ , attracted somewhat from the centre by  $A'$ . If, lastly,  $A'$  and  $A$  be drawn asunder to their original great distance, the electrion will not regain its original position in  $A'$ , but will come to the centre of  $A$  and rest there. Here then we have another illustration of the tendency found in § 9, of the smaller atom to take electrions from the larger.

§ 14. In preventing the two atoms from rushing together by holding them against the attractive force of the electrion, we shall have gained more work during the approach than we afterwards spent on the separation; and we have now

\* Denoting by  $\zeta$  the distance between the centres, and by  $X$  the force on  $E$  when its distance from  $C'$  is  $x'$ , we have

$$X = e^2 \left[ \frac{1}{(\zeta - x')^2} - \frac{x'}{a'^3} \right].$$

Hence for equilibrium  $\frac{1}{(\zeta - x')^2} = \frac{x'}{a'^3}$ . This is a cubic for  $x'$  of which the proper root (the smallest root) for the case  $\zeta = 2a'$  is  $\cdot38a'$ . The formula for  $X$  has a minimum value when  $\zeta - x' = a' \sqrt[3]{2}$ , which makes

$$X = \frac{e^2}{a'^2} \left[ \frac{3}{2} \sqrt[3]{2} - \frac{\zeta}{a'} \right].$$

Hence the value of  $x'$  for equilibrium coincides with the value of  $X$ , a minimum, and the equilibrium becomes unstable, when  $\zeta$  is diminished to

$$\frac{3}{2} \sqrt[3]{2} a' = 1\cdot890 a'. \text{ For this, the value of } x' \text{ is } \frac{\sqrt[3]{2}}{2} a' = \cdot63 a'.$$

† "On the Production of Wave Motion in an Elastic Solid," Phil. Mag. Oct. 1899, § 44.

‡ "On the Motion of Ponderable Matter through Space Occupied by Ether," Phil. Mag. Aug. 1900, §§ 15, 17.

left the system deprived of the further amount of energy carried away by ethereal waves into space.

§ 15. The system in its final state with the electrion at the centre of the smaller atom has less potential energy in it than it had at the beginning (when the electrion was at the centre of A'), by a difference equal to the excess of the work which we gained during the approach above that which we spent on the final separation of A' and A, plus the amount carried away by the ethereal waves. All these items except the last are easily calculated from the algebra of the footnote on § 13; and thus we find how much is our loss of energy by the ethereal waves.

§ 16. Very interesting statical problems are presented to us by consideration of the equilibrium of two or more electrions within one atom, whether a polyelectronic atom with its saturating number, or an atom of any electric strength with any number of electrions up to the greatest number that it can hold. To help to clear our ideas, first remark that if the number of electrions is infinite, that is to say if we go back to Aepinus' electric fluid, but assume it to permeate freely through an atom of any shape whatever and having any arbitrarily given distribution of electricity of the opposite kind fixed within it, the greatest quantity of fluid which it can take is exactly equal to its own, and lodges with density equal to its own in every part. Hence if the atom is spherical, and of equal electric density throughout as we have supposed it, and if its neutralizing quantum of electrions is a very large number, their configuration of equilibrium will be an assemblage of more and more nearly uniform density from surface to centre, the greater the number. Any Bravais homogeneous assemblage whatever would be very nearly in equilibrium if all the electrions in a surface-layer of thickness a hundred times the shortest distance from electrion to electrion were held fixed; but the equilibrium would be unstable except in certain cases. It may seem probable that it is stable if the homogeneous assemblage is of the species which I have called\* equilateral, being that in which each electrion with any two of its twelve next neighbours forms an equilateral triangle. If now all the electrions in the surface-layer are left perfectly free, a slight rearrangement among themselves and still slighter among the neighbouring electrions in the interior will bring the whole multitude (of thousands or millions) to equilibrium. The subject is of

\* "Molecular Tactics of a Crystal," § 4, being the Second Robert Boyle Lecture, delivered before the Oxford University Junior Scientific Club, May 16, 1893 (Clarendon Press, Oxford).

extreme interest, geometrical, dynamical, and physical, but cannot be pursued further at present.

§ 17. To guide our ideas respecting the stable equilibrium of moderate numbers of electrions within an atom, remark first that for any number of electrions there may be equilibrium with all the electrions on one spherical surface concentric with the atom. To prove this, discard for a moment the atom and imagine the electrions, whatever their number, to be attached to ends of equal inextensible strings of which the other ends are fixed to one point C. Every string will be stretched in virtue of the mutual repulsions of the electrions; and there will be a configuration or configurations of equilibrium with the electrions on a spherical surface. Whatever their number there is essentially at least one configuration of stable equilibrium. Remark also that there is always a configuration of equilibrium in which all the strings are in one plane, and the electrions are equally spaced round one great circle of the sphere. This is the sole configuration for two electrions or for three electrions; but for any number exceeding three it is easily proved to be unstable, and is therefore not the sole configuration of equilibrium. For four electrions it is easily seen that, besides the unstable equilibrium in one plane, there is only the stable configuration, and in this the four electrions are at the four corners of an equilateral tetrahedron.

