

# River Bends and Meanders\*

## Mechanical Analysis of the Forces at Work Suggests That Meanders Result from "Buckling"

By Colonel Hoc

THE very remarkable tendency exhibited by water courses to assume in certain cases a sinuous form—a form often repeated quite rhythmically—seems not to have been studied so thoroughly as it deserves to be. The explanations given of this "serpentine" motion in technical works are still both inexact and incomplete. They are often referred merely to the inequality of resistance in the bed of the stream and to the reflection of the current from one bank of the stream towards the other. However, it may be pointed out that the phenomenon often occurs on the one hand in territory of the most regular character, and on the other that it may be observed in low lying gravel-covered areas whose tendency is to check transverse speed far more than to deflect it in its full force.

It is pointed out, also, that wherever there is even a slight concavity in one bank of a stream centrifugal force comes into play; the liquid mass rushes thither causing a raising of the level and a consequent formation of eddies which possess a very considerable degree of erosive power. But this explanation, however accurate it may be in many cases, does not indicate why the tendency to serpentine motion is so variable, sometimes exhibiting a maximum amount in placid streams and again failing to appear in those of swift current. Furthermore, close observation reveals many instances in which it is the convex bank which is eroded, in which case the stream tends to straighten its bed instead of curving it in the direction in which the centrifugal force is exerted.

Finally, the phenomenon is sometimes based upon

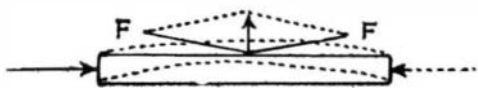


Fig. 1—Composition of forces acting when a railway car "buckles"

the deposits of gravel and sand made by streams, but these considerations are applicable only in certain cases.

The phenomenon of "buckling" is well known. Every body having one of its dimensions greater than the other is in unstable equilibrium when subjected to external forces exerted along the preponderating dimension, and in such a case every transverse curvature exhibits a tendency to be accentuated. A post which supports a ceiling tends to warp in the horizontal direction, and the arch of a bridge tends to warp in a direction perpendicular to its plane. As soon as the two forces of compression  $F$   $F'$  (Fig. 1) undergo any accidental deviation with respect to the axis, they give rise to a transverse component, and the greater the absolute value of the two forces, the greater the value of this component.

But the theory of buckling is applicable to a mobile system, as we shall see further on. To simplify the matter let us first consider a system which is less complex than that of the liquid current; e. g., a train made up of cars of the same type drawn down a long incline by the force of gravity in a straight line. If the degree of inclination of the slope is constant and likewise the resistance to the motion, then all the cars will have the same normal speed, neither compression nor tension being perceptible in the couplings. But

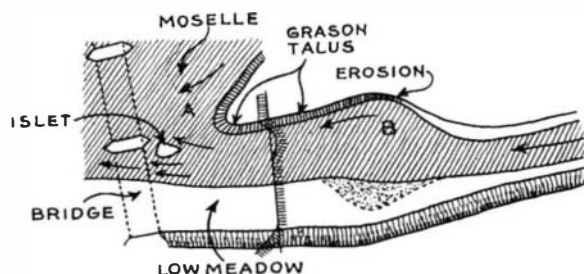


Fig. 2—Effect of high water at A on the course of the tributary B

if the speed of the lower cars tends to diminish either because their resistance is augmented by means of the brakes or because they have reached a less inclined slope, it is obvious that the couplings would be compressed. It is equally obvious that this compression would facilitate any transverse displacement which might tend to be produced accidentally either

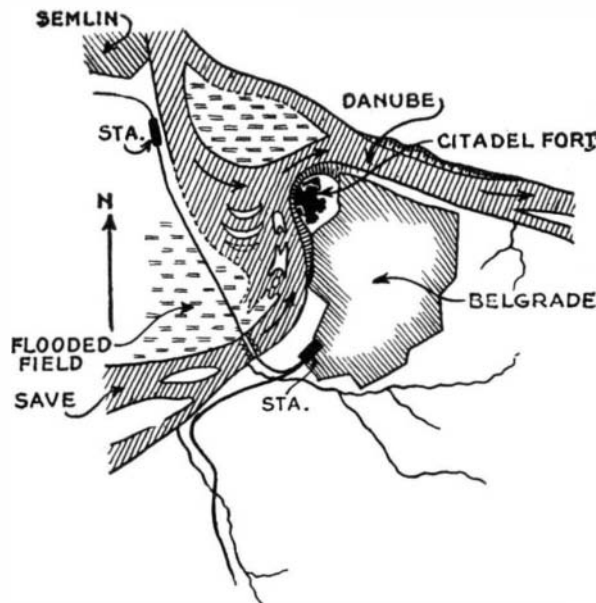


Fig. 3—Confluence of the Save and the Danube—contracted waterway and voluminous affluent

within the limits of the play of the wheels or by the spreading of the tracks. Here we see clearly that just as in the case of a flexible bar compression would tend to produce undulation or buckling.

