

THE
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ON THE THEORY OF RELATIVITY: PHILOSOPHICAL
ASPECTS.

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I. INTRODUCTION.

THOSE who look on physics from the outside not infrequently have the feeling that it has forgotten some of its philosophical foundations. And among physicists themselves this condition of their science has not entirely escaped notice.¹

The physicist who, above all other men, has to deal with space and time has fallen into conventions concerning them of which he is often not aware. It may be true that these conventions are exactly the ones which he should make. It is certain, however, that they should be made only by one who is fully conscious of their nature as conventions and not as absolute realities beyond the power of the investigator to modify.

Likewise, a question arises as to what element of convention is involved in our usual conceptions of mass, energy, etc.; that the question is not easily answered will become apparent on reflection.

These and many other considerations suggest the desirability of a fresh analysis of the foundations of physical science. Now it is a ground of gratulation for all those interested in this matter that there has arisen within modern physics itself a new movement capable of contributing most effectively to the construction of a more satisfactory foundation for its superstructure of theory. I refer to the recent widespread interest in the principle of relativity and the rapid developments to which it has led.

It is at once admitted that the theory of relativity is not yet established on an experimental basis which is satisfactory to all persons; in fact, some of those who dispute its claim to acceptance are among the most

¹ Compare the discussion given by Crew, *PHYSICAL REVIEW*, 31, 79-92.

eminent men of science of the present time. On the other hand there is a large and growing body of workers who are rapidly pushing forward investigations the inspiration for which is afforded by the theory of relativity.

This state of affairs will probably give rise to a considerable controversial literature. If the outcome of this controversy is the acceptance in the main of the theory of relativity, then this theory will afford just the means needed to arouse in investigators in the field of physics a lively sense of the philosophical foundations of their science. If the conclusions of relativity are refuted this will probably be done by a careful study of the foundations of physical science and a penetrating analysis of the grounds of our confidence in the conclusions which it reaches. This of itself will be sufficient to correct the present tendency to forget the philosophical basis of the science.

It follows that in any event the theory of relativity is certain to force a fresh study of the foundations of physical theory. If it accomplishes no more than this it will have done well.

The object of this paper is to point out some of the more immediate and fundamental conclusions of a philosophical nature to which the theory of relativity gives rise, especially those related to the foundations of physics. An attempt is made to present the discussion in such form as to be intelligible to one who is unacquainted with the theory of relativity; this is done for the convenience of those who have not hitherto been interested in the matter. It will probably be found desirable, however, to have as much previous knowledge of the subject as may be obtained from Comstock's paper in *Science*, N. S., 31 (1910), 767-772.¹

II. THE FUNDAMENTAL BASIS OF RELATIVITY.

The fundamental basis of the theory of relativity may be set forth in a few statements each of which is a generalization from experiment. In so far as these statements are generalizations they are of course unproved experimentally. But they are nevertheless supposed to be the natural teachings of experiment; and as such are to be accepted with a good degree of confidence unless it can be shown that they are in contradiction to other known experimental facts.

The first of these statements gives expression to a law which is suggested by the famous Michelson-Morley experiment,² the object of which was to try out a certain theory concerning the ether by ascertaining

¹ One who desires to go further into the subject might consult with profit the author's previous papers in the *PHYSICAL REVIEW*, Vol. 35, Series 1, pp. 153-176; Vol. I., Series 2, pp. 161-178.

² See *American Journal of Science* (3), 34 (1887), 333-345.

whether a result predicted by it would be found by actual test. For a long time it had been supposed that the known facts about light, electricity and magnetism required for their explanation the theory that the ether of space is stationary. Such a conclusion led to the belief that the velocity of the earth through the ether could be determined by optical experiments. Thus it was predicted that the time which would be required for a beam of light to pass a given distance and return would be different in the two cases when the path of light was parallel to the direction of the motion and when it was perpendicular to this direction. The object of the Michelson-Morley experiment, as we have said, was to put this prediction to a crucial test.

The experiment was a bold one, seeing that the velocities to be measured were so little different; and yet it was carried out in such a brilliant way as to permit no serious doubt of the accuracy of the results. The difference of velocity predicted by theory was found by experiment not to exist; there was not the slightest difference of time in the passage of light along two paths of equal length, one in a direction parallel to the earth's motion and the other in a direction perpendicular to it.