§ 18. For five electrions we have clearly stable equilibrium with three of them in one plane through C, and the other two at the ends of the diameter perpendicular to this plane. There is also at least one other configuration of equilibrium: this we see by imagining four of the electrions constrained to remain in a freely movable plane, which gives stable equilibrium with this plane at some distance from the centre and the fifth electrion at the far end of the diameter perpendicular to it. And similarly for any greater number of electrions, we find a configuration of equilibrium by imagining all but one of them to be constrained to remain in a freely movable plane. But it is not easy, without calculation, to see, at all events for the case of only five electrions, whether that equilibrium would be stable if the constraint of all of them but one to one plane is annulled. For numbers greater than five it seems certain that that equilibrium is unstable.

§ 19. For six we have a configuration of stable equilibrium with the electrions at the six corners of a regular octahedron; for eight at the corners of a cube. For ten, as for any even number, we should have two configurations of equilibrium (both certainly unstable for large numbers) with two halves

of the number in two planes at equal distances on the two sides of the centre. For twelve we have a configuration of stable equilibrium with the electrions at positions of the twelve nearest neighbours to C in an equilateral homogeneous assemblage of points\*; for twenty at the twenty corners of a pentagonal dodecahedron. All these configurations of § 19, except those described for ten electrions, are stable if, as we are now supposing, the electrions are constrained to a spherical surface on which they are free to move.

§ 20. Except the cases of § 18, the forces with which the strings are stretched are the same for all the electrions of each case. Hence if we now discard the strings and place the electrions in an atom on a spherical surface concentric with it, its attraction on the electrions towards the centre takes the place of the tension of the string, provided it is of the proper amount. But it does not secure, as did the strings, against instability relatively to radial displacements, different for the different electrions. To secure the proper

amount of the radial force the condition is  $\frac{ie^2r}{\alpha^3} = T$ ; where

$i$  denotes the number of electrions;  $e$  the electric quantity on each (and therefore, § 8,  $ie$  the electric quantity of vitreous electricity in the atom);  $r$  denotes the radius of the spherical surface on which the electrions lie;  $\alpha$  the radius of the atom; and  $T$  the tension of the string in the arrangement of § 17.

We have generally  $T = q \frac{e^2}{r^2}$  where  $q$  is a numeric depending on the number and configuration of the electrions found in each case by geometry. Hence we have  $\frac{r}{\alpha} = \sqrt[3]{\frac{q}{i}}$  for the ratio of the radius of the smaller sphere on which the electrions lie to the radius of the atom. For example, take the case of eight electrions at the eight corners of a cube.  $T$  is the resultant of seven repulsions, and we easily find

$$q = \frac{3}{4} \left( \sqrt{3} + \sqrt{\frac{3}{2}} + \frac{1}{3} \right) \text{ and finally } \frac{r}{\alpha} = .6756.$$

Dealing similarly with the cases of two, three, four, and six electrions, we have the following table of values of  $\left(\frac{r}{\alpha}\right)^3$  and  $\frac{r}{\alpha}$ ; to which is added a last column showing values of

\* "Molecular Tactics of a Crystal," § 4.

$$v^2 \frac{3\alpha^2 - r^2}{2\alpha^2} - \sum \frac{\alpha}{D},$$

being  $\frac{\alpha}{e^2}$  of the work required to remove the electrions to infinite distance.

Number of Electrions.	Configuration.	$\left(\frac{r}{a}\right)^3$ .	$\frac{r}{a}$ .	$\frac{\alpha}{e^2} \times$ work required to remove the electrions to infinite dis- tance, = $w$ .
2	At the ends of a diameter	$\frac{1}{8}$	5000	4.500
3	At the corners of an equi- lateral triangle	$\frac{1}{3\sqrt{3}}$	5774	9.000
4	At the corners of a square	$\frac{\sqrt{2}}{8} + \frac{1}{16}$	6208	14.750
4	At the corners of an equi- lateral tetrahedron	$\frac{3}{16} \sqrt{\frac{3}{2}}$	6124	15.000
6	At the corners of an equi- lateral octahedron	$\frac{1+4\sqrt{2}}{24}$	6522	33.335
8	At the corners of a cube	$\frac{3}{32} \left( \sqrt{3} + \sqrt{\frac{3}{2}} + \frac{1}{3} \right)$	6756	52.180

§ 21. In the configurations thus expressed the equilibrium is certainly stable for the cases of two, three, and four electrions. It seems to me, without calculation, also probably stable for the case of six, and possibly even for the case of eight. For the case of twenty at the corners of a pentagonal dodecahedron the equilibrium is probably not stable; and even for the cases of twelve electrions and ten electrons, the equilibrium in the configurations described in §§ 18, 19 may probably be unstable, when, as now, we have the attraction of the atom towards the centre instead of the inextensible strings.

§ 22. In fact when the number of electrions exceeds four, we must think of the tendency to be crowded out of one spherical surface, which with very large numbers gives a tendency to uniform distribution throughout the volume of the atom as described in § 16 above. Thus, in the case of five electrions, § 18 shows a configuration of equilibrium in which the two electrions lying in one diameter are, by the mutual repulsions, pushed very slightly further from the

centre than are the three in the equatorial plane. In this case the equilibrium is clearly stable. Another obvious configuration, also stable, of five electrions within an atom is one at the centre, and four on a concentric spherical surface at the corners, of a tetrahedron. From any case of any number of electrions all on one spherical surface, we may pass to another configuration with one more electrion placed at the centre and the proper proportionate increase in the electric strength of the atom. Thus from the cases described in § 19, we may pass to configurations of equilibrium for seven, nine, eleven, thirteen, and twenty-one electrions. All these cases, with questions of stability or instability and of the different amounts of work required to pluck all the electrions out of the atom and remove them to infinite distances, present most interesting subjects for not difficult mathematical work; and I regret not being able to pursue them at present.