If, on the other hand, the speed of the lower cars tended to increase either because of diminished resistance or because of an increase in the slope, the couplings would be subjected to tension; in this case the transverse strain would no longer tend to augment any existing or commencing curvature, but would tend on the contrary to straighten the train by dragging it towards the short radius.

The two phenomena exhibited by liquid currents are as follows, according to the circumstances:

(a) A tendency to bend inwards through the erosion of one of the banks, when the velocity of the current decreases by reason of an increase in the resistance of the bed of the stream, from alteration in width of the waterway or from decrease in the grade. If the curvature of the banks already exists this action will tend to augment it by reason of the superposed centrifugal force.

(b) A tendency to straighten out by eroding the convex bank (that of short radius), when from opposite causes the current is accelerated.

When threads of rain water run down the surface of a slightly oily pane of glass such as the window of a car, we constantly see them leave the vertical direction and form an abrupt curve, which is, however promptly straightened out. This phenomenon is the simplest form of the one described above. The thread of water being slowed up at one point by some resistance undergoes in its downward path the transverse inflection of a compressed rod.

If we turn now to the case of a river we find above bridges and sand-banks a strong tendency to inflection as is indicated in Figs. 2, 3, 4 and 5.

Fig. 2 represents a canal which is tributary to the Moselle at Toul at a period when the water in the latter is at a medium height. The high level of the river creates a resistance to the flow of the canal whose water-way is carried forward from its habitual emplacement A to below the pillar of the bridge. At B we can readily observe the inflection of the current with the erosion of the bank referred to above.

Fig. 3 represents the confluence of the Danube and the Save at Belgrade as it looked in July, 1914. At the foot of the rock upon which the citadel stands, the mass of the Danube hurls itself against the double obstacle of a contracted water-way and of a voluminous affluent. The observer obtains a definite impression that if this mass is inflected it is because of the said resistance and for no other reason. The crescent-shaped alluvial deposits which separate the various arms have too slight a relief and are too mobile to lend themselves to the usual explanations by reflections, etc.

Fig. 4 is taken from the Inn River in the vicinity of St. Moritz. Fig. 5 represents the strongly marked curve formed by erosion in the left bank of the Durance above the Brillanne-Oraison bridge. It exhibits the traces of an analogous curve upon the right bank. Sim-

ilar erosions are repeated above several other bridges over the same river.

It is doubtless true in these last two cases that the obstruction of the bed by banks of gravel deposited in the retarded water above the obstacle help to throw the current toward the banks. This is a cause which exerts its influence in the same manner as the tendency to buckling. It even acts alone in times of low water when the depths are slight and when the pressures count for little by the side of the other forces present, in a current with a steep declivity. But this obstruction is not sufficient to explain an incurved winding of more than 180°, like that in Fig. 5.

If there remained any doubt that something more than the deposit of gravel is required to explain the phenomenon the example of the glaciers cited below would remove all uncertainty in this respect.

It appears to be true, therefore, in general, of a stream of water as well as for a train of cars, that a local resistance determines up-stream a tendency to lateral inflection, i. e. to winding, because of the compression between the said resistance and the force of the up-stream inertia.

In the course of a tranquil stream this cause acts alone at the beginning, but is reinforced by centrifugal force as soon as the first curvature has occurred. In the course of a rapid stream it is also reinforced

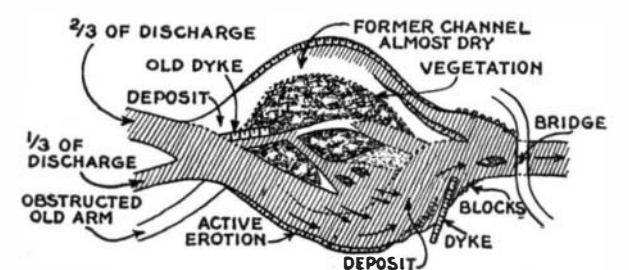


Fig. 4—Buckling effect of a contraction at the bridge (Inn at St. Moritz)

by the obstruction of the bed which results from the deposition of gravel above the obstacle.