There are different points of view from which one may look at this experiment. In the theory of relativity it is taken in the light of an attempt to detect the earth's motion through space by means of the effect of this motion on terrestrial phenomena. So far as the experiment goes, it indicates that such motion cannot be detected in this way. Furthermore, no one has yet been able to devise an experiment by means of which the earth's motion through space can be detected by observations made on the earth alone. The question arises: Is it possible to have any such experiment at all? In the theory of relativity this question is answered in the negative. The Michelson-Morley experiment and other experiments are thus generalized into one of the fundamental laws or postulates of relativity, which may be stated as follows:

A. The uniform translatory motion of any system cannot be detected by an observer traveling with the system and making observations on it alone.

Another part of the fundamental basis of the theory of relativity is a principle which has long been familiar in the theory of light and has never been found in disagreement with experimental facts. It may be stated thus:

B. The velocity of light in free space is independent of the velocity of the source of light.

By means of laws *A* and *B*, taken in connection with certain principles universally accepted in the classical mechanics, it may be shown¹ that

¹ See a treatment by the author in the first paper referred to above.

the velocity of light is independent of the direction of motion of the observer. Thus we have:

C. THEOREM. The velocity of light in free space as measured by any observer is independent of the direction of motion of that observer.

We are thus led to inquire further as to whether the velocity of light in free space is independent of the absolute value of the velocity of the observer. It appears to be impossible to prove that it is so independent; and yet one is unable to conceive of any way in which it could be dependent on the absolute value without at the same time depending on the direction of the velocity also. But, accepting *A* and *B*, we prove *C* which asserts that such dependence does not exist. There seems, then, nothing left to do but to *assume* that dependence on absolute value does not exist; and this is what is done in the theory of relativity. In the absence of experimental information in contradiction to this assumption it is undoubtedly the natural one to make. Any other procedure would be out of harmony with the usual methods of science. This assumption, therefore, is one which we make as the most natural teaching of experimental facts; and as such we treat it as a "law of nature" so long as fresh experiment does not invalidate it. This law may be stated as follows:

D. The velocity of light in free space as measured by any observer is independent of the absolute value of the velocity of the observer.

The law which we have stated as *A* above is often referred to as the first postulate of relativity. *B*, *C* and *D* taken together constitute the second postulate of relativity. We have broken this postulate into parts in order to state clearly just how it depends on experiment. Combining the parts we may state its essential content as follows:

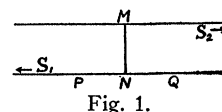
E. The velocity of light is independent of the relative velocity of the source of light and the observer.

The theory of relativity consists of those conclusions which can be derived by logical process (that is, mathematically) from *A* and *E* in conjunction with certain principles which are universally accepted in the classical mechanics—at least, this may be taken as a tentative definition of the theory of relativity. If one agrees that experiment has been properly generalized in *A*, *B* and *D* one has therefore the alternative of accepting the conclusions of relativity or of giving up almost the whole of the usual system of mechanics. That one should take the first horn of the dilemma hardly admits question.

III. FUNDAMENTAL CONCLUSIONS OF THE THEORY OF RELATIVITY.

For the sake of convenience in stating some of the fundamental conclusions of the theory of relativity let us suppose that we have two obser-

vers A and B placed on platforms moving with respect to each other. Let us suppose that A is on a platform denoted by S_1 and that B is on a platform denoted by S_2 . Suppose that to A the platform S_2 appears to move with the velocity v in the direction indicated by the arrow at S_2 ; then to B the platform S_1 will appear to move with velocity v in the direction indicated by the arrow at S_1 . Suppose further that the units of length employed by A and B are such that they arrive at the same numerical results in measuring the length MN , MN being perpendicular to the line of relative motion of S_1 and S_2 . The platform S_1 and the instruments employed by A for measuring time and length will be spoken of as the system of reference S_1 . Similarly, we shall speak of the system of reference S_2 . For convenience we shall use β to denote the ratio v/c , where c is the velocity of light.



The two systems of reference S_1 and S_2 being thus defined, the question arises as to how the units of length and of time of S_1 are related to the corresponding units of S_2 . If we accept as true the laws stated in A and E of the preceding section—as, in fact, is done in the theory of relativity—we are led to some very remarkable conclusions. We state first the relation of the time units:

To an observer A on S_1 the time unit of S_1 appears to be in the ratio $\sqrt{1 - \beta^2} : 1$ to that of S_2 , while to an observer B on S_2 the time unit of S_2 appears to be in the ratio $\sqrt{1 - \beta^2} : 1$ to that of S_1 .

Thus if A compares his clocks with those of B it will appear to A that A 's clocks are running faster than B 's clocks; on the other hand, if B compares his clocks with those of A it will appear to B that B 's clocks are running faster than A 's clocks. Thus the two observers are in hopeless disagreement as to the measurement of time. An analysis of this divergence is made below in section V.