§ 23. Consider now the electric properties of a real body, gaseous, liquid, or solid, constituted by an assemblage of atoms with their electrions. It follows immediately from our hypothesis, that in a monatomic gas or in any sufficiently sparse assemblage of single atoms, fixed or moving, Faraday's "*conducting power for lines of electric force,*" or what is now commonly called the *specific electro-inductive capacity*, or the *electro-inductive permeability*, exceeds unity by three times the ratio of the sum of the volumes of the atoms to the whole volume of space occupied by the assemblage, whether the atoms be monelectronic or polyelectronic, and however much the electrion, or group of electrions, within each atom is set to vibrate or rotate with each collision, according to the kinetic theory of gases. To prove this, consider, in a uniform field of electrostatic force of intensity  $F$ , a single atom of radius  $\alpha$ , and, at rest within it, a group of  $i$  electrions in stable equilibrium. The action of  $F$  produces simply displacements of the electrions relatively to the atom, equal and in parallel lines, with therefore no change of shape and no rotation; and,  $x$  denoting the amount of this displacement, the equation

for the equilibrium of each electrion is  $\frac{ie x}{\alpha^3} = F$ . This gives

$ie x = \alpha^3 F$  for the electric moment of the electrostatic polarization induced in the atom by  $F$ . In passing, remark that  $\alpha^3 F$  is also equal to the electric moment of the polarization produced in an insulated unelectrified metal globe of radius  $\alpha$ , when brought into an electrostatic field of intensity  $F$ ; and conclude that the electric inductive capacity of a uniformly dense assemblage of fixed metallic globules, so sparse that

their mutual influence is negligible, is the same as that of an equal and similar assemblage of our hypothetical atoms, whatever be the number of electrions in each, not necessarily the same in all. Hence our hypothetical atom realizes perfectly for sparse assemblages Faraday's suggestion of "small globular conductors, as shot" to explain the electro-polarization which he discovered in solid and liquid insulators. (Experimental Researches, § 1679.)

§ 24. Denoting now by  $N$  the number of atoms per unit volume we find  $NV\alpha^3F$  as the electric moment of any sparse enough assemblage of uniform density occupying volume  $V$  in a uniform electric field of intensity  $F$ . Hence  $N\alpha^3$  is what (following the analogy of electromagnetic nomenclature) we may call the electro-inductive susceptibility\* of the assemblage; being the electric moment per unit bulk induced by an electric field of unit intensity. Denoting this by  $\mu$ , and the electro-inductive permeability by  $\omega$ , we have (Electrostatics and Magnetism, § 629 (14))

$$\omega = 1 + 4\pi\mu = 1 + 3 \left( N \frac{4\pi\alpha^3}{3} \right)$$

which proves the proposition stated at the commencement of § 23.

§ 25. To include vibrating and rotating groups of electrions in the demonstration, it is only necessary to remark that the time-average of any component of the displacement of the centre of inertia of the group relatively to the centre of the atom will, under the influence of  $F$ , be the same as if the assemblage were at rest in stable equilibrium.

§ 26. The consideration of liquids consisting of closely packed mobile assemblages of atoms or groups of atoms with their electrions, forming compound molecules, as in liquid argon or helium (monatomic), nitrogen, oxygen, &c. (diatomic), or pure water, or water with salts or other chemical substances dissolved in it, or liquids of various complex chemical constitutions, cannot be entered on in the present communication, further than to remark that the suppositions we have made regarding forces, electric and other, between electrions and atoms, seem to open the way to a very definite detailed dynamics of electrolysis, of chemical affinity, and of heat of chemical combination. Estimates of the actual magnitudes concerned (the number of molecules per cubic centimetre of a gas, the mass in grammes of an atom of any substance, the diameters of the atoms, the absolute value of the electric quantity in an electrion, the effective mass or

\* Suggested in my 'Electrostatics and Magnetism,' §§ 628, 629.

inertia of an electrion) seem to show that the intermolecular electric forces are more than amply great enough to account for heat of chemical combination, and every mechanical action manifested in chemical interactions of all kinds. We might be tempted to assume that all chemical action is electric, and that all varieties of chemical substance are to be explained by the numbers of the electrions required to neutralize an atom or a set of atoms (§ 6 above); but we can feel no satisfaction in this idea when we consider the great and wild variety of quality and affinities manifested by the different substances or the different "chemical elements"; and as we are assuming the electrions to be all alike, we must fall back on Father Boscovich, and require him to explain the difference of quality of different chemical substances by different laws of force between the different atoms.

§ 27. Consider lastly a solid; that is to say, an assemblage in which the atoms have no relative motions, except through ranges small in comparison with the shortest distances between their centres\*. The first thing that we remark is that every solid would, at zero of absolute temperature (that is to say all its atoms and electrions at rest), be a perfect insulator of electricity under the influence of electric forces, moderate enough not to pluck electrions out of the atoms in which they rest stably when there is no disturbing force. The limiting value of  $F$  here indicated for perfect insulation, I shall for brevity call the disruptional force or disruptional intensity. It is clear that this disruptional force is smaller the greater the number of electrions within an atom.