Let us now imagine for a moment a water-course which is rectilinear and with a constant degree of fall running over a bottom which is capable of being undermined. It would be impossible for the dragging of the materials of the bottom to be uniform at all points. Certain portions would be excavated and others would fill up according to their comparative solidity and because of accidental inequalities of speed. After the lapse of a certain period of time the bed would exhibit a succession of troughs and crests. Each bank would form an obstacle in the sense of the preceding discussion and would create a tendency to transverse deviation in the area above, namely that of the trough. The longitudinal profile and the plan would then resemble more or less closely the diagrammatic forms shown in Fig. 6.

Everything would concur in accentuating these forms.

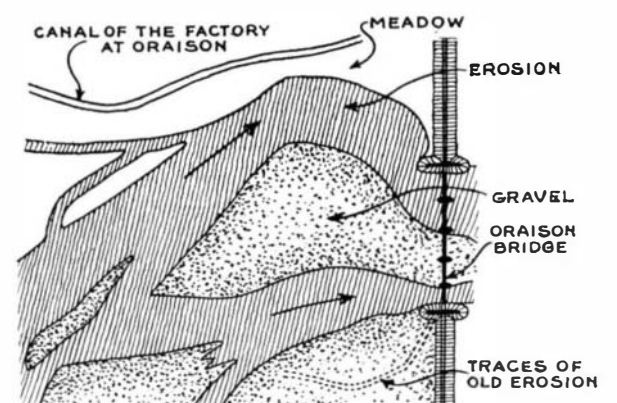


Fig. 5—Another meander due to grading above a bridge (the Durance at Brillanne-Oraison bridge)

It is a well known fact that the transportation of materials by streams is particularly favored by eddies, which are especially apt to form in the curves (where the liquid streamlets have unequal rates of speed) and near shored up banks. The excavation would therefore increase both in width and in depth toward the concave bank of the curve, under the simultaneous effects of the transverse component shown in Fig. 1,

\*Translated for the SCIENTIFIC AMERICAN SUPPLEMENT from *Le Génie Civil* (Paris).

of the eddies, of centrifugal force, and finally, of the contraction of the bed by means of the deposits against the convex bank (see Fig. 8); at the same time the deposits would be increased upon the crests, where these causes would cease to operate.

We can easily imagine the regularity of aspect that would be assumed by such an ensemble in the bottom of a homogeneous valley in which the debris banks would be distributed at more or less constant distances according to the circumstances which condition the transportation so that the windings would assume uniform curvature.

In other cases the said gravel banks would be occasioned by deposits from affluent stream and would consequently be more irregular. This is doubtless what took place in the Theiss River already alluded to in reference to windings, since the course it traverses is

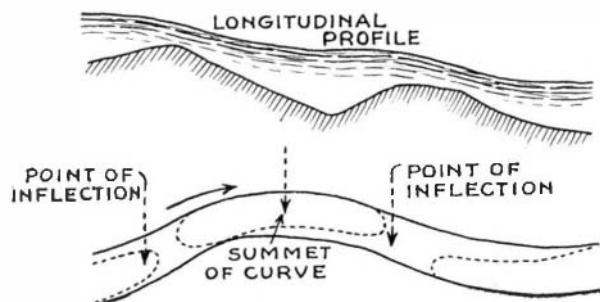


Fig. 6—Diagrams of points of cutting and depositing. Dotted lines are isobaths

more than twice as long as its length as the crow flies. The Theiss receives a series of tributaries coming from the Carpathian mountains and bringing with them such large quantities of alluvial deposits that it has displaced its bed a hundred kilometers towards the west since the beginning of the geologic era. It is very probable that the said alluvial deposits have been the cause of such an exceptional development of sinuosity by reason of the obstacles which they have put in the way of the current.

At first glance one is tempted to seek an explanation of the windings which characterize the Seine below Paris in similar declivities or obstacles. But this is incorrect. These windings are not of recent formation but fall into the category of what geographers term incised meanders, or those which are ancient in contrast to the modern ones which are known as divergent meanders. The Seine had as its precursor a river without any perceptible fall, through which the waters of the ancient lake of Beauce emptied themselves, describing meanders upon the surface of the tertiary massif around a middle northwest direction. But the bottom of the lake became progressively higher in its northwest portion, attaining an altitude of 260 meters in the forest of Villers-Cotterets, and 150 meters in Normandy. The water courses already formed between the Oise and the Yonne are obliged to make their beds descend through the sinuous *massif* in proportion as its relief is accentuated.