Another matter of fundamental importance in the measurement of time will be brought to attention by the question: When are two events which happen at different places to be considered simultaneous? The nature of the difficulty can be seen from the following result in the theory of relativity, an analysis of which is to be found below in section VI.:

If an observer A on S_1 places two clocks at a distance d apart in a line parallel to the line of relative motion of S_1 and S_2 (say at P and Q respectively) and adjusts them so that they appear to him to mark the same time: then to an observer B on S_2 the clock on S_1 which is forward in point of motion appears to be behind in point of time by the amount

$$\frac{v}{c^2} \frac{d}{\sqrt{1 - \beta^2}}.$$

Let us consider the units of length in S_1 and S_2 . If A and B make measurements of length in the direction MN their results are in perfect agreement in the theory of relativity as in the classical mechanics; but the state of matters is very different when measurement is made in a line parallel to the line of relative motion of S_1 and S_2 , as one sees from the following theorem:

Let l denote a line parallel to the line of relative motion of S_1 and S_2 . Then to an observer A on S_1 the unit of length of S_1 along l appears to be in the ratio $\sqrt{1 - \beta^2} : 1$ to that of S_2 while to an observer on S_2 the unit of length of S_2 along l appears to be in the ratio $\sqrt{1 - \beta^2} : 1$ to that of S_1 .

Thus it appears that when A and B are measuring length in a line parallel to their line of relative motion they are in hopeless disagreement. What this result signifies we shall attempt to explain in section IV.

From the results stated above it may be shown that:

The velocity of light is a maximum which the velocity of a material body may approach but can never reach.

If one brings into consideration the mass of a moving body another result, essentially equivalent to that just given, may also be obtained; this may be stated as follows (see further discussion in section VIII. below):

No finite force is sufficient to give a material particle a velocity as great as that of light.

In classical mechanics it is customary to assume that the mass of a given body is constant and that it is independent of the direction in which the mass is measured. But if the principle of relativity is accepted it follows that neither of these conclusions is valid. It turns out that the mass of a body depends on its velocity and also on the direction, relative to the line of motion of the body, along which that mass is measured. For convenience in distinguishing these measurements of mass we shall speak of the "transverse mass" as that with which we must reckon when we consider motion in a line perpendicular to the line of relative motion of the systems S_1 and S_2 ; when the motion is parallel to this line we shall speak of the "longitudinal mass" of the body.

The general conclusions of the theory of relativity concerning mass may now be stated as follows:

Let m_0 denote the mass of a body when at rest relative to a system of reference S . When it is moving with a velocity v relative to S , denote by t_v its transverse mass, that is, its mass in a line perpendicular to its line of motion. Similarly, denote by l_v its longitudinal mass. Then we have

$$t_v = \frac{m_0}{(1 - \beta^2)^{\frac{1}{2}}},$$

$$l_v = \frac{m_0}{(1 - \beta^2)^{\frac{1}{2}}}.$$

IV. THE NOTION OF LENGTH.

In the preceding section we saw that two observers A and B on relatively moving systems of reference S_1 and S_2 respectively are in disagreement as to units of length along a line l parallel to their line of relative motion. This disagreement is of a very peculiar character. To A it appears that B 's units are longer than his own. On the other hand, it seems to B that his units are shorter than A 's. In the two cases the apparent ratio is the same; more precisely, the unit which appears to either observer to be the shorter seems to him to have the ratio $\sqrt{1 - \beta^2} : 1$ to that which appears to him to be the longer. Although they are thus in disagreement there is yet a certain symmetry in the way in which their opinions diverge.

Let us suppose that these two observers now undertake to bring themselves into a closer agreement in measurements of length along the line l . Suppose that B agrees arbitrarily to shorten his unit so that it will appear to A that the units of A and B are of the same length. Then, so far as A is concerned, all difficulty has disappeared. How is B affected by this change? We see that the difficulty which he experienced is not disposed of; on the other hand it is greater than before. Already, it seemed to him that his unit was shorter than A 's. Now, since he has shortened his unit, the divergence appears to him to be increased. Moreover, the symmetry which we found in the former case is now absent.

Furthermore, if any other changes in the units of A and B are made we shall always find difficulties as great as or greater than those which we encountered in the initial case. There is no other conclusion than this: We are face to face with an essential difficulty—one that is not to be removed by any mere artifice. What account of it shall we render to ourselves?

This much is already obvious: The length of an object is not an absolute something; it depends upon the measurer in an essential way.

We have just spoken of the length of an object, a *material object*. One can hardly refrain from raising the question of the *abstract* notion of length as apart from any material thing having length. But this problem has new difficulties of its own. All lengths of which I have experience are lengths of material objects or lengths between material

objects. Can one formulate in the mind the notion of length as apart from these material things which are measured; and, if so, how can such a notion of length be applied experimentally to the measurement of material things?