§ 28. The electro-inductive permeability of a solid at zero temperature is calculable by the static dynamics of § 24, modified by taking into account forces on the electrions of one atom due to the attractions of neighbouring atoms and the repulsion of their electrions. Without much calculation it is easy to see that generally the excess of the electro-inductive permeability above unity will be much greater than three times the sum of the volumes of the electric atoms per unit volume of space, which we found in § 24 for the electro-inductive permeability of an assemblage of single atoms, sparse enough to produce no disturbance by mutual actions. Also without much calculation, it is easy to see that now the induced electric moment will not be in simple proportion to

\* I need scarcely say that it is only for simplicity in the text that we conveniently ignore Roberts-Austen's admirable discovery of the inter-diffusion of solid gold and solid lead, found after a piece of one metal is allowed to rest on a piece of the other for several weeks, months, or years.

F, the intensity of the electric field, as it was rigorously for a single atom through the whole range up to the disruptional value of F; but will tend to increase more than in simple proportion to the value of F; though for small practical values of F the law of simple proportion is still very nearly fulfilled.

§ 29. Raise the temperature now to anything under that at which the solid would melt. This sets the electrions to performing wildly irregular vibrations and rotations, so that some of them will occasionally be shot out of their atoms. Each electrion thus shot out will quickly either fall back into the atom from which it has been ejected, or will find its way into another atom. If the body be in an electric field F, a considerable proportion of the electrions which are shot out will find their way into other atoms in the direction in which they are pulled by F; that is to say, the body which was an infinitely perfect insulator at zero absolute temperature has now some degree of electric conductivity, which is greater the higher the temperature. There can be no doubt that this is a matter-of-fact explanation of the electric conductivity, which so nearly perfect an insulator as the flint glass of my quadrant electrometer at atmospheric temperature shows, when heated to far below its melting-point, (according to Prof. T. Gray \*  $\cdot 98 \cdot 10^{-24}$  at  $60^{\circ}$  cent.;  $4 \cdot 9 \cdot 10^{-24}$  at  $100^{\circ}$ ;  $8300 \cdot 10^{-24}$  at  $200^{\circ}$  cent.); and of the enormous increase of electric conductivity of rare earths at rising temperatures above  $800^{\circ}$  C., so admirably taken advantage of by Professor Nernst in his now celebrated electric lamp.

§ 30. If the hypotheses suggested in the present communication are true, the electric conductivity of metals must be explained in the same way as that of glass, guttapercha, vulcanite, Nernst filament, &c., with only this difference, that the metallic atom must be so crowded with electrions that some of them are always being spilt out of each atom by the intermolecular and electronic thermal motions, not only at ordinary atmospheric temperatures, and higher, but even at temperatures of less than  $16^{\circ}$  centigrade above the absolute zero of temperature. I say  $16^{\circ}$  because in Dewar's Bakerian Lecture to the Royal Society of London, June 13, 1901, "The Nadir of Temperature," we find that platinum, gold, silver, copper, and iron have exceedingly high electric conductivity at the temperature of liquid hydrogen boiling under 30 mms. of mercury, which must be something between  $20^{\circ} \cdot 5$ , the boiling-point of hydrogen at 760 mms. pressure,

\* Proc. Roy. Soc. Jan. 12, 1882.

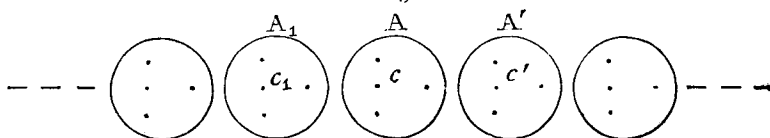
and  $16^{\circ}$ , the temperature of melting solid hydrogen, both determined by Dewar with his helium thermometer. There is no difficulty in believing that the electrons in each of the metallic atoms are so numerous that, though they rest in stable equilibrium within the atoms, closely packed to constitute the solid metal at  $0^{\circ}$  absolute, and may move about within the atom with their wildly irregular thermal motions at  $1^{\circ}$  of absolute temperature, they may between  $1^{\circ}$  and  $2^{\circ}$  begin to spill from atom to atom. Thus, like glass or a Nernst filament below  $300^{\circ}$  absolute, a metal may be an almost perfect insulator of electricity below  $1^{\circ}$  absolute: may, like glass at  $333^{\circ}$  absolute, show very notable conductivity at  $2^{\circ}$  absolute: and, like glass at  $473^{\circ}$  absolute as compared with glass at  $333^{\circ}$  absolute, may show 8000 times as much electric conductivity at  $2^{\circ}8$  as at  $2^{\circ}$ . And, like the Nernst filament at  $1800^{\circ}$  or  $2000^{\circ}$  absolute, our hypothetical metal may at  $6^{\circ}$  absolute show high conductivity, comparable with that of lead or copper at ordinary temperatures. The electric conductivity in the Nernst filament goes on increasing as the temperature rises till the filament melts or evaporates. Nevertheless it is quite conceivable that in our hypothetical metal with rising temperature from  $2^{\circ}$  to  $16^{\circ}$  absolute the electric conductivity may come to a maximum and decrease with further rise of temperature up to and beyond ordinary atmospheric temperatures. In fact, while some extent of thermal motions is necessary for electric conductivity, too much of these motions must mar the freedom with which an electron can thread its way through the crowd of atoms to perform the function of electric conduction. It seems certain that this is the matter-of-fact explanation of the diminution of electric conductivity in metals with rise of temperature.