This condition is frequently met with (as in the Meuse below Charleville, etc.), and imparts difficulty to the apparently easy study by means of maps of the circumstances connected with the formation of meanders.

Various laws to which meanders are subject, especially as regards navigation, such as the relations between the outline and the depth, stability, and so forth, have been formulated by MM. Fargue and Girardon. They are closely related to the subject here treated, and would deserve to be quoted in their entirety, but that they can be found in all treatises upon applied hydraulics, we shall merely recall the importance of a gradual and successive variation of the curvature for the production of a regular and stable profile along the *thalweg*.

M. Clavel has pointed out, elsewhere, in a certain section of the Garonne, a regular displacement of the meanders, in the course of time, in the down-stream direction (Fig. 7). He explains this fact by the observation that the corrosion is greatest as the greatest depth; but, according to one of the laws of Fargue this is always at some distance down stream from the apex of the concave curve. The attack on the banks progresses down stream, therefore, along the axis of the *ensemble* of the valley. It would be interesting to ascertain whether this phenomenon is general—that is, of course, when the banks present no fixed point.

Let us make now a final observation with respect to successive meanders, which has the advantage of being entirely detached from the questions of rapidity and of gravel deposits which complicate the subject in the case of water courses. We would speak of glaciers, those slow-moving rivers of ice whose flow presents so

many curious analogies with that of streams of water.

Tyndall pointed out that in glaciers the thread of the current is displaced alternately to the right and to the left of the median line, forming approximate loops which succeed each other, now on one side and now on the other. In other words, the mass tends to be borne toward the outside of each curve, and we are

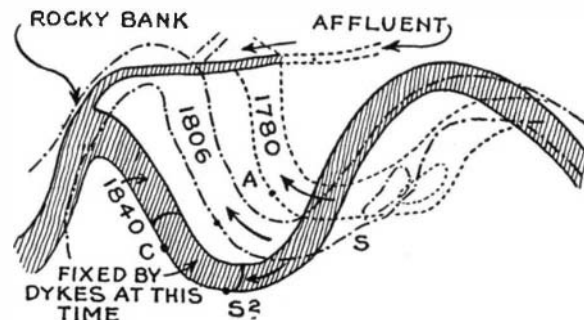


Fig. 7—Systematic down-stream shifting of a meander in the Garonne

able to recognize the phenomenon of buckling, though there can be no question here of a reflection of the current or of the operation of centrifugal force since the speed of flow is imperceptible.

Let us now examine cases analogous to that of the train of cars with a constantly increasing speed, it being understood of course that in a water course the traction is replaced by a tumultuously flowing current.

Fig. 8 was taken from the Moselle, between Thaon and Girmont, shortly after a rise in the river. The current follows, as is usual, the foot of the concave bank with steep slopes. But we observe, opposite the convex bank a trough *DC*, excavated in the deposits of gravel *ABC*, such as is ordinarily found at low water. We are accordingly obliged to conclude that during the rise of the waters the principal current was carried to this side, approaching a straight line as nearly as possible. If the rise of the waters had lasted long enough the gravel deposit would undoubtedly have been entirely excavated; the excavation began along the line of trough close to the resisting border, by reason of the phenomenon referred to above, namely the formation of eddies against the solid banks.

Similar instances of the deflection of the main current and of the straightening out of curves during a period of rising waters occur quite frequently. The comparative importance of the obstacles is modified, in fact, by the increase in the height of the water and in its rapidity of flow, and some of them are overcome by the kinetic force without an increase in the static pressure above them being required, as before. The influence of these obstacles upon the curvature of the current is consequently greatly reduced.

The formation of a great many islands, e. g. those in the Seine below Paris (Billancourt, etc.), can be explained by these returns of the main current towards the interior side in periods of rising waters, with the excavation of troughs like *DC* in Fig. 8.