Questions of this nature we shall lay aside and shall return to the consideration of the length of material objects.

We have certain intuitive notions concerning the nature of matter which it is necessary for us to examine now. We have usually supposed that to revolve a steel bar, for instance, through an angle of ninety degrees has no effect upon its length. Let us suppose for the moment that this is not so; but that the bar is shorter when pointing in some directions than in others, so that its length is the product of two factors one of which is its length in a certain initial position and the other of which is a function of the direction in which the body points relative to that in the initial position. Suppose that at the same time all other objects experience precisely the same change for varying directions. It is obvious that in this case we should have no means of ascertaining this dependence of length upon the direction in which the body points.

To an observer placed in a situation like this it would be natural to assume that the length of the steel bar is the same in all directions. In other words, in arriving at his definition of length he would make certain conventions to suit his convenience.

Now suppose that the system of such an observer is set in motion with a uniform velocity v relative to the previous state of the system; and that at the same time all bodies on his system undergo simultaneously a continuous dilatation or contraction. This observer would have no means of ascertaining that fact; and accordingly he would suppose that his steel bar had the same length as before. In other words, he would unconsciously introduce a new convention concerning his measurement of length.

There is no *a priori* reason why our actual universe should not be such as the hypothetical one just described. To suppose it so unless our experience demands such a supposition would be unnatural; because it would introduce an unnecessary inconvenience. But suppose that in our growing knowledge of the universe there should come a time when we could more conveniently represent to ourselves the actual facts of our experience by supposing that all material things are subject to some such deformations as those which we have indicated above; there is certainly no *a priori* reason why we should not conclude that such is the essential nature of the structure of the universe.

Naturally we would not come to this conclusion without due considera-

tion. We would first enquire earnestly if there is not some more convenient way by which we can reconcile all experimental facts; and only in the event of a failure to find such a way would we be willing to so profoundly modify our views of the material world.

Now, if we agree to suppose that our actual universe is subject to a certain (appropriately defined) deformation of the general type discussed above it would follow that observers A and B on the respective systems S_1 and S_2 would be in just such disagreement as to units of length as that which exists, according to the theory of relativity. Therefore, that which at the outset seemed to be of such essential difficulty is easily enough explained, *if we are willing to modify so profoundly our conception of the nature of material bodies.*

Whether in the present state of science experimental facts demand such a radical procedure is a question which will be answered differently by different minds. To one who accepts the postulates of relativity there is indeed no other recourse; one who refuses to accept them must find some other satisfactory way to account for experimental facts. The Lorentz theory of electrons gives striking evidence in favor of supposing that matter is subject to some such deformations as those mentioned above; and this evidence is the more important and interesting in that the deformations (as conceived in this theory) were assumed to exist simply in order to be able to account directly for experimental facts.

V. THE MEASUREMENT OF TIME.¹

That two observers in relative motion are in hopeless disagreement as to the measurement of length in their line of relative motion is a conclusion which is probably (at first) sufficiently disconcerting to most of us; and some no doubt have the feeling that any explanation of it so far offered is at best artificial and does not reach the root of the matter. But it is an even greater shock to intuition to conclude, as we are forced to do according to the theory of relativity, that there is a like ineradicable disagreement in the measurement of time. A discussion similar to that in the preceding section brings out the fact that our observers A and B cannot possibly arrive at consistent means of measuring intervals of time. The treatment is so far similar to the preceding discussion for length that we need not repeat it; we shall content ourselves with a brief discussion of conclusions to be drawn from the matter.

Why is this inability of A and B to agree in measuring time received

¹ In connection with this section and the following one the reader should compare the excellent and interesting treatment of the problem of measuring time to be found in Chapter II. of Poincaré's *Value of Science* (translated into English by Halsted).

in our minds with such a distinct feeling of surprise and shock? It is doubtless because we have such a lively sense of the passage of time. It seems to be a thing which we know directly, and the conclusion in question is contrary to our unsophisticated intuition concerning the nature of time.

But what is it that we know directly? We have an immediate perception of what it is for two conscious phenomena to coexist in our mind; and consequently we perceive immediately the simultaneity of events in our minds. Further, we have a perfectly clear sense of the order of succession of events in our own consciousness. Is that not all that we know directly?

The difficulties which *A* and *B* experience in correlating their measurements of time grow out of two things, of neither of which we have direct perception.