§ 31. Regretting much not to be able (for want of time) to include estimates of absolute magnitudes in the present communication, I end it with applications of our hypothesis to the pyro-electricity and piezo-electricity of crystals. A crystal is a homogeneous assemblage of bodies. Conversely, a homogeneous assemblage of bodies is not a crystal if the distance between centres of nearest neighbours is a centimetre or more; it is a crystal if the distance between nearest neighbours is  $10^{-8}$  of a cm. or less. Pyro-electricity and piezo-electricity are developments of vitreous and resinous electric forces such as would result from vitreous and resinous electrification on different parts of the surface of a crystal, produced respectively by change of temperature and by stress due to balancing forces applied to the surfaces.

§ 32. To see how such properties can or must exist in

crystals composed of our hypothetical atoms with electrions, consider first merely a row of equal tetraelectric atoms in a straight line, each having its quantum of four electrions within it. Fig. 6 shows a configuration of stable equilibrium of the electrions not, however, truly to scale. The sets of

Fig. 6.



three dots indicate trios of electrions at the corners of equilateral triangles, the middle dot in each row being alternately on the far side and the near side of the plane of the paper, which contains the centres of the atoms and the remaining electrion of each four. Let  $C_1$ ,  $C$ ,  $C'$ , be the centres of the atom  $A_1$ ,  $A$ ,  $A'$ . An easy calculation shows that the quartet of electrions within  $A$ , regarded for the moment as a group of four material points rigidly connected, is attracted to the left with a less force by  $A_1$  than to the right by  $A'$  (in making the calculation remember that  $A_1$  attracts all the electrions within  $A$  as if it were a quantity  $e$  of vitreous electricity collected at  $C_1$ , and similarly in respect to  $A'$ ). There are corresponding smaller differences between the opposite attractions of the more and more remote atoms on the two sides of  $A$ . Let  $\delta$  denote the excess of the sum of the rightwards of these attractions above the leftwards. The geometrical centre of the electrions within  $A$  is displaced rightwards to a distance,  $l$ , from  $C$  equal to  $\frac{a^3\delta}{16e^2}$ .

§ 33. Imagine now a crystal or a solid of any shape built up of parallel rows of atoms such as those of § 32. The amount of the displacing force on each quartet of electrions will be somewhat altered by mutual action between the rows, but the general character of the result will be the same; and we see that throughout the solid, except in a thin superficial layer of perhaps five or ten atoms deep, the whole interior is in a state of homogeneous electric polarization, of which the electric moment per unit of volume is  $4eNl$ ; where  $N$  is the number of atoms per unit volume, and  $l$  is the displacement of the geometrical centre of each quartet from the centre of its atom. This is the interior molecular condition of a di-polar pyro-electric crystal, which I described in 1860\* as probably accounting for their known pyro-electric quality,

\* 'Collected Mathematical and Physical Papers,' vol. i. p. 315.

and as in accordance with the free electro-polarities of fractured surfaces of tourmaline discovered by Canton\*. If a crystal, which we may imagine as given with the electrons wholly undisturbed from their positions according to § 32, is dipped in water and then allowed to dry, electrons would by this process be removed from one part of its surface and distributed over the remainder so as to wholly annul its external manifestation of electric quality. If now either by change of temperature or by mechanical stress the distances between the atoms are altered, the interior electro-polarization becomes necessarily altered ; and the masking superficial electrification got by the dipping in water and drying will now not exactly annul the electrostatic force in the air around the solid. If at the altered temperature or under the supposed stress the solid is again dipped in water and dried, the external electric force will be again annulled. Thus is explained the pyro-electricity of tourmaline discovered by Aepinus.

§ 34. But a merely di-polar electric crystal with its single axis presents to us only a small, and the very simplest, part of the whole subject of electro-crystallography. In boracite, a crystal of the cubic class, Haüy found in the four diagonals of the cube, or the perpendiculars to the four faces of the regular octahedron, four di-polar axes : the crystal on being irregularly heated or cooled showed as it were opposite electricities on the surfaces in the neighbourhood of opposite pairs of corners of the cube, or around the centres of the opposite pairs of triangular faces of the octahedron. His discoveries allow us to conclude that in general the electric æolotropy of crystals is octopolar with four axes, not merely dipolar as in the old-known electricity of the tourmaline. The intensities of the electric virtue are generally different for the four axes, and the directions of the axes are in general unsymmetrically oriented for crystals of the unsymmetrical classes. For crystals of the optically uniaxial class, one of the electro-polar axes must generally coincide with the optic axis, and the other three may be perpendicular to it. The intensities of the electro-polar virtue are essentially equal for these three

\* Wiedemann (*Die Lehre von der Elektrizität*, Second Edition, 1894, vol. ii. § 378) mentions an experiment without fully describing it by which a null result, seemingly at variance with Canton's experimental discovery and condemnatory of my suggested theory, was found. Interesting experiments might be made by pressing together and reseparating fractured surfaces of tourmaline, or by pressing and rubbing polished surfaces together and separating them. It would be very difficult to get trustworthy results by breakages, because it would be almost impossible to avoid irregular electrifications by the appliances used for making the breakage. The mode of electric measurement followed in the experiment referred to by Wiedemann is not described.