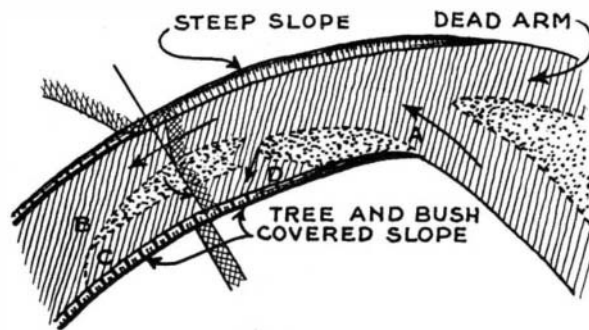


Fig. 8—The Moselle between Thaon and Gismont. Effect of high water currents

In streams of very steep grades the formation of meanders due to "buckling" is hardly ever exhibited except at moderate stages. During periods of low water the current rambles through a bed much too wide for it and is obstructed by deposits of gravel left by the last period of high water. It winds its way among troughs excavated by these rising waters opposite the banks, and it is deflected most of all by the accumulations in its bed.

[The second part of Colonel Hoc's article is largely occupied with a theoretical study of the meanderings of rivers, executed by means of mathematical calculations, and consists mainly of the study of elastic stability, particularly the phenomenon known as buckling. Buckling may be described as mechanical instability of the apparent form with a tendency to increase of all transverse incurvation. An example of this is the

tendency to flexion of a rubber pipe traversed by water under pressure. Another example is the tendency to assume a meandering or serpentine form exhibited by rivers when their banks are capable of being deformed by erosion. If there are fixed points which may be represented by piles, rocks, etc., the water course is comparable to a bar of metal two points of which are held fast transversely, with the peculiarity that theoretically there is no minimum distance below which the buckling will cease, since the formula of Euler is not applicable, the constant elasticity being nil. In practice the river bed remains sufficiently rectilinear if the fixed points are near enough to each other. If there are no fixed points, which indeed is the most usual case, the importance of the action upon the banks of the river will depend upon the variation of the pressure and we can determine by calculation the various positions of equilibrium assumed by a rod of indefinite length when the pressure is constant, when the pressure is maximum at a given point, and when the pressure is minimum at the said point.

The calculations in question were made at Colonel Hoc's request by Captain Jacquart, an engineer in the Department of Roads and Bridges of the French army, and were found to agree with the facts empirically observed. Since they are too technical to quote here in detail, we confine ourselves to reproducing the following final paragraphs dealing with the practical consequences involved.]

There is a tendency to the formation of a meander when the rapidity of the current is diminished either because of a decrease in the degree of declivity or because of resistance offered down stream as by a sand bank or a contraction of the channel. This tendency is sometimes to be attributed to deposits of gravel but is most frequently due to a resultant of pressures and of the forces of inertia acting laterally in a manner closely analogous to the buckling of solid bars when compressed.

The incurvation when once begun is increased by the effect of the centrifugal force of the eddies produced within the curves and by the transport of the materials of the bed of the river to the point where the obstacle occurs. The engineer, therefore, who plans to build a bridge or any other construction offering resistance must take into consideration not only the raising of the level of high water but also the possible erosion of the banks of the stream above the said obstacle. Consequently the only practical guidance is to be found by a study of what has occurred in the remainder of the same water course.

The repetition of the phenomenon described above in a stream of water which presents an alternation in the degree of slope or a succession of sand or gravel banks will occasion a series of successive meanders exhibiting a longitudinal profile like that of a staircase as represented in Fig. 6.

The protection of the banks need not be continuous. It may consist of submerged piles properly spaced, which bear a certain analogy with the fixed points arranged in a bar of metal to prevent buckling.

The question is often asked whether it would be preferable when regularizing the course of streams to make the new bed follow a straight line or a curve. The foregoing remarks indicate that the straight line involves the minimum of stress as concerns the erosion of the banks, consequently there is no reason why it should not be used, as is very frequently done, in fact, in Switzerland (Upper Rhone, Aar, etc.).

However, from the navigator's point of view the straight course is open to the objection of causing an increase in the declivity and consequently of the excavation of the bottom. Upon the French Rhone (where the piles, moreover, constitute a *regularization* of the existing bed rather than a *correction*), it is estimated that a channel having a well marked but not excessive sinuosity is easier to maintain in a stable state than one which is nearly rectilinear, since the latter may wander in times of low water in the smaller bed. This reminds us of the preference often given to a curve of large radius over a tangent in the planning of railway tracks.

When the declivity increases in the down stream direction the tendency to buckling is replaced by a tendency to straighten. The main current abandons the concave or external side of the curve in spite of the action of centrifugal force and establishes itself near the convex or interior side. The same thing occurs at a great many points during periods of high water the relative importance of the obstacles being changed while the variations of superficial slope sometimes undergo a change of direction. The current then digs a trough opposite the convex border and this at times causes an island to be formed or transforms a meander into a dead arm.