In the first place there are two consciousnesses involved; and what reason have we to suppose that succession of events is the same for these two? This question we shall not treat, assuming that the principal matter can be put into such impersonal form as to obviate this difficulty altogether. (As a matter of fact, so far as anything characteristic of the theory of relativity is concerned this can be done.)

The other difficulty has to do with the measurement of time as opposed to the mere psychological experience of its passage. In this matter we are absolutely without any direct intuition to guide us. We have no immediate sense of the equality of two intervals of time. Therefore, whatever definition we employ for such equality will necessarily have in it an important element of convention. To keep this well in mind will facilitate our discussion.

Our problem is this: How shall we assign a numerical measure of length to a given time-interval; say to an interval in which a given physical phenomenon takes place? We shall arrive at the answer by asking another question: Why should we seek to measure time-intervals at all, seeing that we have no immediate consciousness of the equality of such intervals? There can be only one answer: we seek to measure time as a matter of convenience to us in representing to ourselves our experiences and the phenomena of which we are witnesses. In such a way we can render to ourselves a better account of the world in which we live and of our relation to it.

Now, since our only reason for attempting to measure time is in a matter of convenience, the way in which we measure it will be determined by the dictates of that convenience. The system of time measurement which we shall adopt is just that system by means of which the laws of nature may be stated in the simplest form for our comprehension.

Let us return to the case of the two observers A and B of section III. Suppose that each of them has chosen a system of measuring time that suits his convenience in the interpretation of the laws of nature on his system. There is no *a priori* reason why the two observers should measure time-intervals in the same way. In fact, since there is an arbitrary element in the case of each method of measurement and since the two systems are in a state of relative motion, it is not at all unnatural that the units of A and B should differ.

Now it is to be noticed that each of the observers A and B is in just the situation in which we find ourselves. We have chosen a method of measuring time which seems to us convenient. In so far as that method depends on convenience it is relative to us who are observers, and therefore it has in it something which is arbitrary. There is no doubt that it would be desirable for us to know what it is which is arbitrary, which is relative to us who observe; but it is equally obvious that it must be difficult for us to determine what this arbitrary element is.

The theory of relativity makes a contribution to the solution of this problem. We suppose that two observers on different systems find the laws of nature the same as we find them; or, more exactly, we suppose that they find certain specific laws the same as we find them. Then we inquire as to their agreement in measuring time and see that they differ in a certain definite way. This difference is due to things which are relative to the two observers; and thus we begin to get some insight into the ultimate basis of our own method of measurement.

The matter will become clearer if we speak of the simultaneity of events which happen at different places; and therefore we turn to a discussion of this topic.

VI. SIMULTANEITY OF EVENTS HAPPENING AT DIFFERENT PLACES.

What shall we mean by saying that two events which happen at different places are simultaneous?

First of all it should be noticed that we have no direct sense of what this statement should mean. I have a direct perception of the simultaneity of two events in my own consciousness. I consider them simultaneous because they are so interlocked that I cannot separate them without mutilating them. If two things happen which are far removed from each other I do not have a direct perception of both of them in such way that I perceive them as simultaneous. When should I consider such events to be simultaneous?

To answer this question we are forced to the same considerations as those which we met in the preceding section. There can be no absolute

criterion by which we shall be able to fix upon a definition as the only appropriate one. We must be guided by the demands of convenience, and by this alone.

In view of these considerations there is nothing unthinkable about the conclusion (in the theory of relativity) concerning simultaneity which we have given in section III. An observer A on one system of reference regulates clocks so that they appear to him to be simultaneous. It is apparent that to him the notion of simultaneity appears to be entirely independent of position in space. His clocks, even though they are separated by space, appear to him to be running together, that is, to be together in a sense which is entirely independent of all considerations of space.

But when B from another system of reference observes the clocks of A 's system they do not appear to him to be marking simultaneously the same hour; and their lack of agreement is proportional to their distance apart, the factor of proportionality being a function of the relative velocity of the two systems.

Thus instants of time at different places which appear to A to be simultaneous in a sense which is entirely independent of all considerations of space appear to B in a very different light; namely, as if they were different instants of time, one preceding the other by an amount directly proportional to the distance between the points in space at which events occur which mark these instants. Even the order of succession of events is in certain cases different for the two observers, as one can readily verify.

It thus appears that the notion of simultaneity is relative to the system on which it is determined. In other words, *there is no such thing as the absolute simultaneity of events which happen at different places*. The only meaning which simultaneity can have is that which is given to it by convention.

Remark.—The difficulties which we have investigated in this and the preceding section are not peculiar to the theory of relativity. The fact that the quantitative measurement of time is relative to the observer who measures it has been insisted upon by philosophers, notably by Poincaré in his *Value of Science* already referred to. The considerations on which these conclusions have been based have been largely of a speculative character.