axes : it may be null for each of them: it may be null or of any value for the so-called optic axis. Haüy found geometrical differences in respect to crystalline facets at the two ends of a tourmaline ; and between the opposite corners of cubes, as leucite, which possess electropolarity. There are no such differences, there are only the chiral differences, between the two ends of a quartz crystal (hexagonal prism with hexagonal pyramids at the two ends) but there are differences (visible or invisible) between the opposite edges of the hexagonal prism. The electropolar virtue is null for the axis of the prism, and is proved to exist between the opposite edges by the beautiful piezo-electric discovery of the brothers Curie, according to which a thin flat bar, cut with its faces and its length perpendicular to two parallel faces of the hexagonal prism and its breadth parallel to the edges of the prism, shows opposite electricities on its two faces, when stretched by forces pulling its ends. This proves the three electropolar axes to bisect the  $120^\circ$  angles between the consecutive plane faces of the prism.

§ 35. For the present let us think only of the octopolar electric æolotropy discovered by Haüy in the cubic class of crystals. The quartet of electrions at the four corners of a tetrahedron presents itself readily as possessing intrinsically the symmetrical octo-polar quality which is realized in the natural crystal. If we imagine an assemblage of atoms in simple cubic order, each containing an equilateral quartet of electrions, all similarly oriented with their four faces perpendicular to the four diagonals of each structural cube, we have exactly the required æolotropy ; but the equilibrium of the electrions all similarly oriented would probably be unstable ; and we must look to a less simple assemblage in order to have stability with similar orientation of all the electronic quartets.

§ 36. This, I believe, we have in the doubled equilateral homogeneous assemblage of points described in § 69 of my paper on "Molecular Constitution of Matter," republished from the Transactions of the Royal Society of Edinburgh for 1889 in volume iii. of my 'Collected Mathematical and Physical Papers' (p. 426) ; which may be described as follows for an assemblage of equal and similar globes :—Beginning with an equilateral homogeneous assemblage of points, A, make another similar assemblage of points, B, by placing a B in the centre of each of the similarly oriented quartets of the assemblage of A's. It will be found that every A is at the centre of an *oppositely* oriented quartet of the B's. To understand this, let  $A_1, A_2, A_3, A_4$  be an equilateral quartet of the A's ; and imagine  $A_2, A_3, A_4$  placed on a

horizontal glass plate\* with  $A_1$  above it. Let  $B_1$  be at the centre of  $A_1, A_2, A_3, A_4$ , and let  $B_1, B_2, B_3, B_4$  be a quartet of the B's similarly oriented to  $A_1, A_2, A_3, A_4$ . We see that  $B_2, B_3, B_4$  lie below the glass plate, and that the quartet  $B_1, B_2, B_3, B_4$  has none of the A's at its centre. But the vertically opposite quartet  $B_1, B'_2, B'_3, B'_4$  contains  $A_1$  within it; and it is oppositely oriented to the quartet  $A_1, A_2, A_3, A_4$ . Thus we see that, while the half of all the quartets of A's which are oriented oppositely to  $A_1, A_2, A_3, A_4$  are void of B's, the half of the quartets of B's oppositely oriented to  $A_1, A_2, A_3, A_4$  have each an A within it, while the other half of the quartets of the B's are all void of A's.

§ 37. Now let all the A's and all the B's be centres of equal and similar spherical atoms, each containing a quartet of electrions. The electrions will be in stable equilibrium under the influence of their own mutual repulsions and the attractions of the atoms, if they are placed as equilateral quartets of proper magnitude, concentric with the atoms, and oriented all as any one quartet of the A's or B's. To see that this is true, confine attention first to the five atoms  $A_1, A_2, A_3, A_4, B_1$ . If the electrions within  $A_1, A_2, A_3, A_4$  are all held similarly oriented to the quartet of the centres of these atoms, the quartet of electrions within  $B_1$  must obviously be similarly oriented to the other quartets of electrions. If, again, these be held oriented oppositely to the quartet of the atoms, the stable configuration of the electrions within  $B_1$  will still be similar to the orientation of the quartets within  $A_1, A_2, A_3, A_4$ , though opposite to the orientation of the centres of these atoms. If, when the quartets of electrions are all thus similarly oriented either way, the quartet within  $B_1$  is turned to reverse orientation, this will cause all the others to turn and settle in stable equilibrium according to this reversed orientation. Applying the same consideration to every atom of the assemblage and its four nearest neighbours, we have proof of the proposition asserted at the commencement of the present section. It is most interesting to remark that if, in a vast homogeneous assemblage of the kind with which we are dealing, the orientation of any one of the quartets of electrions be reversed and held reversed, all the others will follow and settle in stable equilibrium in the reversed orientation.

§ 38. This double homogeneous assemblage of tetra-electronic atoms seems to be absolutely the simplest† molecular structure in which Haüy's octo-polar electric quality can

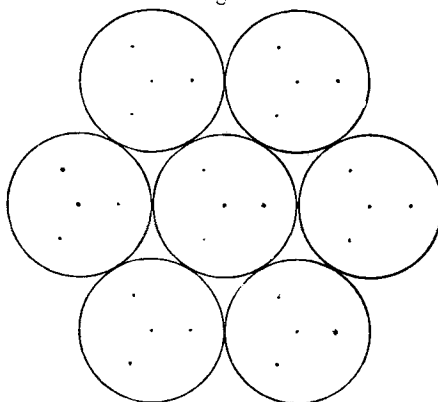
\* Parallel glass plates are useful auxiliaries for graphic construction and illustrative models in the molecular theory of crystals.

† Not the simplest. See § 40 below.

exist. To see that it has octo-polar electric quality, consider an octahedron built up according to it. The faces of this octahedron, taken in proper order, will have, next to them, alternately points and triangular faces of the electronic quartets within the atoms. This itself is the kind of electric æolotropy which constitutes octo-polar quality. Time prevents entering fully at present on any dynamical investigation of static or kinetic results.

§ 39. [Added Oct. 23.] Since what precedes was written, I have seen the explanation of a difficulty which had prevented me from finding what was wanted for octo-polar electric æolotropy in a homogeneous assemblage of single atoms. I now find (§ 40 below) that quartets of electrions will rest stably in equilibrium, under the influence of the mutual repulsion between electrion and electrion and attraction between atom and electrion, in an equilateral homogeneous assemblage in the configuration indicated in fig. 7. The

Fig. 7.



quartets of electrions are supposed to have their edges parallel to the six lines of symmetry of the assemblage. The plane of the paper is supposed to be that of the centres of the seven atoms. The central point in each circle represents a simple electrion which is at distance  $r$ , according to the notation of § 20 above, from the plane of the paper on the near side; and therefore the other three are at the corners of an equilateral triangle at distance  $\frac{1}{3}r$  on the far side to make the electric centre of gravity of the quartet coincide with the centre of its atom. The radius of circle on which these three lie is  $\frac{2\sqrt{2}}{3}r$  or  $\cdot 94r$ . The diagram is drawn correctly to scale

according to the value  $\cdot 612\alpha$  given for  $r$  in the table of § 20, on the supposition that the circles shown in the diagram represent the electric spheres of the atoms in contact.

§ 40. Imagine now the electrions of each quartet to be rigidly connected with one another and given freedom only to rotate about an axis perpendicular to the plane of the paper. To all of them apply torques; turning the central quartet of the diagram slowly and keeping all the others at rest. It is clear that the first  $60^\circ$  of turning brings the central quartet to a position of unstable equilibrium, and  $60^\circ$  more to a position of stable equilibrium corresponding to the first position, which we now see was stable when the others are all held fixed. We are now judging simply from the mutual actions between our central quartet and the six shown around it in the diagram; but it may be easily proved that our judgment is not vitiated by the mutual action between the central quartet and all around it in space, including the six in the diagram. Similarly we see that any one quartet of the assemblage, free to turn round an axis perpendicular to the plane of the paper while all the others are fixed, is in stable equilibrium when oriented as are those shown in the diagram. And similarly again we see the same conclusion in respect to three other diagrams in the three other planes parallel to the faces of the tetrahedrons or corresponding octahedrons of the assemblage. Hence we conclude that if the axial constraints are all removed, and the quartets left perfectly free, every one of them rests in stable equilibrium when oriented either as one set or as the other set of equilateral tetrahedral quartets of the assemblage. It is interesting to remark that if, after we turned the central quartet through  $60^\circ$ , we had held it in that position and left all the others free to rotate, rotational vibrations would have spread out among them from the centre; and, after losing in waves spreading through ether outside the assemblage the energy which we gave by our torque acting on the central quartet, they would come to stable equilibrium with every one of them turned  $60^\circ$  in one direction or the other from its primitive position, and oriented as the central quartet in the position in which we held it.

§ 41. We have thus found that an equilateral homogeneous assemblage of atoms each having four electrions within it, arranges these electrions in equilateral quartets all oriented in one or other of two ways. The assemblage of atoms and electrions thus produced is essentially octo-polar. Of the two elementary structural tetrahedrons, of the two orientations, one will have every one of its electronic quartets pointing towards, the other from, its faces. The elementary

structural octahedron has four of its faces pointed towards, and four pointed from, by its electronic quartets. This is essentially a dynamically octo-polar\* assemblage; and it supplies us with a perfect explanation of the piezo-electric quality to be inferred from the brothers Curie's experimental discovery, and Voigt's mathematical theory.

§ 42. Look at the diagram in § 39; and remember that it indicates a vast homogeneous assemblage consisting of a vast number of parallel plane layers of atoms on each side of the plane of the paper, in which seven atoms are shown. The quartets of electrions were described as all similarly oriented, and each of them equilateral, and having its geometrical centre at the centre of its atom; conditions all necessary for stable equilibrium.