The interest of the subject from the point of view of the theory of relativity is that here we have an experimental basis for the conclusions which were previously reached by speculative considerations. Starting out from certain laws which we have (tentatively) accepted as demonstrated by experiment we have by logical processes alone reached these

remarkable conclusions concerning the nature of time as conceived by us. Thus we have experimental demonstration of important results previously reached only by speculative considerations.

In the very nature of things speculation must often outrun experiment. The philosopher vaguely and boldly conceives a truth which it requires years of patient labor to establish on a firm and satisfactory foundation.

It is one of the glories of modern science that things hitherto of a speculative nature are being brought under the domain of experimental fact. In the process many speculations are overturned and thrown to the winds; but that which is really of value we may believe will always be safe and will some day find its justification in results achieved in the laboratory.

VII. TIME AS A FOURTH DIMENSION.

I have no intention of asserting that time is a fourth dimension of space in the sense in which we ordinarily employ the word "dimension"; such a statement would have no meaning. I wish to point out rather that it is in some measure connected with space, and that in many formulæ it must enter as it would if it were essentially and only a fourth dimension.

This will come out clearly if we consider the Einstein formulæ of transformation from one system of reference to another. These have been worked out in detail in several different places,¹ and the result will be assumed here. It may be stated as follows:

Let us consider three mutually perpendicular axes Ox, Oy, Oz of which Ox is in the line of relative motion of two systems of reference S and S' . Likewise let $O'x', O'y', O'z'$ be three mutually perpendicular axes parallel respectively to Ox, Oy, Oz . Let the first be fixed to S and the second to S' . At time $t = 0$ let O and O' coincide. Then if $t, x, y, z; t', x', y', z'$ are the time and space coördinates on S and S' respectively, we have

$$\begin{aligned}t' &= \frac{1}{\sqrt{1 - \beta^2}} \left(t - \frac{v}{c^2} x \right), \\x' &= \frac{1}{\sqrt{1 - \beta^2}} (x - vt), \\y' &= y, \\z' &= z,\end{aligned}$$

where c is the velocity of light, v the relative velocity of S and S' , and $\beta = v/c$.

In these formulæ the time variable t enters in a way precisely analogous to that in which the space variables x, y, z enter.

¹ Compare PHYSICAL REVIEW, Vol. 35, pp. 171-173.

Suppose now that the law of some phenomenon as observed on S' is given by the equation

$$F(x', y', z', t') = 0$$

and we desire to know the expression of this law on S . We substitute for x', y', z', t' , their values in terms of x, y, z, t given above; and thus obtain an equation stating the law in question.

From these considerations it appears that in many of our problems, namely, in those which have to do at once with two or more systems of reference, the time and space variables taken together play the rôle of four variables each having to do with one dimension of a four-dimensional continuum.

This conclusion alone raises philosophical questions of profound importance concerning the nature of space and time; but into these we cannot enter here.

VIII. A MAXIMUM VELOCITY FOR MATERIAL BODIES.

There are several ways by which it may be shown that a material body cannot have a velocity as great as that of light. One of the simplest is that which comes from a consideration of mass. Let us consider the equation

$$l_v = \frac{m_0}{(1 - \beta^2)^{\frac{1}{2}}},$$

where m_0 is the mass of a body at rest relative to a given system of reference S , l_v is the longitudinal mass of the body moving with a velocity v with respect to S . If we consider larger and larger values of the velocity v we see that l_v increases and becomes infinite as v approaches c . This is equivalent to saying that the longitudinal mass of any material body becomes infinite as the velocity of that body approaches c . Therefore it would require an infinite force to give to a material body the velocity c ; that is, c is a maximum velocity which the velocity of a material body may approach but can never reach.

We may obtain the same result in a different way, and thus arrive at a deeper understanding of the matter. From the first formula of transformation given in the preceding section we have

$$t' \sqrt{1 - \beta^2} = t - \frac{v}{c^2} x.$$

Now suppose that $v/c > 1$ and that t and x are real. Then t' is imaginary. Hence, if two systems have a relative velocity greater than that of light, the time measurement in one of them is expressed as an imaginary

number; which is clearly absurd. Hence the relative velocity of the two systems is less than c , since $v = c$ obviously leads to an absurdity.

This conclusion concerning the maximum velocity of a material body brings up important considerations concerning the essential nature of mass and material things. How shall we conceive of matter so that it should have this astonishing property?

In the present state of science any answer to this question must necessarily be of a speculative nature; but it is probably worth while to mention briefly a theory of mass which is consistent with the existence of a maximum velocity for a material body.