§ 43. Let now the assemblage of atoms be homogeneously stretched from the plane on both sides to any extent, small or great, without any component motions of the centres of the atoms parallel to the planes of the layers. First let the stretch be very great; great enough to leave undisturbed by the other layers the layer for which the centres of atoms are, and the geometrical centres of the quartets were, in the plane of the paper. The geometrical centres of the quartets are not now in the plane of the paper. The single electrions on the near side seen in the diagram over the centres of the circles are drawn towards the plane of the paper; the equilateral triangles on the far side are also drawn nearer to the paper; and the equilateral triangles are enlarged in each atom by the attractions of the surrounding atoms. The contrary inward movements of the single atoms on one side of the plane, and of the triplets on the other side, cannot in general be in the proportion of three to one. Hence the geometrical centres of gravity of the quartets are now displaced perpendicularly to the plane of the paper to far side or near side; I cannot tell which without calculation. The calculation is easy but essentially requires much labour; involving as it does the determination of three unknowns, the length of each side of the equilateral triangle seen in the diagram, the distance of each of its corners from the electrion on the near side of the paper, and the displacement of the geometrical centre of

\* The octo-polar *pyro-electricity*, which is supposed to have been proved by Haüy's experiment, must have been due to something æolotropic in the heating. Uniform heating throughout a regular cube or octahedron could not give opposite electric manifestations in the four pairs of alternate corners of the cube, or alternate faces of the octahedron. Nevertheless the irregular finding of electric octo-polarity by Haüy is a splendid discovery; of which we only now know the true and full significance, through the experimental and mathematical labours of the brothers Curie, of Friedel, and of Voigt.

gravity of the four to one side or other of the plane. Each one of the three equations involves summations of infinite convergent series, expressing force-components due to all the atoms surrounding any chosen one in the plane. A method of approximation on the same general plan as that of the footnote to § 9 above would give a practicable method of calculation.

§ 44. Return to § 42, and consider the diagram as representing a crystal in its natural unstressed condition, consisting of a vast train of assemblages of atoms with centres in the plane of the paper, and in parallel planes on each side of it. We now see that the forces experienced by the electrions of one quartet from all the surrounding atoms in the plane of the paper would, if uncompensated, displace the geometrical centre of gravity of the quartet to one side or other of the plane of the paper, and we infer that the forces experienced from all the atoms on the two sides of this plane give this compensation to keep the centre of gravity of the quartet in the plane. Stretch now the assemblage to any degree equally in all directions. The quartets remain equilateral with their centres of gravity in the plane of the paper and parallel planes. Lastly stretch it farther equally in all directions parallel to the plane of the paper, with no component motion perpendicular to this plane. This last stretching diminishes the influence of all the atoms whose centres are in the plane of the paper tending to displace the centres of gravity of their electrions in one direction from this plane; and therefore leaves all the atoms out of this plane to predominate, and to cause a definite calculable displacement of the centres of gravity of all the quartets in the contrary direction to the former.

§ 45. To realize the operations of § 44, cut a thin hexagonal plate from the middle between two opposite corners of a cubic crystal, or parallel faces of an octahedron. Fix clamps to the six edges of this plate, and apply forces pulling their pairs equally in contrary directions. The whole material of the plate becomes electro-polar with electric moment per unit bulk equal to  $4Nex$ ; of which the measurable result is uniform electrostatical potentials\* in vacuous ether close to the two sides of the plate, differing by  $4\pi \cdot 4Nex t$ , where  $t$  denotes the thickness of the plate,  $x$  the calculated displacement of the centre of gravity of each quartet from the centres of the atoms parallel to the two faces of the plate,  $e$  the electric mass of an electrion, and  $N$  the number of atoms per cubic

\* See my 'Electrostatics and Magnetism,' § 512, cor. 3.

centimetre of the substance. This, for a crystal of the cubic class, is, in Voigt's mathematical theory, the analogue to the electric effect discovered in quartz by the brothers Curie, and measured by aid of thin metal foils attached to the two faces of the plate and metallically connected to the two principal electrodes of an electrometer.

XXX. *The Rate of Recombination of Ions in Gases under Different Pressures.* By R. K. McCUN, M.A., formerly Demonstrator of Physics, McGill University, Montreal; 1851 Exhibition Scholar, McGill University, Trinity College, Cambridge\*.

ANY gas which has been exposed to Röntgen rays, retains the ionization thus produced for a short time after the source of radiation has been removed. The negative and positive ions take an appreciable time to recombine with one another; and it is necessary in some cases to take into consideration the rate at which these ions disappear by recombination. This question of the rate of recombination of ions has been investigated by Prof. Rutherford (Phil. Mag. Nov. 1897) for air and some other gases at atmospheric pressure. In that paper he points out that if, when the rate of production becomes equal to the rate of recombination, the rays be then stopped, the rate at which the conducting particles decay is given by the equation

$$\frac{dn}{dt} = -\alpha n^2, \quad \dots \quad (1)$$

where  $n$  is the number of ions per c.c. in the gas at any time  $t$  after the rays have ceased, and  $\alpha$  is a constant for any given gas at ordinary atmospheric pressure. If  $N$  be the maximum number of ions per c.c. when a steady state has been reached before the cessation of the rays, then the above equation becomes by integration

$$\frac{1}{n} - \frac{1}{N} = \alpha t. \quad \dots \quad (2)$$

#### *Object of Investigation.*

At the suggestion of Professor Rutherford, the present research was undertaken in order to ascertain the conditions which exist in regard to the recombination of the ions in gases under pressures other than atmospheric pressure, and to find the relation between the rates of recombination at different pressures. The objects of the present investigation have been therefore :—

- (1) To determine whether the law of recombination, as set

\* Communicated by Prof. E. Rutherford.