Let us suppose that the mass of a piece of matter is due to a kind of strain in the ether, and that this strain is principally localized in a relatively small portion of space, but that from this center of localization there go out to infinity in all directions lines of strain which belong essentially to the piece of matter. (We make no assumption as to how this strain is set up; it may be due largely or entirely to the motion of electrons in the molecules of the matter.) Suppose that these lines of strain, except in the immediate neighborhood of the center of localization, are of such nature as to escape detection by our usual methods. Suppose further that when the piece of matter is moved, that is, when the center of localization is displaced, these lines of strain have a corresponding displacement, but that the ether of space resists this displacement, the degree of resistance depending on the velocity.

If the mass of matter is due to such a strain in the ether it is natural to suppose that mass is a measure of the amount of that strain. But, on our present hypothesis, we see that when matter is moved through space there is an increase of the strain on the ether due to such motion. This manifests itself to us in the way of an increase in the mass of the given piece of matter.

Moreover, when the body is in motion it is natural to suppose that these lines of strain are not distributed evenly in all directions. On account of this fact it would not be a matter for surprise if the mass of a moving body were different in different directions.

It thus appears that appropriate hypotheses (which have nothing in them inherently unnatural) would lead us to expect the same descriptive properties of mass as those which are actually found to exist if one accepts the postulates of relativity. Hence we conclude that there is nothing *a priori* improbable in the conclusions of relativity concerning the nature of mass. Therefore if we find satisfactory grounds for accepting the initial postulates of relativity, we shall not throw them overboard because of the strange conclusions concerning mass to which they have led us.

IX. ABSOLUTE REST. ABSOLUTE POSITION IN SPACE.

To any one who has examined critically the foundations of the theory of relativity it has certainly become apparent that the unexpected conclusions to which one is led concerning the relations of systems of reference is intimately connected with the point of view which the observer on each system takes in making his measurements. Each observer assumes his system to be at rest; and there seems to be no reason for preferring one assumption to the other or for replacing both of them by a third.

If, however, there were such a thing as absolute rest, or absolute position in space, the matter would be different. Each observer should reckon his velocity relative to that absolute; and the reason for divergence which existed before would no longer be found. We should, however, find it necessary to revise many of our "laws of nature," among them the postulates of relativity, if these laws are to be stated with reference to an absolute.

Since it appears to the present writer that absolute rest and absolute position are undefinable, this matter will be dismissed without further consideration.

X. RELATION OF THE THEORY OF RELATIVITY TO THE PHILOSOPHICAL CONTROVERSY CONCERNING THE ONE AND THE MANY.

The author is probably due the reader an apology for injecting into this paper any remarks concerning the abstruse and difficult question as to whether the universe is monistic or pluralistic—a question which has engaged philosophers from the time of the earliest Greek thinkers down to our own day. But if any one of the special sciences has some light to throw on a question of such profound general importance it seems desirable that those engaged in the study of that science should make it known.

If we accept the statement of one of the most eminent philosophers of our day that the universe is one to us in so far (and only in so far) as we know the connections by which it is bound into a One, it will doubtless be interesting to us to examine in what way the theory of relativity leads us to observe new connections. To speak briefly of these is my purpose in the present section.

1. We have already seen that time is not something apart from one who measures it and the system to which he belongs, and that (so far as we are concerned, at least) there is an intimate interlocking of time and space in an essential manner so that we are not able absolutely to extricate the one from the other. That is to say, there is a mutual interdependence of space and time for us who measure them so that we have

to consider them not as mutually exclusive "forms" of thought but as parts of a larger matrix, a sort of four-dimensional continuum, which at once includes both of them and binds them into a unity higher than that of either of them.

2. The theorem which we stated in section III. concerning units of length in the line of relative motion of two systems of reference not at rest relative to each other leads to the conclusion that when a material body is set in motion it undergoes a shortening in the direction of the line of motion and that this deformation takes place as it were automatically without the expenditure of work. This brings to light a very peculiar connection between material bodies and the "empty space" in which they lie or through which they move. Compare the discussion of mass in section VIII.

3. In the classical mechanics the mass of a body is an invariable quantity. In particular, it is independent of the velocity with which the body moves. Its measure is the same in whatever direction it is measured. In section III. we have already seen that none of these statements are true in the theory of relativity. The mass of a body is intimately connected with the absolute value of the velocity with which it moves relative to a system of reference and also (what is stranger still) with the direction in which the mass is measured relative to the direction of motion of the body.

4. It may also be shown¹ that the measure of gravitation depends in a similar way on the motion of the system on which it is measured.

5. Again, the total amount of energy² which a body possesses is a simple linear function of its transverse mass, so that energy and mass are connected in a most intimate manner. In fact, changes in energy and changes in transverse mass are proportional, the factor of proportionality being the square of the velocity of light.

Thus we have enumerated several important and striking connections, brought to light by the theory of relativity, where we have heretofore supposed that there were no connections whatever. If we look at these a little more closely we shall be able to perceive a still more profound connection underlying those which we have already mentioned. A material body in motion has kinetic energy. The connections which we treated depend on relative motion. It is then not far to the conclusion that these connections depend essentially on the energy which is involved in this relative motion. Thus all the interlocking relations mentioned might be thought of as due to the energy states involved in the several cases.

¹ See Bumstead, *American Journal of Science* (4), 26 (1908), 501.

² See *PHYSICAL REVIEW*, Vol. I., Series 2, p. 176.

This idea seems to me to be one fruitful of important conclusions; but it cannot be taken up in detail at this place.

Less abstruse connections than those mentioned above are also being brought to light. Of these one only will be considered. Page¹ has shown how, on the basis of the theory of relativity, the fundamental relations of electrodynamics may be derived from those of electrostatics. That is to say, where in the classical theory we have the *two* subjects of electrostatics and electrodynamics to be developed separately we have in the theory of relativity the *single* subject which is the synthesis of these two. This synthesis is of an essential nature; it is not a juxtaposition of things which belong apart, but a real unifying of two formerly distinct subjects into one larger and unseparated whole.

This may be taken as a special case of the way in which the theory of relativity enables us to obtain the laws of a moving system from those of a system at rest. The further development of this branch of the subject, one would believe, will lead to important results.

In this way it is obvious that the theory of relativity will introduce a much greater unity into physical science; the value of such a unifying factor can hardly be overestimated.

From the foregoing considerations we conclude finally that the theory of relativity has already made and is yet making important contributions toward the great philosophical problem concerning the one and the many.

XI. ON THE NATURE AND POSSIBILITY OF EXPERIMENTAL PROOF OF THE THEORY OF RELATIVITY.

There are at least two ways in which it may be possible to demonstrate experimentally the accuracy of the theory of relativity.

The first one consists in the direct proof by experiment of the postulates on which the theory is based. These proved, the whole theory then follows by logical processes alone. In my first paper referred to above I have already given a sufficient discussion of this method.

A second method would be as follows: Among the consequences of the theory of relativity seek out one which has the property that if it is *assumed* the postulates of relativity may themselves be then deduced by logical processes alone. If then this assumption is proved experimentally this is sufficient to establish the postulates of relativity, and hence the whole theory. Or, one may find such experimental results as lead to all the essential conclusions of relativity, whence one naturally concludes to the accuracy of the whole theory. A discussion of proofs of this kind is to be found in my second paper referred to above.

¹ American Journal of Science (4), 34 (1912), 57-68.

This indirect method of proof is open to an objection of a kind which does not obtain in the case of the direct method previously mentioned. In the indirect method some auxiliary law, as for instance the law of conservation of electricity, must usually be employed in deducing the relativity postulates or essential conclusions from the new assumption which one attempts to justify by experiment. There is always the possibility that the auxiliary law is itself wrong; and consequently one's confidence in the accuracy of the relativity postulates as thus deduced can be no stronger than that in the truth of the auxiliary law. The same objection can also be raised against many conclusions which we are accustomed to accept with confidence.

To many persons it appears that the first method of proof mentioned above has been carried out successfully and satisfactorily. But if one does not share this opinion it is still legitimate to accept the theory of relativity as a working hypothesis, to be proved or disproved by future experiment. It is an historical fact, patent to every student of scientific progress, that many of our fundamental laws have been accepted just in this way. Take, for instance, the law of conservation of energy. There is no experimental demonstration of this law; and in the very nature of things it is hard to see how there could be. On the other hand, it is at variance with no known experimental fact. Moreover, it furnishes us a very valuable means of systematizing our known facts and representing them to our minds as an ordered whole. In other words, it is the most *convenient* hypothesis to make in the face of the phenomena which we have observed. Similarly, even if one does not believe that the theory of relativity has been conclusively demonstrated, should he not accept that theory (tentatively at least) provided it furnishes him with the most convenient means of representing external phenomena to his mind?

Finally, it should be said that every supposed proof of the theory of relativity is of such character that objections can be raised to it; likewise, every supposed disproof of the theory is in the same state. In the mean time, though we cannot accept the theory with all confidence, we can at least use its conclusions to suggest experiments which otherwise would not have been conceived. Therefore, whether true or false, the theory will be useful in the advancement of science. On this account, if on no other, it should appeal to every person interested in scientific progress.